UNIT 10 DIFFRACTION GRATING

Structure

10.1 INTRODUCTION

You have learnt about Fraunhofer diffraction produced by a single slit aperture. When a narrow vertical slit is illuminated by a distant point source, the Fraunhofer diffraction pattern consists of a series of spots situated symmetrically about a central spot, along a horizontal line. The intensity of the central spot is maximum and it decreases rapidly as we move away from the central spot. For a circular aperture, the diffraction pattern consists of concentric rings with a bright central disc. You might have learnt in your school physics that diffraction phenomenon limit the ability of optical devices to form sharp and distinct images of distinct objects. This restriction at one time hampered the spectroscopic work particularly for substances whose spectrum consisted of doublets. The problem could be overcome by increasing the number of diffracting slits, that is, by using a diffraction grating.

In Sec. 10.2 we have listed qualitative features of the observed double slit diffraction pattern and compared these with those of a single slit pattern. For this we will consider the source of light as a slit. A distinct feature of double slit pattern is that it consists of bright and dark fringes similar to those observed in interference experiments. In Sec. 10.3 we have derived the equation for the resultant intensity distribution. This mathematical analysis is extension of what you have already learnt for single slit. You will learn that the intensity of the central maximum is four times the intensity due to either slit at that point. However, the interference maxima are diffused (broader). These results are generalised for the case of N equally spaced, identical slits in Sec. 10.4.

You will observe that as the number of slits increases, interference maxima get narrower (sharper). For sufficiently large value of N, interference maxima become narrow lines. For this reason, diffraction gratings are an excellent tool in spectral analysis. The occurrence of diffraction grating effects in nature is surprisingly common. Do you know that the green on the neck of a male mallard duck, blue appearance of wings of Morpho butterflies and the beautiful colours of the 'eye' pf the peacock's feathers are also due to diffraction grating effects? The layered structure in cat's retina acts as reflection grating and is responsible for mettalic green reflection at night.

Objectives

After studying this unit, you should be able to

- state salient features of the double slit diffraction pattern
- qualitatively compare double slit and single-slit diffraction patterns
- compare the double and N-slit patterns
- derive equation for the intensity distribution for the double slit pattern
- extend the double-slit calculation for N equally spaced slits
- describe the use of a diffraction grating in spectral analysis, and
- solve numerical examples.

10.2 OBSERVING DIFFRACTION PROM TWO VERTICAL SLITS

Refer to Fig. 10.1. It shows the experimental arrangement for observing diffraction from two vertical parallel slit - apertures in an opaque screen. Both slits have the same width b and height h. The width of the intervening opaque space between the two slits is a.

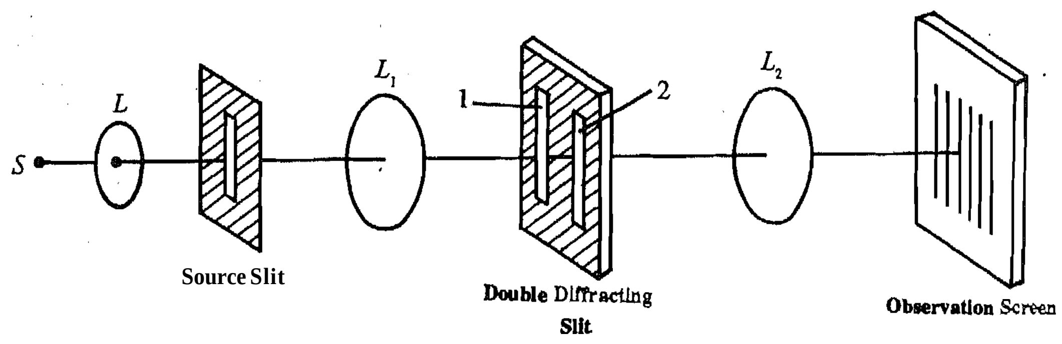


Fig.10.1: Experimental arrangement for observing diffraction from two identical vertical slits

Therefore, the distance between two similar points in these apertures d = b + a. Have you noticed that diffracting apertures are illuminated by a slit source rather than a point source of light? We have used this arrangement because this corresponds more nearly to the actual conditions under which an experiment is performed. That is, the diffraction pattern

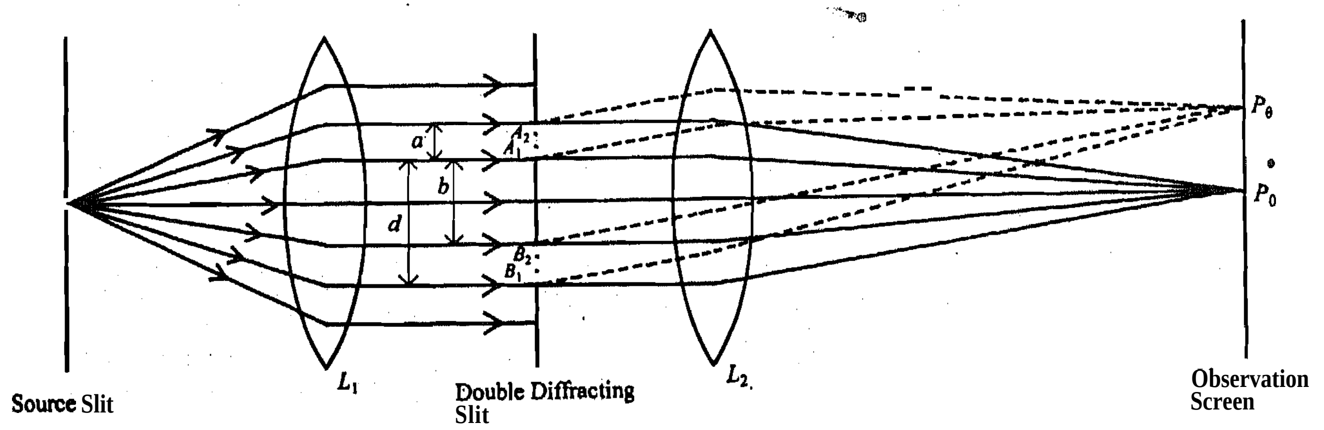


Fig.10.2: Ray geometry of experimental arrangement shown in Fig. 10.1

from a slit source is of greater practical importance than from a point source. The ray geometry of Fig. 10.1 for observing Fraunhofer diffraction from a double slit illuminated by a slit source is shown in Fig. 10.2. The length of the source slit in the arrangement should be adjusted to be parallel to the lengths of the diffracting slits.

Suppose we block one of the diffracting slits, say slit 1, shown in Fig. 10.1 and observe the diffraction pattern on the screen. Obviously, you should expect the single slit diffraction pattern (due to slit number 2 which has not been blocked). Next, uncover slit 1 and block the other. You should again expect single slit diffraction pattern with exactly the same intensity distribution. But what may surprise you at the first glance is that both diffraction patterns are not only identical, they are located at the same position. Were you not expecting these diffraction patterns to be laterally displaced? These patterns are not laterally shifted with respect to one another because of the (well corrected) lens L_2. This is true even for N identical vertical slits. The diffracted wavefronts originating from any slit and travelling along the axis of lens L_2 are focussed at P_0, which forms the peak of the central spot. The diffracted wavelets originating from either slit and moving at an angle θ are focussed at P_θ.

In a well corrected lens consider parallel beams of light travelling in a direction inclined to the axis and falling on different pans of ihe lens. They ate all brought to focus on the back focal plane at a point which is located by the beam passing through the optical centre of the lens.

