UNIT 5 INTERFERENCE BY DIVISION OF WAVEFRONT

Structure

5.1 INTRODUCTION

Anyone with a pan of water can see how the water surface is disturbed in a variety of characteristic patterns, which is due to interference between water waves. Similarly, interference occurs between sound waves as a result of which two people who hum fairly pure tones, slightly different in frequency, hear beats. But if we shine light from two torches or flashlights at the same place on a screen, there is no evidence of interference. The region of overlap is merely uniformly bright. Does it mean that there is no interference of light waves? The answer is 'No'.

The interference in light is as real an effect as interference in water or sound waves, and there is one example of it familiar to everybody — the bright colours of a thin film of oil spread out on a water surface. There are two reason why the interference of light is observed'in some cases and not in others? Firstly, light waves have very short wavelengths — the visible part of the spectrum extends only from 400 mm for violet light to 700 mm for red light. Secondly, every natural source of light emits light waves only as short trains of random pulses, so that any interference that occurs is averaged out during the period of observation by the eye, unless special procedures are used,

Like standing waves and beats, the phenomenon of interference depends on the superposition of two or more individual waves under rather strict conditions that will soon be clarified. When interest lies primarily in the effects of enhancement or diminution of light waves, these effects are usually said to be due to the interference of light. When enhancement (or constructive interference) and diminution (or destructive interference) conditions alternate in a spatial display, the interference is said to produce a pattern of fringes as in the double slit interference pattern. The same condition may lead to enhancement of one colour at the expense of the other colour, producing interference colours as in the case of oil slicks and soap film about which you will study in next unit.

In this unit, we will consider'the interference pattern produced by waves originating from two point sources. However, in case of light waves, one cannot observe interference between the waves from two independent sources, although the interference does take place: Thus, one tries to derive the interfering waves from a single wave so that the constant phase difference is maintained between the interfering waves. This can be achieved by two methods. In the first method a beam is allowed to fall on two closely spaced holes, and the two beam emanating from the holes interfere. This method is known as division of wavefront and will be discussed in detail in this unit. In the other method, known as division of amplitude, a beam is divided at two or more reflecting surfaces, and the reflected beams interfere. This will be discussed in the next unit.

As the phenomenon of interference can be successfully explained by treating light as a wave motion, it is necessary to understand the fundamentals of wave motion. Although you have learnt about this in your class XII and also in the PHE–02 course "Oscillations and Waves", we will begin this unit with study of wave motion which will serve as a recapitulation.

In the next unit we will study how interference takes place by division of amplitude of light wave.

Objectives

After studying this unit, you should be able to

- use the principle of superposition to interpret constructive and destructive interference,
- distinguish between coherent and incoherent sources of light,
- desrrihe the origins of the interference pattern produced by double slit,
- describe the intensity distribution in interference pattern,
- express the fringe-width in terms of wavelength of light,
- describe various arrangements for producing interference by division of wavefront,
- appreciate the difference between Biprism and Lloyd's mirror fringes.

5.2 WAVE MOTION

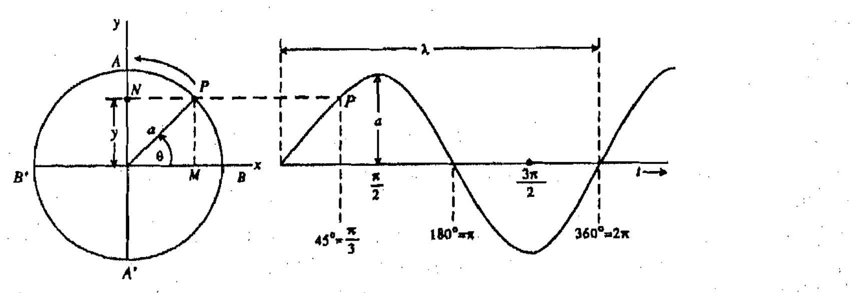
Study **Comment**

You may find it useful to go through the Unit 6 of PHE–02 course in "Oscillation and Wave".

Simple Harmonic **Motion**

A simple harmonic motion is defined as the motion of a particle which moves back and forth along a straight line such that its acceleration is directly proportional to its displacement from a fixed point in the line, and is always directed towards that point.

The best and elementary way to represent a simple harmonic motion is to consider the motion of a particle along a reference circle (See Fig. 5.1). Suppose a particle P travels in a circular path, counterclockwise, at a uniform angular velocity ω. The point N is the perpendicular projection of P on the diameter AOA' of the circle. When the particle P is at point B, the perpendicular projection is at O. As the particle P starts from B, and moves round the circle, N moves from O to **A**, A to A' and then returns to O. This back and forth motion of N is simple harmonic. Let us obtain expressions for displacement, velocity and acceleration and define few terms.



Displacement

Suppose the particle P starts from B and traces an angle θ in time t. Then its angular velocity ω is

$$\omega = \frac{\theta}{t}$$

where the angle θ is measured in radians. The displacement, y, of N from O at time t, is thus given by

$$y = ON = OP \sin NPO$$

$$= a \sin \theta \qquad [\because \angle NPO = \angle POB = \theta]$$

But $\omega = \frac{\theta}{t}$, so that $\theta = \omega t$

$\therefore$ $\boxed{y = a \sin \omega t}$ ...(5.1)

This is the equation of simple harmonic motion.

SAQ **1**

See Fig. 5.1. If you have studied the motion of the point M, which is the foot of the perpendicular from the point P on the x–axis, then write down the equation of simple harmonic motion.

Velocity: The velocity of N is given by

$$\frac{dy}{dt} = a\omega \cos \omega t = \omega \sqrt{a^2 - y^2} \qquad ...(5.2)$$

Acceleration: The acceleration of N is

$$\frac{d^2y}{dt^2} = -\omega^2 a \sin \omega t = -\omega^2 y \qquad ...(5.3)$$

Periodic Time: The periodic time, T, of N is time taken by N to make one complete vibration. Thus

$$T = \frac{2\pi}{\omega} \qquad ..(5.4)$$

Amplitude: Amplitude of vibration is equal to the radius of the reference circle i.e., a.

SAQ 2

A particle is executing simple harmonic motion, with a period of 3s and an amplitude of 6 cm. One-half second after the particle has passed through its equilibrium position, what is its (a) displacement, (b) velocity, and (c) acceleration?

Phase: The phase of a vibrating particle represents its state as regards

i) the amount of displacement suffered by the particle with respect to its mean position, and

ii) the direction in which the displacement has taken place.

In Fig. 5.1, we had conveniently chosen $t = 0$ as the time when P was on the x-axis. The choice of the time $t = 0$ is arbitrary, and we could have chosen time $t = 0$ to be the instant when P was at P' (see Fig.5.2). If the angle P'OX $= \theta$ then the projection on the y-axis at any time t would be given by

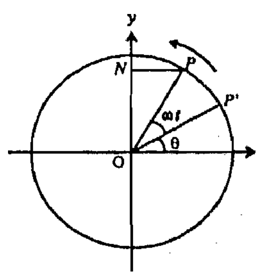


Fig. 5.2: At $t = 0$, the point P is at P' and, therefore, the initial phase is θ

$$y = a \sin(\omega t + \theta) \quad ...(5.5)$$

The quantity $(\omega t + \theta)$ is known as the phase of the motion and θ represents the initial phase. It is obvious from the discussion that the value of 8 is quite arbitrary, and depends on the instant from which we start measuring time.

We next consider two particles, P and Q rotating on the circle with the same angular velocity wand P' and Q' are their respective positions at $t = 0$. Let the angle $\angle P'OX$ and $\angle Q'OX$ be 8 and ϕ respectively (see Fig. 5.3).

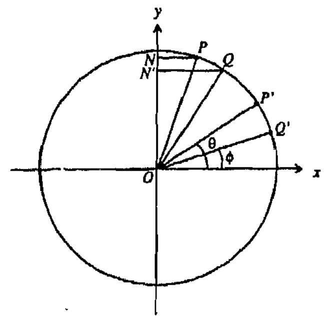


Fig.5.3: **The points Nand N' execute simple harmonic mntlon with the same frequency ω. The initial phases of N nnd N' are θ and ϕ respectively.**

Clearly at an arbitrary time t the distance of the foot or perpendiculars from the origin would be

$$y_P = a \sin(\omega t + 8) \quad ...(5.6a)$$

$$y_Q = a \sin(\omega t + \phi) \quad ...(5.6b)$$

The quantity

$$(\omega t + \theta) - (\omega t + \phi) = \theta - \phi \quad ...(5.7)$$

represents the phase difference between the two simple harmonic motions and if $8 - \phi = 0$ (or an even multiple of π) the motions are said to be in phase, and if $8 - \phi = \pi$ (or an odd multiple of π) the motions are said to be out of phase. If we choose a different origin of time, the quantities θ and ϕ would change by the same additive constant; consequently, the phase difference $(8 - \phi)$ is independent of the choice of the instant $t = 0$

Energy: A particle performing simple harmonic motion possesses both types of energies; potential and kinetic. It possesses potential energy on account of its displacement from the equilibrium position and kinetic energy on account of its velocity. These energies vary during oscillation, however, their sum is conserved provided no dissipative forces are present. Since the acceleration of vibrating particle is $\omega^2 y$, the force needed to keep a particle of mass m at a distance y from O is m $\omega^2 y$. If the particle is to be displaced through a further distance dy, the work to be done will be ω^2 mydy. Now the potential energy of the particle at a displacement y is equal to the total work done to displace the particle from O through a distance y.

$$\therefore \quad \text{P.E.} = \int_0^y \omega^2 my\, dy = \frac{1}{2} m \omega^2 y^2 \quad ...(5.8)$$

Using Eq. (5.2), the kinetic energy of the particle is given by

$$\text{K.E.} = \frac{1}{2} m \left(\frac{dy}{dt}\right)^2 = \frac{1}{2} m\omega^2 (a^2 - y^2) \quad ...(5.9)$$

The total energy of the particle at any distance y from O is given by

Total energy = K.E. + P B

$$= \frac{1}{2} m\omega^2 (a^2 - y^2) + \frac{1}{2} m\omega^2 y^2$$

$$= \frac{1}{2} m\omega^2 a^2 \qquad \text{...(5.10)}$$

Therefore, total energy (intensity) is proportional to (amplitude)^2, and, since $\omega = 2\pi n$, n being the frequency, the energy is also proportional to (frequency)^2.

