UNIT 1 NATURE OF LIGHT

Structure

1.1 INTRODUCTION

You all know that light is responsible for our intimate contact with the universe through one of our sense organs. We are able to admire the wonders'of the world and appreciate the beauty of nature only when there is light. The reds of the sun or the ruby, the greens of the grass or emerald and the blues of the sky or sapphire involve light. In a way light plays a vital role in sustaining life on earth. Even so, we are strangely unaware of its presence. We see not light but objects, (shapes, colours, textures and motion) as constructed by the brain from information received by it.

Have you ever thought : What is light ? How light behaves when it reaches our eyes ? And so on. These questions proved very difficult even for the genius of the class of Newton and Einstein. In fact, search for answers to these gave birth to a new branch of physics: Optics, which is extremely relevant to the modem world. It occupies a prominent place in various branches of science, engineering and technology. Optical studies have contributed to our understanding of the laws of nature. With the development of lasers, fibre optics, holography, optical communication and computation, optics has emerged as a fertile area of practical applications. It is therefore important for you to understand the language and vocabulory of optics very thoroughly.

In this unit you will learn some important facts and developments which were made to unfold the nature of light. However, before you do so you should revise second block of PHE-02 course and fourth block of PHE-07 course. In Sec. 1.2 you will learn about corpuscular (particle) model of light. In Sec. 1.4 we have discussed the wave model of light, with particular reference to electromagnetic waves. You may now be tempted to ask: Does light behave like a particle or a Wave ? You will learn that it is like neither!

Objectives

After going through this unit you should be able to

- name phenomena distinguishing corpuscular and wave models of light
- derive an expression for the velocity of electromagnetic waves
- specify the frequency ranges of different portions of electromagnetic spectrum, and
- explain the importance of Poynting Vector.

1.2 THE CORPUSCULAR MODEL

You must have read in your school physics course that corpuscular model is due to Newton. Contrary to this popular belief, the credit should be given to Descartes, although the earliest speculations about iight are attributed to Pythagoras.

The corpuscular model is perhaps the simplest of the models of light. According to it, **light consists of minute invisible stream of particles called corpuscles. A** luminous body sends corpuscles out in all directions. These particles travel without being affected by earth's gravitation. Newton emphasized that corpuscles of different sizes stimulate sensation of different colours at the ratina of our eye.

In your physics courses at school you must have learnt about evidences in favour of this model. Can you recall them? The two most important experimental evidences are:

(i) Light travels in straight lines. This rectilinear propagation of light is responsible for formation of sharp (perfectly dark) shadows. If we illuminate a barrier in front of a white screen, the region of screen behind the barrier is completely dark and the region outside the barrier is completely lit. This suggests that light does not go around comers. Or does it?

(ii) Light can propagate lhrough vacuum, i.e., light does not require any material medium, as does sound, for propagation.

The speed of propagation of light has been measured by a variety of means. The earliest measurement made by Roemer in 1676 made use of observations of the motion of the moons of Jupiter and apparent variations in the periods of their orbits resulting from the finite speed of propagation of light from Jupiter to earth. The first completely terrestrial measurement of the speed of light was made by Fizeau in 1849.

We can also predict the correct form of the laws of reflection and refraction using the corpuscular model. However, a serious flaw in this theory is encountered in respect of the speed of light. Corpuscular model predicts that light travels faster in a denser medium. This, as you now recognise, contradicts the experimental findings of Fizeau. Do you expect the speed of light to depend on the nature of the source or the medium in which light propagate? Obviously, it is a properly of the medium. This means that the speed of light has a definite value for each medium. The other serious flaw in the corpuscular model came in the form of experimental observations like interference (re-distribution of energy in the form of dark and bright or coloured fringes), diffraction (bending around sharp edges) and polarization.

You may now like to answer an **SAQ.**

SAQ 1

Spend 5 min

Grimaldi observed that the shadow of a very small circular obstacle placed in the palh of light is smaller than its actual size. Discuss how it contradicts corpuscular model.

In the experiment described in **SAQ 1,** Grimaldi also observed coloured fringes around the shadow. This, as we now know, is a necessary consequence of the wavelike character of light. It is interesting to observe that even though Newton had some wavelike conception of light, he continued to emphasize the particle nature. You will learn about the wave model of light in the following section.

