UNIT 6 INTERFERENCE BY DIVISION OF AMPLITUDE

Structure

6.1 INTRODUCTION

We have all seen the marvellous rainbow colours that appear in soap bubbles and thin oil films. When a soapy plate drains, coloured reflections often occur from it. A similar effect occurs when light is reflected from wet pavements that has an oil slick on it. Have you ever wondered what causes the display of colours when light is reflected from such thin oil film or soap bubble?

All these effects are due to interference of light reflected from the opposite surfaces of the film. Thus the phenomenon owe its origin to a combination of reflection and interference.

In the last unit, we discussed the interference of light, but there, the two interfering light waves are produced by division of wavefront. For example, in Young's double slit experiment, light coming out of a pin hole was allowed to fall into two holes, and the light waves emanating from these two holes interfered to produce the interference pattern, But the interference of light waves, which is responsible for the colour of thin films, involves two light beams derived from a single incident beam by division of amplitude of the incident wave. When a light wave falls on a thin film, the wave reflected from the upper surface interferes with the wave reflected from the lower surface. This gives rise to beautiful colours. However, one must initially consider how the phase of a light wave is affected when it is reflected.

In the last unit, you noted that in Lloyd's mirror, the interference takes place between waves coming direct from the source and those reflected from an optically denser medium. As a consequence of this, the central fringe is round to be 'dark' instead of 'bright'. This was explained by assuming the fact that a phase change of π takes place when light waves are reflected at the surface of a "denser" medium. We will begin this unit by giving proof of the statement made above; this proof will be based on the principle of reversibility of light.

It is also possible to observe interference using multiple beams. This is known as multiple beam interferometry, and it will be discussed in the next unit. It will be shown there that multiple beam interferometry offers some unique advantages over two beam interferometry.

Objectives

After studying this unit, you should be able to

- prove that when a light wave is reflected at the surface of an optically denser medium, it suffers a phase change of π.

- describe the origin of the interference pattern produced by a thin film,
- describe the formation, shape and location of interference fringes obtained from a thin wedge-shaped film,
- describe how Newton's rings are used to determine the wavelength of light,
- explain why a thin coating of a suitable substance minimizes the reflection of light from a glass surface,
- distinguish between fringes of equal inclination and fringes of equal thickness.

6.2 STOKES' ANALYSIS OF PHASE CHANGE ON REFLECTION

To investigate the phase change in the reflection of light at an interface between two media, Sir G.C. Stokes used the principle of optical reversibility. This principle states that a light ray, that is reflected or refracted, will retrace its original path, if its direction is reversed, provided there is no absorption of light.

Fig. 6.1(a) shows the surface MN separating media 1 and 2, the lower one being denser. Suppose medium 1 is air and medium 2 is glass.

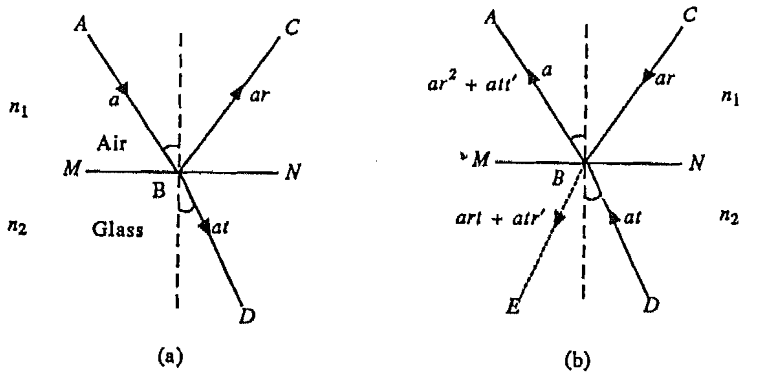


Fig. 6.1: (a) A ray is reflected and refracted at an air-glass interface. (b) The optically reversed situation; the two rays in the lower left must cancel. In both cases, $n_2 > n_1$ (n_1 and n_2 are the refractive indices of the media).

An incident light wave, *AB*, is partly reflected along *BC* and partly transmitted (refracted) along *BD*. Let a be the amplitude of the incident wave *AB*, r be the fraction of the amplitude reflected, and *t* be the fraction transmitted when the wave is travelling from medium 1 to 2. Then the amplitudes along *BC* and *BD* are ar and at, respectively.

Now, suppose the directions of the reflected and transmitted (refracted) waves are reversed. As shown in Fig. 6.1(b), the wave *BC*, on reversal, gives a reflected wave along *BA*, and a transmitted (refracted) wave along *RE*. The amplitude of reflected wave along *BA* is $ar.r = ar^2$ and the amplitude of transmitted wave along *BE* is art. Similarly, the wave *BD*, on reversal, gives a transmitted wave along *BA* and a reflected beam along *BE*. Let r' and t' be the fractions of amplitude reflected and transmitted when the wave is travelling from medium 2 to medium 1. Then the amplitude of the transmitted wave along *BA* is att' and the amplitude of reflected wave along *BE* is a d . But, according to principle of reversibility of light, the reflected and transmitted waves *BC* and *BD*, when reversed, should give the original ray of amplitude a along *BA* only. Hence, the component along *BE* should be zero and that along *BA* should be equal to a. That is

$$art + atr' = 0 \qquad ...(6.1)$$

and

$$ar^2 + att' = a \qquad ...(6.2)$$

From Eqs. (6.1) and (6.2), we get

$$r' = -r \qquad ...(6.3)$$

and

$$tt' = 1 - r^2 \qquad ...(6.4)$$

Eqs. (6.3) and (6.3) are known as **Stoke's** relations.

You must be aware that a transverse wave in a spring undergoes a 180" phase change when reflected from a rigid support. A similar phase change occurs for the reflection of a light wave from the boundary of a medium, having a greater index of refraction. The optically denser medium corresponds to a rigid support. A light wave reflected from the boundary of a medium whose index of refraction is greater than that of the medium in which the incident wave travels undergoes a 180° phase change.

Now. observe carefully Eq. (6.3). Here r is the fraction of amplitude reflected when incident wave is travelling from a rarer to denser medium, and r'when incident wave is travelling from a denser to a rarer medium. The two fractions are numerically equal but have opposite signs. Hence, these are exactly out of phase with each other, i.e., their phase difference is 'π'. If no phase change occurs when a light wave is reflected by a denser medium then there must be a phase change of π when a light wave is reflected by a rarer medium—and conversely, if no phase change occurs when a light wave is reflected by a rarer medium then there must be a phase change of π when a light wave is reflected by a denser medium. Now, out of the two alternatives mentioned above second one is correct because it has been experimentally observed (See sec 5.6 in connection'with Lloyd's mirror) that the phase change of π occurs when the light strikes the boundary from the side of rarer medium. Hence, light reflected by a material of higher refractive index than the medium in which the rays are travelling undergoes a 180° (or π) phase change.

Reflection by a material of lower refractive index than the medium in which the rays are travelling causes no phase change.

The following SAQ will provide a useful check of your understanding of this section.

SAQ 1

In Fig. 6.2, we have illustrated four situations. In the two examples on the left, the refractive index between the surfaces is higher than that outside; in the two examples on the right, it is lower. This determines whether or not there is a phase change. In Fig. 6.2(a) and (b), we have indicated the phase change taking place at the points marked by an arrow. Redraw the Fig. 6.2(c) and (d), indicating the phase change taking place at the points marked by an arrow.