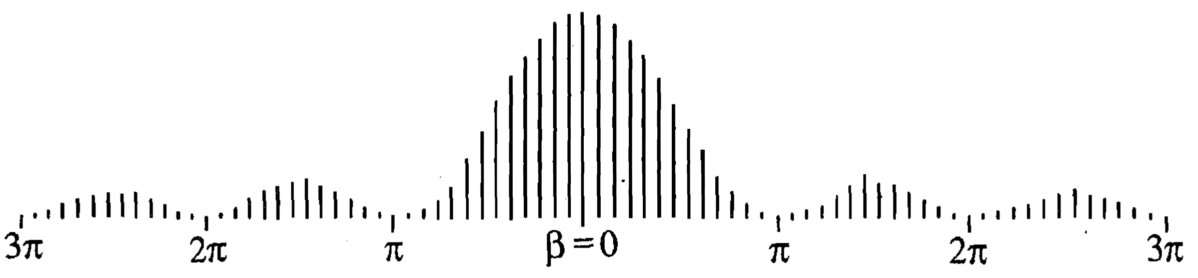


Fig.10.3: Observed double slit diffraction pattern

Now uncover both the slits so that each slit gives its own diffraction pattern. The salient features of the resultant diffraction pattern, shown in Fig. 10.3, are summarised below:

(i) The double slit diffraction pattern consists of a number of equally spaced fringes similar to what is observed in interference experiments.

(ii) The intensities of all fringes are not equal. The fringes are the brightest in the central part of the pattern.

(iii) As we move away on either side of the central fringe, the intensity gradually falls off to zero.

(iv) The fringes reappear with reduced intensity three or four times and become too faint to be observable thereafter.

(v) The intensity at the maximum of double slit pattern is greater than the intensity of principal maximum in single slit pattern.

(vi) The single-slit diffraction pattern acts as an envelope to the double slit pattern.

What is responsible for this pattern? How bright are double slit fringes compared to those in the single slit pattern? You will discover answers to these and other related questions in the following section.

10.3 INTENSITY DISTRIBUTION IN DOUBLE SLIT PATTERN

For calculating the intensity distribution for the arrangement shown in Fig. 10.1 it is sufficient for us lo consider a point source. This is because a point source gives the intensity dislribution along a section perpendicular to the vertical fringes formed from a slit source. For deriving the equation for intensity of double slit pattern, we extend the procedure used for the single slit (Unit 9). Slit 1 acts as a source of diffracted plane wavefronts originating from points A_1, A_2, A,, ... in it. We represent these by a, cos ωt, a, cos $(\omega t - \phi)$, a_0 cos $(\omega t - 2\phi)$, ..., where ϕ is the constant phase difference. The magnitude of field E_1 produced by this slit at the point P_θ is given by (Eq. 9.6):

$$E_1 = A\left(\frac{\sin\beta}{\beta}\right)\cos(\omega t - \beta) \qquad (10.1)$$

where $\beta = \dfrac{\pi\, b \sin\theta}{\lambda}$

For every point like A, in slit 1, we have a corresponding point B_1 in slit 2 at a distance d. The phase difference between diffracted wavefronts reaching P_θ from A, and B_1 is given by

$$6 = \frac{2\pi}{\lambda}(a + b)\sin\theta = \frac{2\pi}{\lambda} d \sin\theta \qquad (10.2)$$

Therefore, the diffracted plane wavefronts starting from points B_1, B,, B_3, ... may be represented as a_0 cos $(\omega t - 6)$, a_0 cos $(\omega t - 6 - \phi)$, a_0 cos $(\omega t - \delta - 2\,\$)$,... And the field E_2 produced by slit 2 at P_θ is given by

$$E_2 = A\left(\frac{\sin\beta}{\beta}\right)\cos[(\omega t - \; - \beta] \qquad (10.3)$$

Since the sources A,, A,, A,, ... and B_1, B_2, B_3, ... are coherent, the magnitude of resultant field at P, due to the double-slit is obtained by the superposition of magnitudes of individual fields:

$$E = E_1 + E_2$$

$$= A\frac{\sin\beta}{p}[\cos(\omega t - \beta) + \cos(\omega t - \beta - 6)]$$

Using the trigonometric identity cos A + cos B = 2 cos $\left(\dfrac{A+B}{2}\right)$ cos $\left(\dfrac{A-B}{2}\right)$, we can rewrite the above expression as

$$E = 24\left(\frac{\sin\beta}{\beta}\right)\cos\left[(s - \beta) - \frac{\delta}{2}\right]\cos\left(\frac{\delta}{2}\right)$$

$$= 24\left(\frac{\sin\beta}{\beta}\right)\cos(\omega t - \; - \; \cos\gamma \qquad (10.4)$$

where $\gamma = \dfrac{\delta}{2} = \dfrac{\pi}{\lambda} d \sin 8$.

The intensity is proportional to the square of the amplitude. So

$$I_\theta = 4A^2\left(\frac{\sin\beta}{\beta}\right)^2\cos^2\gamma \qquad (10.5)$$

For 8 = 0, both β and y vanish so that at the centre (bright fringe), the intensity is

$$I_{,=0} = 4A^2 = 4I_0$$

The expression for intensity of double slit diffraction pattern can be written as

$$I_\theta = 4I_0\left(\frac{\sin\beta}{\beta}\right)^2\cos^2\gamma \qquad (10.6)$$

Since the maximum value of I, is $4I_0$, we see that the double slit provides four times as much intensity in the central maximum as the single slit. This is exactly what you should have expected since the incident beams are in phase and amplitudes superpose.

If you closely examine Eq.(10.6) you will recognise that the term $(\sin^2\beta)/\beta^2$ represents the diffraction pattern produced by a single slit of width b. The $\cos^2$ y term represents the interference pattern produced by two diffracted beams (of equal intensity) having phase difference δ. That is, the intensity of double slit diffraction pattern is product of the irradiances observed for the double-slit interference and single slit diffraction. For $a > b$, the $\cos^2 \gamma$ factor will vary more rapidly than the $(\sin^2 \beta)/\beta^2$ factor. Then we obtain Young's interference pattern for slits of very small widths. In general, the product of sine and cosine factors may be considered as a modulation of the interference pattern by a single slit diffraction envelope. We shall discuss it in detail a little later.

Before we investigate the positions of maxima and minima, let us understand the physical phenomenon that takes place. Diffracted light emerging from these two slits constitutes two coherent beams. These interfere leading to the formation of fringes on the screen. But the intensity of a fringe depends upon the intensities of interfering beams and the phase difference between them when they reach the point under observation. We know that the intensities of diffracted beams are controlled by the diffraction conditions and the direction of observation. Consequently, the intensities of interference fringes are not the same at different points of the screen. In particular, in those directions in which the intensities of diffracted beams are large, the constructive interference will lead to brighter fringes whereas in directions where the two diffracted beams themselves have lower intensities, even their constructive interference will lead to faint fringes.

You should note that we have described the phenomenon as interference between two diffracted beams. How do we distinguish between the two words interference and diffraction which we have used? When secondary wavelets originating from different parts of the same wavefront are made to superimpose, we call it diffraction. Such a case arises when we consider all the wavelets arising from the various points situated in the aperture between the two jaws of a slit. But when two separate beams coming from two different slits are superimposed, we call it interference. It should be clear that in all cases where we apply the principle of superposition, the wavelets have to be coherent in nature to produce an observable pattern.

Before you proceed, you may like to answer an SAQ.

SAQ 1

Spend 2 min

If instead of a monochromatic source of wavelength, we use a source emitting two wavelenths, λ_1 and λ_2 $(< \lambda_1)$, how will the double slit diffraction pattern get influenced?

10.3.1 Positions of Minima and Maxima

To study the position of minima and maxima in the double slit pattern, we use the equation

$$I_\theta = 4I_0 \left(\frac{\sin\beta}{\beta}\right)^2 \cos^2\gamma$$

We note that the intensity I_θ will be zero when either $(\sin\beta / \beta)^2$ or $\cos^2\gamma$ is zero. From Unit 9 you will recall that the diffraction factor $(\sin\beta / \beta)^2$ will be zero for

$$\beta = \frac{\pi b \sin\theta}{\lambda} = \pi,\ 2\pi, 3\pi, \ldots m\pi\ (m \neq 0)$$

or

$$b \sin\theta = \lambda,\ 2\lambda,\ 3\lambda,\ \ldots,\ m\lambda \tag{10.7}$$

This equation specifies the directions along which the available intensity of either beam is zero by virtue of diffraction taking place at each slit.

