(For Counsellor's use only)

Grade . . . . Name . . . .

Evaluated by . . . . Enrolment Number . . . . .

EXPERIMENT 13

MEASUREMENT OF C_P/C_V BY AN ACOUSTIC METHOD

13. Introduction
- Objectives

13.2 Apparatus

13.3 Study **Metarial**
- An Ideal Gas
- First Law of Thermodynamics
- Application to a HydrodynamicSystem
- Application to an Ideal Gas
- Heat Capacities of Various Gases
- Velocity of Sound In a Gas
- Acoustic Resonance in a Tube

13.4 Precautions

13.5 The Experiment
- Setting up the Apparatus
- Measurement of Wavelength In Alr
- Measurement of Wavelength in Carbon-Dioxide
- Calculations and Estimation of Errors

13.6 Conclusions

13.1 INTRODUCTION

The properties of matter depend ultimately on the details way in which atoms and molecules are arranged with respect to each other. It is all the-more remarkable then that the properties **of gases** should largely depend on the simple perfect-gas law, for very many gases of a wide range of composition.

While **the** Law itself is obeyed in large part, the **constants** which appear in the Law do themselves depend ontbe particular gasatoms **or molecules** involved. In this way the properties of the particular gas molecules **are** made evident.

This experiment is designed to let you see how the gas molecules control to some extent one gas property, **namely** the speed with which sound moves **through** a gas. **As** we will see, this in **turn** depends on the way the atoms of the gas **molecule** are arranged. Fortunately some very simple molecules are available to use in the investigation.

OBJECTIVES

At the end of this experiment a student will be able to do the following.

* Setup and use apparatus of a resonant-tube sort, to measure the speed of sound in several simple gases.
* Calculate the statistical errors involved in the measurements.
* Be able to correlate the speed of sound with the molecular structure of the gases, through thermodynamic quantities calculated.

13.2 APPARATUS

One audio-frequency oscillator of low power and variable frequency.
One power amplifier, of the sort used for public- address systems.
One horn driver, from a public-address system, with the horn unscrewed.
One glass tubing about 1.5 cm to 3 cm diameter.
One cap for tube, with gas-admitting arrangement
One meter-scale
One thermometer for measuring room temperature.
One generator for carbon-dioxide (acid - marble. chip).
cotton for ear-plugs.

13.3 STUDY MATERIAL

13.3.1 An Ideal Gas

Thermodynamics takes its importance from the combination of two aspects of physics. One is the rules by which materials and their properties are related. The other is the rules by which processes proceed, that means the way in which quantities change under various conditions.

You have studied in school the way an ideal gas bebaves. The quantities which describe the behavior of an ideal gas are: the temperature, the pressure and the volume of the gas. You have learned that these are related by the following formula.

$$PV = nR\Theta$$

Where the symbols have tbe following meaning.

P = Pressure, in units of pascals (a pascal is one newton per square meter)
V = Volume, in cubic meters.

n = number of moles in the volume of gas.

R = universal gas constant, with the value 8.31 Joules/mol*K-degrees

Θ = ideal-gas temperature, which for our purposes can be considered to be identical to the Kelvin temperature.

It is one of the properties of an ideal gas that the internal energy U is a function only of the temperature Θ. This will turn out to be a useful fact.

13.3.2 First Law of Thermodynamics

If you have studied your course in Heat and Thermodynamics you will be familiar with the followingstatement. If you have not yetstudied that course you should accept for the time being the truth of what follows. For detailed derivations you can refer to pages 77 through 115 in the Reference book.

All material has been found to follow a very important and simple rule, when the thermodynamic quantities for the material are changed little-by-little, and fairly slowly. In those circumstances the following rule applies.

When such a material undergoes a process in which energy is transferred by non-mechanical means, the difference in the internal energy change for the system, and the work done is called HEAT.

This sounds like a strange rule, but when it is properly applied it can help us understand a lot.

13.3.3 Application to a Hydrodynamic System

For the case where pressure and volume are the quantities which are determining the system, this rule can be written in a neat mathematical form as follows.

$$dQ = dU + PdV$$

Where the new symbols have the following meaning.

Q = energy in the form of heat.
U = internal energy of a system (in this case a gas). This will be composed of the kinetic and potential energies of the atoms and molecules which conipose the gas.

