UNIT 4 POLARISATION OF LIGHT

Structure

4.1 INTRODUCTION

In Unit 1 of this block, you learnt that light is a transverse electromagnetic wave. In your school physics curriculum you have learnt that while every wave exhibits interference and diffraction, polarisation is peculiar only to transverse waves. You may even be familiar with basics of polarisation like: What distinguishes the polarised light from unpolarised light? Is light from an ordinary (or natural) source polarised? How do we get polarised light? and so on. In this unit we propose to build upon your this preliminary knowledge.

You must have seen people using antiglare goggles as also antiglare windshields for their cars. Do you know that polarisation of light has something to do with these? Polarisation of light also plays a vital role in designing sky light filters for cameras and numerous optical instruments, including the polarising microscope and polarimeter. You may get opportunity to handle some of these devices if you opt for physics laboratory courses PHE-08(L) and PHE-12(L).

In Sec. 4.2 we have discussed as to what is polarisation. In Sec. 4.3, you will learn about simple states of polariscd light. Sec. 4.4 is devoted to ideal polarisers and Malus' law. In this section you will also learn about double refraction or optical birefringence - a property of materials helpful in producing polarised light. In Sec. 4.5, you will learn about some techniques of producing circularly and elliptically polarised light.

Objectives

After going through this unit you should be able to

- explain what is linearly, circularly or elliptically polarised state of light
- describe how can light be polarised by reflection
- solve simple problems based on Malus' law and Brewster's law
- explain how optical birefringence helps in production of polarised light, and
- explain the production of linearly polarised light by dichroism.

4.2 WHAT IS POLARISATION?

What is polarisation? Why light, not sound, waves are known to polarise? These are some of the basic questions to which we must address ourselves. Polarisation is related to the orientation (oscillations) of associated fields (particles). Refer to Fig. 4.1 which depicts a mechanical wave (travelling along a string). From Fig. 4.1(a) you will note that the string vibrates only in the vertical plane. And vibrations of medium particles are confined to just one single plane. Such a wave is said to be (plane) polarised. How would you classify waves shown in Fig. 4.1(b) and (c)? The wave shown in Fig. 4.1(b) is plane polarised since vibrations are confined to the horizontal plane. But the wave in Fig. 4.1(c) is unpolarised because simultaneous vibrations in more than one plane are present. However, it can be polarised by placing a slit in its path as in Fig. 4.1(d). When the first slit is oriented vertically, horizontal vibrations are cut off. This means that only vertical vibrations are allowed to pass so that the wave is linearly polarised. What happens when a horizontal slit is placed beyond the vertical slit in the path of propagation of the wave? Horizontal as well as vertical components (of the incident wave) will be blocked. And the wave amplitude will reduce to zero.

Let us now consider visible light. The light from a source (bulb) is made to pass through a polaroid (P), which is just like slit one in Fig. 4.1. The intensity of light is seen to come down to about 50%. Rotating P in its own plane introduces no

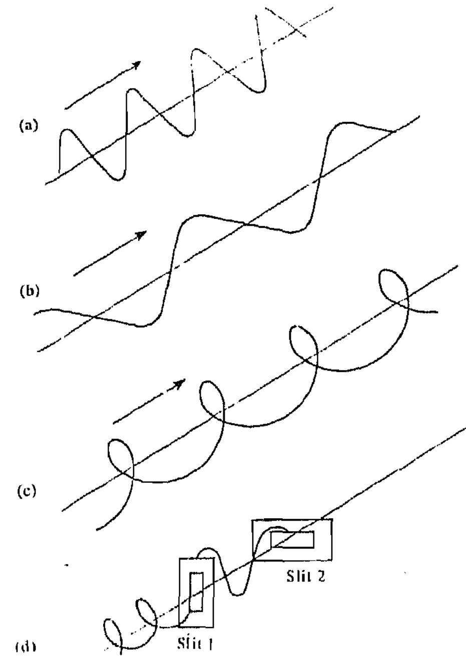


Fig. 4.1: (a) A vertically plane polarised wave on a string (b) A horizontally plane polarised wave (c) an unpolarised wave. (d) The wave in (c) becomes plane polarised after passing through slit one; the wave amplitude reduces to zero if another slit oriented perpendicular to slit one is introduced.

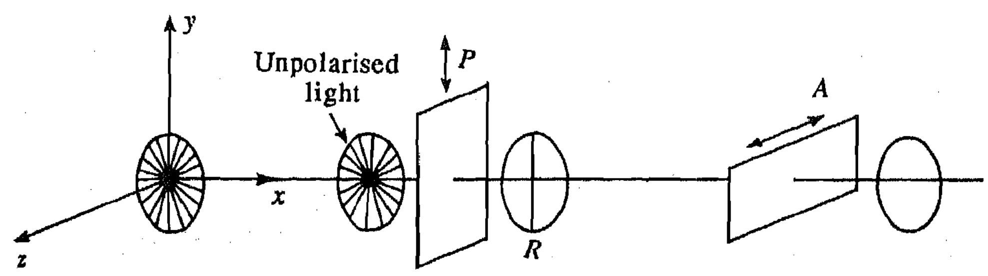


Fig. 4.2: Schematics of the apparatus for observing polarisation of light

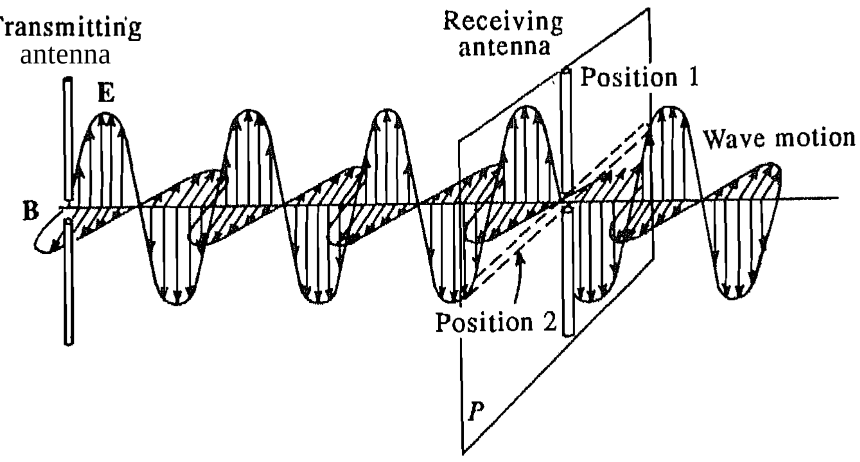
further change in light intensity. Now if a second identical polaroid (A) is introduced in the path of light so that it is parallal to P, the intensity of light from the bulb remains unaffected. But rotating A in its own plane has a dramatic effect! For 90° rotation, the light is nearly cut-off.

You can analyse this result in terms of electromagnetic theory, which demands complete description of associated electric vector and the way it oscillates with respect to the direction of propagation. For the arrangement shown in Fig. 4.2, the electric vector at the source has all orientations in the yz plane. The wave propagates as such till it reaches the polaroid P, which allows essentially unhindered passage of electric vector oriented parallel to its transmission axis. If the transmission axis is along y-axis, the electric field along y-direction (E_y) passes through it unaffected. In addition, the y-components of electric field vectors inclined to y-axis can also pass through P. Thus, after passing through the polaroid P, the electric vectors oriented only along y-axis will be present. **When electric vector oscillates along a straight line in a plane perpendicular to the direction of propagation, the light is said to be plane polarised.** The plane polarised wave further travels to the polaroid A, which is identical to P. When A is at 90° with respect to P, it can allow only the z-components of E to pass. Since only y-components of E are present in the wave incident on A, no light is transmitted by A.

We may now conclude that

1. No polarisation of longitudinal waves occurs as the vibrations are along the line of transmission only.
2. The transverse nature of light is responsible for their polarisation.

An important menifestation of this result arises in TV reception. You may have seen that the TV antenna on your roof-tops arc fixed in horizontal position. Have you ever thought about it.? This is because the TV **signal transmission in our country** is through horizontally oriented transmitting antenna. The explanation for this lies in the observation that the pick up by the receiving antenna is maximum when it is oriented parallel to the transmitting antenna. This is illustrated in Fig. **4.3** for a vertical (dipole) transmitting antenna.



Pig. 4.3: Polarisation of an electromagnetic wave. The antenna responds to the vertical electric field strength of the wave. Reception is maximum in Position 1 and minimum in Position 2.