1.3 THE WAVE MODEL

The earliest systematic theory of lighl was put forward by a contemporary of Newton, Christian Huygens. You have learnt about it in **PHE-02.** Using the wave model, Huygens was able to explain the laws of reflection and refraction. However, the aulhority and eminence of Newton was so great that no one reposed faith in Huygens' proposition. In fact, wave model was revived and shaped by Young through his interference experiments.

Young showed that the wavelength of visible light lies in the range 4000 Å to 7000 Å (Typical values of wavelength for sound range from 15 cm for a high- pitched whistle lo **3** m for a deep male voice.) This explains why the wave character of light goes unnoticed (on a human scale). Interference fringes can be seen only when the spacing belween two light sources is of the order of the wavelength of light. That is also why diffraction effects are small and light is said to approximately travel in straight lines. (A ray is defined as the path of energy propagation in the limit of $\lambda \rightarrow 0$). A satisfactory explanation of diffraction of light was given by Fresnel on the basis of the wave model. An important part in establishing wave model was played by polarisation- a subtle property of light. It established that light is a transverse wave; the oscillations are perpendicular to the path of propagation. But what is it that oscillates? The answer was provided by Maxwell who provided real physical significance and sound pedestal to the wave theory. Maxwell identified light with electromagnetic waves. A light wave is associated with changing electric* and magnetic fields. You will learn these details now.

1.4 LIGHT AS AN ELECTROMAGNETIC WAVE

From the PHE-07 course on Electric and Magnetic Phenomena you will recall that a varying electric field gives rise to a time and space varying magnetic field and vice-versa. This interplay of coupled electric and magnetic fields results in the propagation of three-dimensional electromagnetic waves. To show this, we first recall Maxwell's field equations:

$$\nabla . \mathbf{D} = \rho \quad (1.1a)$$

$$\nabla . \mathbf{B} = 0 \quad (1.1b)$$

$$\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0 \quad (1.1c)$$

and

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \quad (1.1d)$$

where ρ and J denote the free charge density and the conduction current density, respectively. E, D, B, and H respectively represent the electric field, electric displacement, magnetic induction and the magnetic field. These are connected through the Eollowing constitutive relations:

$$\mathbf{D} = \varepsilon \mathbf{E} \quad (1.2a)$$

$$\mathbf{B} = \mu \mathbf{H} \quad (1.2b)$$

and

$$\mathbf{J} = \sigma \mathbf{E} \quad (1.2c)$$

where E, μ and a respectively denote the (dielectric) permittivity, magnetic permeability and the electrical conductivity of the medium.

For simplicity, we consider the field equations in vacuum so that $\rho = 0$ and J = O. Then, if we use connecting relations [Eqs. (1.2a-c)], Eq. (1.1a-d) reduce to

$$\nabla . \mathbf{E} = 0 \quad (1.3a)$$

$$\nabla . \mathbf{H} = 0 \quad (1.3b)$$

$$\nabla \times \mathbf{E} = -\mu_0 \frac{\partial \mathbf{H}}{\partial t} \quad (1.3c)$$

and

$$\nabla \times \mathbf{H} = \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \quad (1.3d)$$

where μ_0 and ε_0 are the magnetic permeability and permittivity of free space. Taking the curl of Eq. (1.3c), we get

$$\nabla \times \nabla \times \mathbf{E} = -\mu_0 \nabla \times (\partial \mathbf{H}/\partial t)$$

$$= -\mu_0 \frac{\partial}{\partial t}(\nabla \times \mathbf{H}) \qquad (1.4)$$

since $\frac{\partial}{\partial t}$ is independent of $\nabla \times$ operation.

To simplify the left hand side of this equation, we use the vector identity

$$\nabla \times \nabla \times \mathbf{E} = \nabla(\nabla . \mathbf{E}) - \nabla^2 \mathbf{E}$$

Since $\nabla . \mathbf{E} = 0$ in view of Eq. (1. 3a), we find that Eq. (1.4) reduces to

$$-\nabla^2 \mathbf{E} = -\mu_0 \frac{\partial}{\partial t}(\nabla \times \mathbf{H})$$

The 3-D wave equation has the form

$$\nabla^2 \psi = \frac{1}{v^2}\frac{\partial^2 \psi}{\partial t^2}$$

where ψ is a physical quantity which propagates wavelike with speed v.