The second factor $(\cos^2\gamma)$ is an interference term and will be zero when

$$\gamma = \frac{\pi d \sin\theta}{\lambda} = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots \left(n + \frac{1}{2}\right)\pi$$

or

$$d \sin\theta = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}, \ldots \left(n + \frac{1}{2}\right)\lambda \tag{10.8}$$

This gives the angles for the intensity to be zero by virtue of destructive interference between two beams. You may recall that this is the same as the condition for the minimum of the interference pattern between two point sources. Eqs. (10.7) and (10.8) specify the direction when the intensity is zero.

We cannot obtain the exact positions of the maxima by any simple relation. This is because we have to find the maximum of a function which is product of two terms. But we can find their approximate positions if we assume that $(\sin\beta / \beta)$ does not vary appreciably over a given region. We are quite justified in making this approximation if the slits are very narrow. Note that we observe the maxima near the centre of the pattern. Under these conditions, the positions of maxima are solely determined by the $\cos^2\gamma$ factor. You know that this factor defines maxima for

$$\gamma = 0, \pi, 2\pi, ..., n\pi$$

or

$$d \sin\theta = 0, \lambda, 2\lambda, ..., n\lambda \qquad (10.9)$$

We know that d sine represents the path difference between the corresponding points in the two slits. When this path difference is a whole number of wavelengths, constructive interference occurs between the two beams. Then we get a maximum which leads to the formation of a series of bright fringes. The central fringe corresponds to $d \sin\theta = 0$. The nth fringe (on either side) occur: when d sine = $n\lambda$. We therefore say that n represents the **order** of interference.

10.3.2 Missing Orders

In the intensity expression $I_\theta = 4I_0 \left(\frac{\sin\beta}{\beta}\right)^2 \cos^2\gamma$, we have $\beta = \frac{\pi b \sin\theta}{\lambda}$ and $\gamma = \frac{R\, d \sin\theta}{\lambda}$.

Thus we see that β and yare not independent. These are connected to each other through the relation

$$\frac{\gamma}{\beta} = \frac{\pi d \sin\theta}{\pi b \sin\theta} = \frac{d}{b} = \frac{a+b}{b} \qquad (10.10)$$

Cases of special interest arise when d is an integral multiple of' 0, say it is an integer p so that $d = pb$. This will happen when the opaque portion a is an integral multiple of the transparent part b. The possibilities are: $a = b$, $a = 2b$ or $a = 30$ etc. so that $d/b = p = 2, 3, 4, ...$ in these cases. Under these conditions, the directions of diffraction minimum and interference maximum will necessarily coincide. To show this, let us assume that a direction of diffraction minimum is given by

$$b \sin\theta = mh$$

We will automatically have the interference maximum in this direction when $d = pb$ since

$$d \sin\theta = (pb) \sin\theta = p\,(b \sin\theta)$$

$$= pm\,\lambda = n\,\lambda$$

where n = pm. The possible values of p are 2, 3, 4,... and those of m are 1, 2, 3,... Thus the nth order interference fringes for which $n = pm$ will have zero intensity since the intensity of both beams is zero by virtue of diffraction condition. As a result their constructive interference also leads to net zero intensity. These are usually known as **missing orders.** For example, when p = 2, we will have 2, 4, 6, 8... orders missing for m values of 1, 2, 3,... etc. Similarly, when p = 3, we will have 3, 6, 9... orders missing and so on.

The special case when dlb = 1 means that the opaque part a = 0 and the two slits exactly join one another. Then we find that all the interference orders are missing. Actually this means that we now have a single slit of double width and consequently we get a single slit diffraction pattern (with no interference fringes).

These ideas are illustrated in the following example.

Example 1

Consider a double slit arrangement with b = 7.0×10^{-3}cm, d = 3.5×10^{-2}cm and λ = 6300Å. How many interference minima will occur between the diffraction minima on either side of the central maximum? If a screen is placed at a distance of 5m from the diffracting aperture, what is the fringe width?

Solution

The first diffraction minima on either side of central maximum 0 = 0 will occur when b $\sin\theta = \pm \lambda$ .That is, for sin8 = $\pm \lambda / b = 9\times10^{-3}$. The interference minima will occur when Eq. (10.8) is satisfied, i.e. when

$$d \sin \theta = \left(n + \frac{1}{2}\right)\lambda$$

On substituting the given values, we find that

$$\sin\theta = \left(n + \frac{1}{2}\right)\frac{\lambda}{d} = \left(n + \frac{1}{2}\right) 1.8 \times 10^{-3} \qquad n = 0, 1, 2, ...$$

i.e.

$$\sin 8 = 0.9 \times 10^{-3}, 2.7 \times10^{-3}, 4.5 \times10^{-3}, 6.3 \times 10^{-3} \text{ and } 8.1 \times 10^{-3}$$

Thus there will be ten minima between the two first order diffraction minima. If θ is small we may write 8, = 0.9×10^{-3} rad, $\theta_2 = 2.7 \times 10^{-3}$ rad, $\theta_3 = 4.5 \times 10$'' rad, 8, = 6.3×10^{-3} rad, 0, = 8.1×10^{-3} rad and the angle between successive interference minima is 1.8×10^{-3} rad.

Thus the fringe width f $\Delta\theta$ is

$$(500 \text{ cm}) \times 1.8 \times 10^{-3} = 0.9 \text{ cm}$$

10.3.3 Graphical Representation

We will now plot $\cos^2 y$, ($\sin^2 \beta / \beta^2$), and their product separately to study the double slit pattern. Before doing that we must decide on the relative scale of the abscissas y and β since the shape of the pattern will depend upon this choice. We have already shown that y /β is equal d/b. Let us say that in a particular case $\gamma / \beta = d/b = 3$. We must then plot the proposed curves for γ = 3P. In Fig. 10.4, the curves (a) and (b) are plotted to the same

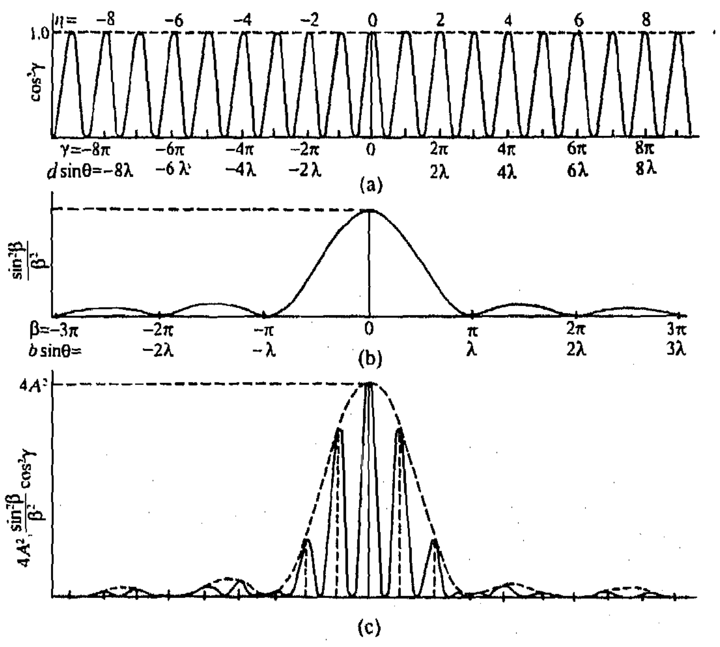


Fig. 10.4: Intensity curves for double slit. We have taken $\gamma = 3\beta$

scale of θ, Fig. 10.4(a) depicts the curve for $\cos^2\gamma$ which gives a set of equidistant maxima of equal intensity located at $\beta = 0, \pm\pi, \pm 2\pi, \pm 3\pi, ...$ In Fig. 10.4(b) we have plotted $(\sin\beta/\beta)^2$ which gives a maximum at $\beta = 0$ and minima at $\beta = \pm\pi, \pm 2\pi, ...$ In Fig. 10.4(c) we have plotted their product. What do you observe? The intensity of the fringes in the resultant pattern is not the same as it was in Fig.10.4(a). It is modulated (reduced) by the factor $\frac{\sin^2\beta}{\beta^2}$

This means that the central fringe or the zeroth fringe is the brightest and the successive two fringes are of decreasing intensity until we reach the point $\beta = \pi$ where the intensity is zero. Thus the third fringe corresponding to $\cos^2\gamma = \pm 3\pi$ falls at $\beta = \pi$ or $-\pi$ and their product is zero. Therefore, the third fringe on either side of the central maxima has zero intensity and its location at the angle θ satisfies simultaneously

$$\beta = \pm\pi \text{ and } \gamma = \pm 3\pi$$

or

$$b \sin\theta = \pm\lambda \text{ and } d \sin\theta = \pm 3\lambda$$

This third fringe will therefore be missing. We will observe the 4th and 5th fringes. We can argue in a similar manner that for 6th fringe

$$\beta = \pm 2\pi \text{ and } \gamma = \pm 6\pi$$

which will therefore have zero intensity and thus be missing.