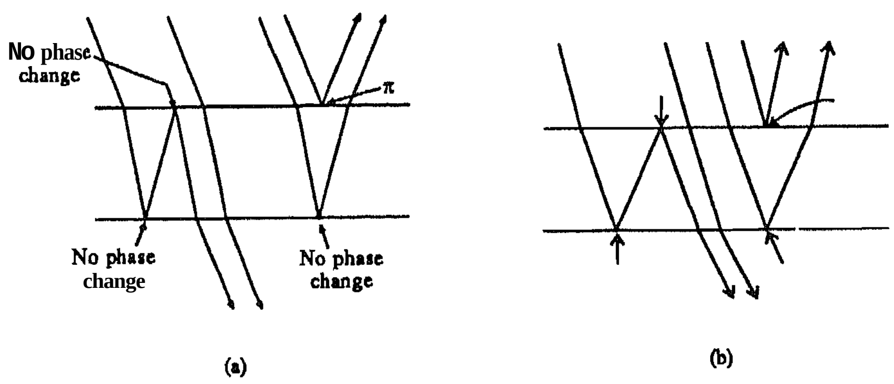


Fig. 6.2

6.3 INTERFERENCE IN THIN FILMS

Suppose a ray of light from a source S strikes a thin film of soapy water, at ***A***, see Fig. 6.3(a) . Part of this will be reflected as ray (1) and part refracted in the direction AB. Upon arrival at B, part of the latter will be reflected to C, and part refracted along BT_1. At C, the ray will again get partly reflected along CD and refracted as ray (2) along CR_2. A continuation of this process yields two sets of parallel rays, one on each side of the film. In each of these sets, of course, the amplitude decreases rapidly from one ray to the next. Considering only the first two reflected rays (1) and (2) we find that these two rays are in a position to interfere. This is because, if we assume S to be a monochromatic point source, the film serves as an amplitude-splitting device, so that ray (1) and (2) may be considered as arising from two coherent virtual sources S'and S" lying behind the film, that is, the two images of S formed by reflection at the top and bottom surfaces of the film, as shown in Fig. 6.3 (b). If the set of parallel reflected rays is now collected by a lens, and focussed at P, each ray has travelled a different distance, and the phase relationship between them may be such as to produce destructive or constructive interference at P. It is such interference that produces the colours of this film when seen by naked eyes.

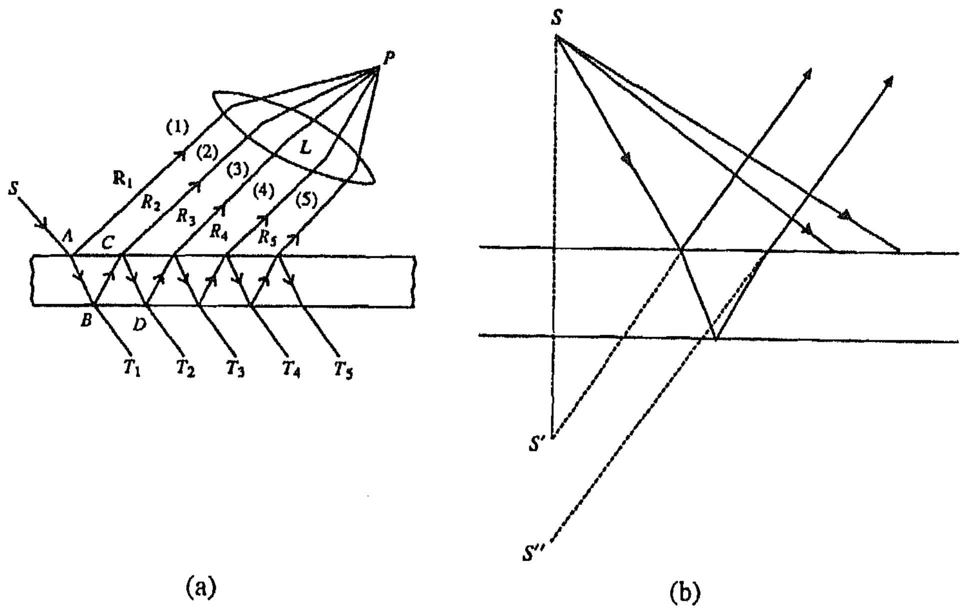


Fig. 6.3: (a) Multiple reflection in a soap film. (b) The interference pattern produced due to rays (1) and (2) is approximately the same as would have been produced by two coherent point sources S' and S''.

Now, we know that the two rays reinforce each other, if the path difference between them is an integral multiple of λ, where λ is the wavelength of light, which is being used to illuminate the film. Hence, let us first find out the path difference between the reflected rays (1) and (2).

Path Difference in Reflected Light

Suppose the ray of light falling on the thin film of soapy water at A be incident at an angle i, as shown in Fig. 6.4. Let the thickness of the film be t and refractive index be μ (>1). At A it is partly reflected along AR_1 giving the ray (1) and partly refracted along AB at an angle r. At B it is again partly reflected along BC and partly refracted along BT_1. Similar reflections and refractions occur at C. Since, the rays AR_1 and CR_2, i.e. ray (1) and say (2) have been derived from the same incident ray, they are coherent and in a position to interfere. Let CN and BM be perpendiculars to AR_1 and AC. As the paths of the rays AR_1 and CR_2 beyond CN are equal, the path difference between ray (1) and (2) is given by

(path ABC in film-path AN in air)

$$\therefore \text{ path difference} = \mu\,(AB + BC) - AN \qquad \ldots(6.5)$$

Here $\qquad AB = BC = \dfrac{BM}{\cos r} = \dfrac{t}{\cos r}$,

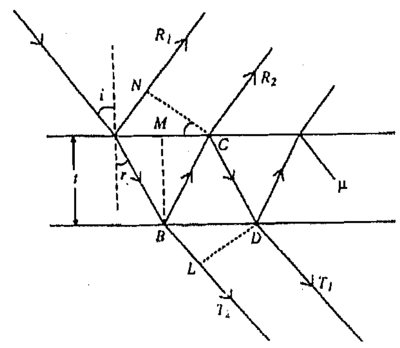


Fig. 6.4: Optical path difference between two consecutive rays in n multiple reflection.

and AN $= AC \sin i$

Now, AC $= AM + MC$

$= BM \tan r + BM \tan r$

$= 2t \tan r$

$\therefore$ AN $= 2t \tan r \sin i$

$$= 2t \frac{\sin r}{\cos r} (\sin i)$$

$$= 2t \frac{\sin r}{\cos r} (\mu \sin r) \qquad \left[\because \frac{\sin i}{\sin r} = \mu\right]$$

$$= 2\mu t \frac{\sin^2 r}{\cos r}$$

Substituting these values of AB, BC and AN in Eq. (6.5) we get,

$$\text{path difference} = \mu\left(\frac{t}{\cos r} + \frac{t}{\cos r}\right) - 2\mu t \frac{\sin^2 r}{\cos r}$$

$$= \frac{2\mu t}{\cos r}(1 - \sin^2 r)$$

$$\therefore \quad \text{path difference} = 2\mu t \cos r \qquad \ldots(6.6)$$

At A, the ray is reflected while going from a rarer to a denser medium and suffers a phase change of π. At B, the reflection takes place when the ray is going from a denser to a rarer medium, and there is no phase change.