On substituting the value of $\nabla \times \mathbf{H}$ from Eq. (1.3d), we get

$$\nabla^2 \mathbf{E} = \mu_0 \varepsilon_0 \frac{\partial^2 \mathbf{E}}{\partial t^2} \qquad (1.5)$$

You can similarly show that

$$\nabla^2 \mathbf{H} = \mu_0 \varepsilon_0 \frac{\partial^2 \mathbf{H}}{\partial t^2} \qquad (1.6)$$

Spend 5 min

SAQ 2

Prove Eq. (1.6)

Do you recognise Eqs.(1.5) and (1.6)? These are identical in form to 3-D wave equation derived in Unit 6 of the Oscillations and Waves course (PHE-02). This means that each component of E and H satisfies a wavelike equation. The speed of propagation of an electromagnetic wave in free space is given by

$$v = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} \qquad (1.7)$$

This remarkably simple result shows that the speed of an electromagnetic wave depends only on μ_0 and ε_0. This suggests that all e.m. waves should, irrespective of frequency or amplitude, share this speed while propagating in free space. We can easily calculate the magnitude of v by noting that for free space

$$\varepsilon_0 = 8.8542 \times 10^{-12}\, C^2 N^{-1} m^{-2}$$

and

$$\mu_0 = 4\pi \times 10^{-7}\, N s^2 C^{-2}.$$

Thus

$$v = \frac{1}{[(8.8542 \times 10^{-12}\, C^2 N^{-1} m^{-2}) \times (4 \times 10^{-7}\, Ns^2 C^{-2})]^{1/2}}$$

$$= 2.99794 \times 10^8\, ms^{-1}$$

This is precisely the speed of light! It is worthwhile to mention here that using the then best known value of ε_0, Maxwell found that electromagnetic waves should travel at a speed of 3.1074×10^8 ms^{-1}. This, to his amusement, was very close to the speed of light measured by Fizeau(3.14858×10^8 ms^{-1}). Based on these numbers, Maxwell proposed the electromagnetic theory of light. In his own words

> "This velocity is so nearly that of light, that it seems we have strong reason to believe that light itself is an electromagnetic disturbance in the form of waves propagated through the electromagnetic field according to electromagnetic laws."

We cannot help but wonder at such pure gold having come out of his researches on electric and magnetic phenomena. It was a rare moment of unveiled exuberance – a classic example of the unification of knowledge towards which science is ever striving. With this one calculation, Maxwell brought the entire science of optics under the umbrella of electromagnetism. Its significance is profound because it identifies light with structures consisting of electric and magnetic fields travelling freely through free space.

The direct experimental evidence for electromagnetic waves came through a series of brilliant experiments by Hertz. He found that he could detect the effect of electromagnetic induction at considerable distances from his apparatus. His apparatus is shown in Fig. 1.1. By measuring the wavelength and frequency of electromagnetic waves, Hertz calculated their speed. He found it to be precisely equal to the speed of light. He also demonstrated properties like reflection. refraction, interference, etc and demonstrated conclusively that light is an electromagnetic wave.

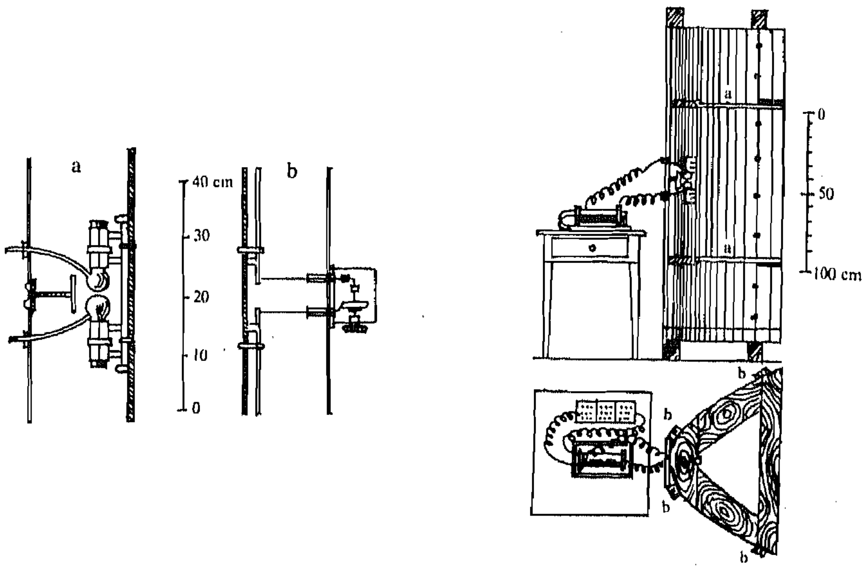

Fig.1.1: Hertz's apparatus for the generation and detection of electromagnetic waves

