UNIT 7 INTERFEROMETRY

Structure

7.1 INTRODUCTION

An instrument designed to exploit the interference of light and the fringe patterns that result from optical path differences, in any of a variety of ways, is called an optical 'interferometer. In this unit, we explain the functioning of the Michelson and the Febry-Perot interferometers, and suggest only a few of their many applications.

In order to achieve interference between two coherent beams of light, an interferometer divides an initial beam into two or more parts that travel diverse optical paths and then superpose to produce an interference pattern. One criterion for broadly classifying interferometers distinguishes the manner in which the initial beam is separated. Wavefront division interferometers sample portions of the same wavefront of a coherent beam of light, as in the case of Young's double slit, Lloyd's mirror or Fresnel's biprism arrangement. Amplitude-division interferometers, instead, use some type of beam-splitter that divides the initial beam into two parts. The Michelson interferometer is of this type. Usually the beam splitting is managed by a semi-reflecting metallic film. In this interferometer, the two interfering beams are widely separated, and the path difference between them can be varied at will by moving the mirror or by introducing a refracting material in one of the beams. Corresponding to these two ways of changing the optical path, there are two important applications of this interferometer, which we will study in this unit.

There is yet another means of classification that distinguishes between those interferometers that function by the interference of two beams, as in the case of the Michelson interferometer, and those that operate with multiple beams, as in the Fabry-Perot interferometer. In this unit, we will show that the fringes so formed are sharper than those formed by two beam interference. Therefore, the interferometers involving multiple beam interference have a very high resolving power, and, hence, find applications in high resolution spectroscopy.

Objectives

After studying this unit, you should be able to

- understand how Michelson interferometer produces different types of fringes, viz., circular, localised (or straight) and white light fringes,

- describe few applications of Michelson interferometer,
- relate the intensity of the transmitted light to the reflectance of the plate surface in Fabry-Perot interferometer, and
- understand the difference between Michelson interferometer and Fabry-Perot interferometer.

7.2 MICHELSON INTERFEROMETER

It is an excellent device to obtain interference fringes of various shapes which have a number of applications in optics. It utilizes the arrangements of mirrors and beam splitter.

Construction: Its configuration is illustrated in Fig. 7.1.

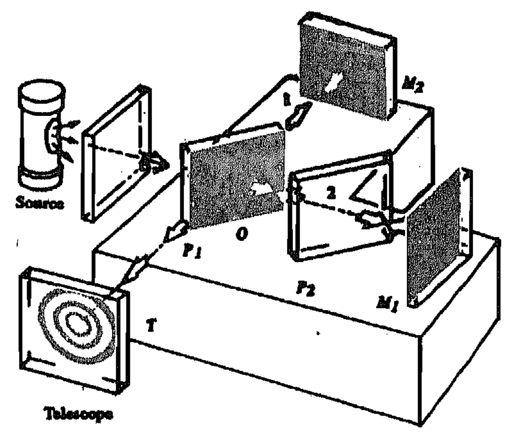

Fig. 7.1: Michelson Interferometer.

Its main optical parts are two plane mirrors M_1 and M_2 and two similar optically-plane parallel glass plates P_1 and P_2. The plane mirrors M_1 and M_2 are silvered on their front surfaces and are mounted vertically on two arms at right angles to each other. To obtain fringes; the mirrors M_1 and M_2 are made exactly perpendicular to each other by means of screws shown on mirror M_1. The mirror M_2 is mounted on a carriage which can be moved in the direction of the arrows. The plates P_1 and P_2 are mounted exactly parallel to each other, and inclined at 45^0 to M_1 and M_2. The surface of P_1 towards P_2 is partially silvered. The plate P_1 is called beam splitter.

Working: An extended source (e.g., a diffusing ground glass plate illuminated by a discharge lamp) emits lightwaves in different directions, part of which travels to the right and falls on P_1. The light wave incident on P_1 is partly reflected and partly transmitted. Thus, the incident wave gets divided into two waves, viz., the transmitted wave 1 and the reflected wave 2. These two waves travel to M_1 and M_2 respectively. After reflection at M_1 and M_2 the two waves return to P_1. Part of the wave coming from M_2 passes through P_1 going downward towards the telescope, and part of the wave coming from M_1 gets reflected by P_1 toward the telescope. Since the waves entering the telescope are derived from the same incident wave, they are coherent, and, hence, in a position to interfere. The interference fringes can be seen in the telescope.

In contrast to the Young double slit experiment, which uses light from two very narrow sources, the Michelson interferometer uses light from a broad spreadout source.

You must be eager to know the purpose of the plate P_2, because till now we have not mentioned anything about P_2.

Function of the plate P_2: Note that if reflection at P_1 occurs at the rear surface at point O, as shown in Fig. 7.1, the light reflected at M_2 will pass through P_1 three times while the light reflected at M_1 will pass through only once. Thus, the paths of waves 1 and 2 in glass are not equal. Consequently, each wave will pass through the same thickness of glass only when a compensator plate P_2, of the same thickness and inclination at P_1, is inserted in the path of wave 1. The compensator plate is an exact duplicate of P_1 with the exception that it is not partially silvered. With the compensator in place, any optical path difference arises from the actual path difference.