You may now like to know: Do natural (or ordinary) light sources emit polariscd light? Answer to this question is 'yes' as well as 'no'! Is this answer not funny? You know that emission of light involves a large number of randomly oriented atomic (or molecular) emitters. Every individual excited atom radiates polarised waves lor about 10^{-8} s. These waves form a resultant wave of given polarisation which persists for the lil'etime of the cxcitcd atom. At the same timc, other atoms (molecules) also emit waves, whose resultant states of polarisation may be quite different. Because of this randomness, every orientation of electric vector in space is equally probable. That is, electric vectors associated with light waves from a source are oriented in all directions in space and thus there is a completely unpredictable change in the overall polarisation. Moreover, due to such rapid changes, individual resultant polarisation slates become almost indiscernible. The light is then said to be **unpolarised**.

In practice, visible light does not correspond to either of these extremes. The oscillations of electric field vectors arc neither completely regular nor completely irregular. That is, light from any source is partially polarised. We ascribe a degree of polarisation to partially polarised light. The degree of polarisation is one for completely polarised light and zero for unpolarised light.

The next logical step perhaps would be to know various types of polarised light. Let us learn about this aspect now.

4.3 SIMPLE STATES OF POLARISED LIGHT

In a right handed coordinate system if a right handed screw is turned so that it rotates the x-axis towards the y-axis, the direction of advance of the screw represents the positive z-axis.

You now know that in e.m. theory, light propagation is depicted as evolution of electric field vector in a plane perpendicular to the direction of transmission. For unpolarised light, spatial variation of electric field at any given time is more or less irregular. For plane polarised light, the tip of electric vector oscillates up and down in a straight line in the same plane. The space variation of E for linearly polarised wave is shown in Fig. 4.4 (a). The diagram on the left shows the path followed by the tip of the electric vector as time passes. You will know that the tip of E cxccutes one lull cycle as one full wave length passes through a reference plane. There arc two other states of polarisation: circular polarisation and elliptical polarisation. The path followed by the tip of E, as the time passes, for these is shown in Fig. 4.4 (b) and (c), respectively.

The yz-plane (or x = 0 plane) in Fig. 4.4 is the plane of polarisation of the weve. We can identify other states of polarisation by **looking at the trajectories of the tip of the electric field vector** as the wave passes through the reference plane. You should always look at the reference plane from the side away from the source (looking back at the source) for the definitions to be unique.

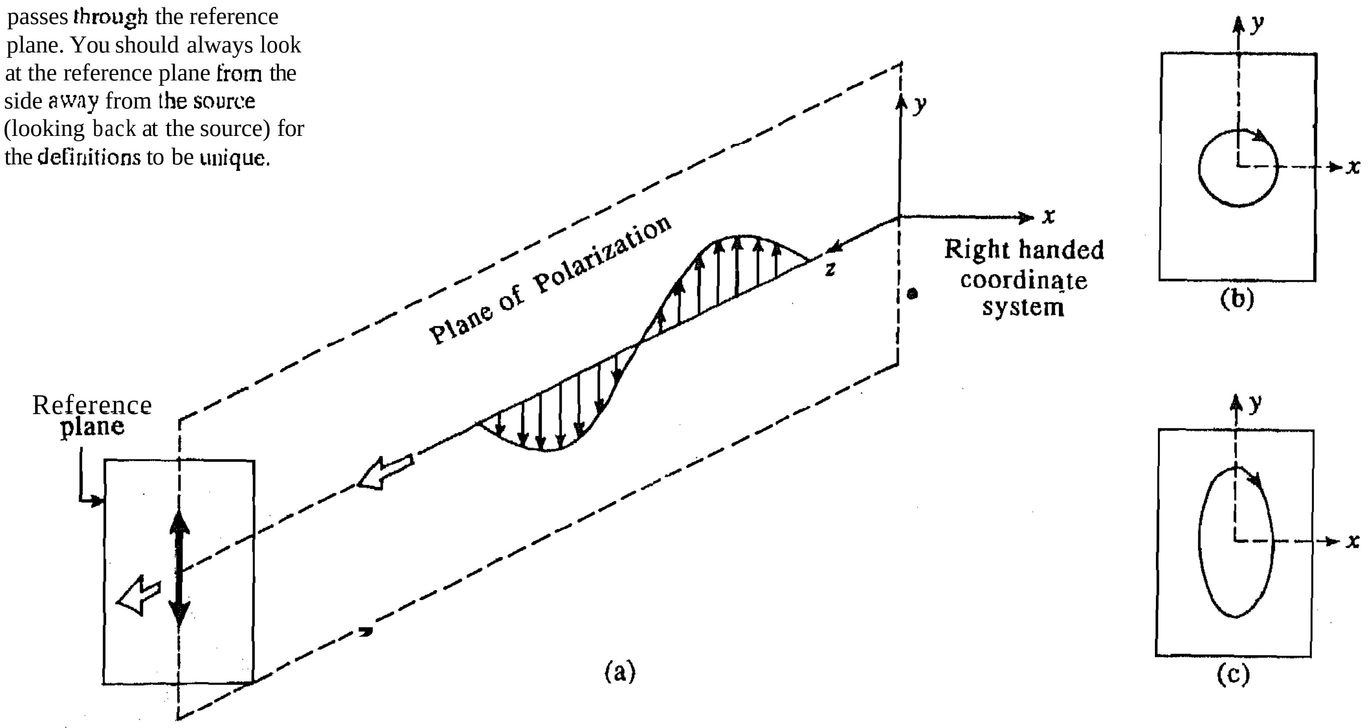


Fig. 4.4: Spatial variation of electric field vector for (a) linearly polarised light. The diagram on the left show the path taken by the tip of the electric vector as time veries. (b) and (c) show the path taken by the tip of the electric vector for circularly and elliptically polarised light.

Let us now mathematically analyse how superposition of two plane polarised light waves of same frequency moving in the same direction gives rise to linearly, circularly or elliptically polarised light.

4.3.1 Linear Polarisation

Suppose that two light waves are moving along the z-direction. Let their electric field vectors be mutually perpendicular, i.e. we choose these along the x and y axes and can represent them respectively in the form

$$\mathbf{E}_1(z,t) = \hat{e}_x E_{01}\cos(kz-\omega t) \tag{4.1}$$

$\mathbf{E}_2(z,t)$ lags $\mathbf{E}_1(z,t)$ for $\varphi > 0$ and vice-versa.

and

$$\mathbf{E}_2(z,t) = \hat{e}_y E_{02}\cos(kz-\omega t+\phi) \tag{4.2}$$

Here $\hat{\mathbf{e}}_x$ and $\hat{\mathbf{e}}_y$ are unit vectors along the x and y-axes respectively. (These are also called polarisation vectors.) ϕ is the phase difference between the two waves.

We expect that the nature of the resultant wave will be determined by the phase difference between them and the value of the ratio E_{02}/E_{01}. Mathematically, we can write the vector sum of these as

$$\mathbf{E}(z,t) = \mathbf{E}_1(z,t) + \mathbf{E}_2(z,t)$$

$$= \hat{\mathbf{e}}_x E_{01}\cos(kz-\omega t) + \hat{\mathbf{e}}_y E_{02}\cos(kz-\omega t+\phi) \tag{4.3}$$

Resultant Amplitude

$$= \sqrt{(E_0)^2+(E_0)^2}$$
$$= \sqrt{2}\,E_0$$

and

$$\tan\theta = \frac{E_{02}}{E_{01}} = \frac{E_0}{E_0} = 1$$
$$= \tan 45^\circ$$

or

$$\theta = 45^\circ$$

Let us first take the simplest case where ϕ is zero or an integral multiple of $\pm 2\pi$. That is, when in - phase waves are superposed, Eq. (4.3) takes the form

$$\mathbf{E}(z,t) = (\hat{\mathbf{e}}_x E_{01} + \hat{\mathbf{e}}_y E_{02})\cos(kz-\omega t) \tag{4.4}$$

The amplitude $\sqrt{E_{01}^2+E_{02}^2}$ and the electric field oscillations in the reference frame make an angle $\theta = \tan^{-1}(E_{02}/E_{01})$ with the x-axis.