If I represents the intensity associated with a light wave then

$$I \propto a^2$$

where a represents the amplitude of the wave.

Wave-motion

So far we considered a single particle, P, executing simple harmonic motion. Let us consider a number of particles which make a continuous elastic medium. If any one particle is set in vibration, each successive particle begins a similar vibration, but a little later than the one before it, due to inertia. Thus, the phase of vibration changes from particle to particle until we reach a particle at which the disturbance arrives exactly at the moment when the first particle has completed one vibration. This particle then moves in the same phase as the first particle. This simultaneous vibrations of the particles of the medium together make a wave. Such a wave can be represented graphically by means of a displacement curve drawn with the position of the particles as abscissae and the corresponding displacement at that instant as ordinate. If the particles execute simple harmonic motion, we obtain a sine curve as shown in Fig. 5.4.

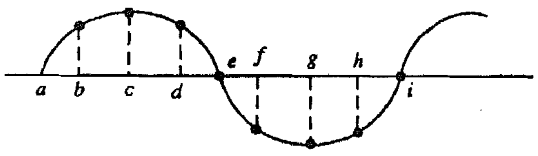


Fig. 5.4. Graphical representation of a wave

It will be seen that the wave originating at a repeates itself after reaching i. The distance ***ai***, after travelling which the wave-form repeats itself, is called the **wavelength** and is denoted by λ. It is also evident that during the time T, while the panicle at a makes one vibration, the wave travels a distance λ. Hence the velocity v of the wave is given by

$$v = \frac{\lambda}{T}$$

If n is the frequency of vibration then $n = 1/T$.

Hence, we have

$$v = n\lambda \qquad \text{...(5.11)}$$

Particles in **Same Phase**

Particles a and i have equal displacements (= zero) and both are tending to move upwards. They are said to be in the same phase. The distance between them is one wavelength. Hence, wavelength is the distance between two nearest particles vibrating in the same phase. Two vibrating particles will also be in the same phase if the distance between them is n λ, where n is an integer.

Particles in **Opposite Phase**

Particles a and e, both have the same displacement (= zero), but while a is tending to go up, e is tending to move downwards. They are said to be in opposite phase The distance between them is $\frac{\lambda}{2}$. The particles are out of phase if the distance between them is $(2n-1)\frac{\lambda}{2}$, where n is an integer.

Equation of a **Simple Harmonic Wave**

Fig. 5.5 shows the wave travelling in the positive x-direction. The displacement y of the

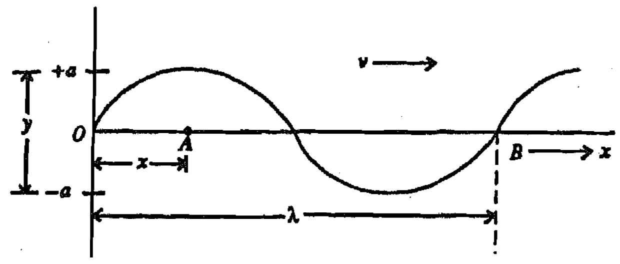


Fig. 5.5. **A simple harmonic wave travelling towards right.**

particle at O at any time t is given by

$$y = a \sin \omega t \qquad \ldots(5.11)$$

Let v be the velocity of propagation of the wave. Then the wave starting from O would reach at a point A, distant x from O in x/v seconds. Hence the particle at A must have started its vibration x/v seconds later than the particle at O. Consequently, the displacement at A at the time t would be same as was at O at time $\frac{x}{v}$ seconds earlier **i.e.** at time $t - \frac{x}{v}$. Substituting $t - \frac{x}{v}$ for t in Eq. (5.1) we obtain the displacement at A **at** time t, which is given by

$$y = a \sin \omega \left(t - \frac{x}{v} \right)$$

Using the relation $\omega = 2\pi/T$ and $v = \frac{4}{T}$ we get

$$y = a \sin \frac{2\pi}{a} (vt - x) \qquad \ldots(5.12)$$

This equation represents the displacement of a particle at a distance x from a fixed point at a time t. This is, **therefore,** the equation of the wave. The wave shown in Fig. 5.5 is generated along a stretched string and in a rope. Such type of waves are called transverse waves. From Unit 4 of Block–1 , you already know that light travels in the form of transverse waves, therefore Eq. (5.12) represents a light wave.

Relation between Phase Difference and Path Difference

The equation of simple harmonic wave is given by Eq. (5.12). If there are two particles P_1 and P_2 at distance x_1 and x_2 from the origin, then,

the phase angle of P_1 at a time $t = \frac{2\pi}{a} (vt - x_1)$

and the phase angle of P_2 at a time $t = \frac{2\pi}{a} (vt - x_2)$

$\therefore$ phase difference between P_1 and P_2

$$= \frac{2\pi}{a} (vt - x_1) - \frac{2\pi}{1} (vt - x_2)$$
$$= \frac{2\pi}{\lambda} (x_2 - x_1)$$

The expression phase difference = $\frac{2\pi}{\lambda}$ (path difference) can be obtained in a less formal manner by membering that a difference in phase of 2π corresponds to a path difference of one wavelength and calculating the required phase difference by proportion.

But $(x_2 - x_1)$ is the path difference between P_2 and P_1.

$$\therefore \text{Phase difference} = \frac{2\pi}{a} \times (\text{path difference}) \qquad \ldots(5.13)$$

When two or more sets of waves are made to overlap in some region of space, interesting effects are observed. For example, when two stones are dropped simultaneously in a quiet pool, two sets of waves are created. In the region of crossing, there are places where the disturbance is almost **zero,** and others, where it is greater than that given by either wave alone. These effects can be explained using a very simple law known as principle of superposition. We will use this principle in investigating the disturbance in regions, where two or more light waves are superimposed. Let us now briefly study this principle.

5.3 PRINCIPLE OF SUPERPOSITION

In any medium, two or more waves can travel simultaneously without affecting the motion of each other. Therefore, at any instant the resultant displacement of each particle of the medium is merely the vector sum of displacements due to each wave separately. This principle is known as "principle of superposition". It has been observed that when two sets of waves are made to cross each other, then after the waves have passed out of the region of crossing, they appear to have been entirely uninfluenced by the other set of waves. Amplitude, frequency and all other characteristics of the waves are as if they had crossed an undisturbed space.

As a simple example, we consider a long stretched string AB (see Fig. **5.6).** The end A of the string is made to vibrate up and down. This vibration is handed down from particle to particle of the string. Suppose the string is vibrating in the form of a triangular pulse, which propagates to the right with a certain speed v. We next assume that from the end B an identical pulse is generated which starts moving to the left with the same speed v.

Fig. **5.6(a)** shows the position of pulse at $t = 0$. At a little later time, each pulse moves close to the other as shown in Fig. **5.6(b),** without any interference. Fig. **5.6(c)** represents the position at an instant when the two pulses interfere; the dashed curves represent the profile of the suing, if each of the impulses were moving all by itself, whereas the solid curve shows the resultant displacement obtained by algebraic addition of each displacement. Shortly later in Fig. **5.6(d)** the two pulses overlap each other and the resultant displacement is zero everywhere. At a much later time, the impulses cross each other (Fig. **5.6(e))** and move as if nothing had happened. This could hold provided the principle of superposition is true.

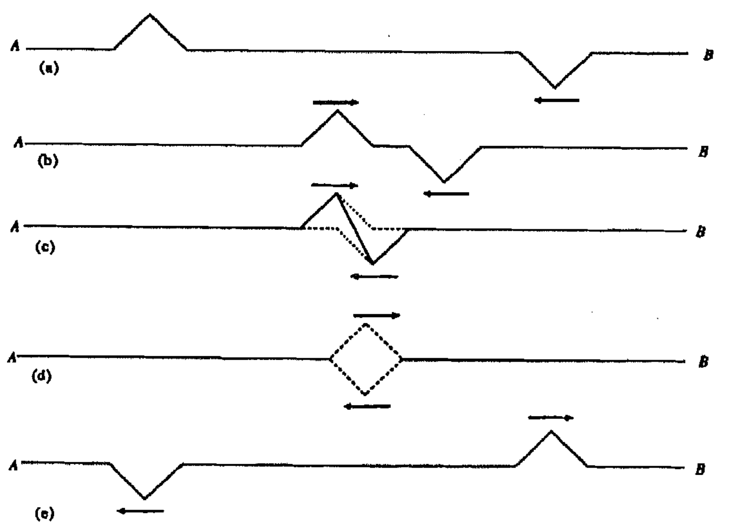


Fig.5.6: The propagation of two triangular pulses in opposite directions in a stretched string. The solid line gives the actual shape of the string; (a), (b), (c), (d) and (e) correspond to different instants of time.

Let us consider the following case of superposition of waves.