Form of fringes: The form of the **fringes** depends on the inclination of M_1 and M_2. To understand how fringes are formed, refer to the Fig. 7.2, where the physical components are represented somewhat differently. An observer at the position of the telescope will, simultaneously, see both mirrors M_1 and M_2 **alongwith** the source L, formed by reflection in the partially silvered surface of the glass plate P_1. Accordingly, we can redraw the interferometer as if **all** the elements were in a straight line. Here M_1' corresponds to the image of mirror M_1 formed by reflection at the silvered surface of the glass plate P_1 so that OM, = OM;. Depending on the positions of the mirrors, image M_1' may be in front of, behind or exactly coincident with mirror M_2. The surfaces L_1 and L_2 are **images** of the source L in mirrors M_1 and M_2 respectively. If we consider a single **point** S on the source L, emitting light in all directions, then on reaching O, it gets split, and thereafter its segments get reflected by M_1 and M_2. In Fig 7.2, we represent this by reflecting the ray off both M_1 and M_2. Thus, the interference fringes may be regarded to be formed by light reflected from the surface of M_1' and M_2. Here, S_1 and S_2 act as coherent point sources, because to an observer at D the two reflected rays will appear to have come from the image points S_1 and S_2. The mirror M_2 and the virtual image of M_1 play the same roles as the two surfaces of the thin film, discussed in unit 6, and the same sort of interferences fringes result from the light reflected by these surfaces.

Now, let us discuss the various types of fringes, **viz.**, circular fringes, localized fringes and white light fringes.

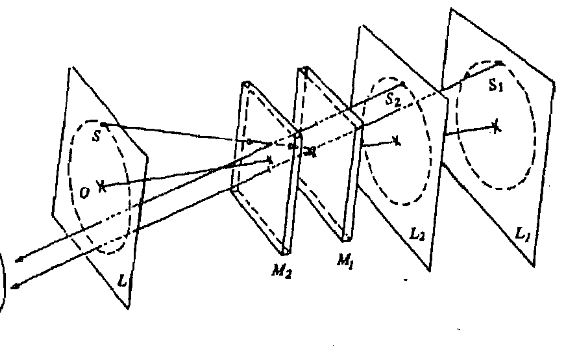

Fig. 7.2: A conceptual rearrangement of the Michelson Interferometer.

7.2.1 Circular Fringes

These fringes are observed when M_1 is exactly perpendicular to M_2. In this situation, the distance of the mirrors M_1 **and** M_2 from the plate P_1 can be varied.

Let us consider the various possible positions of the mirrors M_1 and M_2, and, eventually, see how it gives rise to circular fringes. (i) If the two mirrors have the same axial distance from the rear face of P_1, and if they are perpendicular to each other, the image M_1 is coincident with M_2. At the coincidence position, the two paths **are** of equal length. Thus, we expect the waves to reinforce each other and to form a maximum, But this is not so, because of nphase change, which occurs on external (air-to-glass) reflection only. No phase change occurs on internal (glass-to-air) reflection, and none occurs on transmission or refraction. Look again at Fig. 7.1 and note that it is the **light** that comes from M_1 and goes to **the** observer that is reflected, air-to-glass, at **O**, and undergoes the **π change.** This means that at the coincidence position there will be a minimum: **the centre** of the **field will be dark.**

(ii) Now, we move one of the mirrors. If the mirror is moved through a quarter of wavelength, d = $\lambda/4$, the path length (because if d is separation between M_1 and M_2, then $2d$ is the separation between S_1 and S_2) changes by $\lambda/2$, the two waves getting out of phase by 180^0, the phase change compensates, and we have a maximum. Moving the mirror by another $\lambda/4$, gives minimum, another $\lambda/4$ another maximum and so on. Thus,

$$2d = m\lambda, \quad \text{where } m = 0, 1, 2, \qquad \dots(7.1)$$

is the **Michelson's interferometer** equation.

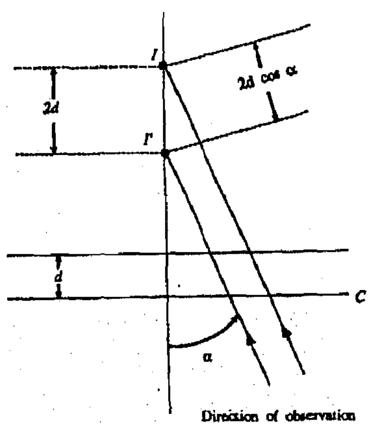

Fig. 7.3: Looking off-axis into the Michelson Interferometer.

(iii) **Next, assume** that **we** look obliquely into the interferometer and that our line of sight makes an angle a with the axis. Ordinarily, the two planes M_1 and M_2 are at a distance d apart, and the two virtual images; I and I' separated by $2d$. But for oblique incidence, as we see from Fig. 7.3, the path difference between the two lines of sight becomes less and instead of Eq. (7.1), we get

$$2d \cos \alpha = m\lambda \; ; \quad \text{where } m = 0, 1, \ldots \qquad \ldots(7.2)$$

For a given mirror separation d, and a given order m, wavelengtli λ and angle a is constant. The maxima will lie in the form of circles about the foot of the perpendicular from the eye to the mirrors. These circular fringes will look like the ones shown in Fig. 7.4. Fringes of this kind, where parallel beams are brought to interfere with a phase difference determined by the angle of inclination 0, are referred to as fringes of equal

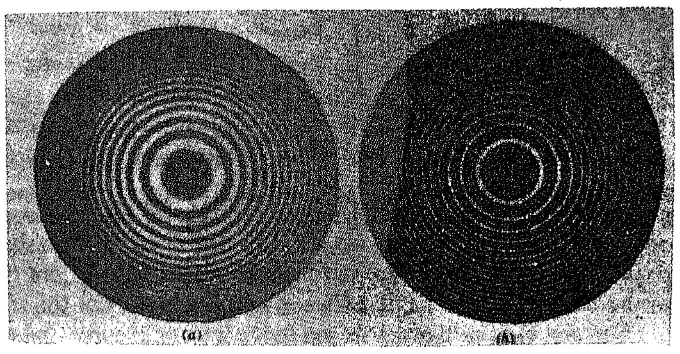

Fig. 7.4: Fringes observed using (a) Michelson interferometer, (b) Fabry-Perot Interferometer.