For the special case of in-phase waves of equal amplitude ($E_{01} = E_{02} = E_0$), the resultant wave has amplitude equal to $\sqrt{2}\,E_0$ and the associated electric vector is oriented at 45° with the x- axis. So we may conclude that when two in-phase linearly polarised light waves are superposed, the resultant wuve has fixed

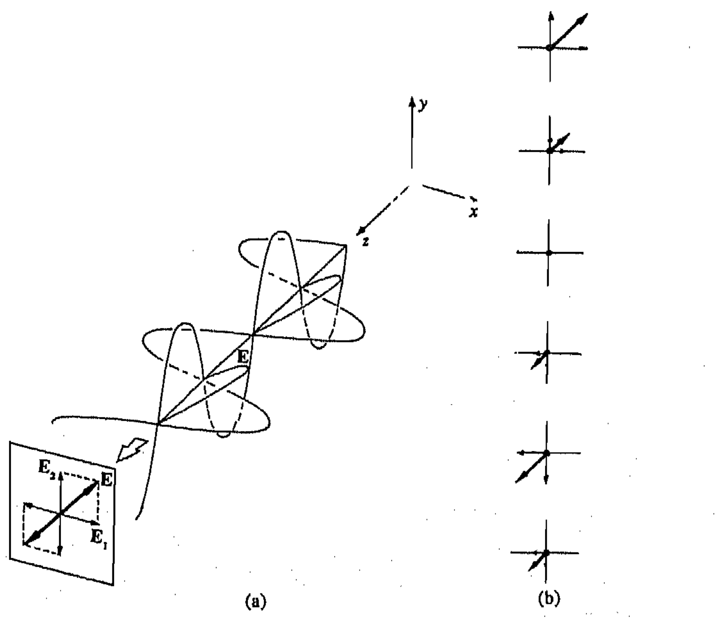


Fig. 4.5: Schematic representation of a plane polarised light wave

orientation as well as amplitude. That is, it too is linearly polarised, as depicted in Fig. 4.5 (a). In the plane of observation, you will see a single resultant **E** oscillating cosinusoidally in time along an inclined line (Fig. 4.5 (b)). The E - field progresses through one complete cycle as the wave advances along the z- axis through one wavelength.

If we reverse this process, we can say that **any linearly polarised light can be visualised as a combination of two linearly polarised lights with planes of polarisation parallel to $x = 0$ and $y = 0$ planes.** (This is similar to resolving a vector in a plane along two mutually perpendicular direclions.) In the subsequent sections, you will use this result frequently.

If the phase difference between lwo plane polarised light waves is an odd integral multiple of $\pm\ \pi$, the resultant wave will again be linearly polarised:

$$\mathbf{E}(z,t) = (\hat{\mathbf{e}}_x E_{01} - \hat{\mathbf{e}}_y E_{02})\cos(kz - \mathrm{w}t) \tag{4.5}$$

What is the orientation of the resultant electric vector in the reference plane'? To know the answer of this question, work-out the following SAQ.

SAQ **1**

Spend 5 min

Depict the orientation of electric vector defined by Eq. (4.5) in the reference (observation) plane.

4.3.2 Circular Polarisation

$\cos(-\theta) = \cos\theta$

$\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$

We now investigate the nature of the resultant wave arising due to superposition of two plane polarised waves whose amplitudes are equal ($E_{01} = E_{02} = E_0$) but phases differ by $\pi/2$, i.e. their relative phase difference $\phi = 2n \cdot \frac{\pi}{2}$; $n = 0, \pm\ 1, \pm\ 2, \ldots$ For $n = 0$, we can rewrite Eqs. (4.1) and (4.2) as

$$\mathbf{E}_1(z,t) = \hat{\mathbf{e}}_x E_0 \cos(kz - \omega t) \tag{4.6a}$$

$$\mathbf{E}_2(z,t) = \hat{\mathbf{e}}_y E_0 \sin(kz - \omega t) \tag{4.6b}$$

The resultant wave is given by

$$\mathbf{E}(z,t) = E_0 [\hat{\mathbf{e}}_x \cos(kz - \omega t) + \hat{\mathbf{e}}_y \sin(kz - \omega t)] \tag{4.7}$$

You may note that the scalar amplitude of' E is constant ($= E_0$) bul its orientation varies with time. To determine the trajectory along which the lip of **E** moves, we can readily combine Eqs.(4.6a) and (4.6b) to yield

$$\left(\frac{E_1}{E_0}\right)^2 + \left(\frac{E_2}{E_0}\right)^2 = 1 \tag{4.8}$$

which is the equation of' a circle. That is, the orientation of resultant electric vector changes continuously and its tip moves along a circlc as the wave propagates (time passes). This means that E is not restricted to a single plane. The question now arises: What is the direction of rotation? Obviously thcrc arc two possibilities: Clockwise and counterclockwise. To know which of these is relevant here, you should tabulate E at different space points at a given time, $t = 0$ say:

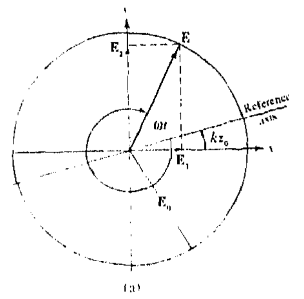
Location in space	$z = 0$	$z = \frac{h}{8}$	$z = \frac{\lambda}{4}$	$z = \frac{3h}{8}$	$z = \frac{h}{2}$	$z = \frac{Sh}{8}$	$z = \frac{3h}{4}$	$z = \frac{7h}{8}$	$z = \lambda$
Electric field	$\hat{e}_x E_0$	$\frac{\hat{e}_x + \hat{e}_y}{\sqrt{2}} E_0$	$\hat{e}_y E_0$	$\frac{\hat{e}_y - \hat{e}_x}{\sqrt{2}} E_0$	$-\hat{e}_x E_0$	$-\frac{\hat{e}_x + \hat{e}_y}{\sqrt{2}} E_0$	$-\hat{e}_y E_0$	$\frac{\hat{e}_x - \hat{e}_y}{2} E_0$	$\hat{e}_x E_0$

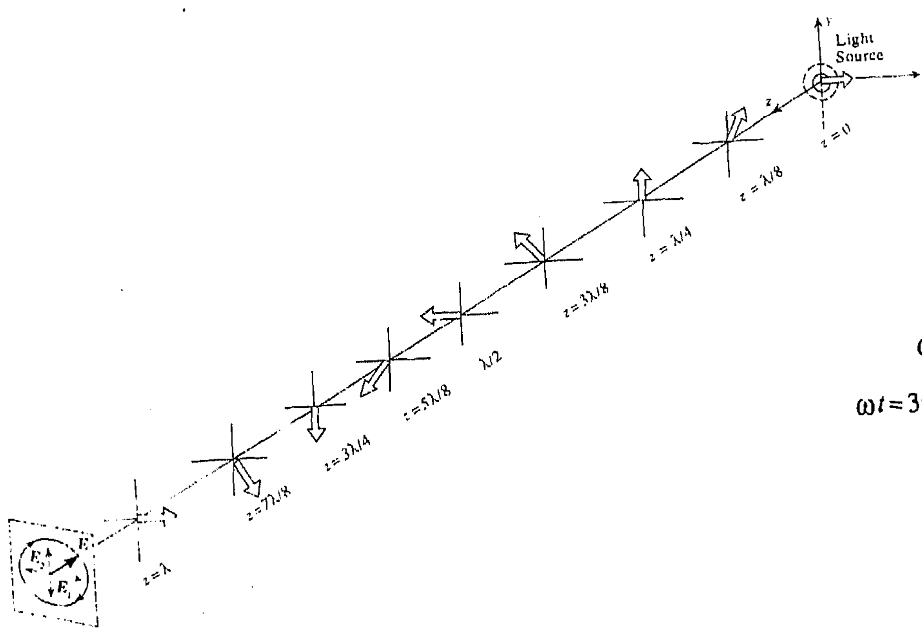
These are depicted in Fig. 4.6. If you position yourself in the reference plane and observe the evolution of **E** from $z = \lambda$ to $z = 0$ (bachward towards source), you will find that the tip of E rotates clockwise. Such a light wave is said to be **right circular wave.** The electric field makes one complete rotation as the wave advances through one wavelength.

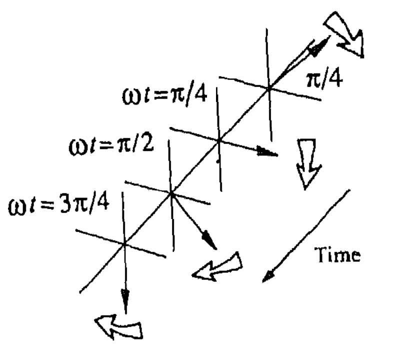
Alterntively, we may fix an arbitrary point $z = z_0$ and nbserve evolution of E as Lime passes. The figure below depicts what is happening at some arbitrary point z_0 on the axis.

In case the phase difference $\phi = 2n + \frac{\pi}{2}$ with $n = 0, \pm 1, \pm 2, ...$, Eq. (4.7) is modified to

$$E(z,t) = E_0 \left[\hat{e}_x \cos(kz - \omega t) - \hat{e}_y \sin(kz - \omega t) \right] \tag{4.9}$$







(b)

Fig. 4.6: Rotation of the electic vector in a right-cirular wave. For consistency, we have used a right handed system.