Superposition of Two Waves of Same Frequency but having Constant Phase Difference

Consider two waves of same frequency but having constant phase difference, say δ. Since they have same frequency, **i.e.** same angular velocity, we write

$$y_1 = a_1 \sin \omega t$$

and $$y_2 = a_2 \sin (\omega t + \delta)$$

where a_1 and a_2 are two different amplitudes, and ω is common angular frequency of the two waves. By the principle of superposition, the resultant displacement is

$$Y = y_1 + Y_Z$$
$$= a, \sin \omega t + a_2 \sin (\omega t + \delta)$$
$$= a_1 \sin \omega t + a_2 \sin \omega t \cos \delta + a_2 \cos \omega t \sin 6$$
$$= \sin \omega t (a_1 + a_2 \cos \delta) + \cos \omega t (a_2 \sin 6)$$

Let us write

$$a_1 + a_2 \cos S = A \cos \theta \qquad \text{... (5.14a)}$$

and $$a_2 \sin 6 = A \sin \theta \qquad \text{...(5.14b)}$$

where A and θ are new constants. This gives

$$y = \sin \omega t \, A \cos \theta + \cos \omega t \, A \sin \theta$$

or $$y = A \sin (\omega t + 8)$$

Hence the resultant displacement is simple harmonic and of amplitude A. Squaring and adding Eq. 5.14a and 5.14b, we get

$$A^2 \cos^2\theta + A^2 \sin^2\theta = (a_1 \, t \, a_2 \cos\delta)^2 + (a_2 \sin \delta)^2$$

or, $$A^2 = a_1^2 + a_2^2 + 2a_1 a_2 \cos\delta$$

Thus; the resultant intensity I which is proportional to the square of the resultant amplitude, is given as

$$I = A^2 = a_1^2 + a_2^2 + 2a_1 a_2 \cos\delta \qquad \text{...(5.15)}$$

(Here we have taken the constant of proportionality as 1, for simplicity).

Thus, we find that the resultant intensity is not equal to the sum of the intensities due to separate waves i.e., $(a_1^2 + a_2^2)$. Since the intensity of wave is proportional to square of amplitude, I_1 a a_1^2 and I_2 a a_Z^2 as before, taking the proportionality constant as 1, we can rewrite Eq. (5.15) as

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \delta \qquad \text{...(5.16)}$$

In Example 1, see how Eq. (5.16) has been used to find the resultant intensity.

Example **1**

Consider interference due to two coherent waves of same frequency and constant phase difference having intensities I and **41,** respectively. What is the resultant intensity when the phase difference between these **two** waves is $\pi/2$ and π?

Solution

According to Eq. {S.16)

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \delta$$

Given: $I_1 = I$ and $I_2 = 41$, so

$$I = 5I + 2I\sqrt{4} \cos \delta$$
$$= 5I + 4I \cos \delta$$

Hence $$I_{\pi/2} = 51 + 41 \cos 90^0 = 5I$$
$$I_\pi = 51 + 41 \cos \pi = I$$

Thus there is a variation of intensity due to interference phenomenon.

Refer again to Eq. (5.16). The intensity I is maximum when $\cos \delta = +1$, that is, when phase difference is given by

$$\delta = 2n\pi \text{ (even multiple of } \pi).$$

From Eq. (5.16)

$$I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2}$$

The resultant intensity is, thus. greater than the sum of the two separate intensities. If

$$I_1 = I_2 \text{ then } I_{max} = 4I_1$$

The intensity I is minimum when $\cos \delta = -1$, i.e., when δ is given by

$$\delta = (2n + 1)\pi \text{ (odd multiple of } \pi).$$

We have from Eq. (5.16)

$$I_{min} = I_1 + I_2 - 2\sqrt{I_1 I_2}$$

The resultant intensity is thus less than the sum of two separate intensities. If $I_1 = I_2$, then $I_{min} = 0$, which means that there is no light.

SAQ 3

Two waves of same frequency and constant phases difference have intensities in the ratio **81:1.** They produce interference fringes. Deduce the ratio of the maximum to minimum intensity.

In general, for the two waves of same intensity and having a constant phase difference of δ, the resultant intensity is given by

$$I = 2I_1 + 2I_1 \cos\delta \qquad (\because I_1 = I_2)$$

$$= 2I_1 (1 + \cos \delta)$$

$$= 4I_1 \cos^2 \frac{\delta}{2} \qquad \ldots(5.17)$$

Therefore, we find that when two waves of the same frequency travel in approximately the same direction and have a phase difference that remains constant with the passage of time, the resultant intensity of light in not **distributed** uniformly in space. The **non-**uniform distribution of the light intensity due to the superposition of two waves is called interference. At some points the intensity is maximum and the interference at these points is called constructive interference. At some other points the intensity is minimum and the interference at these points is called destructive interference.

Usually, when two light waves are made to interfere, we get alternate dark and bright bands of a regular or irregular shape. These are called *Interference* fringes.

SAQ 4

Fig. 5.7 shows two situations where waves emanating from two sources, A and B, arrive at point C and interfere. Which of the two situations indicate constructive interference and destructive interference? Give reasons. (Eq. (5.13) will help you in answering this **question.)**

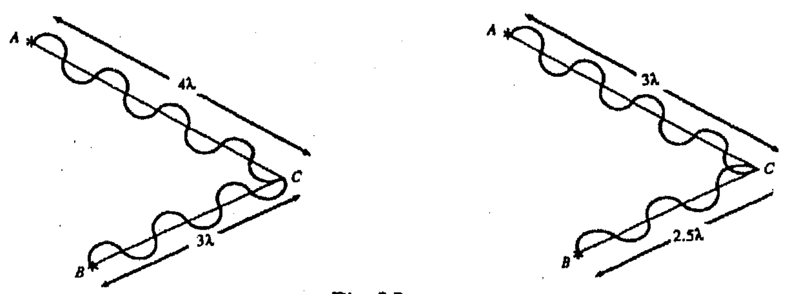


Fig. 5.7:

After solving the above SAQ one can infer that:

for constructive interference,

$$\text{path difference} = n\lambda, \text{ where } n = 0, 1, 2, 3 \qquad \ldots(5.18)$$

for destructive interference,

$$\text{path difference} = m\frac{\lambda}{2}, \text{ where } m = 1, 3, 5, 7 \qquad \ldots(5.19)$$

For the production of stationary interference patterns, i.e. definite regions of constructive and destructive interference, the interfering waves must have (1) the same frequency, and (2) a constant phase difference (and they must be travelling in the same or nearly the same direction). If these conditions are satisfied, we say the wave sources and the waves are coherent. Sources can readily be found with the same vibrating frequency; however, the phase relationship between the waves may vary with time. In the case of light, the waves are radiated by the atoms of a source. Each atom contributes only a small part to the light emitted from the source and the waves bear no particular phase relationship to each other; the atoms randomly emit light, so the phase "constant" of the total light wave varies with time. Hence, light waves brought together from different light sources are coherent over very short periods of time and does not produce stationary interference patterns. Light from two lasers (about this you will study in Block 4) can be made to form stationary interference patterns, but the lasers must be phase-locked by some means. How, then, was the wave nature of light originally investigated, since lasers are a relatively recent development ? In the following sections we will discuss the various arrangements, which provide coherent sources and enable us to observe interference phenomenon. Thomas Young had first demonstrated the interference of light. In the next section we will describe the experiment done by him.

5.4 YOUNG'S DOUBLE-SLIT EXPERIMENT

One of the earliest demonstrations of such interference effect was first done by Young in 1801, establishing the wave character of light. Young allowed sunlight to fall on a pinhole S_0, punched in a screen A as shown in Fig. 5.8. The emergent light spreads out and falls on pinholes S_1 and S_2, punched in the screen B. Pinholes S_1 and S_2 act as coherent sources. Again, two overlapping spherical waves expand into space to the right of screen B. Fig. 5.8 shows how Young produced an interference pattern by allowing the waves from pinholes S_1 and S_2 to overlap on screen C.

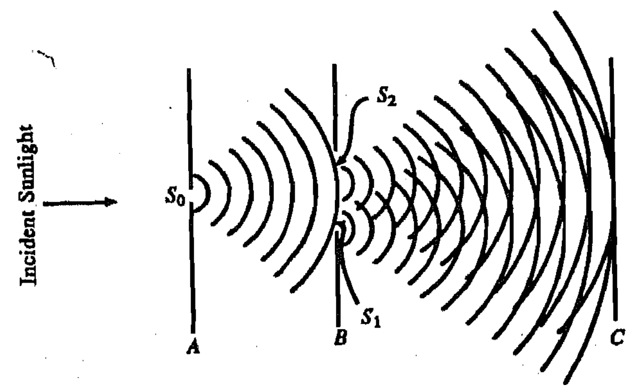


Fig.5.8: Young's double slit experiment. The pinholes S_1 and S_2 act as coherent sources and an interference pattern is observed on the screen C.

Fig. 5.9 shows the section of the wavefront on the plane containing S_0, S_1, and S_2. Since the waves emanating from S_1 and S_2 are coherent, we will see alternate bright and dark curves of fringes, called interference fringes. The interference pattern is symmetrical about a bright central fringe (also called maximum), and the bright fringes decrease in intensity, the farther they are from the central fringe.

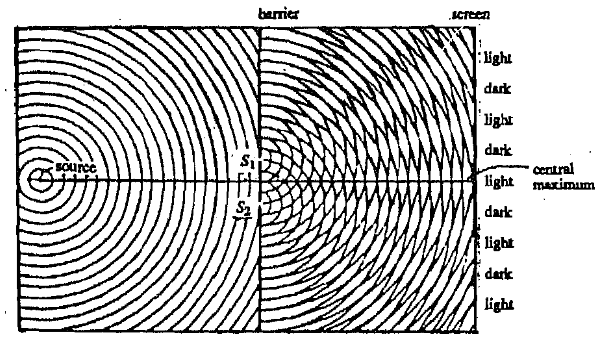


Fig. 5.9: Sections of the spherical wavefronts emanating from S_0, S_1 and S_2.