inclination. These fringes are also known as Haidinger fringes. They differ from the fringes of equal inclination considered in Unit 6, only in that, here there are no multiple reflections so that the intensity distribution is in accordance with Eq. (5.17)

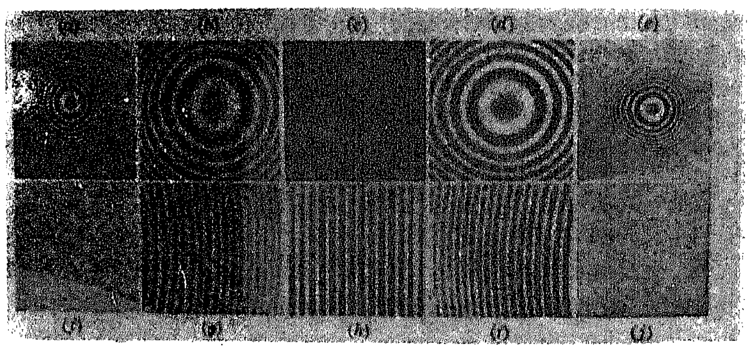

Fig. 7.5: Appearance of the various types of fringes observed In the MIchelson Interferometer. Upper row shows circular frlnges whereas lower row shows, localized frlnges. Path difference Increases outward, In both dlrectlons, from the centre.

The upper part of the Fig. 7.5 shows how the circular fringes look under different conditions. When M_2 is few centimeters beyond M_1, the fringe system will have the general appearance shown in (a) with the rings very closely spaced. If M_2 is now moved slowly toward M_1, so that d is decreased, Eq. (7.2) shows that a given ring, characterized by a given value of the order m, must decrease its radius, because the product $2d$ cos a must remain constant. The rings, therefore, shrink and vanish at the centre, a ring disappearing each time $2d$ decreases by λ, or d by $\lambda/2$. This follows from the fact at the centre cos $\theta = 1$, so that Eq. (7.2) becomes

$$2d = m\lambda$$

which is Eq. (7.1).

To change m by unity, d must change by $\lambda/2$. Now as M_2 approaches M_1, the rings become more widely spaced as indicated in Fig. (7.5b), until we reach a critical position, where the central fringe has spread out to cover the whole field of view, as shown in Fig. 7.5 (c). This happens when M_2 and M_1 are exactly coincident, for it is clear that under these conditions the path difference is zero for all angles of incidence. If the mirror is moved still farther, it effectively passes through M_1, and new widely spaced fringes appear, growing out from the centre. These will gradually become more closely spaced, when the path difference increases, as indicated in (d) and (e) of the Fig. 7.5.

7.2.2 Localized Fringes (Straight Fringes)

If the mirrors M_1 and M_2 are not exactly parallel, the air film between the mirrors is wedge-shaped, as indicated in Fig. 7.6.

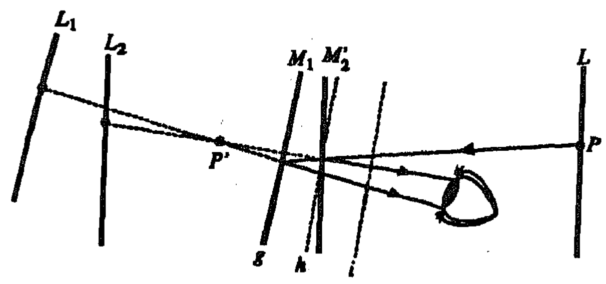

Fig. 7.6: The formation of fringes with inclined mirrors in the Michelson Interferometer.

The two rays reaching the eye from point P on the source are now no longer parallel, but appear to diverge from point P' near the mirrors. For various positions of P on the extended source, the path difference between the two rays remains constant, but the distance of P' from mirrors changes. If the angle between the mirrors is not too small, the latter distance is never great, and hence, in order to see these fringes clearly, the eye must be focused on or near the rear mirror M_2. The localized fringes are, practically, straight, because the variation of the path difference across the field of view is now due primarily to the variation of the thickness of the "air film" between the mirrors. With a wedge-shaped film, the locus of point of equal thickness is a straight line, parallel to the edge of the wedge. The fringes are not exactly straight, if d has an appreciable value, because there is also some variation of the path difference with angle. They are, in general, curved and are always convex toward the thin edge of the wedge. Thus, with a certain value of d, we might observe fringes shaped like those of Fig. 7.5(g). M_2 could then be in position such as g of Fig. 7.6. If the separation of the mirrors is decreased, the fringes will move to the left across the field, a new fringe crossing the centre each timed changes by $\lambda/2$. As we approach the zero path difference, the fringes become straighter until the point is reached where M_2 actually intersects M_1, when they are perfectly straight, as in Fig. 7.5(h). Beyond this point, they begin to curve in the opposite direction, as shown in Fig. 7.5 (i). The blank fields shown in Fig. 7.5 (f) and (j) indicate that this type of fringe cannot be observed for large path differences. As the principle variation of path difference results from a change of the thickness d, these fringes have been termed fringes of equal thickness.

7.2.3 White Light Fringes

If a source of white light is used, no fringes will be seen at all except for a path difference so small that it does not exceed a few wavelengths. In observing these fringes, the mirrors are tilted slightly as for localized fringes, and the position of M_2 is found where it intersects M_1. With white light there will then be observed a central dark fringe, bordered on either side by 8 or 10 coloured fringes. This position is often rather troublesome to find, using white light only. It is best located approximately before hand by finding the place where the localized fringes in monochromatic light become straight. Then a very slow motion of M_1 through this region, using white light, will bring these fringes into view.