However, we must take account of the fact that ray (1) undergoes a phase change of π at reflection while ray (2) does not, since it is internally reflected (See **SAQ** 1). The phase change of π is equivalent to a path difference of $\frac{\lambda}{2}$. Hence, the effective path difference between ray (1) and rays (2) is

$$2\mu t \cos r - \frac{\lambda}{2} \qquad \ldots(6.7)$$

The sign of the phase change is immaterial, Here we have chosen the negative sign to make the equation a bit simpler in form.

As you know from Unit 5, if this path difference is an odd multiple of $\frac{\lambda}{2}$, we might expect rays (1) and (2) to be out of phase, and produce a minimum of intensity. Thus the condition

$$2\mu t \cos r - \frac{\lambda}{2} = (2n-1)\frac{\lambda}{2}, \text{ where } n = 1, 2, \ldots$$

or

$$2\mu t \cos r = n\lambda \qquad \ldots(6.8)$$

becomes a condition for destructive interference as far as rays (1) and (2) are concerned.

Next, we examine the phases of the remaining rays, (3), (4), (5),..... Since the geometry is the same, the path difference between rays (3) and (2) will also be given by Eq. (6.6). But, here, only internal reflections are involved, so the effective path difference will still be given by Eq. (6.6). Hence, if the condition given by Eq. (6.8) is fulfilled, ray (3) will be in the same phase as ray (2). The same holds true for all succeeding pairs, and so we conclude that, under the condition given by Eq. (6.8), rays (1) and (2) will be out of phase, but rays (2), (3), (4),....., will be in phase with each other. Now, since ray (1) has considerably greater amplitude than ray (2), we might think that they will not completely annul each other, that is, the condition given by Eq. (6.8) may not produce complete darkness. But it is not so. We will now prove that the addition of rays (3), (4), (5)....., which are all in phase with ray (2), will give a net amplitude, just sufficient to make up the difference and to produce complete darkness. Fig. 6.5 shows the amplitude of successive rays in multiple reflection.

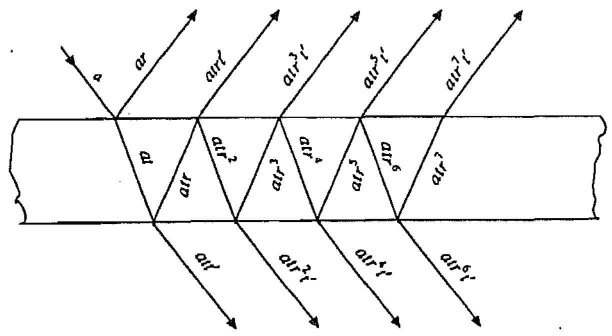


Fig. 6.5: Amplitude of successive rays in multiple reflection.

Adding the amplitudes of all the reflected rays but the first, on the upper side of the film we obtain the resultant amplitude:

$$A = atrt' + atr^3t' + atr^5t' + atr^7t' + \ldots$$
$$= atrt'(1 + r^2 + r^4 + r^6 + \ldots\ldots)$$

Since r is, necessarily, less than 1, the geometrical series in parentheses has a finite sum equal to $1/(1 - r^2)$, giving

$$A = atrt' \frac{1}{(1 - r^2)}$$

But from Stoke's treatment, Eq. (6.4), $tt' = 1 - r^2$, we obtain

$$A = ar \qquad \ldots(6.9)$$

This is just equal to the amplitude of the first reflected ray, hence, we conclude that under the condition of Eq. (6.8), there will be complete destructive interference. On the other hand, if the path difference given by Eq. (6.7) is an integral multiple of λ, i.e., when

$$2\mu t \cos r - \frac{\lambda}{2} = n\lambda, \text{ where } n = 0, 1, 2, \ldots \text{ etc.}$$

or

$$2\mu t \cos r = (2n + 1)\frac{\lambda}{2} \qquad \ldots(6.10)$$

then ray (1) and (2) will be in phase with each other and gives a condition of **constructive interference.** But rays (3), (5), (7),... will be out of phase with rays (2), (4), (6),.... Since (2) is more intense than (3), (4) is more intense than (5), etc., these pairs cannot cancel each other. As the stronger series combines with ray (1), the strongest of all. There will be maximum of intensity.

Thus, when a thin film is illuminated by monochromatic light, and seen in reflected light, it appears bright or dark according as $2\mu t$ cos r is odd multiple of $\frac{\lambda}{2}$ or integral multiple of λ, respectively.

$$2\mu t \cos r = (2n + 1) \text{ — } \quad \text{(condition of maxima)} \qquad \ldots(6.11a)$$

(condition of minima) ...(6.11b)

Before moving further, answer the following SAQ.

SAQ 2

Using Eq. (6.7), state whether the following statement is true or false. Give reasons,

"An **excessively thin** film seen in reflected light appears perfectly black".

Now we are in a position to know the reason of the production of colours in thin film of soap water.

Colours in Thin Films

The eye looking at the film receives rays of light reflected at the top and bottom surfaces of the film. These rays are in a position to interfere. The path difference between the interfering rays, given by Eq. (6.7), depends upon t (thickness of the film) and upon r, and, hence, upon inclination of the incident rays (the inclination is determined by the position of the eye relative to the region of the film, which is being looked at). The sunlight consists of a continuous range of wavelengths (colours). At a particular point of the film, and for a particular position of the eye (i.e., for a particular t and a particular r), the rays of only certain wavelengths will have a path difference satisfying the condition of maxima. Hence, only these wavelengths (colours) will be present with the maximum intensity. While some others, which satisfy the condition of the minima will be missing. Hence, the point of the film being viewed will appear coloured.

We arc working out an example so that the phenomenon of production of colours in thin film is clear to you.

Example 1

A thin film of 4×10^{-5}cm thickness is illuminated by white light normal to its surface ($r = 0°$). Its refractive index is 1.5. Of what colour will the thin film appear in reflected light?

Solution

The condition for constructive interference of light reflected from a film is

$$2\mu t \cos r = (2n + 1) \frac{\lambda}{2}, \text{ where. } n = 0,1,2\cdots$$

Here $\mu = 1.5$; $t = 4 \times 10^{-5}$ cm and $r = 0°$ (since light falls normally) so that $\cos r = 1$.

$$\therefore \qquad 2 \times 1.5 \times 4 \times 10^{-5} = (2n + 1) \frac{\lambda}{2}$$

or

$$\lambda = \frac{2 \times 2 \times 1.5 \times 4 \times 10^{-5}}{2n + 1}$$

$$\lambda = \frac{24 \times 10^{-5} \text{ cm}}{2n + 1} = \frac{24{,}000}{2n + 1} \text{ Å}$$

Taking n = 0, 1, 2, 3, ... we get

$$\lambda = 24000 \text{ Å}, 8000 \text{ Å}, 4800\text{Å}, 3431\text{Å} \ldots\ldots\ldots$$

These are the wavelengths reflected most strongly. Of these, the wavelength lying in the visible region is 4800Å (blue).

So far we have considered viewing of thin film in reflected light. Suppose the eye is now situated on the lower side of the film, shown in Fig. 6.3 and Fig. 6.5. The rays emerging from the lower side of the film can also be brought together with a lens and made to interfere.

Let us find out what colours will arise, when the film is viewed in this position. For this, we have to first calculate the path difference between the rays in transmitted light.