To analyze the interference pattern and investigate the spacing of the interference fringes, consider the geometry in Fig. 5.10. Let S be a narrow slit illuminated by monochromatic light, and S_1 and S_2 two parallel narrow slits very close to each other and equidistant from S. The light waves from S arrive at S_1 and S_2 in the same phase. Beyond S_1 and S_2, the waves proceed as if they started from S_1 and S_2 with the same phase because the two slits are equidistant from S.

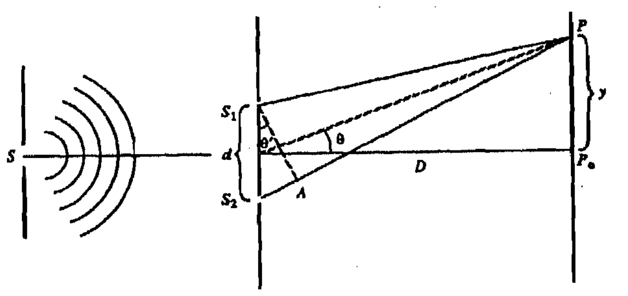


Fig.5.10: The geometry of Young's experiment: The path difference of the light from the slits arriving at P on the screen is d sinθ.

It is assumed that the waves start out at the same phase, because the two slits S_1 and S_2 are equidistant from S. Furthermore, the amplitudes are the same, because S_1 and S_2 are the same size slits and very close to each other. (So the amplitude does not vary very much.) Hence these waves produce an interference pattern on a screen placed parallel to S_1 and S_2. .

To find the intensity at a point P on the screen, we join S_1P and S_2P. The two waves arrive at P from S_1 and S_2 having traversed different paths S_1P and S_2P. Let us calculate this path difference $S_2P - S_1P$. Let,

y = distance of P from P_0, the central point on the screen

d =,separation of two slits S_1 and S_2.

D = distance of slits from the screen.

The corresponding path difference is the distance S_2A in Fig.'5.10, where the line S_1A has been drawn to make S_1 and A equidistant from P. As Young's experiment is usually done with D >> d or y, the angle θ and θ' are neatly same and they are small.

Hence, we may assume triangle $S_1A\ S_2$ as a right-angled mangle and S_2A = d sin θ' = d sin 0 = d tanθ, as for small θ, sin0 = tan 0. As can be seen from the Fig. 5.10, tan $\theta = y/D$.

$$\therefore \quad S_2P - S_1P = S_2A = d\frac{y}{D} \qquad \text{...(5.20)}$$

Now the intensity at the point P is a maximum or minimum according as the path difference $S_2P - S_1P$ is an integral multiple of wavelength or an odd multiple of half wavelength (See Eq. 5.18 and Eq. 5.19). Hence, for bright fringes (maxima),

$$S_2P - S_1P = \frac{yd}{D} = 0, \lambda, 2\lambda, 3\lambda.... = m\lambda$$

where m = 0, 1, 2.....

$$\therefore \quad y = mD\,\lambda/d \quad \text{(bright fringes)} \qquad \text{...(5.21)}$$

The number m is called the order of the fringe. Thus the fringes with $m = 0, 1, 2, ...$ etc. are called zero, first, second...etc. orders. The zeroth order fringe corresponds to the central maximum, the first order fringe (m =1) corresponds to the first bright fringe on either side of the central maximum, and so on. For dark fringes (minima),

$$S_2P - S_1P = \frac{yd}{D} = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2} ... = \left(m + \frac{1}{2}\right)\lambda$$

where m =0, 1, 2, ...

$$. \quad y = \left(m + \frac{1}{2}\right)\frac{\lambda D}{d} \quad \text{(dark fringes)} \qquad \text{...(5.22)}$$

Eq. (5.21) or Eq. (5.22) can be used to find out the distance y_n of the nth order bright (or dark) fringe. Try to solve the following SAQ.

SAQ 5

Monochromatic light passes through two narrow slits 0.40 mm apart. The third-order bright fringe of the interference pattern, observed on a screen 1.0 meter from the slits, is 3.6 mm from the cenlre of the central maximum. What is the wavelength of the light ?

Fringe Width

If y_n and y_{n+1} denote the distances of nth and (n+1)th bright fringes, then

$$y_n = \frac{D}{d}n\lambda$$

and
$$y_{n+1} = \frac{D}{d}(n+1)\lambda$$

The spacing between the nth and (n+1)th fringes (bright) is given by

$$y_{n+1} - y_n = \frac{D}{d}(n+1)\lambda - \frac{D}{d}n\lambda = D\lambda/d$$

It is independent of n. Hence, the spacing between any two consecutive bright fringes is the same. Similarly, it can be shown that the spacing between two dark fringes is also $\frac{D}{d}\lambda$. The spacing between any two consecutive bright or dark fringes is called the fringe-width, which is denoted by β. Thus

$$\beta = \frac{D}{d}\lambda \qquad \text{...(5.23)}$$

One **also finds, by experiment, that fringe-width**

i) varies directly as D,

ii) varies directly as the wave-length of the light used, and

iii) inversely as the distance d between the slits

The fringe-widths are so fine that to see them, one usually uses magnifier or eye-piece.

To make certain that you really understand the meaning of the fringe width, try the following SAQs.

SAQ 6

In a two-slit interference pattern with $\lambda = 6000$ Å, the zero order and tenth order maxima fall at 12.34 mm and 14.73 mm respectively. Find the fringe width.

SAQ 7

If in the SAQ *6*, A is changed to *5000* Å, deduce the positions of the zero order and twentieth order fringes, other arrangements remaining the same.

Shape of the Interference Fringes

In Fig. 5.11, suppose S_1 and S_2 represent the two coherent sources. At the point P, there is maximum or minimum intensity according as

$$S_2P - S_1P = n\lambda$$

or

$$S_2P - S_1P = \left(n + \frac{1}{2}\right)\frac{\lambda}{2}$$

Thus for a given value of n, the locus of points of maximum or minimum intensity is given by

$$S_2P - S_1P = \text{constant},$$

which is the equation of a hyperbola with S_1 and S_2 as foci. In space, the locus of points of maximum or minimum intensity for a particular value of n will be a hyperboloid of revolution, obtained by revolving the hyperbola about the line S_1S_2.

In practice, fringes are observed on a screen XY in a plane normal to the plane of the figure and parallel to the line joining S_1S_2. Hence the fringes that are observed arc simply the sections of the hyperboloids by this plane, i.e. they are hyperbolae. Since the wavc-length of light is extremely small (of the order of 10^{-5}cm), the value of $(S_2P - S_1P)$ is also of that order. Hence these hyperbolae appear, more or less, as straight lines.

Intensity Distribution in the Fringe-System

To find the intensity, we rewrite Eq. (5.15), taking a, = a_2, as follows

$$I = A^2 = 2a^2\,(1 + \cos 6)$$
$$= 4a^2 \cos^2 \frac{\delta}{2}$$

If the phase difference is such that $\delta = 0, 2\pi, 4\pi$......, this gives $4a^2$ or 4 times the intensity of either beam. If $6 = \pi, 3\pi, 5\pi$,......, The intensity is zero.

In between the intensity varies as $\cos^2 \delta/2$. Fig. 5.12 shows a plot of the intensity against the phase difference. When the two beams of light arrive at a point on the screen, exactly out of phase, they interfere destructively, and the resultant intensity is zero. One may well ask what becomes of the energy of the two beams, since the law of conscrvadon of energy

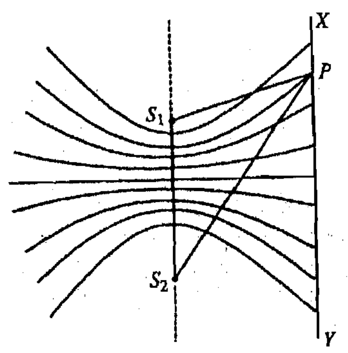


Fig. 5.11 : Shape of the fringes.

tells us that it cannot be destroyed. The answer to this question is that the energy, which apparently disappears at the minima, is actually still present at the maxima, where the intensity is greater than would be produced by the two beams acting separately. In other words, the energy is not destroyed, but merely redistributed in the interference pattern, The average intensity on the screen is exactly what would exist in the absence of interference. Thus, as shown in Fig. 5.12, the intensity in the interference pattern varies between $4A^2$ and zero. Now each beam, acting separately, would contribute A^2, and so, without interference, we would have a **uniform** intensity of $2A^2$, as indicated **by** the broken **line.** Let us obtain the average intensity on the screen for π fringes. We have

$$I_{\text{average}} = \frac{\int_0^{\pi} I \, d\delta}{\int_0^{\pi} d\delta}$$

$$= \frac{\int_0^{\pi} \left(4A^2 \cos^2 \frac{\delta}{2}\right) d\delta}{\int_0^{\pi} d\delta}$$

$$= \frac{\int_0^{\pi} (2A^2 + 2A^2 \cos 6) \, d\delta}{\int_0^{\pi} d\delta} \qquad \left(\because 1 + \cos\theta = \cos^2 \frac{}{2}\right.$$

$$= \frac{[2A^2 \, \delta + 2A^2 \sin \delta]_0^{\pi}}{[\delta]_0^{\pi}}$$

$$= \frac{2A^2 \, \pi}{\pi}$$

$$= 2A^2$$

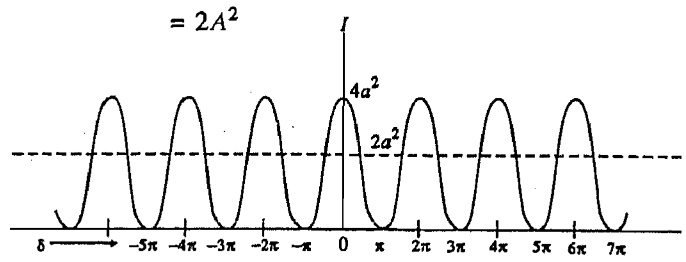


Fig. 5.12: Intensity distribution for the interference fringes from two waves of the same frequency.