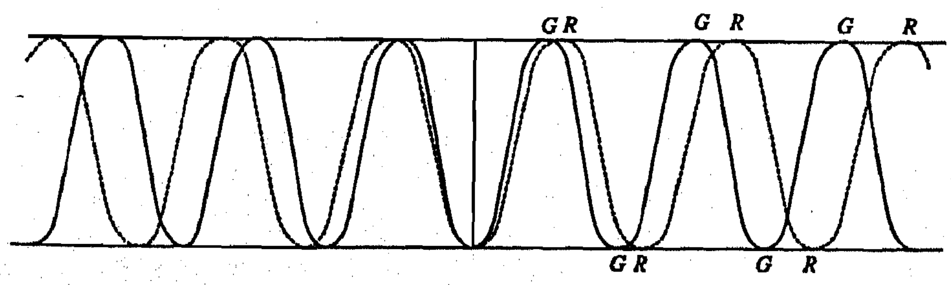

Fig. 7.7: The formation of white light fringes with a dark fringe at the centre.

The fact, that only a few fringes are observed with white light, is easily accounted for when we remember that such light contains all wavelengths between 400 and 750 mm. The fringes for a given colour are more widely spaced, the greater the wavelength. Thus, the fringes in different colours will only coincide for d = 0, as indicated in Fig. 7.7. The solid curve represents the intensity **distribution** in the fringes for the green light, and the broken curve for the red light. Clearly, only the central fringe will be uncoloured, and the fringes of different colours will begin to separate at once an either side. After 8 or 10 fringes, **so** many colours are present at a given point that the resultant colour is essentially white. White light fringes are, particularly, important in the Michelson interferometer, where they may be used to locate the position of zero path difference, as we shall see later.

7.2.4 Adjustment of the Michelson's Interferometer

i) **For** Localised Fringes: The distance of the mirrors M_1 and M_2 from the silvered surface of P_1 are **first** made as nearly equal as possible by moving the movable mirror M_2. A pin-hole is placed between the lens and the plate P_1 (Fig. 7.8). If M_1 is not perpendicular to M_2, four images of the pin-hole are obtained, two by **reflection** at the semi-silvered surface of P_1 and the other two by reflection at the other surface of P_1.

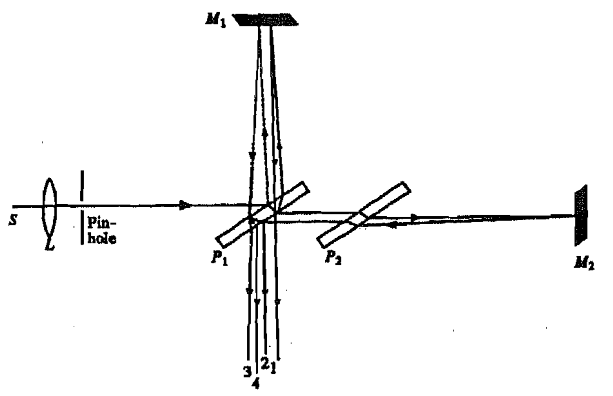

Fig. 7.8: Adjustment of Michelson interferometer.

The former pair is, naturally, brighter than the latter. The small screws at the back of the mirror, M_1, are then adjusted until the two bright images appear to coincide. The pin-hole is now removed. If the coincidence of the images was apparent, the air-film between M_2 and M_1 would be **wedge-shaped,** and the localised fringes would appear.

ii) **For** White light Localised Fringes: First, the localised fringes with monochromatic light are obtained. The mirror M_2 is then moved until the fringes become straight. Monochromatic light is replaced by white light. M_2 is further moved in the same direction until the central achromatic fringe is obtained in the field of view.

iii) For Circular **Fringes:** After localised fringes are obtained, the screws of M_1 are adjusted so that the spacing between these fringes increases. This happens when the angle of the wedge decreases. If this adjustment be continued, at one stage, the angle of the wedge will become zero, and the film will be of constant thickness. At this stage, circular fringes will appear. Finer adjustment is made until on moving the eye side ways or up and down, the fringes do not expand or contract.

7.2.5 Applications

There are three principal types **of** measurement that can be made with Michelson interferometer: (i) wavelengths of light (ii) width and fine structure of spectrum lines (iii) refractive indices. As explained in the sub-section **7.2.3,** when a certain spread of wavelengths is present in the light source, the fringes become indistinct and, eventually, disappear as the path difference is increased. With white light they become invisible when d is only a few wavelengths, whereas the circular fringes obtained with the light of single spectrum line can still be seen after the mirror has been moved several centimeters. Therefore, for making these measurements with this interferometer, it is adjusted for circular fringes.

a) Determination of Wavelength of Monochromatic Light

After having adjusted interferometer for circular fringes, adjust the position of M_2 to obtain a bright spot at the centre of the field of view. If d be the thickness of the film and n the order of the spot obtained, we have

$$2d \cos \alpha = n \lambda \quad (7.3)$$

But at the centre a = 0; so that cos a = 1. Therefore

$$2d = n\lambda \quad (7.4)$$

If now M_2 be moved away form M_1 by $\lambda/2$, $2d$ increases by A. Therefore n + 1 replaces n in Eq. (7.4). Hence, $(n + 1)$th bright spot now appears at the centre (see sec. 7.2.1). Thus, each time M_2 moves through a distance $\lambda/2$, next bright spot appears at the centre. Suppose, during the movement of M_2 through a distance x, N new fringes appear at the centre of the field. Then we have

$$x = N \frac{\lambda}{2}$$

$$\therefore \quad \lambda = \frac{2x}{N} \quad (7.5)$$

Thus, by measuring the distance x with the micrometer and counting the number N, the value of λ can be obtained.

The determination of A by this method is very accurate, because x can be measured to an accuracy of 10^{-4}mm, and the value of N can be sufficiently increased, as the circular fringes can be obtained up to large path differences.

SAQ 1

When the movable mirror of Michelson's interferometer is shifted through 0.0589 mm, a shift of 200 fringes is observed. What is the wavelength of light used? Give the answer in Angstrom units.