The path difference between the transmitted rays BT_1 and DT_2 is given by Eq. (6.6), i.e.,

$$(BC + CD) - BL = 2\mu t \cos r$$

In this case, there is no phase change due to reflection at B or C, because in either case the light is travelling from denser to rarer medium (See SAQ 1). Hence, the effective path difference between BT_1 and DT_2 is also $2\mu t \cos r$.

The two rays BT_1 and DT_2 reinforce each other, if

$$2\mu t \cos r = n\lambda \text{ (condition of maxima)} \qquad \text{...(6.12a)}$$

where n = 1, 2, 3.

In this case, the film will appear bright in the transmitted light.

The two rays will destroy each other if

$$2\mu t \cos r = (2n + 1)\frac{\lambda}{2} \text{ (condition of minima)} \qquad \text{..(6.12b)}$$

where n = 0, 1, 2,.... and the film appears dark in transmitted light.

A comparison of Eqs. (6.11a), (6.11b), (6.12a) and (6.12b) shows that the conditions for the maxima and minima, in the reflected light are just the reverse of those in transmitted light. Therefore, only those colours will be visible in transmitted light, which were missed in reflected light. Hence, the film which appears bright in reflected light will appear dark in transmitted light and vice versa. In other words, the appearances of colours in the two cases is complimentary to each other.

Interference fringes produced by thin films can be classified into two: Fringes of equal inclination and fringes of equal thickness.

Fringes of Equal Inclination

If the lens used in Fig. 6.3 to focus the rays has a small aperture, interference fringes will appear on a small portion of the film. Only the rays leaving the point source that are reflected directly into the lens will be seen (see Fig. 6.6a). For an extended source, light will reach the lens from various directions, and the fringe pattern will spread out over a large area of the film, as shown in Fig. 6.6b.

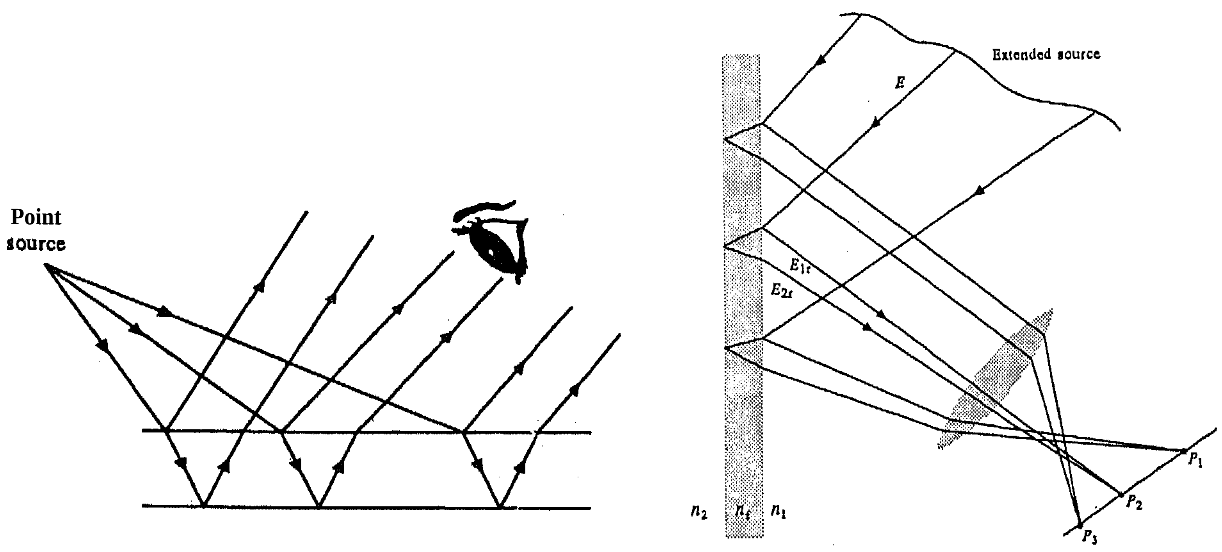


Fig.6.6: (a) Fringes seen In a small portion of the film.(b) Fringes seen on a large region of the film.

The angle i or equivalently r, determined by the position P, will, in turn, control the path difference. The fringes appearing at-points P_1 and P_2 in Fig. 6.7 are, accordingly, known as **fringes of equal inclination.**

Notice that as the film becomes thicker, the separation AC in Fig. 6.4 between ray (1) and (2) also increases, since AC = 2t tan r. When only one of the two rays is able to enter the pupil of the eye, the interference pattern will disappear. The larger lens of a telescope could then, be used to gather in both rays, making the pattern visible. The separation can also be reduced by reducing r, and, therefore, *i*, i.e., by viewing the film at nearly normal incidence.

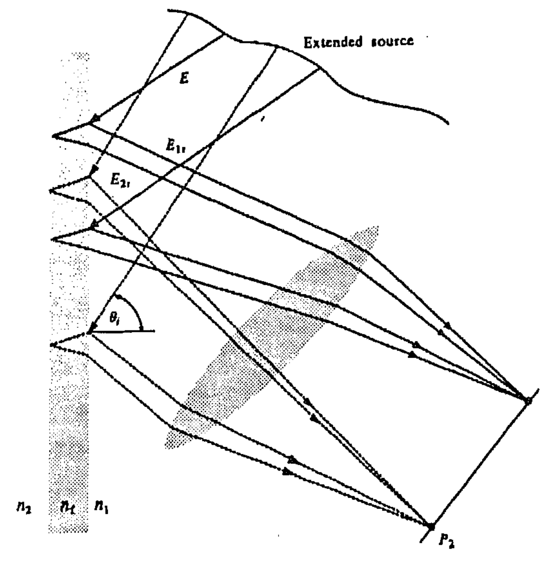


Fig. 6.7: All rays inclined at the same angle arrive ut the same point.

The equal inclination fringes that are seen in this manner for thick plates are known as **Haidinger** fringes. With an extended source, the symmetry of the set up requires that the interference pattern consists of a series of concentric circular bands centered on the perpendicular drawn from the eye to the film, as shown in Fig. 6.8.

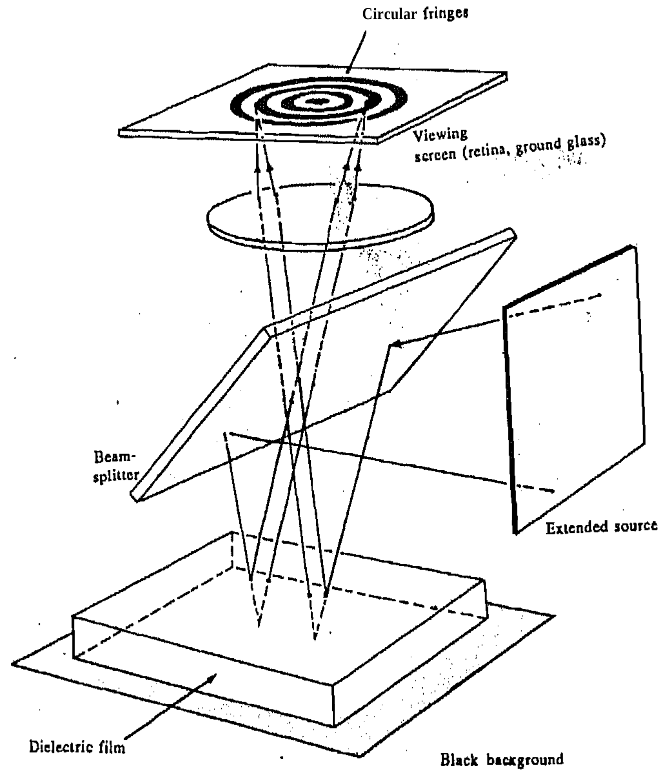


FIg.6.R: Circular Haidinger fringes centcrcd on the lens axis.