It shows that the **E**-vector rotates counter-clockwise in the reference frame. (Before proceeding further you should convince yourself by tabulating the values of **E** at $t = 0$ for different space point.) such a wave is referred to as **left-circular wave.**

Can you now guess as to what will happen if two oppositely polnrised circular waves of equal amplitude are superposed'? Mathematically, you should add Eqs. (4.8) and (4.9). Then you will find that

$$E = 2E_0 \hat{e}_x \cos(kz - wt) \tag{4.10}$$

This equation is similar to Eq.(4.1) which represents a linearly polarised light wave. Thus, we may concluclc that **superposition of two oppositely polarised circular waves (of same amplitude) results in a linearly or plane polarised light wave.**

4.3.3 Elliptical Polarisation

Let us now consider the most general case where two orthogonal linearly polarised light waves of unequal amplitudes and having an arbitrary phase difference ϕ are superposed. Physically we expect that beside its rotation, even the magnitude of resultant electric field vector will change. This means that the tip of E should trace out an ellipse in the reference plane as the wave propagates. To analyse this mathematically, we write the scalar part of Eq.(4.2) in expanded form:

$$\frac{E_2}{E_{02}} = \cos(kz - \omega t)\cos\phi - \sin(kz - \omega t)\sin\phi$$

On combining it with Eq. (4.1) we find that

$$\frac{E_2}{E_{02}} = \frac{E_1}{E_{01}}\cos\phi - \sin(kz - \omega t)\sin\phi$$

or

$$\frac{E_2}{E_{02}} - \frac{E_1}{E_{01}}\cos\phi = -\sin(kz - \omega t)\sin\phi \qquad (4.11)$$

It follows from Eq.(4.1) that

$$\sin(kz - \alpha t) = \left[1 - (E_1/E_{01})^2\right]^{\frac{1}{2}}$$

so that Eq. (4.11) can be rewritten as

$$\frac{E_2}{E_{02}} - \frac{E_1}{E_{01}}\cos\phi = -\left[I - (E_1/E_{01})^2\right]^{\frac{1}{2}} \text{sine}$$

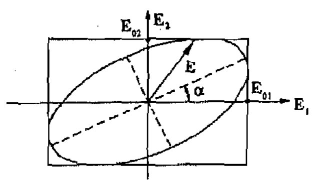


Fig 4.7: Schematics of elliptically polarised light

On squaring both sides and re-arranging terms, we have

$$\left(\frac{E_2}{E_{02}}\right)^2 + \left(\frac{E_1}{E_{01}}\right)^2 - 2\left(\frac{E_2}{E_{02}}\right)\left(\frac{E_1}{E_{01}}\right)\cos\phi = \sin^2\phi \qquad (4.12)$$

Do you recognise this equation? It defines an ellipse whose principal axis is inclined with the (E_1, E_2) coordinate system (Fig. 4.7). The angle of inclination, say α, is given by

$$\tan 2\alpha = \frac{2E_{01}E_{02}\cos\phi}{E_{01}^2 - E_{02}^2} \qquad (4.13)$$

For $\alpha = 0$ or equivalently $\phi = \pm\pi/2, \pm 3\pi/2, \ldots$, Eq. (4.12) reduces to

$$\left(\frac{E_1}{E_{01}}\right)^2 + \left(\frac{E_2}{E_{02}}\right)^2 = 1 \qquad (4.14)$$

which defines an ellipse whose principal axes are aligned with the coordinate axes. We would now like you to solve an SAQ.

SAQ 2

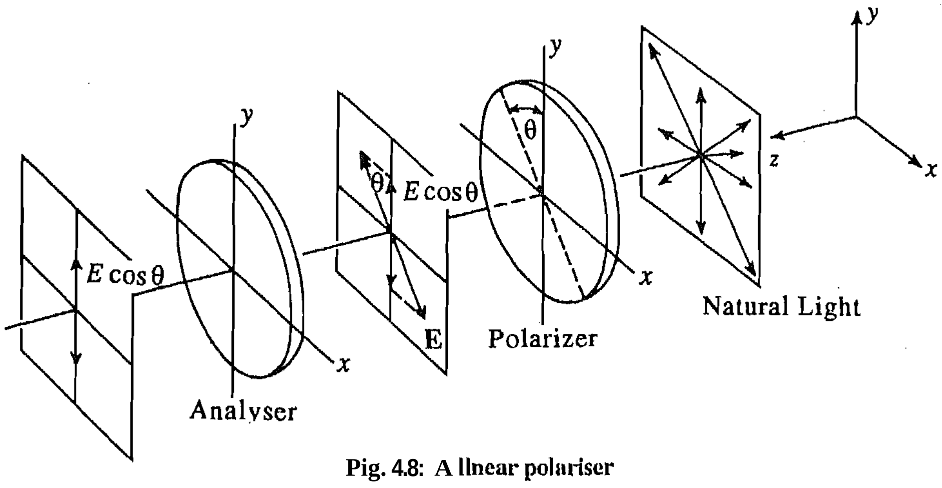
Spend 5 mitt

Starting from Eq. (4.12) show that linear and circular polarisation states are special cases of elliptical polarisation.

Now that you understand what polarised light is, the next logical step is to know techniques used to get polarised light. You will learn some of these now.

4.4 PRINCIPLES OF PRODUCING LINEARLY POLARISED LIGHT

The most important optical device in any polarised light producing arrangement is a polariser. It changes (input) natural light to some form of polarised light (output). Polarisers are available in several configurations. An ideal polariser is one which reduces the intensity of an incident unpolarised light beam by exactly 50 percent. When unpolarised light is incident on an ideal polariser, the outgoing light is in a definite polarisation state (P-state) with an orientation parallel to the transmission axis of the polariser. That is, the polariser somehow discards all except one particular polarisation state. How do we determine whether or not a device is a linear polariser? The law which provides us necessary tool is Malus' law. Let us learn about it now.



Pig. 4.8: A llnear polariser

4.4.1 Ideal Polnriser: Malus' Law

Refer to Fig. 4.8. Unpolarised light is incident on an ideal polariser, whose transmission axis makes an angle 9 with y-axis. For this arrangement, only a P-state parallel to the transmission axis of the polariscr will be transmitted, This light is incident on an identical ideal polariser, called analyser, whose transmission axis is vertical. Suppose that there is no absorption of light. **Then,** if **E** is the electric field transmitted by the polariser, only its component Ecos θ parallel to the transmission axis of the analyser would pass through. The intensity of the polarised light reaching the detector is given by

$$I(\theta) = I(0)\cos^2\theta \qquad (4.15)$$

where θ is the angle between the transmission axes of the polariser and the analyser. The maximum intensity I(0) occurs when the transmission axis of the polariser and the analyser are parallel.

Eq. (4.15) constitutes what is known as **Malus' law.** To use it to check whether an optical device is an ideal linear polariser or not, you may like to solve an SAQ.

SAQ 3.

Unpolarised light falls on two polarising sheets placed one over another. What must be the angle between their transmission axes if the intensity of light transmitted finally is one-third the intensity of the incident light'? Assume that each polarising sheet acts as an ideal polariser.

Spend 5 min

So far we have confined to a linear ideal polariser. Polarisers are available in several configurations. (We can have circular or elliptical polarisers as well.) They are based on one of the following physical mechanisms: reflection, birefringence or double refraction, scattering and dichroism or selective absorption. You will now learn about some of these in detail.

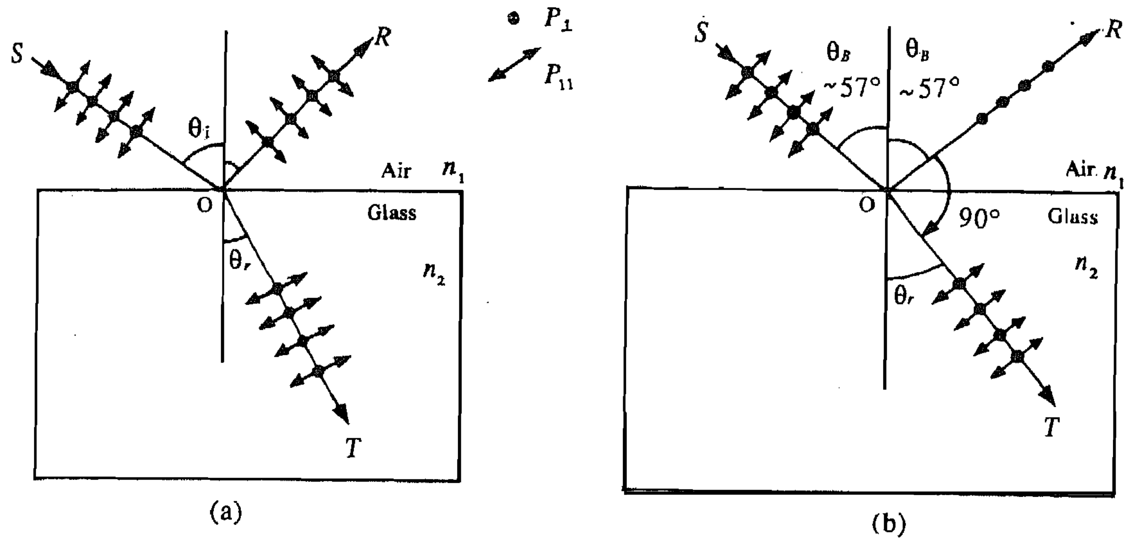
4.4.2 Polarisation by Reflection: Brewster's Law

Reflection of light from a dielectric like plastic or glass is one of the most common methods of obtaining polarised light. You may have noticed the glare across a window pane or the sheen on the surface of a billiard ball or book jacket. It is due to reflection at the surface and the light is partially polarised. To understand its theoretical basis we will consider laboratory situations.