(b) Determination of **difference** in **Wavelength :** When the source of light has two wavelengths λ_1 and λ_2 very close together (like D_1 and D_2 lines of sodium), each wavelength produces its own system of rings. Let $\lambda_1 > \lambda_2$. When the thickness of the film is small, the rings due to λ_1 and λ_2 almost coincide, since λ_1 and λ_2 are nearly equal. The mirror M_2 is moved away. Then, due to different spacing between the rings of λ_1 and λ_2, the rings of λ_1 are gradually separated from those of λ_2. When the thickness of the air-film becomes such that dark rings of λ_1 coincides with bright rings of λ_2 (due to closeness of λ_1 and λ_2, the dark rings due to λ_1 will practically coincide with bright rings due to $\lambda/_2$ in the entire field of view), the rings have maximum indistinctness.

The mirror M_2 is moved further away through a distance, say, x until the rings, after becoming most distinct, once again become most indistinct. Clearly, during this movement, n fringes of λ_1 and (n + 1) fringes of λ_2 have appeared at the centre (because then the dark rings of λ_1 will again coincide with the bright rings of λ_2). Now, since the movement of the mirror M_2 by λ_2 results in the appearance of one new fringe at the centre, we have

$$x = n \frac{\lambda_1}{2} = (n + 1) \frac{\lambda_2}{2}$$

or

$$n = \frac{2x}{\lambda_1} \text{ and } (n + 1) = \frac{2x}{\lambda_2}$$

$$\therefore \quad \frac{2x}{\lambda_2} - \frac{2x}{\lambda_1} = 1$$

or

$$\frac{2x(\lambda_1 - \lambda_2)}{\lambda_1 \lambda_2} = 1$$

or

$$\lambda_1 - \lambda_2 = \frac{\lambda_1 \lambda_2}{2x}$$

Since λ_1 and λ_2 are close together, $\lambda_1 \lambda_2$ can be replaced by λ^2 where λ is the mean of λ_1 and λ_2.

$$\lambda_1 - \lambda_2 = \frac{\lambda^2}{2x} \quad (7.6)$$

Thus if we measure the distance moved by M_2 between two consecutive positions of disappearance of the fringe pattern and the mean wavelength is known, we can determine the difference $(\lambda_1 - \lambda_2)$.

SAQ 2

In Michelson's interferometer, the reading for a pair of maximum indistinctness were found to be 0.6939 mm and 0.9884 mm. If the mean wavelength of the two components of light be 5893Å, deduce the difference between the wavelengths of the components.

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c) Determination of Refractive Index of a Thin Plate

If a thickness t of a substance having an index of refraction μ is introduced into the path of one of the interfering beams in the interferometer, the optical path in this beam is increased because of the fact that light travels more slowly in the substance, and consequently, has a shorter wavelength. The optical path is now μt through the medium, whereas it was practically t through the corresponding thickness of air ($\mu = 1$). Thus, the increase in the optical path due to insertion of the substance is $(\mu - 1)t$.

In practice, the insertion of a plate of glass in one of the beams produces a discontinuous shift of the fringes so that the number of fringes cannot be counted. With monochromatic fringes, it is impossible to tell which fringe in the displaced set corresponds to one in the original set. With white light, the displacement in the fringes of different colours is very different. This illustrates the necessity of adjusting the interferometer to produce straight white light fringes. After having adjusted so, the cross-wire is set on the achromatic fringe, which is perfectly straight. The given plate is now inserted in the path of one of the interfering waves. This increases the optical path of the beam by $(\mu - 1)t$. Since the beam traverses the plate twice, an extra path difference of 2 $(\mu - 1)t$ is introduced between the two interfering beams. The fringes get shifted. The movable mirror M_2 is moved till the fringes are brought back to their initial positions so that the achromatic fringe again coincides with the cross wire. If the displacement of M_2 is x, then

$$2x = 2(\mu - 1)t$$

or

$$x = (\mu - 1)t. \quad (7.7)$$

Alternatively, if N be the number of fringes shifted then

$$2(\mu - 1)t = N\lambda \quad (7.8)$$

Thus, measuring x, t, may *be* calculated if μ is known, or μ may be calculated if t is known.

This method can be used to find the refractive index of a gas. The gas is introduced into an evacuated tube placed along the axis of one of the interfering beams, and the experiment is carried out as described above.

SAQ 3

A transparent film of glass of refractive index 1.50 is introduced normally in the path of one of the interfering beams of a Michelson's interferometer, which is illuminated with light of wavelength 4800Å. This causes 500 dark fringes to sweep across this field, Determine the thickness of the film.

There is yet another type of interferometer, called Fabry-Perot interferometer, which produces fringes much sharper than those produced by Michelson interferometer. In the next section, let us study this interferometer and see how it is used as a powerful spectrometer.

7.3 FABRY-PEROT INTERFEROMETER

It is based on the principle of multiple beam interference. It is a high resolving power instrument, which makes use of the 'fringes of constant inclination' produced by the transmitted tight after multiple reflections between two parallel and highly-reflecting glass plates.

It consists of two optically-plane glass plates A and B (Fig. 7.9) with plane surfaces. The inner surfaces are coated with partially transparent films of high reflectivity and placed accurately parallel to each other. Screws are provided to secure parallelism if disturbed. The two uncoated surfaces of each plate are made to have a slight angle between them in order to avoid unwanted fringes formed due to multiple reflections in the plate itself.

One of the two plates is kept fixed, while the other can be moved to vary the separation of the two plates. In this configuration, the instrument is called a Fabry-Perot interferometer. Sometimes both the plates are at a fixed separation with the help of spacers. The system with fixed spacing is known as Fabry-Perot etalon. The Fabry-Perot interferometer (or etalon) is used to determine wavelengths precisely, to compare two wavelengths, to calibrate the standard metre in terms of wavelength, etc.