Such fringes are formed at infinity, and are observed by a telescope focussed at infinity. These fringes are observed in Michelson interferometer, about which we will study in next unit.

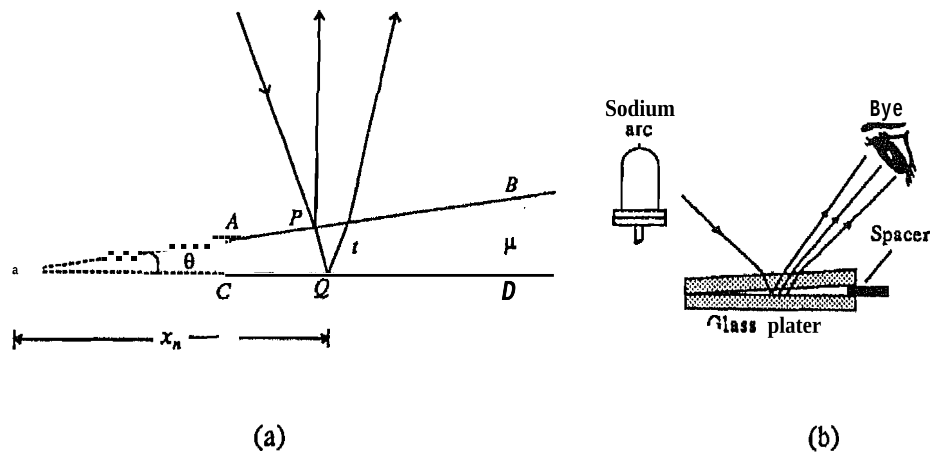
Fringes of Equal Thickness

Interference fringes, for which thickness t is the dominant parameter rather than r, are referred to as **fringes of equal thickness.** Each fringe is the locus of all points in the film for which thickness is a constant. Such fringes are localised on the film itself, and are observed by **a** microscope focussed on the film. Fringes due to the wedge-shaped film belong to this class of fringes, which you will study in the next section.

Fringes of equal thickness can be distinguished from the circular pattern of Haidinger's fringes by the manner in which the diameters of the rings vary with order n. The central region in the Haidinger pattern corresponds to the maximum value of n, whereas just the opposite applies to fringes of equal inclination.

6.4 INTERFERENCE BY A WEDGE-SHAPED FILM

So far, we have assumed the film to be of uniform thickness. We will now discuss the interference pattern produced by a film of varying thickness, i.e., a film which is not plane-parallel. Such a film may be produced by a wedge, which consists of two non-parallel plane surfaces, as shown in Fig. 6.9a and 6.9b. Observe that the interfering raysdo not enter the eye parallel to each other but appear lo diverge from a point near the film.



Pig. 6.9: Fringes of equal thickness: (a) method of visual observations. (b) a parallel beam of light incident on a wedge.

Let us consider a thin wedge-shaped film of refractive index μ, bounded by two plane surfaces AB and CD, inclined at an angle θ as shown in Fig. 6.9b. Let the film be illuminated by a monochromatic source of light from a slit held parallel to the edge of the wedge (the edge is the line passing through the point O and perpendicular to the plane of the paper). Interference occurs between the rays reflected at the upper and lower surfaces of the film. In this case the path difference for a given pair of rays is practically that given by Eq. (6.6). But, if it is assumed that light is incident almost normally at a point P on the film, the factor cos r may be considered equal to 1. Thus, the path difference between the rays reflected at the upper and lower surfaces is $2\mu t$, where t is the thickness of the film at P. **An** additional path difference of $\frac{a}{2}$ is introduced in the ray reflected from the upper surface. The effective path difference between the two rays is

$$2\mu t - \frac{\lambda}{2} \qquad ...(6.13)$$

Hence the condition for bright fringes becomes

$$2\mu t - \frac{\lambda}{2} = n\lambda$$

or $$2\mu t = (2n + 1)\frac{\lambda}{2}$$ ...(6.14)

The condition for dark fringe is

$$2\mu t = n\lambda$$ ...(6.15)

It is clear that for a bright or dark fringe of a particular order, t must remain constant. Since in the case of a wedge-shaped film, t remains constant along lines parallel to the thin edge of the wedge, the bright and dark fringes are straight lines parallel to the thin edge of the wedge. Such fringes are commonly referred to as "fringes of equal thickness". At the thin edge, where $t = 0$, path difference $= \frac{\lambda}{2}$, which is a condition for minimum intensity. Hence, the edge of the film is dark. The resulting fringes resemble the localized fringes in the Michelson interferometer (this you will study in next unit) and appear to be formed in the film itself.

Spacing between **Two** Consecutive **Bright** (or **Dark)** Fringes

For the nth dark fringe, we have

$$2\mu t = n\lambda$$

Let this fringe be obtained at a distance x_n from the thin edge. Then $t = x_n \tan\theta = x_n \theta$ (when θ is small and measured in radians).

$$\therefore \quad 2\mu x_n\theta = n\lambda$$ ...(6.16)

Similarly, if the $(n + 1)$th dark fringe is obtained at a distance x_{n+1} from the thin edge, then

$$2\mu x_{n+1}\theta = (n + 1)\lambda$$ ...(6.17)

Subtracting Eq. (6.16) from Eq. (6.17), we get

$$2\mu\theta (x_{n+1} - x_n) = \lambda$$

Hence the fringe width β is

$$\beta = x_{n+1} - x_n = \frac{\lambda}{2\mu\theta}$$ ...(6.18)

where θ is measured in radians.

Similarly, it can be shown that the spacing between two consecutive bright fringes (fringe width) is $\frac{\lambda}{2\mu\theta}$.

SAQ 3

Using sodium light (λ = 5893 Å), interference fringes are formed by reflection from a thin air wedge. When viewed perpendicularly, 10 fringes are observed in a distance of 1 cm. Calculate the angle of the wedge.

If the fringes **of** equal thickness are produced in the air film between a convex surface of a long-focus lens and a plane glass surface, the fringes will be circular in shape because the thickness **of** the air film remains constant on the circumference of a circle. The ring-shaped fringes, thus produced, were studied by Newton. In the next section, we will study Newton's ring.

6.5 NEWTON'S RINGS

When a plano-convex lens of large radius of curvature is placed with its convex surface in contact with a plane glass plate, air-film is formed between the lower surface of the lens (LOL') and the upper surface of the plate (POQ), as shown in Fig. 6.10. The thickness of the air film is zero at the point of contact O, and it increases as one moves away from the point of contact. If monochromatic light is allowed to fall normally on this film, reflection takes place at both the top and bottom of the film. As a result of' interference between the light waves reflected from the upper and lower surfaces of the air film, constructive or destructive interference takes place, depending upon the thickness of the film. The thickness of the air film increases with distance from the point of contact, therefore, the pattern of bright and dark fringe consists of concentric circles. In Fig. 6.10, 1 and 2, are the interfering rays corresponding to an incident ray AB. As the rings are observed in reflected light, the effective path difference between the interfering rays 1 and 2 is practically that given by Eq. (6.13).