You now know that electromagnetic waves are generated by time varying electric and magnetic fields. So these are described by the amplitudes and phases of these fields. The simplest electromagnetic wave is the plane wave. You may recall that in a plane wave the phases of all points on a plane normal to the direction of propagation are same. And for a plane electromagnetic wave propagating along the $+z-$ direction, the phase is $(kz - \omega t)$, where k is the wave number and ω is the angular frequency of electromagnetic plane wave. And the scalar electric and magnetic fields can be expressed as

$$E = E_0 \exp\left[i(kz - \omega t)\right]$$

$$H = H_0 \exp\left[i(kz - \omega t)\right]$$

where E_0 and H_0 are amplitudes of E and H.

For a wave propagating along the + z-direction, the field vectors E and H are independent of x and y. Then Eqs. (1.3a) and (1.3b) reduce to

$$\frac{\partial E_z}{\partial z} = 0 \qquad (1.8a)$$

and

$$\frac{\partial H_z}{\partial z} = 0 \qquad (1.8b)$$

By the same argument you will find that the time variation of E_z and H_z can be expressed as

$$\frac{\partial E_z}{\partial t} = 0 \qquad (1.9a)$$

$$\frac{\partial H_z}{\partial t} = 0 \qquad (1.9b)$$

To arrive at Eqs. (1.9 a,b), we write the z-components of Eqs. (1.3c) and (1.3d) as

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -\mu_0 \frac{\partial H}{\partial t}$$

and

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = -\varepsilon_0 \frac{\partial E_z}{\partial t}$$

Since E and H are independent of x and y, the LHS will be identically equal to zero.

What do these equations convey? Physically, these imply that the components of E and H along the direction of propagation of an electromagnetic wave (+z-direction in this case) does not depend upon time and the space coordinate z. So we must have

$$E_z = 0 = H_z \qquad (1.10)$$

You should convince yourself why any other constant value of E_z and H_y would not represent a wave. We can now draw the following conclusions:

1. Plane electromagnetic waves have no longitudinal component. That is, they are transverse. This implies that if electric field is along the x- axis, the magnetic field will be along the y-axis so that we may write

$$\mathbf{E} = \hat{\mathbf{x}} E_0 \, e^{i(kz - \omega t)}$$

and

$$H = \hat{\mathbf{y}} H_0 \, e^{i(kz - \omega t)} \qquad (1.11)$$

You may now ask: Are E_0 *and* H_0 connected? If so, what is the relation between them'?To discover answer to this question you have to solve TQ2:

$$H_0 = \frac{k}{\mu_0 \omega} E_0$$

2. Since $\frac{k}{\mu_0 \omega}$ is a real number, the electric and magnetic vectors should be in phase. Thus if E becomes zero (maximum) at some instant, H must also necessarily be zero (maximum) and so on. This also shows that neither electric nor magnetic wave can exist without the other. An electric field varying in

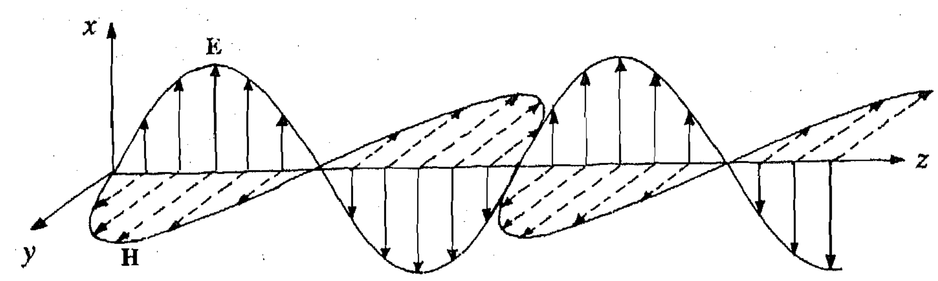

Fig.1.2: The electric and magnetic fields associated with a plane electromagnetic wave

time sets up a space-time varying magnetic field, which, in turn, produces an electric field varying in space and time, and so on. You cannot separate them. This mutually supporting role results in the generation of electromagnetic waves. The pictorial representation of fields of a plane electromagnetic waves (propagating along the + z- direction) is shown in Fig. 1.2. You will note that electric and magnetic fields are oriented at right angles to one another and to the direction of wave motion. Moreover, the variation in the spacing of the field lines and their reversal from one region of densely spaced lines to another reflect the spatial sinusoidal dependence of the wave fields.