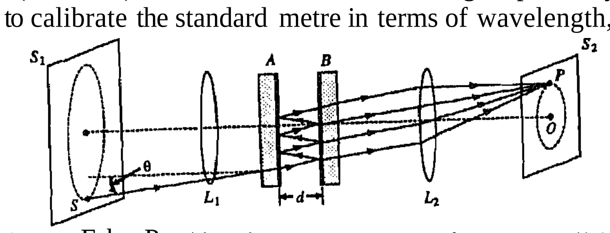

Fig. 7.9: Fnbry-Perut interferometer. S is part uf an external light source.

S_1 is a broad source of monochromatic light and L_1 a convex lens which makes the beam more collimated. An incident ray suffers a large number of internal reflections successively at the two silvered surfaces, as shown. At each reflection a small fractional part of the light is also transmitted. Thus, each incident ray produces a group of coherent and parallel transmitted rays with a constant path difference between any two successive rays. A second convex lens, L_2, brings these rays together at a point P in its focal plane, where they interfere. Hence, the rays from all points of the source produce an interference pattern on a screen S_2 placed in the focal plane of L_2.

Formation of the Fringes: Let **d** be the separation between the two silvered surfaces, and 0 the inclination of particular ray with the normal to the plates. Then the path difference between any two successive transmitted rays corresponding to the incident ray is 2d $\cos\theta$. The medium between the two silvered surfaces is usually air. As you saw, while solving SAQ 1 in Unit 6, that π phase changes occur on both of these (air-to-glass) surfaces, hence, the condition

$$2d \cos\theta = n\lambda,$$

holds for maximum intensity.

Here, n is an integer, called the order of interference, and λ the wavelength of light. The locus of points in the source which give rays of a constant inclination θ is a circle. Hence, with an extended source, the interference pattern consists of a system of bright concentric rings on a dark background, each ring corresponding to a particular value of θ. Fig. 7.4(b) shows the fringes obtained using a Fabry-Perot interferometer. Also shown, in the figure for comparison, arc fringes obtained by using Michelson interferometer (see Fig. 7.4a). It can readily be seen that the Fabry-Perot interferometer, which employs the principle of multiple beam interference, produces much sharper fringes, and could, hence, be used to study hyperfine structure of spectral lines. The intensity distribution of the circular fringes of Fig. 7.4b is not in accordance with Eq. (5.17). To determine how much light is reflected and transmitted at the two surfaces, let us read the following section.

7.3.1 Intensity Distribution

Comment: You are advised to go through the Appendix carefully given at the end of this unit.

We return now to the problem of reflections from a parallel plate, already considered in a two-beam approximation in Unit 6. Fig. 7.10 shows the multiple reflections and transmissions through a plane parallel plate of "air" enclosed between two glass plates of Fabry-Perot interferometer. Here, n'is the refractive index of glass plate and n the refractive index of air enclosed. Suppose a wave is incident at an angle 8, as shown in Fig. 7.10. This incident wave will suffer multiple reflections. Let the reflection and transmission amplitude co-efficient be r and t at an external reflection and r'and t'at an internal reflection.

If the amplitude of incident ray is expressed as a $e^{i\omega t}$, the successive transmitted rays can be expressed by appropriately modifying both the amplitude and phase of the initial wave. Referring to Fig. 7.10, these are

$$A_1 = (tt'\,a)\,e^{i\omega t}$$
$$A_2 = (tt'\,r'^2\,a)\,e^{i(\omega t - \delta)}$$
$$A_3 = (tt'\,r'^4\,a)\,e^{i(\omega t - 2\delta)} \text{ and so on.}$$

A little inspection of these equations shows that

$$A_N = tt'\,r'^{2(N-1)}\,a e^{i\omega t}\,e^{-i(N-1)\delta}$$

The quantities r, r', t, t', are given in terms of n, $n'\,\theta$, θ' by the Fresnel formulae. For our present purpose we do not need these explicit expressions but only relations between them. We have

$$tt' = T \qquad \text{...7.9(a)}$$

and $$r^2 = r'^2 = R \qquad \text{...7.9(b)}$$

where R and T, respectively are the reflectivity and lransmissivity of the plate surfaces. Then, using Eq. 7.9, we have

$$A_1 = aTe^{i\omega t},$$
$$A_2 = aTRe^{i(\omega t - \delta)},$$
$$A_3 = aTR^2e^{i(\omega t - 2\delta)}, \text{ and so on,}$$

By the principle of superposition, the resultant amplitude is given by

$$A = aT + aTRe^{-i\delta} + aTR^2e^{-2i\delta} + aTR^3e^{-3i\delta} + ...$$

Here, we have ignored $e^{i\omega t}$, as it is of no importance in combining waves of the same frequency. Hence,

$$A = aT\,(1 + Re^{-i\delta} + R^2e^{-2i\delta} + R^3e^{-3i\delta} + ...)$$

The infinite geometric series in the parentheses has the common ratio $Re^{i\delta}$ and has a finite sum because $r^2 < 1$. Summing up the series, we obtain

$$A = aT\frac{1}{1 - Re^{-i\delta}}$$

The complex conjugate of A is therefore

$$A^* = aT\frac{1}{1 - Re^{+i\delta}}$$

Hence the resultant intensity I is given by

$$I = AA^* = \frac{a^2T^2}{(1 - Re^{-i\delta})(1 - Re^{+i\delta})}$$

$$= \frac{a^2T^2}{1 - R^2 - 2R\,(e^{i\delta} + e^{-i\delta})} = \frac{a^2T^2}{1 + R^2 - 2R\cos\delta}$$

$$= \frac{a^2T^2}{(1 - R^2) + 2R\,(1 - \cos\delta)} = \frac{a^2T^2}{(1 - R^2) + 4R\sin^2\frac{6}{2}}$$