This effect was studied by Malus. One evening he was examining a calcite crystal while standing at the window of his house. The image of the Sun was reflected towards him from the windows of Luxembourg Palace. When he looked at the image through the calcite crystel, he was amused at disappearance of one of the double images as he rotated the crystal.

Suppose that an unpolarised light wave is incident on an interface between two different media at an angle θ_i as shown in Fig. 4.9.

The reflection coefficients when the electric vector of the incident wave is perpendicular to the plane of incidence or when it lies in the plane of incidence are



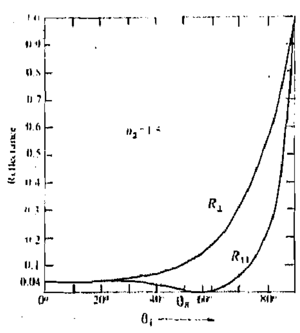
Pig. 4.9: (a) Polnrisation by reflection: the unpolarised light beam has been represented as ←•→ which indicate two electric field vibrations. '•' indicates electric field vibration perpendicular to the page ($P_\perp$) and '↔' indicates electric field vibration In the plane of the paper ($P_{||}$). (b) At Brewsfer's angle, the reflected light is plane polarised.

given by Fresnel's equations (Eqs. (2.21a) and (2.21c)):

$$R_{||} = \frac{\tan^2(\theta_i - \theta_r)}{\tan^2(\theta_i + \theta_r)} \qquad (14.16a)$$

and

$$R_{\perp} = \frac{\sin^2(\theta_i - \theta_r)}{\sin^2(\theta_i + \theta_r)} \qquad (14.16b)$$



Pig. 4.10: Variation of reflectance wlth angle of incidence

where 8, is the angle of refraction. These equations show that whereas $R_\perp$ can never be zero, $R_{||}$ will become zero when 8; + θ_r = $\frac{\pi}{2}$. (The case θ_i = 8, is trivial as it implies continuity of optically identical media.) That is, there will be no reflected light beam with E parallel to the plane of incidence. The angle of incidence for which light is completely transmitted is called **Brewster's angle**. Let us denote it by θ_B. A plot of $R_\perp$ and $R_{||}$ versus θ_B is shown in Fig. 4.10 for the particular case of air-glass interface.

We can represent an incoming unpolarised light as made up of two orthogonal, equal amplitude P-states with electric field vector parallel and perpendicular to the plane of incidence. Therefore, when the unpolarised wave is incident on an interface and the angle of incidence is equal to Brewster's angle, the reflected wave will be linearly polarised with E normal to the incidece plane. This provides us with one of the most convenient methods for production of polarised light. To elaborate, we recall from Snell's law that

$$n_i \sin \theta_B = n_2 \sin 0,$$

where n_1 and n_2 are the refractive indices of the media at whose interface light undergoes reflection. Since $0, = \frac{\pi}{2} - \theta_B$, it readily follows thal

$$n_1 \sin \theta_B = n_2 \cos \theta_B$$

or

$$\tan \theta_B = \frac{n_2}{n_1} \qquad (14.17)$$

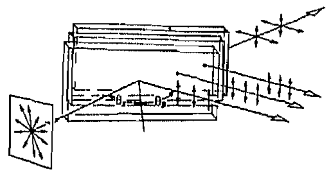
That is, the tangent of Brewster angle is equal to the ratio of the refractive indices of the media at whose interface incident light is reflected. When the incident beam is in air ($n_1 = 1$) and the transmitting medium is glass ($n_2 = 1.5$), the Brewster angle is nearly 56°. Similarly, θ_B for air-water interface, like surface of a pond or a lake is 53°. This means that when the sun is 37° above the horizontal, the light reflected by a calm pond or lake should be completely linearly polarised.

We, however, encounler some problems in utilizing this phenomenon to construct an effective polariscr on account of two reasons:

(i) The reflected beam , although completely polarised, is weak.

(ii) The transmitted beam, although slrong, is only partially polarised.

These sliortcomings are overcome using a **pile of plate** polarisers. You can fabricate such a device with glass plates for the visible, silver chloride plates for the infrared, and quartz for the ultraviolet region. It is an easy matter to construct a crude arrangement of this Sort with a dozen or so microscope slides (Fig. 4.11).The beautiful colours that appear when the slides are in contact is due to interference, which you will study in the next block.



Pig. 4.11: Polarisation of light by a pile of plates

You may now like to solve an SAQ.

SAQ 4

Spend 5 min

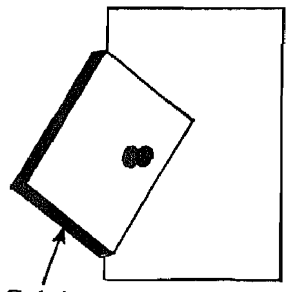
A plate of flint glass is immersed in water. Calculate the Brewster angles for internal as well as external reflection at an interface.

Having studied as to how reflection of light can be used lo produce polarised light, you may be tempted to know whether or not the phenomenon of refraction can also be used for the same? Refraction of light in isotropic crystals like NaCl or non-crystalline substances like glass, water or air docs not lead to polarisation of light. However, refraction in crystalline substances like calcite or cellophane is optically anisotropic becouse it leads to what is known as double refraction or birefringence. This is because anisotropic crystals display two distinct principal indices of refraction, which correspond to the E-oscillations parallel and perpendicular to the optic axis. Let us now learn how birefringence can be used to produce polarised light.

4.4.3 Polarisation by Double Refraction

Mark a **black** dot on a piece of paper and observe it through a glass plate. You will see only one dot. Now use a calcite crystal. You will be surprised at the remarkable observation: instead of one, **two grey dots** appear, as shown in Fig. 4.12. Further, rotation of the crystal will cause one of the dots to remain stationary while the other appears to move in a circle about it. Similarly, if you place a calcite crystal on your book, you will see two images of each letter. It is because the calcite crystal splits the incident light beam into two beams. This phenomenon of splitting of a light beam into two is known as double refraction or birefringence. Materials exhibiting this property are said to be birefringent. We bring you the excitement of Bartholinus, who discovered birefringentce, in his words:

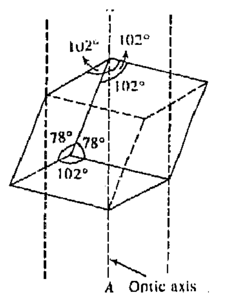
> Greatly prized by all men is the diamond, and many are the joys which similar treasures bring, such as precious stones and pearls ... but he, who, on the other hand, prefers the knowledge of unusual phenomena to these delights, he will, 1 hope, have no less joy in a new sort of body, namely, a transparent crystal, recently brought to us from Iceland, which perhaps is one of the greatest wonders that nature has produced. As my investigation of this crystal proceeded there showed itself a wonderful and extraordinary phenomenon: objects which are looked at through the crystal do not show, as in the case of other transparent bodies, a single refracted image, but they appear double.



Pig. 4.12: Double refraction of a light beam by calcite crystal.

In some of the text books, you may find that ordinary and extraordinary rays are being denoted by **bold** letters O and E. We have used small letters (o- and e-) to avoid confusion with the notation for the electric field.

Before we discuss polarisation of light by double refraction in detail, you should familiarise yourself with some of the concepts related to this phenomenon. The two refracted beams into which incident light splits have different angles of refraction. The distinguishing feature of these two refracted light beams is that one of these obeys the Snell's law. It is called the ordinary ray **(o-ray)** in accordance with the nomenclature given by Bartholinus. The other beam does not obey Snell's law abd is called the extraordinary ray (e-ray). That is, a birefringent crystal displays two distinct indices of refranction. Another important concept is that of optic axis, which signifies some special direction in a birefringent crystal along which two refractive indices are equal (i, e. both o-and e-rays traval in the same direction with the same velocity). When unpolarised light is incident perpendicular to these special directions, both the o-and th e-rays travel in the same direction with different velocities. You may now like to know: Does optic axis refer to any particular line through the crystal? The answer to this question is: It refers to a direction. This means that for any given point in the crystal, an optic axis may be drawn which will be parallel to that for any other point. For example, AA' and broken lines parallel to A A' show the optic axis for a calcite crystal as shown in Fig. **4.13.**



Pig. 4.13: A Calcite crystal. The line AA' shows the direction of the optic axis. For the calcite crystal, the direction of the optic axis Is determined by joining the two blunt corners of the crystal.