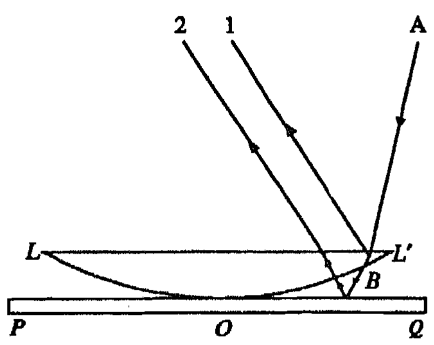


Fig.6.10: An arrangement for observing Newton's rings.

As we have considered an air-film, $\mu = 1$. The condition for the bright ring which is given by Eq. (6.14), is

$$2t = (2n - 1)\frac{\lambda}{2} \quad ...(6.19)$$

and the condition for the dark ring which is given by Eq. (6.15) is

$$2t = n\lambda \quad ...(6.20)$$

Let us find out the relationship between the radii of the rings and the wavelength of the light. Consider Fig. 6.11, where the lens LOL' is placed on the glass plate POQ. Let R be the radius of curvature of the curved surface of the lens. Let r_n be the radius of the nth Newton's ring corresponding to point P, where the film thickness is t. Draw perpendicular PN. Then, from the property of a circle, we have

$$PN^2 = ON \times NE$$

or

$$r_n^2 = t\,(2R - t)$$

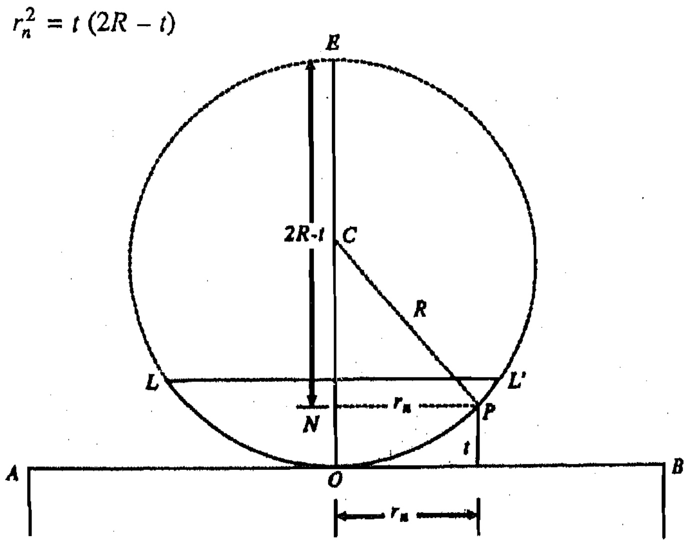


Fig. 6.11: r_n represents the radius of the nth dark ring, tho thickness of air film (where the nth dark ring is formed) is t.

Since t is small compared to R, we can neglect t^2.

Hence, $r_n^2 = 2Rt$

or $$2t = \frac{r_n^2}{R} \quad \ldots(6.21)$$

The condition for a bright ring is

$$2t = (2n - 1)\frac{\lambda}{2}$$

But from Eq. (6.21), $2t = \frac{r_n^2}{R}$

$$\therefore \quad \frac{r_n^2}{R} = (2n - 1)\frac{\lambda}{2}$$

or $$r_n^2 = (2n - 1)\frac{\lambda R}{2} \quad \text{(Bright ring)}$$

If D_n be the diameter of the nth bright ring, then $D_n = 2r_n$ or $r_n = \frac{D_n}{2}$. Substituting this in the last expression, we get

$$D_n^2 = 2\,(2n - 1)\,\lambda R$$

or $$D_n = \sqrt{2\lambda R}\,\sqrt{2n - 1}$$

or $$D_n \propto \sqrt{2n - 1} \quad (\lambda \text{ and R being constant}) \quad \ldots(6.22)$$

This shows that the radii of the rings vary as the square-root of odd natural numbers. Thus the rings will be close to each other as the radius increases, as shown in Fig. 6.12.

Fig. 6.12: Newton's rings as observed In reflected light.

Between the two bright rings there will be **a** dark ring whose radius will be proportional to the square-root of the natural numbers. Attempt the following SAQ and prove the above statement yourself.

SAQ 4

Using Eqs. (6.20) and (6.21), prove that the radius of the dark ring is proportional to the square-root of the natural numbers.

The ring diameters depend on wavelength, therefore, the monochromatic light will produce an extensive fringe system such as that shown in Fig. 6.12.

When the contact between lens and glass is perfect, the central spot is black. **This is direct evidence of the relative phase change of π between the two types of reflection, air-to-glass and glass-to-air, mentioned in Sec. 6.2. If there were no such phase change, the rays reflected from the two surfaces in contact should be in the same phase, and produce a bright spot at the centre.** However, the central spot can be made bright due to slight modification. In an interesting modification of the experiment, due to Thomas Young, if the lower plate is made to have a higher index of refraction than the lens, and the film in between is filled with an oil of intermediate index, then both reflections are at "rare-to-dense" surfaces. In this situation, no relative phase change occurs, and the central fringe of the reflected system is bright.

If D_n is the diameter of the nth bright ring, then

$$D_n^2 = 2(2n-1)\lambda R \qquad \ldots(6.23)$$

If D_{n+p} is the diameter of the $(n+p)$th bright ring, then

$$D_{n+p}^2 = 2\,[2(n+p)-1]\lambda R \qquad \ldots(6.24)$$

Subtracting Eq. (6.23) from Eq. (6.24), we get

$$D_{n+p}^2 - D_n^2 = 2\,[2(n+p)-1]\lambda R - 2\,(2n-1)\lambda R$$

$$= 4\,p\lambda R$$

$$\therefore \qquad \lambda = \frac{D_{n+p}^2 - D_n^2}{4pR} \qquad \ldots(6.25)$$

It may be mentioned here, that the point of contact may not be perfect. As such the nth ring may not be the nth fringe but Eq. (6.25) is almost always valid. On measuring the diameters of the rings and the radius of curvature R, the wavelength λ can be calculated. with the help of the Eq. (6.25). In laboratory, the radius of curvature can be accurately measured with the help of a spherometer.

If a liquid of refractive index μ is introduced between the lens and the glass plate, then the expression for path difference between two interfering rays will also include μ. Then the radii of the dark rings would be given by

$$r_n = \left(\frac{n\lambda R}{\mu}\right)^{1/2} \qquad \ldots(6.26)$$

Thus, when a little water is introduced between the lens and the plate, the rings contract according to the relation

$$\boxed{\frac{\text{diameter of a ring in water-film}}{\text{diameter of the same ring in air-film}} = \frac{1}{\sqrt{\mu}}} \qquad \ldots(6.27)$$

where μ is the refractive index of water.. .

A ring system is also observed in the light transmitted by Newton's ring plates. There are two differences in the reflected and transmitted systems of rings, (i) The rings observed in transmitted light are exactly complementary to those seen in the reflected light, so that the central spot is now bright. (ii) The rings in transmitted light are much poorer in contrast than those in reflected light.

Before moving to the next section, solve the following SAQ.

SAQ 5

If in a Newton's ring experiment, the air in the interspace is replaced by a liquid of refractive inaex 1.33, in what proportion would the diameters of the ring change?