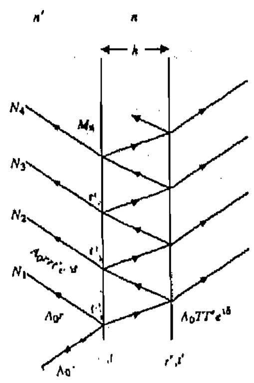

Fig. 7.10: Multiple reflection and transmission in a parallel "air" plate enclosed between the two plates of Fabry–Perot Interferometer.

$$= \frac{a^2T^2}{(1-R)^2} \frac{1}{1 + \frac{4R}{(1-R)^2}\sin^2\frac{\delta}{2}} \qquad \ldots(7.10)$$

The intensity will be a maximum when $\sin^2\frac{\delta}{2} = 0$, i.e. $\delta = 2n\pi$ where $n = 0, 1, 2, \ldots$ Thus

$$I_{max} = \frac{a^2T^2}{(1-R)^2} \qquad \ldots(7.11)$$

Similarly, the intensity will be a minimum when $\sin^2\frac{\delta}{2} = 1$, i.e. $\delta = (2n+1)\pi$ where $n = 0, 1, 2, \ldots$ Thus

$$I_{min} = \frac{a^2T^2}{(1-R)^2} \frac{1}{1 + \frac{4R}{(1-R)^2}} = \frac{a^2T^2}{(1+R)^2} \qquad \ldots(7.12)$$

Eq. (7.10) can now be written as

$$I = \frac{I_{max}}{1 + \frac{4R}{(1-R)^2}\sin^2\frac{\delta}{2}} \qquad \ldots(7.13)$$

or

$$I = \frac{I_{max}}{1 + F\sin^2\frac{\delta}{2}} \qquad \ldots(7.14)$$

Here, $F = \frac{4R}{(1-R)^2}$ is called the coefficient of Finesse. Eq. (7.14) is the intensity expression for the Fabry-Perot fringes.

If we plot I against δ for different values of R (the reflectivity of the plates), a set of curves is obtained (Fig. 7.11). They show that the larger the value of R, the more rapid is the fall of intensity on either side of a maximum. (That is, higher the reflectivity of the plates, sharper are the interference bright fringes.) Further, as Eq. (7.11) and (7.12) show, larger the value of R, greater is the difference between I_{max} and I_{min}. In fact, we obtain a system of sharp and bright rings against a wide dark background.

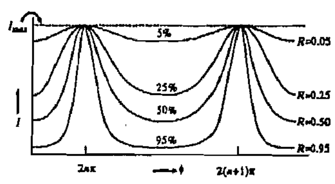

Fig. 7.11: The transmitted intensity as a function of δ showing how the sharpness depends on **reflectance.**

Percentages refer to reflectance of surfaces.

As mentioned in the beginning of the sec. 7.3, Fabry-Perot interferometer is a high resolving power instrument. Its resolving power $\frac{\lambda}{\Delta\lambda}$ is given by

$$\frac{\lambda}{\Delta\lambda} = \frac{4\pi h\cos r\sqrt{F}}{4.147\,\lambda}$$

where h is the thickness of the film enclosed between the two silvered surfaces, r is the angle of refraction inside the film, λ the wavelength of incident light and F is the coefficient of Finesse.

To have an idea of the numerical value of resolving power, let us consider a Fabry-Perot etalon with h = 1cm and F = 80. The resolving power for normal incidence in the wavelength region around λ = 5000 Å would be

$$\frac{\lambda}{\Delta\lambda} = \frac{4\pi\sqrt{80}}{5 \times 10^{-5} \times 4.147} = 5.42 \times 10^5$$

that is, two wavelengths separated by 0.0092 Å can be resolved at λ = 5000Å.

7.3.2 Superiority over Michelson's Interferometer

When the light consists of two or more close wavelengths (such as D_1 and D_2 lines of sodium), then in a Fabry-Perot interferometer each wavelength produces its own pattern, and the rings of one pattern are clearly separated from the corresponding rings of the other pattern. Hence the instrument is very suitable for the study of the fine structure of spectral lines, In Michelson's instrument separate patterns are not produced. The presence of two close wavelengths is judged by the alternate distinctness and indistinctness of the rings when the optical path difference is increased.

7.4 SUMMARY

- The Michelson interferometer uses an extended monochromatic source.
- When M_1 and M_2 are perpendicular to each other, i.e., when M_1 and M_2 are parallel, the fringes given by a monochromatic source are circular and localized at infinity.
- When the mirrors of the interferometer are inclined with respect to each other, i.e., when M_1 and M_2 are not perpendicular to leach other, a pattern of straight parallel fringes are obtained.
- Whether M_1 and M_2 are parallel or inclined, any fringe shift seen in an interferometer may be due to either a change in thickness or a change in refractive index.
- As the movable mirror is displaced by $\frac{\lambda}{2}$, each fringe will move to the position previously occupied by an adjacent fringe. If N is the number of fringes that have moved past a reference point, when the mirror is moved a distance x, then

$$x = N\frac{\lambda}{2}$$

- Michelson interferometer can be used in the measurement of two closely spaced wavelengths.
- Fabry-Perot interferometer, which employs the principle of multiple beam interference, produces much sharper fringes than those produced by Michelson interferometer.
- In the Fabry-Perot interferometer it is the fringe pattern formed by transmitted light that is observed and as such that intensity distribution would be given by

$$I = \frac{I_{max}}{1 + \frac{4R}{(1-R)^2}\sin^2\frac{\delta}{2}}$$

- Resolving power of Fabry-Perot interferometer is given by

$$\frac{\lambda}{\Delta\lambda} = \frac{4\pi h \cos r\sqrt{F}}{4.147\,\lambda}$$

7.5 TERMINAL QUESTIONS

i) When one leg of a Michelson interferometer is lengthened slightly, 150 dark fringes sweep through the field of view. If the light used has A = 480 mm, how far was the mirror in that leg moved?