Thus, the average intensity is equal to the sum of the separate intensities. That is whatever energy apparently disappears at the minima is actually present at the maxima. There is no violation of the law of conservation of energy in the phenomenon of interference.

Till now we have considered interference pattern produced when a monochromatic light from a narrow slit'falls on two parallel slits. What happens if white light is used to illuminate slits? Read the following sub-section.

5.4.1 White-Light Fringes

If white light is used to illuminate the slits we obtain an interference pattern consisting of a **central** 'white' fringe, having on both sides a few coloured fringes and then a general illumination.

A pair of white light coherent sources is equivalent to a number of pairs of monochromatic sources. Each monochromatic pair produces its own system of fringes **with a different fringe-width β, since β depends on $\lambda \left(\beta = \frac{D\lambda}{d}\right)$.**

At the centre of the pattern, the path difference between the interfering waves is zero. Therefore, the path difference is also zero for all wavelengths. Hence, all the different coloured waves of the white light produce a bright **fringe** at the centre. This superposition of the different **colours** makes the central fringe 'white'. This is the 'zero order fringe'.

As we move on either side of the centre, the path difference gradually increases from zero. At a certain point it becomes equal to **half** the wavelength of the component having the smallest wave-length, i.e., violet. This is the position of the first dark fringe of violet. Beyond this, we obtain the first minimum of blue, green, yellow and of red in the last. The inner edge of the first dark fringe, which is the first minimum for violet. receives sufficient intensity due to red, hence it is reddish. The outer edge of the first dark fringe, which is minimum for red, receives sufficient intensity due to violet, and is therefore, violet. The same applies to every other dark fringe. Hence, we obtain a few coloured fringes on both sides of the central fringe.

As we move further away from the centre, the path difference becomes quite large. Then, from the range 7500 – 4000 Å, a large numbcr of wavelengths (colours) will produce maximum intensity at a given point, and an equally large numbcr will produce minimum intensity at that point. For example, at any point P, we may have

$$\text{path difference} \begin{cases} = 11\,\lambda_1 = 12\lambda_2 = 13\lambda_3 = \ldots\text{etc. (maxima)} \\ = \left(11 + \frac{1}{2}\right)\lambda_1' = \left(12 + \frac{1}{2}\right)\lambda_2' = \left(13 + \frac{1}{2}\right)\lambda_3' = \ldots \text{etc. (minima,} \end{cases}$$

Thus, at P, we shall have 11th, 12th, 13th, etc., bright fringes of λ_1, λ_2, λ_3,...etc., and 11th, 12th, 13th,... etc., dark fringes of λ'_1, λ'_2, λ'_3, ...etc. Hence, the resultant colour at P is very nearly white. This happens at all points, for which the path difference is large. Hence, in the region of large path difference uniform white illumination is obtained.

For maxima, path difference $= n\,\lambda$, where $n = 0, 1, 2, ..$

For minima, path difference $= \left(n + \frac{1}{2}\right)\lambda$, where $n = 0, 1, 2, \ldots$

SAQ 8

Let the path difference $S_1P - S_2P = 30 \times 10^{-5}$ cm. What are the λ's for which the point P is a maximum?

In the usual interference pattern with a monochromatic source, a large number of interference fringes are obtained, and it becomes extremely difficult to determine the position of the central fringe. Hence, by using white light as a source the position of central fringe can be easily determined.

5.4.2 Displacement of Fringes

We will now discuss the change in the interference pattern produced when a thin transparent plate, say of glass or mica, is introduced in the path of one of the two interfering beams, as shown in Fig. 5.13. It is observed that the entire fringe-pattern is displaced to a point towards the beam in the path of which the plate is introduced. If the displacement is measured, the thickness of the plate can be obtained provided the refractive index of the plate and the wavelength of the light are known.

Suppose a thin transparent plate of thickness t and refractive index μ is introduced in the path of one of the constituent interfering beams of light (say in the path of S_1P, shown in Fig. 5.13). Now, light from S_1 travel partly in air and partly in the plate. For the light path from S_1 to P, the distance travelled in air is $(S_1P - t)$, and that in the plate is t. Suppose, c and v be the velocities of light in the air and in the plate, respectively. If the time taken by light beam to reach from S_1 to P is, T, then

$$T = \frac{S_1P - t}{c} + \frac{t}{v}$$

or,

$$T = \frac{S_1P - t}{c} + \frac{\mu t}{c} \qquad \left(\because v = \frac{c}{\mu}\right)$$

$$= \frac{S_1P + (\mu - 1)\,t}{c}$$

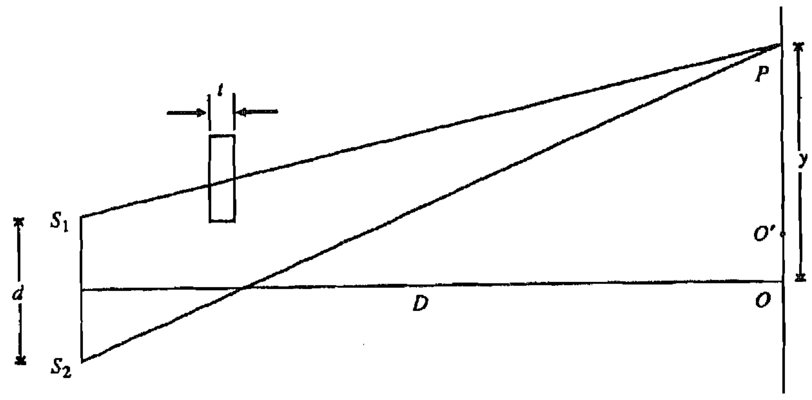


Fig. 5.13: If a thin transparent sheet (of thickness t) is introduced in one of the beams, the fringe pattern gets shifted by a distance $(\mu-1)\ tD/d$.

Thus the effective path in air from S_1 to P is $[S_1P + (\mu-1)t]$, i.e., the air path S_1P is increased by an amount $(\mu-1)t$, due to the introduction of the plate of material of refractive index, μ.

Let O be the position of the central bright fringe in the absence of the plate, the optical paths S_1O and S_2O being equal. On introducing the plate, the two optical paths become unequal. Therefore, the central fringe is shifted to O', such that at O' the two optical paths become equal. A similar argument applies to all the fringes. Now, at any point P, the effective path difference is given by

$$S_2P - [S_1P + (\mu-1)t]$$

$$= (S_2P - S_1P) - (\mu - 1)t$$

From Eq. (5.20), $S_2P - S_1P = \frac{d}{D}\,y$

$\therefore$ Effective path difference at $P = \frac{d}{D}\,y - (\mu - 1)\,t$

If the point P is to be the centre of the nth bright fringe, the effective path difference should be equal to *nil* i.e.,

$$\frac{d}{D}\,y_n - (\mu - 1)\,t = n\lambda$$

or

$$\boxed{y_n = \frac{D}{d}\,[n\lambda + (\mu - 1)\,t]} \qquad \ldots(5.24)$$

In the absence of the plate ($t = 0$), the distance of the nth bright fringe from O is $\frac{D}{d}\,n\lambda$.

$\therefore$ Displacement y_0 of the nth bright fringe is given by

$$y_0 = \frac{D}{d}\,[n\lambda + (\mu - 1)\,t] - \frac{D}{d}\,n\lambda$$

$$\boxed{y_0 = \frac{D}{d}\,(\mu - 1)\,t} \qquad \ldots(5.25)$$

The shift is independent of the order of the fringe, showing that shift is the same for all the bright fringes. Similarly, it can be shown that the displacement of any dark fringe is also given by Eq. (5.25). Thus, the entire fringe-system is displaced through a distance $\frac{D}{d}\,(\mu - 1)\,t$ towards the side on which the plate is placed, The fringe-width is given by:

$$\beta = y_{n+1} - y_n$$

$$= \frac{D}{d}\,[(n + 1)\,\lambda + (\mu - 1)\,t] - \frac{D}{d}\,[n\lambda + (\mu - 1)\,t] \qquad \text{(see Eq. (5.24))}$$

$$= \frac{D\lambda}{d},$$

which is the same as before the introduction of the plate.

Eq, (5.25) enables us to determine the thickness of extremely thin transparent sheets (like that of mica) by measuring the shift of the fringe system.

Now, apply this strategy yourself to SAQ 9.

SAQ 9

In a double slit interference arrangement one of the slits is covered by a thin mica sheet whose refractive index is 1.58. The distances S_1S_2 and AO (see Fig. 5.13) are 0.1 cm and 50 cm, respectively. Due to the introduction of the mica sheet, the central fringe gets shifted by 0.2 cm. Determine the thickness of the mica sheet.