6.6 APPLICATIONS OF THE PRINCIPLE OF INTERFERENCE IN THIN FILM

1. An important and simple application of the principle of interference within film is in the production of coated surfaces. To accomplish this, the glass lens is coated with the film of a transparent substance that has an index of refraction between the refraction indices for air and glass (See Fig. 6.13). The thickness of the film is one quarter of the wavelength of light in the film so that

$$t = \frac{\lambda}{4\mu_1}.$$

If we assume normal incidence, then the path difference between the light wave 'reflected from the upper surface of the film and the light wave reflected from the lower surface of the film is $2\mu_1 t = 2\mu_1 \times \frac{\lambda}{4\mu_1} = \frac{\lambda}{2}$. Both waves undergo a phase change of 180° as reflections at both surfaces are from "rare-to-dense". Thus, the two reflected waves are out of phase because of path difference and, therefore, these interfere desrructively. Such a film is known as non-reflecting film, because it gives zero reflection. However, this does not mean that a non-reflecting film destroys light, but it merely redistributes light so that a decrease of refleclion is accompanied by a corresponding increase of transmission.

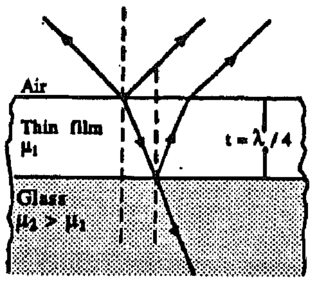


Fig. 6.13: A film coating on a glass lens makes the lens "non-reflecting" when the film thickness is λ/4 for normal incident. The total path difference of the reflected rays is then λ/2, and the waves interfere destructively, i.e., the incident light is totally transmitted.

The practical importance of these films is that by their use one can greatly reduce loss of light by reflection at the various surfaces of lenses or prisms used in binoculars, cameras, etc. Usually, glass is coated with a very thin layer of magnesium fluoride, the refractive index of which ($\mu = 1.38$) is intermediate between those of glass and air.

2. Another important application of thin film interference phenomenon is the converse of the procedure just discussed, viz., the glass surface is coated by a thin film of suitable material to increase the reflectivity. The film thickness is again $\lambda/4\mu_f$, where μ_f represents the refractive index of the film. The film is such that its refractive index is greater than that of the glass, This is because an abrupt phase change of π occurs only at the air-film interface and the beams reflected from the air-film interface and the film-glass interface constructively interfere.

3. The fringes obtain by a wedge-shaped film has important practical applications in the testing of optical surfaces for flatness. An air-film is formed between a perfectly plane surface and the surface under test. If the latter surface is plane, the fringes will be straight and parallel, and, if not, these will be irregular in shape.

4. The accuracy of the grinding of a lens surface can be tested by observing the shape of Newton's rings formed between it and an accurately flat glass surface, using monochromatic light. If the rings are not perfectly circular, the grinding is imperfect.

You should be able to apply whatever you have learnt in this section to solve the following SAQ.

6.7 SUMMARY

- When the light wave is reflected from a boundary, there is an abrupt change of phase. When the light ray is reflected while going from a rarer to a denser medium, it suffers a phase change of π. But there is no phase change when the light ray is reflected while going from a denser to a rarer medium.

- Length l in a medium of refractive index μ is optically equivalent to length μl in a vacuum. μl is called the optical path length of distance l in the medium.

- For a thin film in reflected light, the conditions for constructive and destructive interference are:

  $$2\mu t \cos r = (2n + 1)\frac{\lambda}{2} \quad \text{(maxima)}$$

  $$2\mu t \cos r = n\lambda \quad \text{(minima)}$$

  where μ is the refractive index of the film, t is its thickness and r is the angle of refraction in the film.

- For a thin film in transmitted light, the conditions for constructive and destructive interference are:

  $$2\mu t \cos r = n\lambda \quad \text{(maxima)}$$

  $$2\mu t \cos r = (2n + 1)\frac{\lambda}{2} \quad \text{(minima)}$$

- The basic formula for the path difference between the interfering rays, obtained due to division of amplitude by a film of thickness t and refractive index μ, is $2\mu t \cos r$, where r is the inclination of ray inside the film. If the thickness of the film is uniform, the path difference $2\mu t \cos r$ varies only with inclination r, and gives rise to the "fringes of equal inclination". On the other hand, if the thickness of the film is rapidly varying, the path difference $2\mu t \cos r$ changes mainly due to changes in μ. This gives rise to the "fringes of equal thickness".

- The spacing β between two consecutive bright (or dark) fringes produced by wedge-shaped film is given by

  $$\beta = \frac{\lambda}{2\mu\theta}$$

  where λ is the wavelength of light being used for illuminating the film, μ the refractive index of the film, and θ (measured in radians) the angle between the two plane surfaces, which form the wedge-shaped film.

- The diameters of the bright rings are proportional to the square-roots of the odd natural numbers, whereas the diameters of dark rings are proportional to the square-roots of natural numbers, provided the contact is perfect.

- On measuring the diameters of Newton's rings and the radius of curvature R, the wavelength can be calculated with the help of the following relation:

  $$\lambda = \frac{D_{n+p}^2 - D_n^2}{4pR}$$

- The phenomenon of interference is used in the testing of optical surfaces and producing non-reflecting glasses of reflective coatings.

6.8 TERMINAL QUESTIONS

1) White light is reflected normully from a uniform oil film ($\mu = 1.33$). An interference maximum for 6000 Å and a minimum for 4500 Å, with no minimum in between, are observed, Calculate the thickness of the film.

2) Light (A = 6000 Å) falls normally on a thin wedge-shaped film (μ = 1.5). There art ten bright and nine dark fringes over the length of the film. By how much docs the film thickness change over this length?

3) Two glass plates 12 cm long touch at one end, and are separated by a wire 0.048 mm in diameter at the other. How many bright fringes will be observed over the 12 cm distance in the light (λ = 6800 Å) reflected normally from the plates?

4) Newton's rings are formed in reflected light of wavelength 5895×10^{-8}cm with a liquid between the plane and curved surfaces. The diameter of the fifth ring is 0.3 cm and the radius of curvature of the curved surface is 100 cm. Calculate the refractive index of the liquid, when the ring is (i) bright, (ii) dark.

5) A Newton's rings arrangement is used with a source emitting two wave-lengths

$$\lambda_1 = 6.0 \times 10^{-5} \text{ cm and } \lambda_2 = 4.5 \times 10^{-5} \text{ cm}$$

and it is round that the nth dark ring due to λ_1 coincides with the $(n+1)$th dark ring due to λ_2. If the radius of curvature of the curved surface is 90 cm, find the diameter of the nth dark ring for λ_1.

6.9 SOLUTIONS/ANSWERS

SAQs

1) See Fig. 6.14

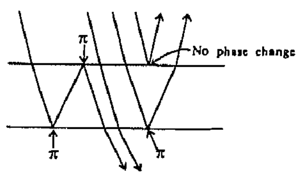


Fig. 6.14

2) According to Eq. (6.7) the path difference between the interfering rays in reflected light is $2\mu t \cos r - \frac{\lambda}{2}$. When the film is excessively thin, t is very small, and $2\mu t \cos r$ is almost zero. Hence the path difference, in such a case becomes $\frac{\lambda}{2}$. This is a condition of minimum intensity. Hence, the film will appear black in the reflected light.