You may now like to answer the following SAQ.

SAQ 2

Spend 2 min

Write down the general condition for missing orders in terms of the ratio d/b.

10.4 FRAUNHOFER PATTERN FROM *N* IDENTICAL SLITS

You now know that interference of waves diffracted by individual slits determines the intensity distribution in the double slit pattern. Let us now consider the diffraction pattern produced by N vertical slits. We use the same experimental arrangement, as shown in Fig. 10.1 for two slits. For simplicity we assume that (i) each slit is of width b and has the same length (ii) all slits are parallel to each other and (iii) the intervening opaque space between any two successive slits is the same, equal to a. Therefore the distance between any two equivalent points in two consecutive slits is $a + b$. Let us denote it by d which we call **the grating element.** As before, we take the source of light to be in the form of a slit and adjust the length of this source slit to be vertical and parallel to the length of N slits. An arrangement consisting of a large number of parallel-equidistant narrow rectangular slits of the same width is called **diffraction grating.** As discussed in the double slit pattern, the diffraction pattern will consist of vertical fringes parallel to the slit source. We now wish to study the intensity distribution in this pattern.

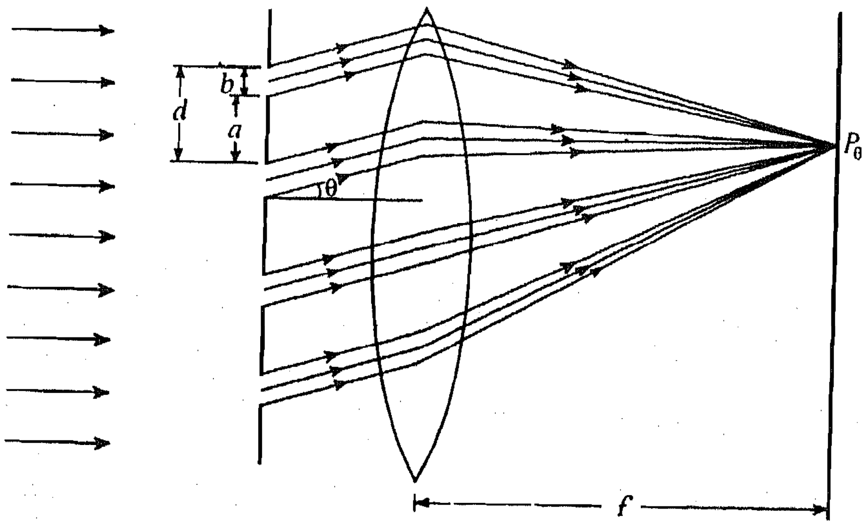


Fig. 10.5: Fraunhofer diffraction of a plane wave incident normally on a multiple slit aperture

10.4.1 Intensity Distribution

To derive an expression for the intensity distribution we will follow the procedure and arguments similar to those used for the double slit. Consider a point source of light which sends out plane waves. That is, a plane wavefront is incident on the arrangement shown in Fig. 10.5. (Speaking in terms of ray-optics, we may say that light rays fall normally on the grating.) You may recall that the intensity distribution along any section perpendicular to the vertical fringes formed from a slit source will be the same as obtained from a point source. Physically, light emerging from N slits after diffraction at each slit results in N diffracted beams. Since these are coherent, interference takes place between them resulting in the formation of fringes. It is important to note that diffraction controls the intensity from each slit in a given direction.

As before, we consider that the diffracted rays proceed towards P_0, where θ is the angle between the diffracted rays and the normal to the grating. Let $E_1, E_2, E_3 ..., E_N$ denote the fields produced by the first, the second, the third ... and the Nth slit at the point P_θ. Then we have

$$E_1 = A\frac{\sin\beta}{\beta}\cos(\omega t - \beta)$$

$$E_2 = A\frac{\sin\beta}{\beta}\cos(\omega t - \beta - \delta)$$

$$E_3 = A\frac{\sin\beta}{\beta}\cos(\omega t - \beta - 2\delta)$$

$$E_N = A\frac{\sin\beta}{\beta}\cos[\omega t - \beta - (N-1)\delta]$$

where various symbols have the same meaning as in Sec. 10.3. Also, we have assumed that the phase changes by equal amount δ from one slit to the next.

The field E at P_θ is obtained by summing these N terms :

$$E = A\frac{\sin\beta}{\beta}\cos(\omega t - \beta) + A\frac{\sin\beta}{\beta}\cos(\omega t - \beta - \delta) + A\frac{\sin\beta}{\beta}\cos(\omega t - \beta - 2\delta) + ... + A\frac{\sin\beta}{\beta}\cos[\omega t - \beta - (N-1)\delta] \quad (10.11)$$

You can write it as

$$E = A\frac{\sin\beta}{\beta}[\cos(\omega t - \beta) + \cos(\omega t - \beta - \delta) + ... + \cos[\omega t - \beta - (N-1)\delta]]$$

You have learnt to sum the series given here [Unit 2, Block 1 of the PHE-02 course on Oscillations and Waves; Eq. (2.38)]. We have reproduced it in the margin. The result is

$$E = A\left(\frac{\sin\beta}{\beta}\right)\frac{\sin N\gamma}{\sin\gamma}\cos\left(\omega t - \beta - \frac{1}{2}(N-1)\delta\right) \quad (10.12)$$

where $\gamma = \frac{\delta}{2} = \frac{\pi}{\lambda} d\sin\theta$.

The intensity of the resultant pattern is obtained by squaring the amplitude of the resultant field in this expression. Therefore

$$I_\theta = A^2\frac{\sin^2\beta}{\beta^2}\frac{\sin^2 N\gamma}{\sin^2\gamma} \quad (10.13)$$

Let us pause for a while and ask: What have we achieved so far? We have obtained an expression for the resultant intensity of diffraction pattern from N- slits. We expect it to be true for any number of slits.

For a single slit. Eq. (11.13) reduces to

$$I_\theta = A^2\frac{\sin^2\beta}{\beta^2}$$

which is the same as Eq. (9.7).

In complex notation,

$\exp(i\theta) = \cos\theta + i\sin\theta$ (i)

so that

$\mathrm{Re}[\exp(i\theta)] = \cos\theta$ (ii)

It means that

$\cos(\omega t - \beta) = \mathrm{Re}[e^{i(\omega t - \beta)}]$

$\cos(\omega t - \beta - \delta) = \mathrm{Re}[e^{i(\omega t - \beta - \delta)}]$

and $\cos(\omega t - \beta - (N-1)\delta) = \mathrm{Re}[e^{i(\omega t - \beta - (N-1)\delta)}]$

$\therefore \cos(\omega t - \beta) + \cos(\omega t - \beta - \delta) + ... = \mathrm{Re}[e^{i(\omega t - \beta)} + e^{i(\omega t - \beta - \delta)} + ... + e^{i(\omega t - \beta - (N-1)\delta)}]$ (iii)

The RHS can be written as

$\mathrm{RHS} = e^{i(\omega t - \beta)}[1 + e^{-i\delta} + e^{-2i\delta} + ... + e^{-i(N-1)\delta}]$ (iv)

This is a geometric series with common factor $e^{-i\delta}$ and can be summed up easily using the formula

$S = \frac{1 - r^n}{1 - r}$;

$\therefore \mathrm{RHS} = e^{i(\omega t - \beta)} \times \frac{1 - e^{-iN\delta}}{1 - e^{-i\delta}}$

$= e^{i(\omega t - \beta)} \times \frac{e^{-iN\delta/2}}{e^{-i\delta/2}}\left(\frac{e^{iN\delta/2} - e^{-iN\delta/2}}{e^{i\delta/2} - e^{-i\delta/2}}\right)$

$= e^{i(\omega t - \beta - (N-1)\delta/2)} \frac{\sin(N\delta/2)}{\sin(\delta/2)}$

Hence LHS of (iii) is recovered by the Real part, which is Eq. (10.12).