Just take this equation as true, if you have not yet progressed to where you can derive it,

Now we need a definition - the definition of the heat capacity of such a system at constant volume. This is given by the following expression, for the case of the restricted dependence mentioned above.

$$C_V = \left(\frac{dU}{d\Theta}\right)_V$$

Now, the I Law of Thermodynamics above may be used at constant volume ($dV = 0$), when $dQ = dU$. Hence the following equation.

$$C_V = \left(\frac{dU}{d\Theta}\right)_V$$

$$dU = C_V d\Theta$$

This expression is substituted in the expression above, for the change in heat of the system when a change occurs. This results in the following important relation.

$$dQ = C_V d\Theta + PdV$$

13.3.4 Application to an Ideal Gas

This expression is good for all kinds of materials. If now we look at the equation for an ideal gas (above) and substitute it in the general expression for dQ then we get the following, considering a process involving very small changes.

$$dQ = (C_V + nR)d\Theta$$

SAQ: Please use the space here to actually fill in the derivation referred to, by substitution. It will take only a few lines!

....

....

....

If the process takes place at constant pressure then this expression, when divided by $d\Theta$ results in the value of C_p, the heat capacity at constant pressure. This results in the following very useful relation.

$$C_P = C_V + nR$$

SAQ: Please fill in here the few lines needed to actually carry out the derivation of the previous equation.

....

....

....

This shows that the heat capacity of a gas is always greater at constant pressure than it is at constant temperature, and the difference is constant and equal to nR. This is indeed a remarkable fact!

Now using the ideal gas law in the expression for dQ, you can get the following other expression for dQ.

$$dQ = C_P \, d\Theta - VdP$$

SAQ: Please actually place here the few lines needed to get this expression.

....

....

....

....

Now these two expressions for dQ can be combined for the case of adiabatic processes (those for which $dQ = 0$) to get the following important expression.

$$\frac{dP}{P} = -\gamma \frac{dV}{V}$$

where $\gamma = C_P/C_V = c_P/c_V$

and c_P and c_V are molar heat capacities.

SAQ:

Please use the space below for doing this coinbination and arriving at the above expression.

....

....

....

13.3.5 Heat Capacities of Various Gases

The subject of statistical mechanics is closely linked with heat and thermodynamics. One important result of that subject is that the molar heat capacities of gases are closely linked with the number of motions which a gas atom can undertake. In fact the rule is as follows.

For every motion a gas atom can have, there is an amount of molar heat capacity at constant volume equal to $(1/2)R$.

So, for a monatomic gas like helium, which can move independently in any of three directions (x, y or z) the following is true.

$c_V = (3/2)R$
$c_P = (3/2)R + R = (5/2)R$ (see an equation above)
$\gamma = 5/3 = 1.67$

A diatomic gas like oxygen or nitrogen (mostly what composes the atmospheric air) has molecules which, in addition to x-y-z motions can rotate about an axis (not the axis which joins the atoms, though!)
In addition, the atoms can vibrate along an axis which joins the atoms. Hence the following.

$c_V = (5/2)R$
$c_P = (5/2)R + R = (7/2)R$ (see an equation above)
$\gamma = 7/5 = 1.4$

A gas like carbon dioxide (CO_2) which has three atoms strung out in almost a straight line, can in addition bend in two independent ways. Hence the following.

$c_V = (7/2)R$
$c_P = (7/2)R + R = (9/2)R$ (see an equation above)
$\gamma = 9/7 = 1.29$

So, by a measurement of γ we can find out something about the atomic and molecular structure of the elements composing a gas! This is exactly what you will do in the experiment.

13.3.6 Velocity of Sound in a Gas

Your reference book (Page 118 and following) has a detailed derivation of a formula by which you can calculate the velocity of sound in a gas. This derivation assumes that sound is transmitted by longitudinal waves using an adiabatic process, which is the same assumption we have used above. The result of this derivation is as follows.

$$W = \left(\frac{1}{\rho} * \frac{1}{\frac{-dV}{dP} * V} \right)^{1/2}$$

$$= \left(\frac{\gamma P}{\rho} \right)^{1/2}$$

Where

w = velocity of suond in m/s

p = gas density in kg/m^3

SAQ:
Please verify using the **equations** of 13.3.4 that the second equality above **follows** from the **first**.

....

....

....

So, you can find out y by finding out *w*, and this is the experimental objective you have now.

13.3.6 Acoustic Resonance in a Tube

The method you are **going** to use to measure the speed of sound in a gas is often called "Kundt's Tube Method", after a **German** physicist: It simply depends on the fact that longitudinal sound waves traveling in a tube closed at one end have a resonant phenomenon.