5.5 FRESNEL'S BIPRISM

With regard to Young's double-slit experiment, objection was raised that the bright fringes observed by Young were probably due to some complicated modification of the light by the edges of the slits and not due to interference. Soon after, Fresnel devised a series of arrangements to produce the interference of two beams of light which was not subject to this criticism. One of the experimental arrangements, known as Fresnel's Biprism arrangement, is shown in Fig. 5.14.

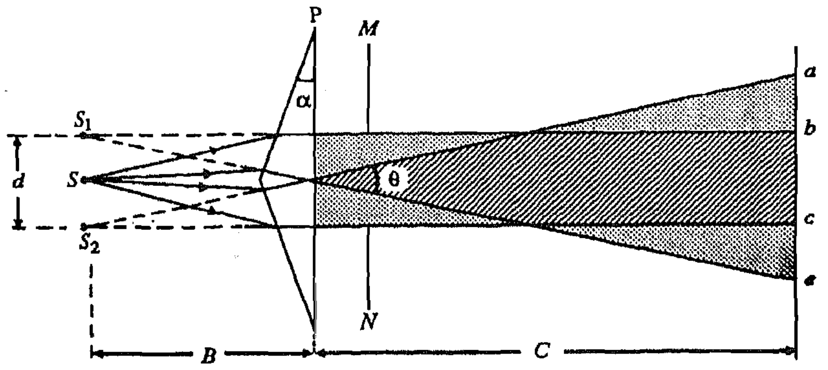


Fig. 5.14: **Diagram of Fresnel's Biprism experiment.**

S is a narrow vertical slit illuminated by monochromatic light. The light from S is allowed to fall symmetrically on the Biprism P, placed at a small distance from S and having its refracting edges parallel to the slit. The light emerging from the upper and lower halves of the prism appears to start from two virtual images, S_1 and S_2 of S, which act as coherent sources. The cones of light bS_1e and aS_2c, diverging from S_1 and S_2, are superposed and the interference fringes are formed in the overlapping region bc.

If screens M and N are placed, as shown in the Fig. 5.14, interference fringes are observed only in the region bc. When the screen ae is replaced by a photographic plate, a picture like the upper one, in Fig. 5.15, is obtained.

The closely spaced fringes in the centre of the photograph are due to interference, while the wide fringes at the edge of the photograph are due to diffraction. These wider bands are due to the vertices of the two prisms, each of which act as a straight edge, giving a pattern of diffraction (about this you will learn in Block 3). When the screens M and N are removed from the light path, the two beams overlap over the whole region oe. The lower photograph in Fig. 5.15 shows for this case the equally spaced interference fringes superimposed on the diffraction pattern, of a wide aperture.

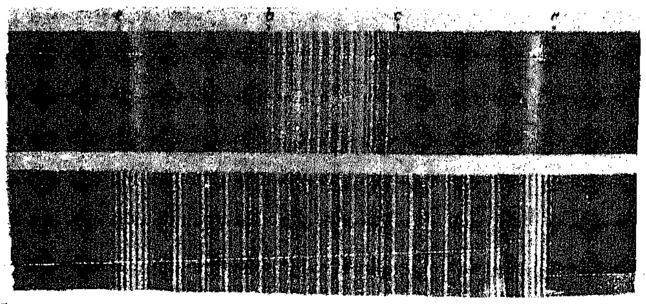


Fig. 5.15: Interference and diffraction fringes produced in the Fresnel Biprism experimental arrangement.

With such an experiment, Fresnel was able to show the interference effect without the diffracted beams through the two slits. Just as in Young's double slit experiment, this arrangement can also be used to determine the wavelength of monochromatic light. The light illuminates the slit S and interference fringes can be easily viewed through the eye-piece. The fringe-width β can be determined by means of a micrometer attached to the eye piece. If D is the distance between source and screen, and d the distance between the virtual images S_1 and S_2, the wave-length is given by

$$\lambda = \frac{\beta d}{D} \qquad \ldots(5.26)$$

The distances d and D can easily be determined by placing a convex lens between the Biprism and the eyepiece. For a fixed position of the eyepiece, there will be two positions of the lens, shown as L_1 and L_2 in Fig. 5.16 where the images of S_1 and S_2 can be seen at the eyepiece. Let d_1 be the distance between the two images, when the lens is

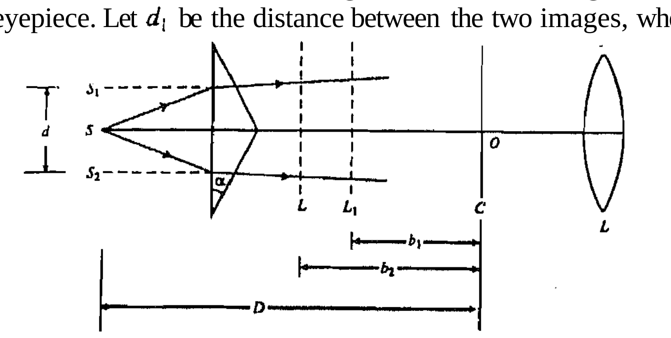


Fig. 5.16: Fresnel's biprism arrangement. C and L represents the position of cross wires and the eyepiece, respectively. In order to determine d a lens Is Introduced between the biprism and cross wires. L_1 and L_2 represent the two positions of the lens where the slits are clearly seen.

at the position L_1 (at a distance b_1 from the eyepiece). Let d_2 and b_2 be the corresponding distances, when the lens is at L_2. Then it can easily be shown that

$$d = \sqrt{d_1 d_2} \qquad \ldots(5.27a)$$

and

$$D = b_1 + b_2 \qquad \ldots(5.27b)$$

Use Eq. (5.26) and (5.27) to solve the following **SAQ.**

SAQ 10

In a Fresnel's Biprism experiment, the eyepiece is at a distance of 100 cm from the slit. A convex lens inserted between the Biprism and the eyepiece gives two images of the slit in two positions. In one case, the two images of the slit are 4.05 mm apart, and in the other case 2.10 mm apart. If sodium light of wavelength 5893 Å is used, find the thickness of the interference fringes.

5.6 SOME OTHER ARRANGEMENT FOR PRODUCING INTERFERENCE BY DIVISION OF WAVEFRONT

Two beams may be brought together in several other ways to produce interference. In **Fresnel's two-mirror** arrangement, light from a slit is reflected in two plane mirrors slightly inclined to each other. The mirror produces two virtual images of the slit, as shown in Fig. 5.17.

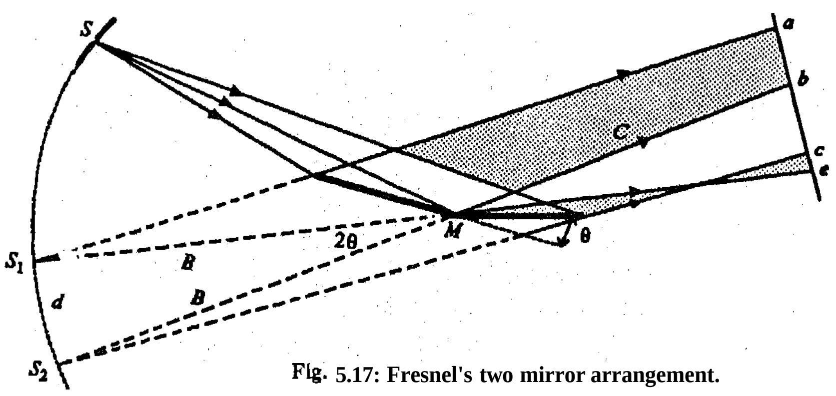


Fig. 5.17: Fresnel's two mirror arrangement.

They are like the images in Fresnel's biprism, and interference fringes are observed in the region bc, where the reflected beams overlap:

Even a simpler mirror method is available. This is known as **Lloyd's** mirror. Here the slit and its virtual image constitute the double source.

At grazing incidence, almost the entire incident light is reflected so that the direct and the reflected beam have nearly equal amplitudes. Hence the fringes have good contrast.

Lloyd's Mirror

It is a simple arrangement to obtain two coherent sources of light to produce a stationary interference pattern. It consists of a plane mirror MN (Fig. 5.18) polished on the front surface and blackened at the back (to avoid multiple reflection). S_1 is a narrow slit, illuminated by monochromatic light, and placed with its length parallel to the surface of the mirror. Light from S_1 falls on the mirror at nearly grazing incidence, and the reflected beam appears to diverge from S_2, which is the virtual image of S_1. Thus S_1 and S_2 act as coherent sources. The direct cone of light AS_1E and the reflected cone of light BS_2C are superposed, and the interference fringes are obtained in the overlapping region BC on the screen.

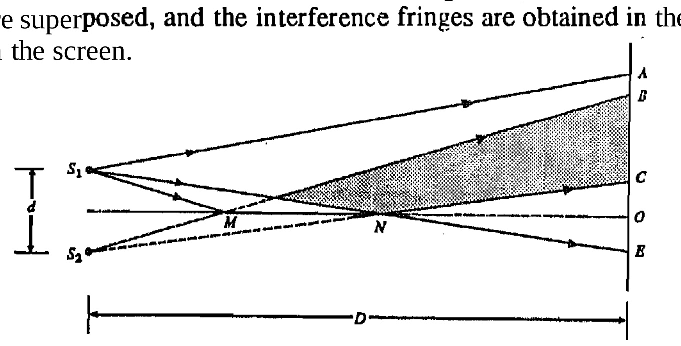


Fig. 5.18: Lloyd's mirror

Zero-Order Fringe

The central zero-order fringe, which is expected to lie at O (the perpendicular bisector of S_1S_2) is not usually seen since only the direct light, and not the reflected light, reaches O. It can be seen by introducing a thin sheet of mica in the path of light from S_1, when the entire fringe system is displaced in the upward direction. (You could see this yourself while solving SAQ 11.)