1.4.1 Energy Transfer: The Poynting Vector

From Unit 6 of PHE-02 course you will recall that a general characteristic of wave motion is: Wave carries energy, **not** matter. Is it true even for electromagnetic waves? To know the answer, you should again consider the two field vectors (E and H) and calculate the divergence of their cross product. You can express it as

$$\nabla.(\mathbf{E}\times\mathbf{H}) = \mathbf{H}.(\nabla\times\mathbf{E}) - \mathbf{E}.(\nabla\times\mathbf{H}) \qquad (1.12)$$

If you now substitute for the cross products on the right-hand side from Maxwell's third and fourth equations respectively for free space, you will get

$$\nabla.(\mathbf{E}\times\mathbf{H}) = -\mathbf{H}.\mu_0\frac{\partial\mathbf{H}}{\partial t} - \mathbf{E}.\varepsilon_0\frac{\partial\mathbf{E}}{\partial t}$$

Recall the identity $\nabla.(\mathbf{A}\times\mathbf{B}) = \mathbf{B}.(\nabla\times\mathbf{A}) - \mathbf{A}.(\nabla\times\mathbf{B})$ from unit 2 of PHE-04 course on Mathematical Methods in Physics-I.

The time derivatives on the right-hand side can be written as

$$\mathbf{H}.\mu_0\frac{\partial\mathbf{H}}{\partial t} = \mathbf{H}.\mu_0\frac{\partial\mathbf{H}}{\partial t} = \frac{1}{2}\mu_0\frac{\partial}{\partial t}(\mathbf{H}.\mathbf{H})$$

and

$$\varepsilon_0\mathbf{E}.\frac{\partial\mathbf{E}}{\partial t} = \varepsilon_0\mathbf{E}.\frac{\partial\mathbf{E}}{\partial t} = \frac{\varepsilon_0}{2}\frac{\partial}{\partial t}(\mathbf{E}.\mathbf{E})$$

so that

$$\nabla.(\mathbf{E}\times\mathbf{H}) = -\frac{\partial}{\partial t}\frac{1}{2}(\varepsilon_0\mathbf{E}.\mathbf{E} + \mu_0\mathbf{H}.\mathbf{H}) \qquad (1.13)$$

Gauss' divergence theorem relates the surface integral of a vector function to the volume integral of the divergence of this same function:

$$\int_S \mathbf{D}.d\mathbf{A} = \int_V \nabla.\mathbf{D}\,dV$$

The surface integral is taken over the closed surface, S bounding the volume, V.

Do you recognise Eq. (1.13)? If so, can you identify it with some known equation in physics? This equation rcseinbles the equation of continuity in hydrostatics. To discover the physical significance of Eq. (1.13), you should integrate it over volume V bound by the surface S and use Gauss' theorem. This yields

$$\int_V \nabla.(\mathbf{E}\times\mathbf{H})\,dV = -\frac{\partial}{\partial t}\int_V \frac{1}{2}(\varepsilon_0\mathbf{E}.\mathbf{E} + \mu_0\mathbf{H}.\mathbf{H})\,dV$$

or

$$\int_S (\mathbf{E}\times\mathbf{H}).d\mathbf{A} = -\frac{\partial}{\partial t}\int_V \frac{1}{2}(\varepsilon_0\mathbf{E}.\mathbf{E} + \mu_0\mathbf{H}.\mathbf{H})\,dV$$

The integrand on the right hand side refers to the time rate of flow of electromagnetic energy in free space. You will note that both E and H contribute to it equally. The vector

$$\mathbf{S} = \mathbf{E}\times\mathbf{H} \qquad (1.14)$$

is called the Poynting Vector. It is obvious that **S, E** and **H** are mutually orthogonal. Physically it implies that S points in the direction of propagation of the

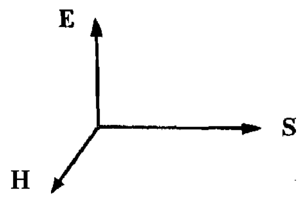

Fig. 1.3: The Poynting Vector

wave since electromagnetic waves are transverse. This is illustrated in Fig. 1.3.

You may now like to know the time-average of energy carried by electromagnetic waves (light) per unit area. If you substitute for E and H in Eq. (1.14) and average over time, you will obtain

$$\langle S \rangle = \hat{z} \frac{k}{2 \mu_0 \omega} E_0^2 \qquad (1.15)$$

Before you proceed, you should convince yourself about the validity of this result. To ensure this we wish you to solve SAQ **3.**

Spend 5 min

SAQ3

Prove Eq.(**1**.15).