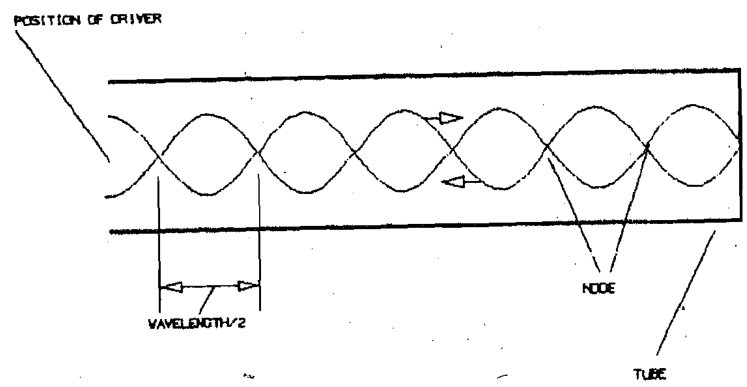

Fig.13.1

The wave which progresses to the right in **Fig.13.1,** and the reflected wave which proceeds to the left are coherent since they have the same source. These **two** pressure waves add up and produce a standing-wave pattern of alternating minima and maxima. The distance between these is $\lambda/2$, where A is the wavelength of the acoustic wave.

Usually the **driver** for the sound wave is at the open end of the **tube,** where the amplitude of the wave is a maximum. If the length **of the** tube is equal to an odd-integral number of wavelengths then the interference will be resonant and the **pressure** maxima and **minima** will be very pronounced.

In your experiment you will put a small amount of very **light** power along the bottom inside of a **glass tube** closed at one end, and introduce sound at the open end. When the frequency is **adjusted** so that resonance is observed, you will see that the strong sound waves displace **the** powder, which accumulates at the resonant **nodes. These** are the places where there is **very little** net motion of the gas in the lube.

With this **observation you** can easily measure **the distance** *L* between nodes **separated by** *N* **half-**wavelengths. This will then allow the calculation of the wavelength **λ.** If the frequency ν is known then the speed of sound *w* can be **calculated** from the following expression.

$$W = \lambda \, \nu$$

This in turn will allow the verification of the ratio if heat capacities, y, which is one **object of** this **experiment.**

13.4 PRECAUTIONS

. The chief precautions in this experiment concern the high levels of sound power you will be using. It is really best to plug your ears with cotton bits before you start the experiment. This will go a long way to avoiding any painful experience, or even the possibility of ear damage.

Next, make sure that you start each section of the experiment with the volume control on the power amplifier set to "zero". You can then turn up the oscillator level to say mid-range value, and slowly and carefully increase the power amplifier volume until you have enough power to mover the powder in the tube.

Remember that when you tune the oscillator to a resonance a lot of sound energy will escape from the tube, so be prepared to turn the power volume down.

Once you have establis hed a resonance with modest power output, you can just quickly increase the power and immediately decrease it. This will boost the powder to the regions of the nodes. Then you can measure the node spacing with the oscillator and power amplifier turned completely down!. This will avoid your becoming unpopular with your fellow-students, from having the sound blasting ALL the time.

13.5 THE EXPERIMENT

13.5.1 Setting up the Apparatus

Please arrange and clamp your glass tube. The tube should have a small amount (one or two "pinches") of a light powder inside and distributed along the whole bottom length. You can do this by rotating the tube and tapping it to move the powder.

The usual powder is 'lycopodium powder', not naturally available in India. Use instead the pollen from pine tree blossoms, or from reed-flower blossoms. Both are excellent but require to be gathered at the right time of year! If no preparation has been made for this, you can even use the chalk dust from the chalkboard rail in your classroom!

Close the end of the tube with a piece of aluminium foil held in place by several layers of black electrical insulation tape. This will give a satisfactory sound reflection, and permit you to introduce other gases besides air just by poking a hole!

The driver from the loudspeaker horn is heavy. Clamp it solidly to a stand. You may be able to force on the threaded portion a short tube which will help you to hold the driver.

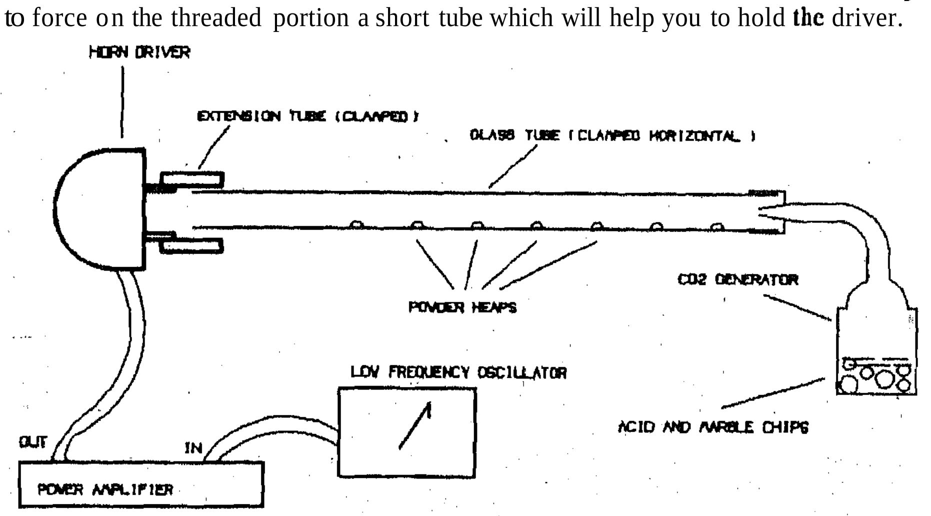

Fig.13.2

Clamp the entire apparatus as shown in Fig.13.2. It is important that the tube be horizontal - otherwise the powder will slowly drift out of the tube as it is agitated by the sound!