Birefringent crystals which posses only one optic axis are called **uniaxial** crystals. Similarly, crystals having two optic axes are called **biaxial** crystals. Calcite, quartz and ice are examples of uniaxial crystals and mica is a biaxial crystal. Most of the polarisation devices are made of uniaxial crystals. Further, the uniaxial crystal for which the refractive index o-ray (n_o) is more than the refractive index for the e-ray (n_e) is called **nigative** uniaxial crystal. On the other hand, if $n_e > n_o$, we have a positive uniaxial crystal. Values of n_o and n_e for some of the birefringent crystals are given in Table **4.1.** The difference $\Delta n = n_e - n_o$ is a measure of birefringence.

4.1: Refractive indices of some uniaxial birefringent crystals for light of wavelength 5893 Å

Crystal	n_o	n_e
Tourmaline	1.669	1.638
Calcite	1.6584	1.4864
Quartz	1.5443	1.5534
Sodium Nitrate	1.5854	1.3368
Ice	1.309	1.313

Let us now enquire how unpolarised light incident on uniaxial crystal gets polarised? We know that when unpolarised light beam enters a calcite crystal, it splits into the o-and the e-rays. The electric field vector of e-ray vibrates in the plane containing the optic axis and the electric field vector of o - ray vibrates perpendicular to it, as shown in Fig. **4.14.** We may, therefore, concude that due to

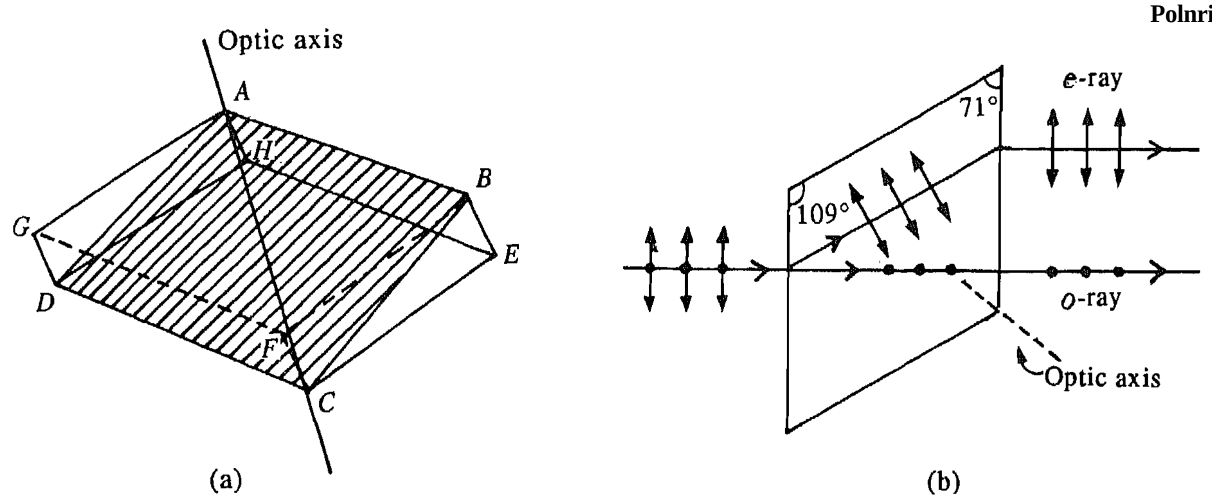


Fig. 4.14 (a) ABCD is one of the principal sections of the calcite crystal; it contains the optic axis and is normal to the cleavage faces *BECF* and *AHDG*. (b) Unpolarised light beam passing through a principal section of the calcite crystal.

double refraction, the unpolarised light beam splits into two components which are **plane polarised.**

Huygens explained many aspects of double refraction in calcite on the basis of wave theory. Since the o-ray obeys Snell's law, it propagates with uniform velocity in all directions in the crystal. As a result, the wave surfaces are spherical. However, the e-ray propagates with different velocities in different directions in the crystal and hence the resulting wave surface is an ellipsoid of revolution, i. e. a spheroid. Further, to reconcile with the fact that both the o-and e-rays travel with the same velocity along the optic axis, both the wave surfaces ware assumed to touch each other at the two extremities of the optic axis. These features are depicted in Fig. 4.15. You may now like to know the nature of wave surfaces for o-and e-waves in positive uniaxial crystals. This is subject matter of TQ 1.

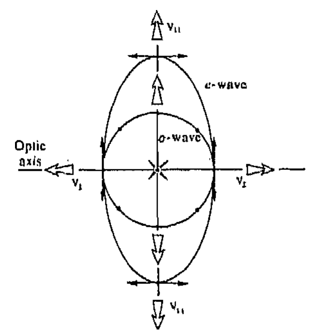


Fig. 4.15: o-and e-wave surfaces in negative uniaxial crystal (calcite).

From the above discussion it follows that in double refraction, an unpolarised light wave splits into o-and e-components with their E-vibrations perpendicular to each other. By selective absorption of one of the P-states, we can produce linearly polarised light. This is readily done by a device, called Nicol prism, by removing the o-ray through total internal reflection. It was designed by William Nicol in 1828. You will learn about it now.

Nicol prism

Nicol prism is made from a naturally occurring crystal of calcite. The length of the crystal is three times its width and the smaller faces *PQ* and *RS* and ground from 71° to a more acute angle of 68° (Fig. 4.16). The crystal is then cut along *PS* by a plane passing through *P* and S and perpendicular to the principal section *PQSR*. The cut surfaces are polished to optical flatness and then cemented together with a layer of (nonrefringent material) Canada balsam.

Can you guess why Canada balsam is used as cementing material? Well, for sodium light, refractive index of Canada balsam is 1.552, which is midway between refractive indices for o-ray (n_o = 1.658) and the e-ray (n_e = 1.486) in calcite. Thus, it is an optically rarer medium with respect to ordinary ray and denser for extraordinary ray. The critical angle for total internal reflection of o-ray is $\sin^{-1}\dfrac{1.552}{1.658} = 69^\circ$.

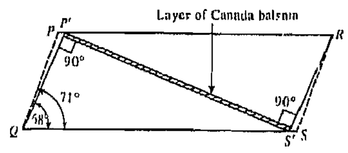


Fig. 4.16: Nicol Prism

So, when incident unpolarised light splits into two rays inside the crystal, the o-ray gets totally reflected at the canada balsam surface when it is incident on it at an angle of 69°. (It is for this reason that the end faces of the crystal are ground so as to make the angles 68° from 71°.) The emergent light will, therefore, be made up only of plane polarised e-component.

Some of the limitations of Nicol prism as polariser are:

1. It can be used for polarisation of visible light only.
2. e-ray also can get totally reflected by the Canada balsam surface if it is travelling along the optic axis. Why? It is so because in this situation the refractive index for e-ray will be same as for o-ray (i. e. greater than the refractive index for Canada balsam).

With time, a number of modifications have been incorporated in the basic design of the Nicol prism to overcome some of these limitations. However. we will not go into these details.

So far you have studied about production of linearly polarised light by reflection and double refraction. Other methods employed to produce linearly polarised light are selective absorption (or dichroism) and scattering. We will here discuss only dichroism and that too in brief.

4.4.4. Selective Absorption: Dichroism

As you know, unpolarised light wave can be regarded as made up of two orthogonal, linearly polarised waves. Many naturally occuring and man made materials have the property of selective absorption of one of these; the other passes through without much attenuation. This property is known as **dichroism.** Materials exhibiting this property are said to be **dichroic materials.** The net result of passing an unpolarised light through dichroic material is the production of linearly polarised light beam. A particularly simple dichroic device is the so-called Wire–Grid polariser. You will learn about it now.