1.4.2 The Electromagnetic Spectrum

Soon after Hertz demonstrated the existence of electromagnetic waves in 1888, intense interest and activity got generated. In 1895, J.C. Bose, working at Calcutta, produced electromagnetic waves of wavelengths in the range 25 mm to 5 m. (In 1901, Marconi succeded in transmitting electromagnetic waves across the Atlantic Ocean. This created public sensation. In fact, this pioneering work marked the beginning of the era of communication using electromagnetic waves.) X-rays, discovered in 1898 by Roentgen, were shown in 1906 to be e.m. waves of wavelength much smaller than the wavelength of light waves. Our knowledge of e.m. waves of various wavelengths has grown continuously since then. The e.m. spectrum, as we know it today, is shown in Fig. 1.4.

The range of wavelengths (and their applications in modern technologies) is very wide. However, the boundaries of various regions are not sharply defined. The visible light is confined to a very limited portion of the spectrum from about

Fig.1.4: The electromagnetic spectrum

4000 Å to 7000 Å. As you know, different wavelengths correspond to different colours. The red is at the long wavelength-cnd of visible region and the violet at the short wavelength -end. For centuries our only information about the universe beyond earth has come from visible light. All electromagnetic waves from 1 m to 10^6 m are referred to as radiowavcs. These are used in transmission of radio and television signals. The ordinary AM radio corresponds to waves with $\lambda = 100$m, whereas FM radio corresponds to 1m. The niicrowaves are used for radar and satellite communications ($\lambda \sim 0.5$m – 10^{-3} m).

Between two radio waves and visible light lies the infrared region. Beyond the visible region we encounter the ultraviolet rays, X-rays and gamma rays. You must convince yourself that all phenomena from radio waves lo gamma rays are essentially the same; they are all electromagnetic waves which differ only in wavelength (or frequency). You may now be tempted to enquire: Why do we attribute different nomenclature to different portions of the electromagnetic spectrum? The distinction is a mere convenience while identifying their practical applications.

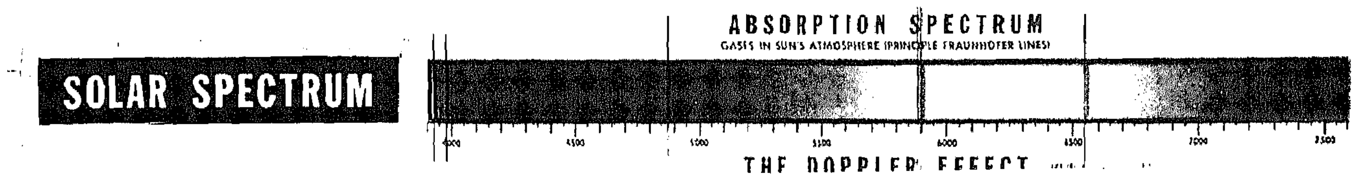

Fig. 1.5: The solar spectruni received on the earth

In our solar system, the sun is the major source of e.m. waves. If you closely examine the solar spectrum received on the earth, you will observe broad continuous spectrum crossed by Fraunhofer dark absorption lines (Fig. 1.5).

Let us now sum up what you have learnt in this unit.

1.5 SUMMARY

- Light is an electromagnetic wave.
- The electric and magnetic fields constituting an electromagnetic wave satisfy the equations

$$\nabla^2 \mathbf{E} = \mu_0 \varepsilon_0 \frac{\partial^2 \mathbf{E}}{\partial t^2}$$

  and

$$\nabla^2 \mathbf{H} = \mu_0 \varepsilon_0 \frac{\partial^2 \mathbf{H}}{\partial t^2}$$

- For a plane electromagnetic wave propagating along the + z- direction, the electric and magnetic fields can be expressed as

$$\mathbf{E} = \hat{\mathbf{x}} E_0 \exp\left[i (kz - \omega t) \right]$$

  and

$$\mathbf{H} = \hat{\mathbf{y}} H_0 \exp\left[i (kz - \omega t) \right]$$

- Th. electromagnetic waves are transverse.
- The pointing vector $\mathbf{S} = \mathbf{E} \times \mathbf{H}$ defines the direction of propagation of an electromagnetic wave.
- The visible light is confined to a very limited portion (4000 Å - 7000 Å) of electromagnetic spectrum.