SAQ 11

Interference bands are obtained with a Lloyd's mirror with light of wavelength 5.45×10^{-5} cm. A thin plate of glass of refractive index 1.5 is then placed normally in the path of one of the interfering beams. The central dark band is found to move into the position previously occupied by the third dark band from the centre. Calculate the thickness of the glass plate.

With white light the central fringe is expected,to be white, but actually it is found to be 'dark'. This is because the light suffers a phase change of π or a path-difference of $\frac{\lambda}{2}$ when reflected from the mirror. Therefore, the path difference between the interfering rays at the position of zero-order fringe becomes $\frac{\lambda}{2}$ (instead of zero), which is a condition for a minimum. Hence the fringe is dark.

Determination of Wavelength

Let **d** be the distance between the cdherent sources S_1 and S_2, and D the distance of the screen from the sources. The fringe–width is then given by

$$\beta = \frac{D\lambda}{d}$$

Thus, knowing β, D and d, the wavelength λ can be determined.

Acromatic Fringes and their Production by Lloyd's Mirror

A system of white and dark fringes, without any colours, obtained by white light are known as 'achromatic fringes'.

Ordinarily, with white light, we obtain a central white fringe, having on either side of it a few coloured fringes (as you have studied in subsection 5.4.1). This is because the fringe-width $\beta = \frac{DL}{d}$ is different for different wavelengths (colours). If however, the fringe-width is made the same for all wavelengths, the maxima of each order for all wavelengths will coincide, resulting into achromatic fringes. That is, for achromatic fringes, we must have

$$\frac{D\lambda}{d} = \text{constant}$$

or

$$\frac{\lambda}{d} = \text{constant}$$

We can easily realise this condition with a Lloyd's mirror by using a slit illuminated by a narrow spectrum of the white light as shown in Fig. 5.19. The narrow spectrum $R_1 V_1$ is produced by a prism, or, preferably, by a plane diffraction grating. The Lloyd's mirror is placed with its surface close to the violet end of the spectrum and such that $R_1 V_1$ is perpendicular to its plane.

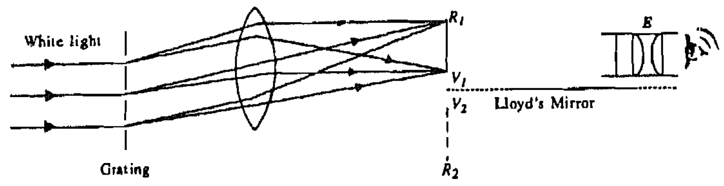


Fig. 5.19 : Achromatic fringes produced by Lloyd's mirror.

$R_1 V_1$, and its virtual image, $R_2 V_2$, formed by the mirror act as coherent sources. They are equivalent to a number of pairs of sources of different colours. Thus, the pair $R_1 R_2$ produces a set of red fringes, and the pair $V_1 V_2$ a set of violet fringes. The intermediate pairs produce the sets of fringes of intermediate colours. The red and violet fringes will be of the same width if

$$\frac{\lambda}{d} = \text{constant}$$

i.e.

$$\frac{\lambda_r}{d_r} = \frac{\lambda_v}{d_v}$$

or

$$\frac{d_r}{d_v} = \frac{\lambda_r}{\lambda_v}$$

where d_r is the distance $R_1 R_2$, and d_v the distance $V_1 V_2$.

Hence, the last expression gives

$$\frac{R_1 R_2}{V_1 V_2} = \frac{\lambda_r}{\lambda_v}$$

Therefore, if the distance of the violet end V_1 from the surface of the mirror is so adjusted by displacing the mirror laterally that the above condition is satisfied, the red and violet fringes will have the same width, and will exactly be superposed on each other. Since, in a grating spectrum, the dispersion is accurately proportional to the wavelength, the condition (λ/d) = constant is simultaneously satisfied for all the wavelengths. Thus, when this adjustment is made, fringes of all colours are superposed on one another. Hence, achromatic fringes are observed in the eyepiece E placed in the over-lapping region.

Difference between Biprism and Lloyd's **Mirror Fringes**

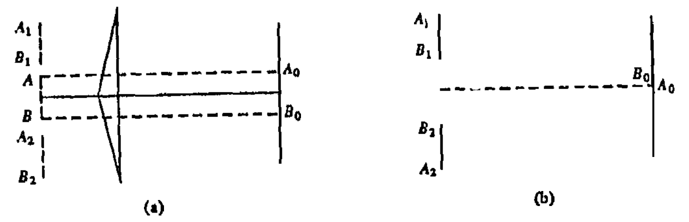
The following are the main points of difference between the biprism and Lloyd's mirror fringes.

1) In biprism, the complete pattern of fringes is obtained. In Lloyd's mirror, ordinarily, only a few fringes on one side of the central fringe are visible, the central fringe itself being invisible.

2) In biprism the central fringe is bright, while in Lloyd's mirror it is dark.

3) The central fringe in biprism is less sharp than that in Lloyd's mirror.

The coherent sources in the biprism are $A_1 B_1$ and $A_2 B_2$ (Fig. 5.20a) the virtual images of a slit AB. In Lloyd's mirror, the coherent sources are a slit $A_1 B_1$ itself and its virtual image $B_2 A_2$ (Fig. 5.20 b). In both cases, A_1 and A_2 form one extreme pair of coherent point-sources, and B_1 and B_2 another extreme pair. In the biprism, the zero-order fringes corresponding to $A_1 A_2$ and $B_1 B_2$ are formed at A_0 and B_0, which lie on the right bisectors of $A_1 A_2$ and $B_1 B_2$, respectively. Hence, the zero-order fringe extends from A_0 to B_0. In Lloyd's mirror, on the other hand, all pair of coherent sources have a common perpendicular bisector, so that zero-order fringes due to all of these are formed in one and the same position. Hence the zero-order fringe is sharp in this case.



Pig. 5.20 : Showing the difference between biprism and Lloyd's mirror fringes.

4) In biprism $A_1 A_2 = B_1 B_2 = d$. Hence, the fringe-width $\beta = \frac{D\lambda}{d}$ is the same for all pairs of coherent sources. In Lloyd's mirror arrangement d is different for different pairs of coherent sources, e.g., $A_1 A_2 > B_1 B_2$. Hence, the fringe-width is different for different pairs of coherent sources.

5.6 SUMMARY

- The relationship between phase difference and path difference is:

  $$\text{phase difference} = \frac{2\pi}{\lambda}\ (\text{path difference})$$

  If two waves of same frequency and of amplitudes a_1 and a_2 and phase difference δ are superposed then, according to principle of superposition, the amplitude A of the resultant wave is given by

  $$A^2 = a_1^2 + a_2^2 + 2a_1 a_2 \cos\delta$$

- Two sources are said to be coherent if they emit light waves with no or constant phase difference.

- When two waves of the same frequency travel in approximately the same direction and have a phase difference that remains constant with time, the resultant intensity of light is not distributed uniformly in space. This non-uniform distribution of the light intensity is due to the phenomenon of interference.

- For constructive interference

  path difference = $n\lambda$, where n = 0, 1, 2, ...

  and for destructive interference

  path difference = $m\frac{\lambda}{2}$, where m = 1, 3, 5, 7

- In an interference pattern, the distance between any two consecutive maxima or minima is given by

  $$\beta = \frac{D\lambda}{d}$$

  where β is called the fringe-width, λ is the wavelength of light used, d is the distance between the two coherent sources, and D is the distance between the sources and the screen.

- When a thin transparent plate of thickness t and refractive index μ is introduced in the path of one of the constituent interfering beams of light, the entire fringe system is displaced through a distance $\frac{D}{d}(\mu - 1)t$.
- Just as in Young's double slit experiment, the wavelength of light can be determined from measurement of fringe-width produced by the biprism by the following relation:

$$\lambda = \frac{\beta d}{D}$$

where $d = \sqrt{d_1 d_2}$ and $D = b_1 + b_2$.

d_1 is the distance between the two images, when the lens is at the position L_1 at a distance b_1 from the eyepiece. d_2 and b_2 are the corresponding distances when the Iens is at L_2.

- Some other devices for producing coherent sources are : Fresnel's two mirror arrangement and Lloyd's mirror.
- Lloyd's mirror produces achromatic fringes.

5.7 TERMINAL QUESTIONS

1) Young's experiments is performed with light of the green mercury line. If the fringes are measured with a micrometer eye-piece 80 cm behind the double slit, it is found that 20 of them occupy a distance of 10.92 mm. Find the distance between two slits. Given that the wavelength of green mercury line is 5460 Å.

2) In a certain Young's experiment, the slits are 0.2 mm apart. An interference pattern is observed on a screen 0.5m away. The wavelength of light is 5000 Å. Calculate the distance between the central maxima and the third minima on the screen.

3) A Lloyd's mirror, of length 5 cm, is illuminated with monochromatic light (λ = 5460 Å) from a narrow slit 0.1 cm from its plane, and 5 cm, measured in that plane, from its near edge. Find the separation of the fringes at a distance of 120 cm from the slit, and the total width of the pattern observed.

5.8 SOLUTIONS AND ANSWERS

SAQs

1) The distance OM is given by a cos 8. Hence the equation is x = a cos 8 or $x = a \cos \omega t$.