3) Let θ radian be the angle of the air-wedge. For normal incidence, the fringe-width is given by

$$\beta = \frac{\lambda}{2\theta} \qquad (\because \mu = 1 \text{ for air})$$

Here $\lambda = 5893 \times 10^{-8}$ cm and β = 1/10 cm.

$$\therefore \quad \theta = \frac{\lambda}{2\beta} = \frac{5893 \times 10^{-8}}{2 \times 1/10} = 2.95 \times 10^{-4} \text{ radian.}$$

4) According to Eq, (6.20). the condition for the dark ring is

$$2t = \text{nil}$$

But from Eq. (6.19), $2t = \frac{r_n^2}{R}$

$$\therefore \quad \frac{r_n^2}{R} = n\lambda$$

If D_n be-the diameter of the nth dark ring, $r_n = \frac{D_n}{2}$

$$\therefore \quad \frac{D_n^2}{4R} = n\lambda$$

or $\quad D_n = \sqrt{4nR\lambda}$

or $\quad D_n = \sqrt{4R\lambda}\,\sqrt{n}$

or $\quad D_n \propto \sqrt{n}$

Thus, the diameters of the dark rings are proportional to the square root of the natural number.

$$\frac{(D_n)^2_{air}}{(D_n)^2_{liquid}} = \mu$$

or $$\frac{D_{liquid}}{D_{air}} = \frac{1}{\sqrt{\mu}} = \frac{1}{\sqrt{1.33}} = 0.867$$

The rings are contracted to 0.867 their previous diameters.

6) In this case of interference in thin films, the situation is somewhat different. The reflections at both the upper and lower surfaces of the material ($\mu = 1.25$) film take place under similar conditions, i.e., when light is going from a rarer to a denser medium. Thus, there is a phase change of π at both reflections, which means no phase difference due to reflection between the two, interfering beams.

The path difference between the two interfering beams is $2\mu t$ for normal incidence, where t is the thickness and μ the refractive index of the film.

The two beams will destroy each other, if the path difference is an odd multiple of $\frac{\lambda}{2}$, i..e, when

$$2\mu t = (2n-1)\frac{\lambda}{2}; \text{ where } n = 1, 2, 3, \ldots$$

This is the condition of minima.

Here $\mu = 1.25$ and $\lambda = 6000$ Å.

$$\therefore \quad 2 \times 1.25 \times t = (2n-1) \times \frac{6000}{2} \text{ Å}$$

Hence the required thickness is given by

$$t = (2n-1)\frac{6000}{2 \times 2 \times 1.25} \text{ Å}$$

$$= (2n-1)\ 1200\text{Å}; \text{ where } n = 1, 2, 3, \ldots$$

TQs

1) The condition for an interference maximum in the light reflected normally from an oil film of thickness t is

$$2\mu t = \left(n + \frac{1}{2}\right)\lambda; \text{ where } n = 0, 1, 2, \ldots$$

and that for an interference minimum is

$$2\mu t = n\lambda; \qquad \text{where } n = 1, 2, 3, \ldots$$

Here $\mu = 1.33$. Now there is a maximum for $\lambda = 6000$Å

We can'write

$$2 \times 1.33 \times t = \left(n + \frac{1}{2}\right) 6000 \text{ Å} \qquad \ldots(i)$$

$$2 \times 1.33 \times t = (n+1)\ 4500\text{Å} \qquad \ldots(ii)$$

In view of eq. (i) we have taken the integer (n + 1) rather than n in eq. (ii) Comparing eq. (i) and (ii), we get

$$\left(n + \frac{1}{2}\right) 6000 = (n+1)\ 4500$$

Substituting $n = 1$ in eq. (i), we get

$$2 \times 1.33 \times t = \frac{3}{2} \times 6000Å$$

$$\therefore \quad t = \frac{3 \times 6000}{2 \times 2 \times 1.33} = 3383Å$$

2) The condition of destructive interfcrence in light reflected from a film is

$$2 \mu t \cos r = n\lambda.$$

Suppose the film thickness changes over this length by At. Lct n be the order of the dark fringe appearing at one end of the film. The order of the **dark** fringe at the other end will be $(n + 9)$. Wc, therefore, have

$$2 \mu t \cos r = n\lambda,$$

and $$2 \mu (t + At) \cos r = (n + 9)\lambda$$

Subtracting, we get

$$2 \mu (At) \cos r = 9\lambda$$

$$\therefore \quad t = \frac{9\lambda}{2\mu \cos r}$$

If thc fringes are seen normally, then cos r =1.

$$\therefore \quad t = \frac{9}{2\mu} = \frac{9 \times 6300}{2 \times 1.5} = 18900 Å$$

$$= 1.89 \times 10^{-4} \text{ cm}.$$

3) Let t bc thc thickness of the wire and l the length of thc wedge, as shown in Fig. 6.15. The wedge angle is

$$\theta = \frac{t}{l} \text{ radian.}$$

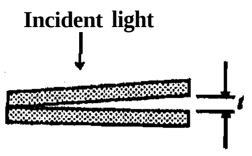


Fig. 6.15

Now, fringe-width $\beta = \frac{a}{2\theta}$

Putting value of θ from above we get

$$\beta = \frac{l\lambda}{2t}.$$

Since N fringes are seen; $1 = N\beta$. Thus

$$\beta = \frac{N\beta\lambda}{2t}$$

$$\therefore \quad N = \frac{2t}{\lambda}.$$

But $\lambda = 6800Å = 6800 \times 10^{-8}$ cm and $t = 0.048$ mm = 0.0048 cm.

$$\therefore \quad N = \frac{2 \times 0.0048}{6800 \times 10^{-8}} = 141.$$

4) i) The diameter D_n of the nth bright ring is given by

$$D_n^2 = \frac{2(2n-1)\lambda R}{\mu}$$

$\therefore \qquad \mu = \dfrac{2\,(2n-1)\,\lambda R}{D_n^2}$

Here n = 5, $\lambda = 5895 \times 10^{-8}$cm, R = 100 cm and D_n = 0.3 cm

$$\therefore \qquad \mu = \frac{2\,(10-1) \times 5895 \times 10^{-8} \times 100}{(0.3)^2} = 1.18$$

ii) The diameter of the nth dark ring is given by

$$D_n^2 = \frac{4n\,\lambda R}{\mu}$$

$$\therefore \qquad \mu = \frac{4n\,\lambda R}{D_n^2} = \frac{4 \times 5 \times 5895 \times 10^{-8} \times 100}{(0.3)^2} = 1.31.$$

5) $D_n^2 = 4nR\lambda$

where D_n = diameter of nth ring, R = the radius of curved surface and λ = the wavelength of light.

If D_n and D_{n+1} be two diameters,.

$$D_n^2 = 4nR\lambda_1 \qquad \text{....(i)}$$

$$D^2_{n+1} = 4\,(n+1)\,R\lambda_2$$

But $\qquad D_n = D_{n+1}$

$\therefore \qquad 4nR\lambda_1 = 4\,(n+1)R\lambda_2$

or $\qquad 4nR\,(\lambda_1 - \lambda_2) = 4R\lambda_2$

or $\qquad n = \dfrac{4R\lambda_2}{4R\,(\lambda_1 - \lambda_2)}$

$$= \frac{\lambda_2}{(\lambda_1 - \lambda_2)}$$

$$= \frac{4.5 \times 10^{-5}}{(6-4.5)\,10^{-5}} = 3$$

Putting n = 3 in (i)

$$D_3 = 4 \times 3 \times 90 \times 6 \times 10^{-5}$$

$$= 648 \times 10^{-4}$$

$$= 25.45 \times 10^{-2} \text{ cm.}$$