1.6 TERMINAL QUESTIONS

1 Derive the wave equation for the propagation of electromagnetic waves in a conducting medium.

2. Starting from Eqs. (1.3c) and (1.3d) show that

$$H_0 = \frac{k}{\mu_0 \omega} E_0$$

3. The energy radiated by the sun per second is approximately 4.0 x 10^{26} Js^{-1}. Assuming the sun to be a sphere of radius 7 x 10^8 m, calculate the value of Poynting vector at its surface. How much of it is incident on the earth? The average distance between the sun and earth is 1.5 x 10^{11} m.

1.7 SOLUTIONS AND ANSWERS

SAQs

1. According to the corpuscular model, light travels in straight lines. As a result, the size of the shadow should be equal to the size of the object. Grimaldi's observation - the size of the shadow is smaller than the size of the obstacle - indicates that light bends around edges, contradicting corpuscular model.

2. Taking the curl of Eq. (1.3d), we get

$$\nabla \times \nabla \times \mathbf{H} = \varepsilon_0 \nabla \times \left(\frac{\partial \mathbf{E}}{\partial t}\right)$$

$$= \varepsilon_0 \frac{\partial}{\partial t} (\nabla \times \mathbf{E})$$

Using the vector identity

$$\text{curl curl H} = \text{grad div H} - \nabla^2 \mathbf{H}$$

we have

$$\nabla(\nabla . \mathbf{H}) - \nabla^2 \mathbf{H} = \mu_0 \varepsilon_0 \frac{\partial}{\partial t}\left(-\frac{\partial \mathbf{H}}{\partial t}\right)$$

Since $\nabla . H = 0$, we **get**

$$\nabla^2 \mathbf{H} = \mu_0 \varepsilon_0 \frac{\partial^2 \mathbf{H}}{\partial t^2}$$

3. From Eq. (1.14), we have for Poynting vector

$$\mathbf{S} = \mathbf{E} \times \mathbf{H}$$

Taking only the real part of Eq. (1.11), the electric and magnetic field vectors can be represented as

$$\mathbf{E} = \hat{\mathbf{x}} E_0 \cos(kz - \omega t)$$

$$\mathbf{H} = \hat{\mathbf{y}} H_0 \cos(kz - \omega t)$$

$$= \hat{\mathbf{y}} \frac{k}{\mu_0 \omega} E_0 \cos(kz - \omega t) \qquad \left(\because H_0 = \frac{k}{\mu_0 \omega} E_0\right)$$

So

$$\mathbf{E} \times \mathbf{H} = (\hat{\mathbf{x}} \times \hat{\mathbf{y}}) \frac{k}{\mu_0 \omega} E_0 \cos^2(kz - \omega t)$$

or

$$\mathbf{S} = \hat{\mathbf{z}}\frac{k}{\mu_0\,\omega}E_0^2\cos^2(kz - \omega t)$$

This gives the amount of energy crossing a unit area perpendicular to z-axis per unit time. Typical frequency for an optical beam is of the order of $10^{15}\,s^{-1}$ and the cosine term will fluctuate rapidly. Therefore, any measuring device placed in the path would record only an average value. The time average of the cosine term, as you know, is 1/2. Hence

$$\langle \mathbf{S} \rangle = \hat{\mathbf{z}}\frac{k}{2\mu_0}\frac{E_0^2}{\omega}$$

TQs

1. While deriving the wave-equation for electromagnetic waves in free space, we assumed that the electric cuurrent density is zero:

$$\mathbf{J} = \sigma\mathbf{E} = 0$$

This is because the conductivity (σ) of the free space was taken to be zero However in case of conducting medium, σ is non-zero. Hence

$$\mathbf{J} = \sigma\mathbf{E}$$

and

$$\mathbf{D} = \varepsilon\mathbf{E}$$

$$\mathbf{B} = \mu\mathbf{H}$$

where symbols have their usual meaning.

With the help of above relations, Maxwell's relations in a conducting medium can be written as

$$\nabla\cdot\mathbf{E} = 0 \qquad (1a)$$

$$\nabla\cdot\mathbf{B} = 0 \qquad (1b)$$

$$\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t} \qquad (1c)$$

and

$$\nabla\times\mathbf{B} = \mu\left(\sigma\mathbf{E} + \varepsilon\frac{\partial\mathbf{E}}{\partial t}\right) \qquad (1d)$$

Taking curl of equation (1c), we get

$$\nabla\times\nabla\times\mathbf{E} = \nabla\times\left(-\frac{\partial\mathbf{B}}{\partial t}\right)$$