The Wire-Grid Polariser

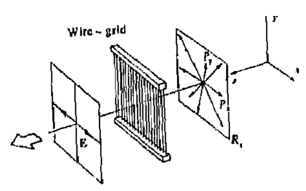


Fig. 4.17: The Wire-grid Polariser

The wire–grid polariser constists of a grid of parallel conducting wires, as shown in Fig. 4.17. Suppose that unpolarised light is incident on the grid from the right. It can be thought as made up of two orthogonal P–states: P_x and P_y in the reference plane R_z. The y–component of the electric field drives the electrons of each wire and generates a current. It produces (Joule) heating of the wire. The net result is that energy is transferred from the field to the wire grid. In addition, electrons accelerating along the *y*–direction radiate in the forward as well as backward directions. The incident wave tends to be cancelled by the wave re–radiated in the forward direction. As a result, transmission ofy–component of field is almost blocked. However, the x–component of field is essentially unaltered as it propagetes through the grid and the light coming out of the wire–grid is linearly polarised. The wire–grid polariser almost completely attenuates the P_y component when the spacing betwen the wires is less than or equal to the wavelength of the incident wave. You must realise that this restriction is rather stringent for the fabrication of a wire–grid polariser for visible light ($\lambda \sim 5\times 10^{-7}$m).

An easy way out of this difficulty in the fabrication of the grid polariser is to employ long chain polymer molecules made up of atoms which provide high electrical conductivity along the length of the chain. These chains of polymer molecules behave similar to the wires in the wire–grid polariser. The alignment of these chains are almost parallel to each other. Because of high electrical conductivity, the electric vector of unpolarised light parallel to the chain gets absorbed. And the P–state perpendicular to these chains passes through. These chemically synthesized polarisers are fabricated in the form of plastic sheets and are known as **polaroids.** Since the spacing between lhese molecular chains in a polaroid is small compared to the optical wavelength, such polaroids are extremely effective in producing linearly polarised light.

Dichroic Crystals

Some naturally occurring crystalline materials are inherently dichroic due to anisotropy in their structure. One of the best known dichroic materials is **tourmaline,** a precious stone often used in jewellery. Tourmalines are essentially boron silicates of differing chemical composition. The component of E perpendicular to the principal axis is strongly absorbed by the sample. Thicker the crystal, more complete will be the absorption. A plate cut from a tourmaline crystal parallel to its optic axis acts as a linear polariser. This is illustrated in Fig. 4.18,

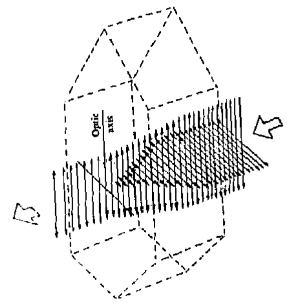


Fig. 4.18: Tourmaline crystal polariser

We shall now consider a class of optical elements known as wave plates which serve to change the polarisation of the incident wave. A wave plate introduces a phase lag between the two P–states by a predetermined amount. That is, the relative phase of the two emerging components is different from its initial value. This concept can be used to convert a given polarisation state into any other and in so doing it is possible even to produce circular or elliptic polarisation as well. This is the subject matter of the next section.

4.5 WAVE PLATES: CIRCULAR AND ELLIPTIC POLARISERS

Consider a plane wave incident on a calcite crystal. It splits in o-and e- waves. Since calcite is a negative uniaxial crystal, $n_o > n_e$ and $v_{||}$ (velocity of e–wave) > $v_\perp$ (velocity of o–wave) implying that the e–ray travels faster than the o–ray. After traversing the calcite crystal of thickness d, the path difference between them is given by

In case of positive uniaxial crystals, $n_e > n_o$ and hence the path difference will be d ($n_e - n_o$). In fact the general expressiopn for the path difference is 3(| $n_o - n_e$ |)

$$\Delta = d\,(n_o - n_e)$$

. and the relative phase difference between o– and e–rays is

$$\delta = \frac{2\pi}{\lambda}\Delta = \frac{2\pi}{\lambda}(n_o - n_e)\,d \qquad (4.18)$$

though while entering both the components were in phase.

The **state of polarisation** of emerging light depends on 6, apart from the amplitudes of incoming orthogonal field components. Let us now consider some specific cases:

(i) When the phase difference, $8 = 2m\pi$; where m is an integer, the ralative path difference is m h. A device which induces a path difference between the two orthogonal field vibrations in integral multiples of λ is called the full wave plate. It introduces no observable effect on the polarisation of the incident beam. That is, the field vibrations of the emergent light will be identical with the field vibrations of the incident light.

(ii) When $6 = (2m+1)\pi$, the relative path differencc will be $(m + \frac{1}{2})\lambda$. Such crystals are called **half–wave plates.**

(iiii) When $6 = (2m+1)\frac{\pi}{2}$, the relative path difference will be $\left(m + \frac{1}{2}\right)\frac{\lambda}{2}$. Such a birefringent sheet is called **quarter–wave plate.** When linearly polarised light traverses a quarter–wave plate, the emergent light will, in general, be elliptical and the axes of the ellipse will coincide with the previleged directions of the thin plate. However, half–wave or full–wave plate leave the state of polarisation unchanged.

Thus, we may conclude that the **path difference between the o– and e–waves in a birefringent device depends on its thickness.**

You should now solve the following SAQ.

SAQ 5

Spend 5 min

Calculate the thickness of a quarter wave-plate for light of wavelength 5890 Å. The refractive indices for o – and e –rays are 1.55 and 1.50 respectively.

We now summarise what you have learnt in this unit.

4.6 SUMMARY

Visible light can be linearly, circularly or elliptically polarised. All these polarisation states arise on superposition of two linearly (or plane) polarised light waves characterised by different amplitudes and phases.

The electric field vectors of two linearly polarised light beams propagating along z-axis can be represented as

$$\mathbf{E}_1(z, t) = \hat{\mathbf{e}}_x E_{01} \cos(kz - \omega t)$$

$$\mathbf{E}_2(z, t) = \hat{\mathbf{e}}_y E_{02} \cos(kz - \omega t + \phi)$$

where E_{01} and E_{02} are the amplitudes of the two waves and ϕ is the phase difference between them. Superposition of these two polarised waves will result in

Linearly polarised light if $\phi = 0$ or an integral multiple of $\pm\, 2\pi$

Circularly polarised light if $\phi = \pi/2$ and $E_{01} = E_{02}$

Elliptically polarised light if $\phi = \pi/2$ and $E_{01} \neq E_{02}$

- According to Malus, when the transmission axes of polariser and the analyser are at an angle θ, the intensity of the polarised light reaching the detector is given by $I(\theta) = I(0)\cos^2\theta$ where, $I(0)$ is the intensity of the polarised light when $\theta = 0$.
- When natural light strikes an interface at Brewster's angle $\theta_B = \tan^{-1}(n_2/n_1)$, where n_1 and n_2 are the refractive indices of medium of incidence and transmission, the reflected light is linearly polarised.
- When light falls on a calcite crystal, it splits into two. The phenomenon is known as double refraction or birefringence. These two refracted beams are known as o– and e-rays. Snell's law holds for o–rays (ordinary rays).
- In a birefringent material, the o– and the e–rays travel in the same direction with same velocity along the optic axis. However, in a direction perpendicular lo the optic axis, they travels with different velocities. The electric field vibrations for o– and the e–rays are mutually perpendicular.
- The phenomenon of double refraction produces linearly polarised light. Nicol prism works on this principle. In the Nicol prism, the o–ray undergo total internal reflection at the inferface and the transmitted beam consists of only electric field vibrations corresponding to e–ray and hence the transmitted beam is linearly polarised.
- Selective absorption (or dichroism) of the electric field component with particular orientalions by material can also be used for producing linearly polarised light. Tourmaline is an example of dichroic material,
- For a calcite crystal of thickness d the path difference between o– and e– rays is given by $\Delta = d\,|n_o - n_e|$

The Corresponding phase difference

$$\delta = \frac{2\pi}{\lambda}\Delta = \frac{2\pi}{\lambda} d(|n_o - n_e|)$$

When the phase difference $6 = 2m\pi$ where m is an **integer,the** relative path difference between the o– and e–rays will be mh. Such crystals are called full–wave plate. When $6 = (2m + 1)\pi$, path difference will be $\lambda/2$ and such crystals act as full–wave plate. And when $\delta = (2m+1)\frac{\lambda}{2}$, path difference will be $\lambda/4$ (for $m = 0$) and such crystals are called quarter–wave plate.

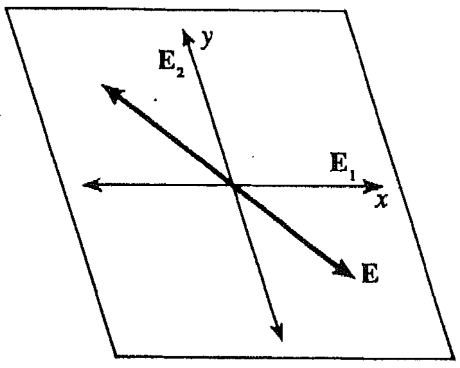
4.7 TERMINAL QUESTIONS

1. In sub–section 4.4.3, you studied about propagation of o– and c– waves in a negative uniaxial crystal (calcite). Draw a diagram and describe the propagation of o– and e–waves in a positive uniaxial crystal (quartz) for normal incidence.
2. For a certain crystal, $n_o = 1.5442$ and $n_e = 1.5533$ for light of wavelength 6×10^{-7} **m.** Calculate the least thickness of a quarter–wave plate made from the crystal for use with light of this wavelength.