2) Circular fringes are observed in a Michclson interferometer illuminated with light of wavelength 5896 Å. When the path difference between the mirrors M_1 and M_2 is **0.3** cm, the central fringe is bright. Calculate the angular diameter of the 7th bright fringe.

7.6 SOLUTIONS AND ANSWERS

SAQs

1) The distance, x, moved by the mirror when N fringes cross the field of view is given by

$$x = N\frac{\lambda}{2}$$

$$\therefore \quad \lambda = \frac{2x}{N}$$

Here, x = 0.00589 cm, and N =200.

$$\therefore \quad \lambda = \frac{2 \times 0.00589}{200} = 0.0000589 \text{ cm} = 5890 \text{ Å}$$

2) If x be the distance moved by the movable mirror between two consecutive positions of maximum indistinctness (or distinctness), we have

$$\Delta\lambda = \frac{\lambda_1 \times \lambda_2}{2x} = \frac{\lambda^2}{2x},$$

Where λ is the average of λ_1 and λ_2.

Here A = 5893Å = 5893 x 10^{-8}cm and x = 0.9884 − 0.6939 = 0.2945 mm = 0.02945 cm.

$$\therefore \quad \Delta\lambda = \frac{(5893 \times 10^{-8})^2}{2 \times 0.02945} = 5.896 \times 10^{-8}\text{cm} = 5.896\text{Å}.$$

3) Let t be the thickness of the film. When it is put in the path of one of the interfering beams of the Michelson's interferometer, an additional path difference of $2(\mu - 1)t$ is introduced. If N be the number of fringes shifted, we have

$$2\,(\mu - 1)\,t = N\lambda$$

$$\therefore \quad t = \frac{N\lambda}{2\,(\mu - 1)}$$

Here N= 500; $\lambda = 4800 \times 10^{-8}$cm, μ = 1.50.

$$\therefore \quad t = \frac{500 \times 4800 \times 10^{-8}}{2\,(1.50 - 1)}$$

$$= \frac{500 \times 4800 \times 10^{-8}}{2 \times 0.50}$$

$$= 0.024 \text{ cm.}$$

TOs

I) Darkness is observed when the light beams from the two legs are 180" out of phase. As the length of one leg is increased by $\frac{\lambda}{2}$, the path length increases by A, and the field of view changes from dark to bright to dark. When 150 fringes pass, the leg is lengthened by an amount

$$(150)\left(\frac{\lambda}{2}\right) = (150)\,(240 \text{ nm}) = 36{,}000 \text{ nm} = 0.036 \text{ mm}$$

2) The expression for the bright circular fringe is

$$2d \cos r = n\lambda$$

At the centre $r = 0$, so that

$$2d = n\lambda \qquad \text{(i)}$$

n now stands for the order of the central bright Cringe. The order of fringes decreases as we move outwards from the centre. Thus the second bright fringe is of (n − 1)th order,..., seventh bright fringe is of (n −6)th order. Hence if θ be the angular radius of 7th bright fringe, we have

$$2d \cos\theta = (n - 6)\lambda \qquad \text{(ii)}$$

Eq. (i) and (ii) give

$$2d\,(1 - \cos\theta) = 6\lambda$$

or $\cos\theta = 1 - \frac{6\lambda}{2d}$

Putting the given values:

$$\cos\theta = 1 - \frac{6 \times (5896 \times 10^{-8}\,\text{cm})}{2 \times 0.3\,\text{cm}}$$

$$= 1 - 0.0005896 \simeq 0.9994$$

∴ $\theta = \cos^{-1}(0.9994) = 2°$.

∴ angular diameter = 4°.

7.7 APPENDIX

Method of Complex Amplitudes

In place of using the sine or the cosine to represent a simple harmonic wave, one may write the equation in the exponential form as

$$y = ae^{(i\omega t - kx)} = ae^{i\omega t}\,e^{-i\delta}$$

where $\delta = kx$ is constant at a particular point in space and represents phase of the wave. The presence of $i = -1$ in this equation makes the quantities complex. We can nevertheless use this representation, and at the end of the problem take either the real (cosine) or the imaginary (sine) part of the resulting expression. The time-varying factor exp $(i\omega t)$ is of no imporlance in combining waves of the same frequency, since the amplitudes and relative phases are independent of time. The other factor, a exp $(-i\delta)$, is called the complex amplitude. It is a complex number whose modulus a is the real amplitude, and whose argument δ gives the phase relative to some standard phase. Negative sign merely indicates that the phase is behind the standard phase. In general, the vector a is given by

$$a = ae^{i\delta} = x + iy = a(\cos\delta + i\sin\delta)$$

Then it will be seen that

$$a = \sqrt{x^2 + y^2}, \quad \tan\delta = \frac{y}{x}$$

Thus, if a is represented as in Fig. (7.12), plotting horizontally its real part and vertically its imaginary part, it will have the magnitude a and will make the angle δ with the x axis, as we require for vector addition.

The advantage of using complex amplitudes lies in the fact that the vector addition of real amplitudes can be written more easily in the form of an algebraic addition of complex amplitudes. For example, consider the real parts of two waves that follow the equations

$$A_1 = A_1 e^{i(\omega t + \delta_1)}$$

and

$$A_2 = A_2 e^{i(\omega t + \delta_2)} \quad ...(7.15)$$

Adding these two equations gives

$$A = A_1 + A_2 = A_1 e^{i(\omega t + \delta_1)} + A_2 e^{i(\omega t + \delta_2)} \quad ..(7.16)$$

We can now take out the common exponent $i\omega t$:

$$A = e^{i\omega t}(A_1 e^{i\delta_1} + A_2 e^{i\delta_2}) \quad ...(7.17)$$