SAQ 3

Spend 2 *min*

Show that for N = 2, Eq. (10.13) reduces to Eq. (10.6) for the double slit.

10.4.2 Positions of Principal Maxima

For obtaining the positions of maxima (as well as minima), let us re-examine Eq. (10.13). We note that the intensity distribution is a product of two terms; the first term ($\sin^2 \beta/\beta^2$) represents the diffraction term produced by a single slit whereas the second term ($\sin^2 N\gamma / \sin^2 \gamma$) represents the interference term (for N slits). The interference term controls the width of interference fringes, while the diffraction term governs their intensities.

As in case of the double slit, we cannot locate the exact positions of maxima; their approximate positions can however be obtained by neglecting the variation of $\sin^2 \beta / \beta^2$. This is quite justified for very narrow slits. Therefore, for obtaining the positions of maxima we consider only the interference term.

We will now show that the maximum value of $\frac{\sin^2 N\gamma}{\sin^2\gamma}$ is N^2 which occurs for $\gamma = 0, \pi, 2\pi, ..., nx$. At the first glance, you will note that the quotient becomes indeterminate at these values. To overcome such a situation, we compute the first derivative of the numerator as well as the denominator separately before inserting the value of argument. Following this procedure you will readily obtain

$$\lim_{\gamma \to n\pi} \frac{\sin N\gamma}{\sin\gamma} = \lim_{\gamma \to n\pi} \frac{N \cos N\gamma}{\cos\gamma} = \pm N$$

so that for $\gamma = 0, \pi, ... n\pi$ we have

$$\left(\frac{\sin N\gamma}{\sin\gamma}\right)^2 = N^2$$

The expression for intensity for $\gamma = n\pi$ takes the form

$$I_\theta = A^2 \frac{\sin^2\beta}{\beta^2} N^2 = N^2 A^2 \frac{\sin^2\beta}{\beta^2} \tag{10.14}$$

where $\beta = \frac{\pi b \sin e}{\lambda}$.

We therefore conclude that the positions of maxima are obtained when

$$\gamma = 0, \pi, 2\pi, ...n\pi \text{ or } N\gamma = 0, N\pi, 2N\pi, ..., Nn\pi \tag{10.15}$$

Physically, at these maxima the fields produced by each of the slits are in phase and the resultant field is N times the field due to each of the slits.

When N is large, the intensity, being proportional to N^2, is very large and we will obtain intense maxima, if only $\sin^2 \beta / \beta^2$ is not too small. Such maxima are known as principal maxima.

The condition of principal maxima ($\gamma = n\pi$) can be rewritten as

$$d \sin e_n = n\lambda \tag{10.16}$$

which is identical to Eq. (10.9). It implies that

1. The principal maxima in N-slit pattern correspond in position to those of the double slit.
2. The relative intensities of different orders are modulated by the single slit diffraction envelope.
3. n cannot be greater than d/λ since $|\sin \theta| \le 1$. Can you imagine the implications of this condition? If you ponder for a while, you will realise that this condition suggests existence of only a finite number of principal maxima, which are designated as the first, second, third, ... order of diffraction. Moreover, there will be as many first order principal maxima as the number of wavelengths in the incident wave.

4. The relation between β and γ obtained for double slit in terms of slit width and slit separation does not change. That is, Eq. (10.10) holds for N-slits as well.

10.4.3 Minima and Secondary Maxima

As discussed in locating the position of maxima, to be able to find the minima in the diffraction pattern, we locate the minima of the interference term. We note that the numerator in $\sin^2 N\gamma/\sin^2 \gamma$ will become zero more often than the denominator. The numerator becomes zero for $N\gamma = 0, \pi, 2\pi, ..., p\pi$, or $\gamma = \frac{p\pi}{N}$. Therefore, $\sin\gamma = \sin\frac{p\pi}{N}$ will not become zero for all integral values of p. It will become zero only for special cases when $p = 0, N, 2N, ...$ so that γ assumes values $0, \pi, 2\pi, ...$ But you will recall that for these special values of γ, both $\sin N\gamma$ and $\sin \gamma$ vanish and the interference term defines the positions of principal maxima already discussed. However, for all other values of p, the numerator vanishes but not the denominator. That is, intensity vanishes when p, though an integer, is not an integral multiple of N. Hence, the condition for minimum is $\gamma = p\,\pi/N$ except when $p = nN$; n being the order. These values correspond to

$$N\gamma = [\pi, 2\pi, ..., (N-1)\pi], [(N+1)\pi, (N+2)\pi, ..., (2N-1)\pi], [(2N+1)\pi, ...]$$

$$\gamma = \left[\frac{\pi}{N}, \frac{2\pi}{N}, ..., \frac{(N-1)\pi}{N}\right], \left[\frac{(N+1)\pi}{N}, \frac{(N+2)\pi}{N}, ..., \frac{(2N-1)\pi}{N}\right], \left[\frac{(2N+1)\pi}{N}, ...\right] \tag{10.17}$$

These values of γ correspond to path difference

$$d\sin\theta_{min} = \left[\frac{\lambda}{N}, \frac{2\lambda}{N}, ..., \frac{(N-1)\lambda}{N}\right], \left[\frac{(N+1)\lambda}{N}, \frac{(N+2)\lambda}{N}, ..., \frac{(2N-1)\lambda}{N}\right], \left[\frac{(2N+1)\lambda}{N}, ...\right] \tag{10.18}$$

We might write this condition in the general form that

$$d\sin\theta_{min} = \frac{q\lambda}{N}$$

where q takes all the integral values except $0, N, 2N, ...$

In other words, $q = (1, 2, ..., (N-1))$, $(N+1, N+2, ...2N-1)$, $(2N+1, ...)$ etc.

You should note that when $q = 0, N, 2N, ..., Nn$ we see that $d\sin\theta$ becomes equal to $0, \lambda, 2\lambda, ..., n\lambda$ so that $d\sin\theta$ becomes equal to $n\lambda$ which represent the principal maxima and are omitted in the values of mimina.

Let us summarise what you have learnt in this unit so far.

> The condition for principal maxima:
>
> $$\gamma = 0, \pi, 2\pi, ..., n\pi$$
>
> and therefore
>
> $$N\gamma = 0, N\pi, 2N\pi, ..., nN\pi$$
>
> **We may write** $\quad \gamma = \frac{\pi d}{\lambda}\sin\theta_{max} = n\pi$ where $n = 0, 1, 2, ...$
>
> In terms of path difference $\quad d\sin\theta_{max} = n\lambda$
>
> The conditions for minima:
>
> $$N\gamma = [\pi, 2\pi, ..., (N-1)\pi], [(N+1)\pi, (N+2)\pi, ..., (2N-1)\pi], ...$$
>
> $$\gamma = \left[\frac{\pi}{N}, \frac{2\pi}{N}, ..., \frac{(N-1)\pi}{N}\right], \left[\frac{(N+1)\pi}{N}, \frac{(N+2)\pi}{N}, ..., \frac{(2N-1)\pi}{N}\right] .... \left[\frac{(2N+1)\pi}{N}, ...\right]$$
>
> In terms of path difference
>
> $$d\sin\theta_{min} = \left[\frac{\lambda}{N}, \frac{2\lambda}{N}, \frac{3\lambda}{N}, ... \frac{(N-1)\lambda}{N}\right], \left[\frac{(N+1)\lambda}{N}, \frac{(N+2)\lambda}{N}, ...\right] ....$$

If you write all possible values of γ, you will find that we have $(N-1)$ positions of minima between any two successive principal maxima. In Eq. (10.17) the first square bracket lists the positions of $(N-1)$ minima between the central and first principal maximum. Similarly the second square bracket lists the positions of $(N-1)$ minima between the first and second principal maxima. In other words, the first minimum oneither side of the nth principal maximum given by $d \sin\theta_{max} = n\lambda$ will be at $d \sin\theta_{min} = n\lambda \pm \frac{\lambda}{N}$.