Using the identity $\nabla\times\nabla\times\mathbf{A}$ = grad div $\mathbf{A} - \nabla^2\mathbf{A}$, we have

$$\text{grad div }\mathbf{E} - \nabla^2\mathbf{E} = -\frac{\partial}{\partial t}(\nabla\times\mathbf{B})$$

Using (1a) and (1d) in this expression, we get

$$-\nabla^2\mathbf{E} = -\frac{\partial}{\partial t}\left[\mu\left\{\sigma\mathbf{E} + \varepsilon\frac{\partial\mathbf{E}}{\partial t}\right\}\right]$$

which implies that

$$-\nabla^2 \mathbf{E} = -\mu\sigma\frac{\partial \mathbf{E}}{\partial t} - \mu\varepsilon\frac{\partial^2 \mathbf{E}}{\partial t^2}$$

or

$$\nabla^2 \mathbf{E} - \mu\sigma\frac{\partial \mathbf{E}}{\partial t} - \mu\varepsilon\frac{\partial^2 \mathbf{E}}{\partial t^2} = 0 \qquad (2)$$

This is the wave equation lor the propagation of the electromagnetic waves in a conducting medium.

2. If we write Eqs. (1.3c) and (1.3d) in component form, we get

$$\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = -\mu_0\frac{\partial H_x}{\partial t} \qquad (a)$$

$$\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = -\mu_0\frac{\partial H_y}{\partial t} \qquad (b)$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -\mu_0\frac{\partial H_z}{\partial t} \qquad (c)$$

and

$$\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} = \varepsilon_0\frac{\partial E_x}{\partial t} \qquad (d)$$

$$\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} = \varepsilon_0\frac{\partial E_y}{\partial t} \qquad (e)$$

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = \varepsilon_0\frac{\partial E_z}{\partial t} \qquad (f)$$

Now let us take x-axis along the electric field vector E. Then

$$E_y = 0 \qquad (g)$$

From Eq. (1.10) it follows for electromagnetic wave travelling along the x-axis that

$$E_z = 0 = H_z \qquad (h)$$

On using these results in Eq. (a) to (f), we get

$$\frac{\partial H_x}{\partial t} = 0 \quad (i) \qquad \frac{\partial H_y}{\partial z} = -\varepsilon_0\frac{\partial E_x}{\partial t} \quad (l)$$

$$\frac{\partial E_x}{\partial z} = -\mu_0\frac{\partial H_y}{\partial t} \quad (j) \qquad \frac{\partial H_x}{\partial z} = 0 \quad (m)$$

$$\frac{\partial E_x}{\partial y} - \mu_0\frac{\partial H_z}{\partial t} \quad (k) \qquad \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \quad (n)$$

The plane wave representation of the electric and magnetic field vectors is given as

$$\mathbf{E} = E_0 \exp\left[i(kz - \omega t)\right]$$

$$H = H_0 \exp\left[i(kz - wt)\right]$$

Substituting these in Eq. (j), we **get**

$$i\,k\,E_0 = i\,\mu_0\,\omega\,H_0 \Rightarrow H_0 = \frac{k}{\mu_0\,\omega}E_0$$

3. We know that Poynting vector denotes the rate at which energy is radiated per unit area. So we can write the total average energy radiated from the surface of sun per unit time as

or

$$E = \langle S \rangle \times 4\pi R^2$$

$$\langle S \rangle = \frac{E}{4\pi R^2} = \frac{4.0 \times 10^{26}\,\text{Js}^{-1}}{4 \times 3.1416 \times (7 \times 10^8\,\text{m})^2}$$

$$= 6.5 \times 10^7\,\text{J m}^{-2}\,\text{s}^{-1}$$

To calculate the energy incident on the earth, we should know the average Poynting vector $\langle S_E \rangle$ at the surface of earth. To do so, we denote the distance between the surface of earth and centre of the sun as $R_E = R_{ES} + \text{R}$ and note that

$$\langle S_E \rangle 4\pi R_E = \langle S \rangle \times 4\pi R^2$$

so that

$$\langle S_E \rangle = \left(\frac{R}{R_E}\right)^2 \langle S \rangle$$

$$= \left(\frac{7 \times 10^8\,\text{m}}{1.5 \times 10^{11}\,\text{m}}\right)^2 \times \left(6.5 \times 10^7\,\text{J m}^{-2}\,\text{s}^{-1}\right)$$

$$= 1.42 \times 10^3\,\text{Jm}^{-2}\,\text{s}^{-1}.$$