Once the apparatus is mechanically clamped please connect the driver to the output terminals of the power amplifier, and the output of the oscillator to the input terminals of the power amplifier. Of course both of these should be OFF when the connections are being made!

13.5.2 Measurement of Wavelength in Air.

Set the oscillator frequency to about 1000 hz. Set the output to about midway to the maximum.

Set the power amplifier volume to ZERO. Switch on the oscillatorand the power amplifier and wait a little while for them to function. After a minute or so, slowly increase the volume of the power amplifier until some sound is heard. If nothing is heard at all, TURN DOWN THE **VOLUME,** and go back to check your connections.

When some sound is heard. slowly increase the frequency of the oscillator. You will notice that at certain frequencies the sound is suddenly louder. These are the resonant frequencies.

STEP 1: Increase the power (the sound level) near such a resonance until you see the powder in the tube begin to "dance" a little. Then tune the oscillator very slowly to maximize the "dancing". Note the frequency of the oscillator and record in Table 1

STEP 2: Now just quickly turn up the volume and immediately turn it down again. This short blast of sound power will move the powder towards the nodes of the resonance. Do this a few times until the powder is in small and clearly-defined heaps. Then turn down the power to zero.

STEP 3: Measure the distance between the most widely-separated clear heaps, as well as the number of gaps between these heaps. Enter these values in Table 1

TABLE 1

	FREQUENCY	NO.OF SPACES	DISTANCE TO COVER THESE SPACES	WAVELENGTH (CALC.)
PART 1				
PART 2				

Average Wavelength = m/s

Statistical error, = m/s

Now turn up the power oscillator a bit and increase the frequency of the oscillator until another resonance is observed.

Repeat Steps 1 - 3, and do this for at least three more frequencies.

The result of these measurements will be a series of values for the speed of sound over the frequency range you covered.

Please now repeat the entire measurement sequence, starting again at about 1000 hz frequency. Enter all the readings in the second part of Table 13.1.

13.5.3 Measurement of Wavelength in Carbon-Dioxide

First you must set up a generator of carbon-dioxide gas. This is easily done with a little chemistry. Take the generator-jar and place a small handful of marble chips in thd bottom. Add hydrochloric acid to cover the chips, and close the apparatus. Carbon-dioxide should be generated for at least 1/2 hour in the small quantities you need.

Connect a rubber tubing to the generator, and a glass nozzle to the other end. Push the sharp glass nozzle through the black-tape-and-aluminium-foil end of the resonant tube. Gas will slowly flow into the tube and fill it after a few minutes. The gas is heavier than air so it will not easily just run out.

Now, repeat all the measurements suggested for'13.5.2, and enter the results in Table .2.

When you have finished; remove the gas generator, flush it with water to remove all traces of acid, and return the marble chip residue to its stock. Clean and dry the generator apparatus for the next student.

TABLE 2

	FREQUENCY	NO.OF SPACES	DISTANCE TO COVER THESE SPACES	WAVELENGTH (CALC.)
PART 1				
PART 2				

13.5.4 Calculations and Estimation of Errors

For each resonant frequency, calculate the speed of the sound wave, as suggested in 13.3.6. Form the average value for air, including all the readings in Part 1 and Part 2 of Table 1. Using all the values and your usual method, find the statistical error of the readings.

If you find a value greater than 2% for the error, then you should look at your readings for possible actual mistakes (wrong number of spaces counted, oscillator calibration no good, etc.). This is an experiment in which you can expect pretty good accuracy – at least 1% or less.

Now repeat the calculations for the readings in carbon-dioxide, and enter in the suitable part of Table 2.

Now using the equation suggested in 13.3.6 you can calculate the values for γ and see if they agree with the values predicted in 13.3.5. You can take the pressure from the daily weather map in your local newspaper, and the value for density you should find from a handbook in your library. Please look these up in advance!

13.6 CONCLUSIONS

You should be in a position now to say whether your experimental experience agrees with the predictions of the study material or not. You will probably find some deviations – the world is often not ideal! Please write here what you feel about the results you have obtained.

....

....

....

....

....

....

....