Further, we know that between any two consecutive minima, there has to be a maxima Such maxima are said to be secondary **maxima.** There will be $(N-2)$ positions of secondary maxima between two consecutive principal maxima. As in single slit diffraction pattern the secondary maxima are not symmetrical, and the intensity of secondary maxima is very small. There are therefore of little practical importance. Fig. (10.6) shows the intensity pattern for $N=8$. Here we have shown principal maxima corresponding to $n=0, 1, 2, 3$ and six secondary maxima between adjacent principal maxima.

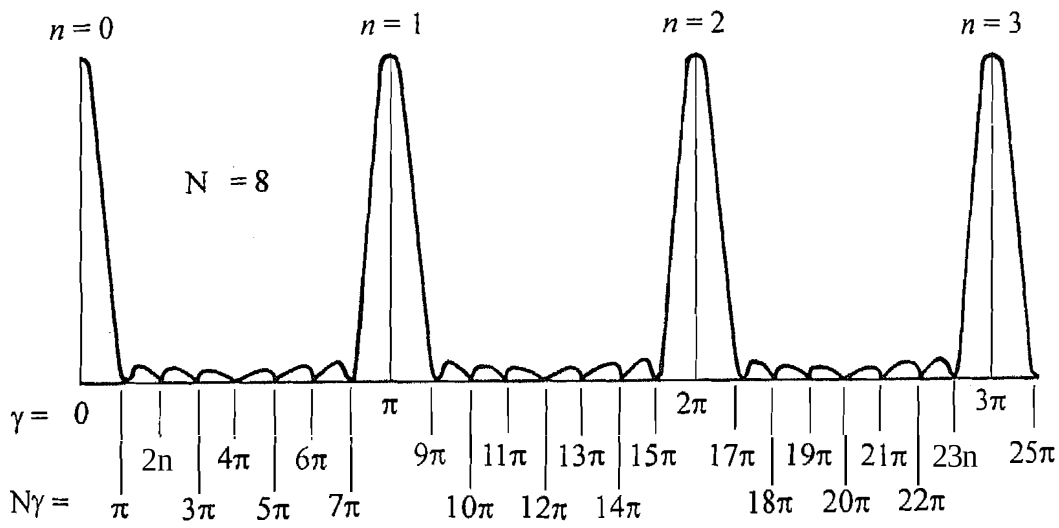


Fig.10.6: Intensity pattern for a diffraction grating of 8 slits.

Example 2

Calculate the maximum number of principal maxima that can be formed with a grating 5000 lines per cm for light of wavelength 5000 Å.

Solution :

Grating element $d = \frac{1\ \text{cm}}{5000} = 2 \times 10^{-4}$ cm

The condition for the formation of principal maxima is $d \sin\theta_{max} = n\lambda$. Since $|\sin\theta| < 1$, we cannot have n greater than $\frac{d}{\lambda}$. In this specific case

$$n = \frac{2 \times 10^{-4}\ \text{cm}}{5000 \times 10^{-8}\ \text{cm}} = 4$$

Therefore, it will be able to show **1st, 2nd,** 3rd and 4th orders of principal maxima.

If, on the other hand, we have a grating with 15000 lines per cm

$$n = \frac{(1/15000\ \text{cm}^{-1})}{5 \times 10^{-5}\ \text{cm}} = \frac{6.6 \times 10^{-5}\ \text{cm}}{5 \times 10^{-5}\ \text{cm}}$$

which is less than 2. Such a grating will show only **1st** order spectrum with $\lambda = 5000$ Å. You can verify this result while observing grating spectrum in your second level physics laboratory course.

10.4.4 Angular Half -width of Principal Maxima

You now know that for N slits

1. The principal maxima occur when $\gamma = n\pi$ and therefore $N\gamma = nN\pi$, i.e. $d \sin\theta_{max} = n\lambda$
2. On either side of the principal maxima, we have a minimum when $N\gamma = Nn\pi \pm \pi$ or

when $\gamma = n\pi \pm \frac{\pi}{N}$. In terms of path diffcrcnce and angle of diffraction, the conditions for principal maxima and the adjaccnt minimum are

$$d \sin\theta_{max} = nh \tag{10.19a}$$

and

$$d \sin\theta_{min} = n\lambda \pm \frac{\lambda}{N} \tag{10.19b}$$

The angle bctwccn θ_{max} and θ_{min} is callcd thc angular half-width of principal maxima. Let us dcnotc it by $\delta\theta$. Wc now procccd to calculatc this angle. Wc can calculate $\delta\theta$ (= | $\theta_{max} - \theta_{min}$ |) by computing θ_{max} and θ_{min} from these equations. Alternatively, by choosing $\theta_{min} > \theta_{max}$, wc substitutc $\theta_{min} = \theta_{max} + \delta\theta$ in Eq. (10.19b) to obtain

$$d \sin(\theta_{max} + \delta\theta) = n\lambda + \frac{\lambda}{N}$$

or

$$d \sin\theta_{max} \cos \delta\theta + d \cos\theta_{max} \sin \delta\theta = n\lambda + \frac{\lambda}{N}$$

For $\delta\theta \to 0$, cos 68 $\to$ 1 and sin 68 $\to$ 68. Hence

$$d \sin\theta_{max} + d \cos\theta_{max}\, \delta\theta = n\lambda + \frac{\lambda}{N}$$

Using Eq. (10.19a), we find that it takcs a compact form:

$$d \cos\theta_{max}\, \delta\theta = \frac{\lambda}{N}$$

so that

$$\delta\theta = \frac{\lambda}{N\, d \cos\theta_{max}} \tag{10.20}$$

which shows that thc principal maximum becomes sharper as N increases. It is for this reason that grating spcctrum is so sharp. You will now learn about it in detail.

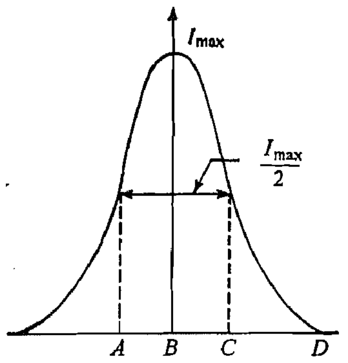
You may now question as to why is $\delta\theta$ called angular half width? It is quitc simple. You know that the principal maximum extends from minimum on one side to minimum on the other side and $\delta\theta$ is half of it. If wc consider the case of 6 slits the first principal maximum extends from

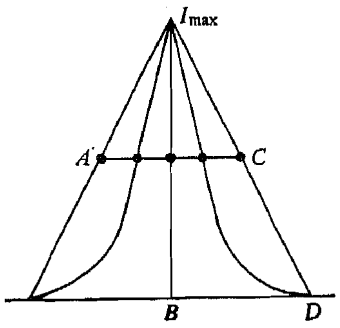
$$N\gamma = 5\pi \text{ to } N\gamma = 7\pi$$

or

$$d \sin\theta_{max} = \frac{5\lambda}{6} \text{ to } \frac{7\lambda}{6}$$

You must note that the term half width of a spectrum line (or a diffraction curve) has a slightly different meaning. The diagram shown below represents the intensity vs θ curve. The half width gives the width of the curve at $\frac{I_{max}}{2}$. It is equal to AC in the diagram. The angular half width on the other hand, is equal to BD. Obviously you can convince yourself that A C is not equal to BD. Only in the extreme case whcn the curve is a triangle is $AC = BD$.