4.8 SOLUTIONS AND ANSWERS

SAQs

1 The plane of vibration of the electric vector defined by Eq. (4.5) is rotated with respect to that shown in the Fig. 4.5. This is signified by the negative sign before $\hat{\mathbf{e}}_y$ in the parentheses and is depicted below



2. We know from Eq. (4.12) that

$$\left(\frac{E_2}{E_{02}}\right)^2 + \left(\frac{E_1}{E_{01}}\right)^2 - 2\left(\frac{E_2}{E_{02}}\right)\left(\frac{E_1}{E_{01}}\right)\cos\phi = \sin^2\phi \qquad \text{(i)}$$

If we choose $\phi = \pi$ in (i), we get

$$\left(\frac{E_2}{E_{02}}\right)^2 + \left(\frac{E_1}{E_{01}}\right)^2 - 2\left(\frac{E_2}{E_{02}}\right)\left(\frac{E_1}{E_{01}}\right) = 0$$

which can be written in a compact form:

$$\left(\frac{E_2}{E_{02}} - \frac{E_1}{E_{01}}\right)^2 = 0$$

or

$$E_2 = \frac{E_{02}}{E_{01}} E_1 \qquad \text{(ii)}$$

This defines a straight line ($y = mx$) with slope E_{02}/E_{01}. In other words, elliptically polarised light reduces to linearly polarised light for $\phi = n\pi$ ($n = 0, \pm 1, 2, \ldots$).

When $\phi = \pi/2$ and $E_{01} = E_{02} = E_0$, **Eq.** (4.12) reduces to

$$\left(\frac{E_2}{E_0}\right)^2 + \left(\frac{E_1}{E_0}\right)^2 = 1$$

which defines a circle ($x^2 + y^2 = a^2$) of radius E_0

3. Since both polarising sheets are ideal, the intensityof the incident unpolarised beam, **I,** will reduce to half after passing throught one of them as shown in the **Fig.4.19**. After passing through the second polarising sheet, we are told that the intensity reduces to one third **of** original value.

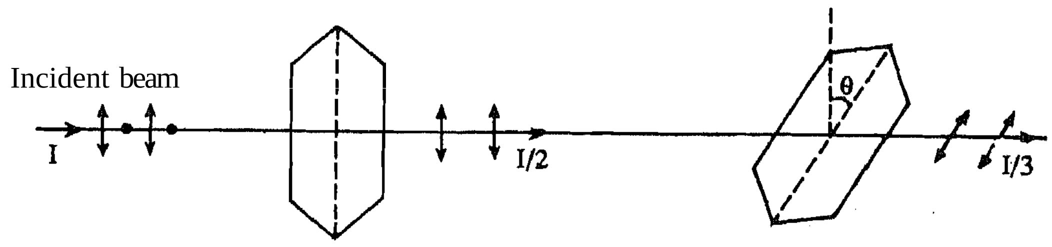


Fig. 4.19: Unpolarised Ught beam of intensity I passing through two polarisers

From Malus' Law we know that

$$I(\theta) = I(0)\cos^2\theta$$

Here $I(\theta) = I/3$ and $I(0) = 1/2$. Therefore

$$\cos^2\theta = (2/3) = 0.666$$

or

$$\theta = \cos^{-1}(.666)^{1/2}$$

$$= 35.3^\circ$$

That is, the **angle** between the **transmissio** axes of two **polarisers** is about 35°

4. For external reflection

$$\tan i_B = \frac{n_2}{n_1} = \frac{1..67}{1.33}$$

$$\Rightarrow \qquad i_B = \tan^{-1}\left(\frac{1.67}{1.33}\right)$$

$$\text{or} \qquad i_B = 51.47^\circ$$

Fro internal reflection

$$\tan i_B = \frac{n_1}{n_2} = \frac{1.33}{1..67}$$

$$i_B = 38.53^\circ$$

5. The path difference produced between the o– and e–rays of birefringent crystal of thiickness d is

$$A = d(|n_0 - n_e|)$$

And corresponding relaltive phase difference is given by

$$\delta = \frac{2\pi}{\lambda}\Delta$$

$$= \frac{2\pi}{\lambda} d(|n_o - n_e|)$$

The phase difference produced bya quarter–wave plate

$$6 = \pi/2$$

On comparing the above expressions for the phase difference, we have

$$d = \frac{\lambda}{4}(n_o - n_e)$$

$$= \frac{5890\ \text{Å}}{4}(1.55 - 1.50)$$

$$= 73.63\ \text{Å}$$

$$= 74\ \text{Å}$$

TQs

1. In case of negative uniaxial.crystal (calcite), e–ray travels faster than the o–ray and hence $n_o > n_e$. Therefore, when a light beam is incident normally upon a calcite crystal, whose optic axis is parallel to the refracting surface and lies in the plane of incidence, o-wave has a spherical wavefront and the e–wave has an spheroidal wavefront.

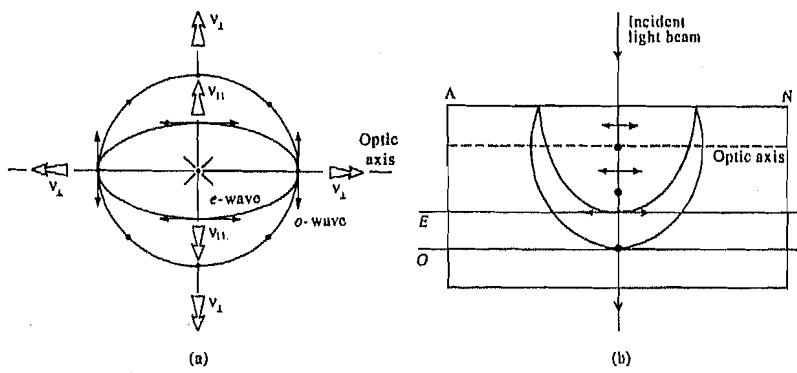


Fig. 4.20: (a) o-and e-wave surfaces in n positive uniaxial crystal (quartz): (b) Propagation of o-and e-waves in quartz.

In case of positive uniaxial crystals like quartz, the e–ray travels slower than the o–ray. Therefore, the spherical wavefront corresponding to o–ray will be outside the spheroidal wavelront corresponding to e–ray (Fig. 4.20a). Since the optical properties of birefringent crystal are symmetrical with respect to its optic axis, the axis of revolution of the sphroid must coincide with the optic axis of the crystal. When a light beam falls on a positive uniaxial crystal, with its optic axis in the plane of incidence and parallel to the refracting surface, the wavefront for o– and e–waves is shown in Fig. 4.20b.

In the above mentioned case, EE' and OO' are the refracted wave–fronts for e– and o–rays respectively at the same instant of time. They are parallel to each other and travel in the same direction which is perpendicular to the refracting surfacee AN.. These two wavefronts, however, will travel with different velocities. As a result, a path difference will be introduced between the o– and the e–ray on emergence, but there is no separation between the two beams. In principle, we can constructs quarter–wave plate, half–wave plate etc. using positive uniaxial crystal as well.

2. **In the** birefringent crystal of thickness d, the path difference between the o– and e–rays is $d \mid n_e - n_o \mid$.

In this problem, $n_e > n_o$, so that we can write $\Delta = d\,(n_e - n_o)$ and the corresponding relative phase difference

$$\delta = \frac{2\pi}{\lambda} d\,(n_e - n_o)$$

For constructing a quarter wave–plate, the path difference should be $\lambda/4$, which corresponds to phase difference of $\pi/2$. Thus, from above equation, we must have, for a quarter wave plate

$$\frac{2\pi}{\lambda} d\,(n_e - n_o) = \pi/2$$

or

$$d = \frac{\lambda}{4\,(n_e - n_o)}$$

We have

$$n_o = 1.5442, n_e = 1.5533, \text{and } \lambda = 6 \times 10^{-7}\ \text{m}.$$

Hence,

$$d = \frac{6 \times 10^{-7}\ \text{m}}{4 \times (1.5533 - 1.5442)}$$

$$= \frac{6 \times 10^{-1}\ \text{m}}{0.0367}$$

$$= 1.65 \times 10^{-5}\ \text{m}.$$

That is, the quarter–wave plate should be 1.65×10^{-5} m thick.