2) $y = a \sin \omega t = a \sin \frac{2\pi}{T} t$

If we replace π by 180°, and put a = 6 cm = 0.06 m, and T = 3 s, we get

$$y = (0.06) \sin \frac{2 \times 180°}{3} t$$

a) Thus displacement after 0.5 sec is,

$$y = 0.06 \sin \frac{2 \times 180°}{3} \times 0.5$$
$$= 0.06 \sin 60''$$
$$= 0.052 \text{ m}$$

b) Velocity, $v = a\omega \cos \omega t$

$$= a \frac{2\pi}{T} \cos \frac{2\pi}{T} t$$
$$= 0.06 \times \frac{2\pi}{3} \cos \frac{2 \times 180''}{3} \times 0.5$$
$$= 0.06 \times \frac{2\pi}{3} \times \cos 60°$$
$$= 0.063 \text{ ms}^{-1}.$$

c) Acceleration. $= \omega^2 y = \left(\frac{2\pi}{T}\right)^2 a \sin \omega t$

$$= \left(\frac{2\pi}{T}\right)^2 \text{ x } 0.06 \text{ x } \sin \frac{2\pi}{T} t$$

$$= \left(\frac{2\pi}{3}\right)^2 \times 0.06 \times \sin \frac{2 \times 180^\circ}{3} \times 0.5$$

$$= 0.228 \text{ ms}^{-2}.$$

We have

$$\frac{I_{max}}{I_{min}} = \frac{I_1 + I_2 + 2\sqrt{I_1 I_2}}{I_1 + I_2 - 2\sqrt{I_1 I_2}} = \frac{\left(\sqrt{I_1} + \sqrt{I_2}\right)^2}{\left(\sqrt{I_1} - \sqrt{I_2}\right)^2}$$

Now $\frac{I_1}{I_2} = \frac{81}{1}$ or $\frac{\sqrt{I_1}}{\sqrt{I_2}} = \frac{9}{1}$ or $\sqrt{I_1} = 9\sqrt{I_2}$

Hence $I_{max}/I_{min} = \frac{\left(9\sqrt{I_2} + \sqrt{I_2}\right)^2}{\left(9 - \sqrt{I_2}\right)^2} = \frac{(10)^2 I_2}{(8)^2 I_2} = \frac{100}{64} = \frac{25}{16}$

The phase difference is related to the path difference by Eq. (5.13) as follows :

$$\text{phase difference} = \frac{2\pi}{a} (AC - BC)$$

$$= \frac{2\pi}{\lambda} (4 - 3)\ \lambda$$

$$= 2\pi$$

This is the condition of maximum intensity. So the waves interfere, constructively, in Fig. 5.7(a).

In case of Fig. 5.7(b)

$$\text{phase difference} = \frac{2n}{a} (AC - BC)$$

$$= \frac{2\pi}{\lambda} (3 - 2.5)\ \lambda$$

$$= \pi$$

This is the condition of minimum intensity.

Here the waves are completely out of phase and destructive interference occurs.

Given : d = 0.40 mm, D = 10^3 mm, y = 3.6 mm, and m = 3. Using Eq. (5.21), we get

$$\lambda = \frac{yd}{mD} = \frac{(3.6)\ (0.40)}{3 \times 10^3} = 4.8 \times 10^{-4} \text{ mm} = 4.8 \times 10^{-5} \text{ cm}$$

Hence, the light is in the blue-green region of the visible spectrum.

With λ = 6000 Å, the distance between zero-order and tenth order fringe is 14.73 mm − 12.34 mm = 2.39mm, so that the fringe width is 2.39 mm/10 = 0.239 mm.

$\beta = \frac{D\lambda}{d}$ · Therefore

$$\frac{(\beta)_{6000}}{(\beta)_{5000}} = \frac{6000 \text{ Å}}{5000 \text{ Å}} = \frac{6}{5}$$

$$\therefore \quad (\beta)_{5000} = \frac{5}{6} \times (\beta)_{6000} = \frac{5}{6} \times 0.239 = 0.199 \text{ mm}$$

Thus, with $\lambda = 5000$ Å, the zero-order fringe will still be at 12.34 mm, while the twentieth order fringe will be at

$$12.34 \text{ mm} + (0.199 \text{ mm} \times 20) = 16.32 \text{ mm}$$

8) For maxima, the path difference = $n\lambda$

or $\quad 30 \times 10^{-5} \text{ cm} = n\lambda$

$\therefore \quad \lambda = \frac{30 \times 10^{-5}}{n} \text{ cm}$

where n = 1, 2, 3, 4 ....

9) y_0 = 0.2 cm; d = 0.1 cm; D = 50 cm

Hence $\quad t = \frac{d\, y_0}{D(\mu - 1)} = \frac{0.1 \times 0.2}{50 \times 0.58}$

$= 6.7 \times 10^{-4}$ cm

10) The fringe-width is given by

$$\beta = \frac{D\lambda}{2d}, \text{ where } d = \sqrt{d_1 \times d_2}$$

Here d_1 = 4.05 mm = 0.405 cm and d_2 = 2.10 mm = 0.210 cm.

$\therefore \quad d = \sqrt{0.405 \times 0.210} = 0.292$ cm

Also D = 100 cm and λ = 5893 Å = 5893 x 10^{-8} cm,

$\therefore \quad \beta = \frac{100 \times 5893 \times 10^{-8}}{0.292} = 0.0202$ cm

11) By introducing a glass plate of thickness t in one of the interfering beams, t cm of air (μ = 1) are replaced by t cm of glass (μ = 1.5). t cm of glass are optically equivalent to μt or 1.5 t cm of air. The, increase in the length of the path = $\mu t - t$ = $0.5t$. This produces a shift of 2 in the interference bands

$\therefore \quad 0.5t = 2\lambda = 2 \times 5.45 \times 10^{-5}$

and $\quad t = \frac{2 \times 5.45 \times 10^{-5}}{0.5} = 21.8 \times 10^{-5}$ cm.

TQs

1) The fringe width β in Young's experiment is $\beta = \lambda D/d$

Since 20 fringes occupy a distance of 10.92 mm, the fringe width β is

β = (10.92/20) mm = (10.92 x 10^{-3}/20) m

Also $\quad$ D = 80 cm = 0.8 m, and $\lambda = 5.460 \times 10^{-7}$ m

$\therefore \quad d = \frac{5.460 \times 10^{-7} \times 0.8 \times 20}{10.92 \times 10^{-3}} \text{ m} = 0.7912 \times 10^{-4}$ m

= 0.07912 mm

2) See Fig. (5.10). Suppose the required distance on the screen is y.

Here $\quad$ d = 2 x 10^{-4} m (slit separation)

λ = 5 x 10^{-7} m (wave length)

D = 5 x 10^{-1} m (distance between slit to screen)

The minima is observed when the phase difference between the two waves is an odd multiple of π, i.e., when

$\delta = \pi, 3\pi, 5\pi, 7\pi, \ldots\ldots\ldots$

At the third minimum, $\delta = 5\pi$.

From Eq. (5.13). path difference $= \frac{2\pi}{\lambda}\delta = \frac{2\pi}{\lambda}(5\pi)$

But from Fig. 5.10, the path difference between the waves arriving at P is d sin θ.

Hence $5\pi = \frac{2\pi}{a}(d \sin\theta)$

or $\sin\theta = \frac{\lambda}{2\pi}\frac{1}{d}(5\pi) = \frac{5\pi \times 5 \times 10^{-7}}{2\pi \times 2 \times 10^{-4}}$

$= 6.25 \times 10^{-3}$

From Fig. 5.10. the required distance on the screen y = D tan θ

$= D \sin\theta = 5 \times 10^{-1} \times 6.25 \times 10^{-3}$ $\because \tan\theta \approx \sin\theta$

$= 3.1$ m

3) Let MM', (Fig. 5.21) the Lloyd's mirror be 5 cm long. The source S_1 is as shown in the figure. The interference pattern is observed in the region AB.

The fringe width β is given by $\beta = \lambda D/d$

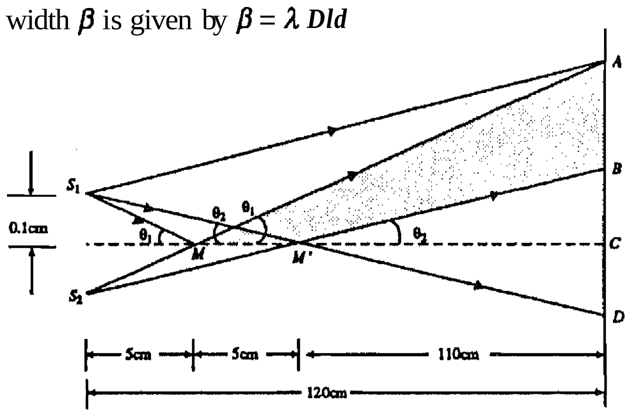


Fig. 5.21.

Given $\lambda = 5460$ Å $= 5.460 \times 10^{-7}$ m; D = 120 cm = 1.20 m and

d = 0.2 cm $= 2 \times 10^{-2}$ m

$\therefore \quad \beta = \frac{5.460 \times 10^{-7} \times 1.20}{2 \times 10^{-3}}$ m $= 3.276 \times 10^{-4}$ m

$= 0.3276$ mm.

The total width of interference pattern is obviously AB. From Fig. (5.21),

$\tan\theta_1 = 0.1/5$, and $\tan\theta_2 = 0.1/10$

Also from rt angled ΔAMC

$AC/MC = \tan\theta_1$ or $AC = 115 \times \tan\theta_1$

$= 115 \times 0.1/5 = 2.3$ cm

From rt angled $\Delta BM'C$

$BC/M'C = \tan\theta_2 = \tan\theta_2$ or $BC = 110 \times (0.1/10) = 1$ cm

$\therefore \quad AB = AC - BC = 2.3 - 1.1 = 1.2$ cm $= 1.2 \times 10^{-3}$ cm