10.5 DIFFRACTION GRATING

You have learnt about the diffraction pattern produced by a system of parallel equidistant slits. An arrangement of a large numbcr of equidistant narrow vertical slits is known as diffraction grating. The first gratings were made by Fraunhofer. He stretched fine silver wire on a frame. His grating had nearly 200 wires to a centimeter. Afterwards gratings were made by ruling fine lincs with a diamond pen on a glass plate. The transparent part between the lincs acted as a slit while the ruling itself acted effectively as the opaque part. Rowland was among the first to rule gratings on a metallic surface. He produced plane as well as concave gratings with nearly 5000 lincs per centimctcr. These gratings are difficult to make and arc expcnsivc but celluloid replicas can be made fairly cheaply and are commonly used in the physics laboratory for spectral analysis. You can make a simple coarse grating for demonstration purposes on a plate by drawing equidistant and parallel scratches on the photographic emulsion. Now-a-days it is possible to produce gratings holographically. Holographic gratings have greater rulings per cm and are definitely better than ruled gratings. You will get an opportunity to learn details about holography in Block-4.

10.5.1 Formation of Spectra

We have seen that for a monochromatic light of wavelength λ_1, the principal maxima are given by the grating equation

$$d \sin\theta_1 = n\,\lambda_1 \qquad n = 0, 1, 2, 3, \ldots$$

With the experimental arrangement described above we will get these principal maxima as one line in each order. Using another source of light which emits a longer wavelength λ_2, we will get a corresponding line in each order at a larger angle θ_2:

$$d \sin\theta_2 = n \lambda_2 \qquad n = 0, 1, 2, 3, \ldots$$

However if the same source of light emits both the colours corresponding to wavelengths λ_1 and λ_2, we will get two lines simultaneously in each order. These two lines will be seen as two spectrum lines separated from each other. This is because except the central maximum (zeroth order). the angles of diffraction for λ_1 and λ_2 are different in various other orders. In the central maximum $\theta = 0$ for all wavelengths and therefore different colours are not separated from each other. What do you expect to observe when we have a white light source? The central image will be white while all other orders will show colours and we will see a continuous spectrum,

We note that in the grating equation, if we know d, θ and n, we can calculate the wavelength of light. Since the grating element (d) is known for a grating and θ can be measured. this arrangement provides a simple and accurate method of measuring λ. This is discussed in the following section.

10.5.2 Observing Grating Spectra

In your second level physics laboratory course, you must have observed grating spectra using a simple spectrometer. This arrangement is depicted in Fig. 10.7. The light from the given source is focussed (with the help of a lens) on the slit of the collimator which sends out a parallel beam of light.

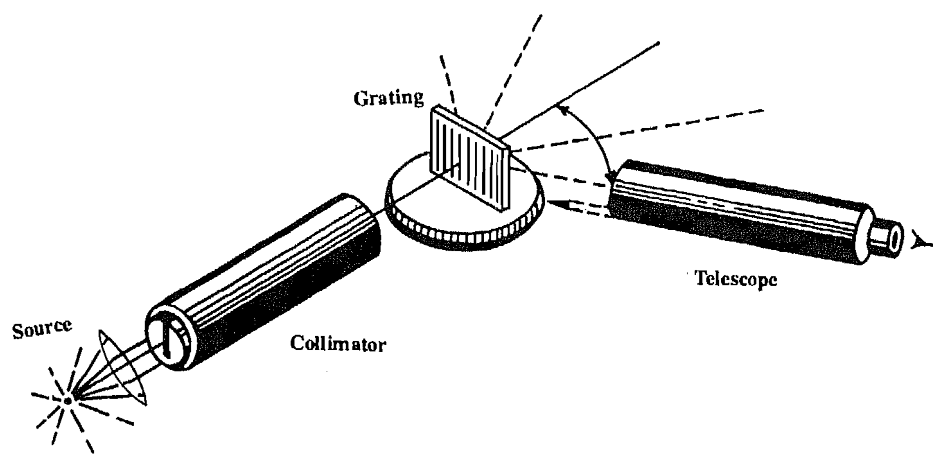


Fig.10.7: A schematic diagram of experimental arrangement for observing grating spectra

The telescope arm is rotated and brought in line with the collimator. This ensures that the parallel beam of light falling on the objective of telescope is focussed at the crosswires, which is in the focal plane of the eye piece. The position of the source of light should be adjusted to get the brightest image. We mount the diffraction grating on the turntable and adjust it so that the light is incident normally on the grating. Next we rotate the telescope arm to the left or right to get the first order spectrum in the field of view. If the source of light is a discharge tube containing sodium, mercury or argon, the spectrum will consist of a series of spectrum lines. Each spectrum line is a diffracted image of the slit, formed by different wavelengths present in the source. To get sharp line images, we adjust the grating so that the diffracting slits are parallel to the collimator slit. This can be done by rotating the grating in its own plane.

To mount a grating for normal incidence, you should follow the steps given below :

1. Take the reading of the turntable when the telescope arm is in line with the collimator. Let it be ϕ.
2. Rotate the telescope to position $\phi \pm 90°$.
3. Mount the grating on the turn table and rotate the turn table till you see the image of the slit after reflection from the grating. At this position the surface of the grating is inclined at 45° to the parallel beam of light emerging from the collimator.
4. Obviously turning the grating through 45° in the proper direction will make the light fall normally on the grating surface.

To measure the wavelength of each line, we set the vertical crosswires at the centre of each spectrum line and note the position of the telescope in each case. The difference between this position of the telescope and the direct position gives the angle of diffraction for each of the lines. To reduce error, the position of the telescope is noted on both sides of the direct position and half of this angle gives the angle of diffraction.

Light emitted by an atom consists of sharp spectrum lines. Light emitted by a molecule consists of a group of lines which when unresolved give a band like appearance and is often called band spectrum, while an incandescent lamp or similar sources will give a continuous spectrum, where various colours merge into one another.

You must have observed that

1. The spectrum exists on both sides of the direct beam.

2. Apart from the first order, the second or even third order spectrum (depending on the grating element) are also present.
3. Different spectrum lines are not equally bright or sharp. This depends on the energy levels and the transitions of the atom giving the spectrum. These concepts are further illustrated in the following example.

Example 3

Rowland ruled 14438 lines per inch in his grating. (i) Calculate the angles of diffraction for violet (λ = 4000 Å) and red (λ = 8000 Å) colours in the first order of spectrum. (ii) What is the largest wavelength which can be seen with this grating in the third order?

Solution

(i) The grating element $d = \frac{2.54 \text{ cm}}{14438} = 0.0001759 \text{cm}$

$$= 1.759 \times 10^{-4} \text{ cm}$$

Suppose that the violet colour (λ = 4000 Å) is diffracted through angle θ_v. Recall the condition For maximum:

$$d \sin\theta_v = n\lambda$$

For first order, on substituting the given values, you will get

$$\sin \theta_v = \frac{4 \times 10^{-5} \text{cm}}{1.759 \times 10^{-4} \text{cm}} = 0.2274$$

Therefore $\theta_v \cong 13°$

Similarly, for red colour (λ = 8000 Å), we have

$$\sin \theta_r = \frac{8 \times 10^{-5} \text{cm}}{1.759 \times 10^{-4} \text{cm}} = 0.4548$$

so that

$$\theta_r = 27°$$

This means that the entire visible spectrum in the first order extends from nearly θ = 13° to θ = 27°, i.e. covers an angle of about 14°.

(ii) $d \sin\theta = 3\lambda_{max}$

According to the given condition, θ = 90° so that $\sin\theta$ = 1 and $d = 3\lambda_{max}$

or

$$\lambda_{max} = \frac{d}{3} = \frac{1.759 \times 10^{-4}}{3} \text{cm} = 5860 \text{ Å}$$

This calculation suggests that in the third order spectrum, the sodium doublet consisting of 5890 Å and 5896 Å will not be visible. Do you recall this from your observations on spectral analysis using a diffraction grating? If you have so far not opted for the second level physics, it will be worthwhile to verify this result.

If you calculate $\sin\theta_v$ and $\sin\theta_r$ for 1st, 2nd and 3rd orders, you will find that for

1st order	$\sin \theta_v = 0.2274 \Rightarrow \theta_v \sim 13°$	$\Rightarrow$ 14° spread	
	$\sin \theta_r = 0.4548 \Rightarrow \theta_r \sim 27°$		
2nd order	$\sin \theta_v = 0.4548 \Rightarrow \theta_v \sim 27°$	$\Rightarrow$ 38° spread	
	$\sin \theta_r = 0.9096 \Rightarrow \theta_r \sim 65°$		
3rd order	$\sin \theta_v = 0.6822 \Rightarrow \theta_v \sim 43°$	$\Rightarrow$ 47° for 4000 Å – 5860 Å	
	$\sin \theta_{max} = 1$ for $\lambda_{max} = 5860$ Å and $\theta_{max} = 90°$		

$\sin \theta_r > 1$ cannot be observed. $\Rightarrow$ entire visible spectrum is not available in 3rd order.

Schematically it is shown below for the spectrum on the left side of the centre. A similar spectrum will be observed on the right side of the central order.

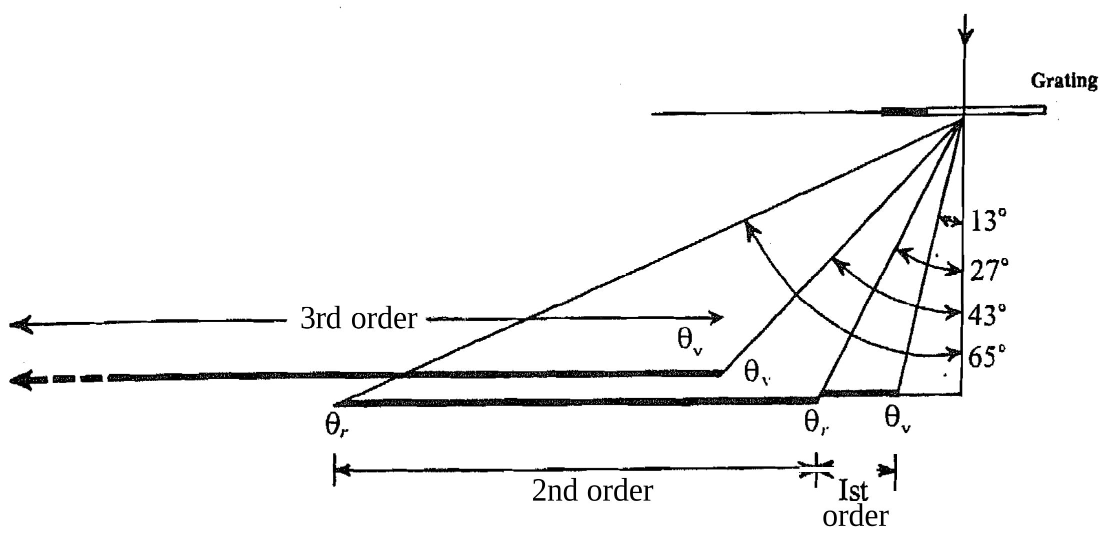


Fig. 10.8: Over-all angular spread of various orders of spectrum

Thus we find that in 1st order red just touches second order violet. (This is because we have selected $\lambda = 4000$ Å and $\lambda = 8000$ Å.) It means that there is essentially no overlapping of first and second order spectra. In third order λ_v begins at $\theta \cong 43°$. If you calculate wavelength λ_x of 2nd order present at $\theta = 43°$ you will find that

$$d \sin 43° = 3\lambda_v = 2\lambda_x \Rightarrow \lambda_x = \frac{3 \times 4000 \text{ Å}}{2} = 6000 \text{Å}.$$

Therefore $\lambda = 6000$ Å of the 2nd order occurs at the same place as $\lambda = 4000$ Å of third order. Therefore, from 6000 Å to 8000 Å will have overlapping colours. This difficulty is usually avoided by using suitable colour filters.

We now summarise what you have learnt in this unit.

10.6 SUMMARY

- The double slit diffraction pattern consists of a number of equally spaced fringes similar to what is observed in interference experiments. These fringes are the brightest in the central part of the pattern.
- In double slit pattern fringes reappear three or four times before they become too faint to observe.
- The central maximum in double slit pattern is four times brighter than that in single slit pattern.
- The intensity of double slit diffraction pattern at an angle θ is given by

$$I_\theta = 4I_0 \frac{\sin^2 \beta}{\beta^2} \cos^2 \gamma$$

  Here $I_0 = A^2$, $\beta = \dfrac{\pi b \sin\theta}{\lambda}$ and $\gamma = \dfrac{\pi}{\lambda} d \sin\theta$, where b is slit width and d is distance between two similar points in these apertures. It is equal to a + b, where a is the width of the intervening opaque space between two slits.
- The intensity of double slit diffraction pattern is product of the irradiances observed for the double slit interference and single slit diffraction. Physically, it arises due to interference between two diffracted beams

- For slits of very small widths, the double slit diffraction pattern reduces to Young's interference pattern.
- The conditions of maxima and minima in double slit (equally spaced) interference pattern are:

$$d \sin\theta = n\lambda \qquad \text{(maxima)}$$

$$d \sin\theta = \left(n + \frac{1}{2}\right)\lambda \qquad \text{(minima)}$$

and the condition for minima for diffraction intensily is

$$b \sin\theta = m\lambda \qquad \text{(minima)}$$

- The intensity distribution in N-slit diffraction pattern is given by

$$I_\theta = A^2 \frac{\sin^2\beta}{\beta^2} \frac{\sin^2 N\gamma}{\sin^2\gamma}$$

- The conditions for maxima and minima in N-slit pattern are given in the box on page 50.
- As the number of slits increases, the maxima get narrower and for sufficiently large values of N, they become very sharp lines. The angular half-width of principal maximum $\delta\theta$ is given by

$$\delta\theta = \frac{\lambda}{N d \cos\theta_{max}}$$

The principal maximum is sharp for large values of N.

10.7 TERMINAL QUESTIONS

1. If we use a while light source in the arrangement shown in Fig. 10.2, how will it affect the fringes?
2. Can there be principal maxima of zero intensity because of diffraction at each slit? If yes, discuss.

10.8 SOLUTIONS AND ANSWERS

SAQs

1. λ_1 will give its diffraction pattern within which we will get the interference fringes. The pattern for λ_2 will be smaller if $\lambda_2 < \lambda_1$. They will both be superimposed on one another coinciding at $\theta = 0$.
2. The general conditions for missing orders in terms of γ and β are: $\gamma = \pm n\pi$ or $d \sin\theta = \pm n\lambda$ and $\beta = \pm m\pi$ or $b \sin\theta = \pm m\lambda$.

   Therefore when $n = pm$ (n, m, p are all integers) we get

   $$\frac{d}{b} = \frac{n}{m} = p$$

   The missing orders occur when d/b is an integer. When $d/b = 1$, i.e. the two slits exactly join, all the interference orders are missing. Physically it means that we have a single slit of double width and consequently no interference.

   For $\frac{d}{b} = 2$, second, fourth, sixth, ... orders will be missing. What do you say about $\frac{d}{b} = 3$?

3. For N = 2, Eq. (10.13) takes the form

$$I_\theta = A^2 \frac{\sin^2 \beta}{\beta^2} \frac{\sin^2 2\gamma}{\sin^2 \gamma}$$

$$= A^2 \frac{\sin^2 \beta}{\beta^2} \frac{(2 \sin \gamma \cos \gamma)^2}{\sin^2 \gamma}$$

$$= 4 A^2 \frac{\sin^2 \beta}{\beta^2} \cos^2 \gamma$$

which is the required result for the double slit.

TQs

1. As before, each wavelength will give its interference fringes. The central fringe for all wavelengths will coincide and hence the central fringe will be white. Fringes of order $n = 1, 2, 3, \ldots$ located on either side of the central fringe, at different θ values given by $d \sin \theta = n\lambda$ for different wavelengths will be coloured.

2. There can be a principal maxima whose intensity is zero because of the diffraction at each slit. These are called missing orders or absent spectra. We know that the relationship between β and γ in terms of slit width and slit separation for N slits is the same as for the double slit. Therefore, the conditions for missing orders remain unaltered. And a particular maximum will be absent if it is formed at the same angle as the minimum of single slit diffraction pattern. This occurs at an angle which satisfies Eq. (10.19a) and (10.19b).