UNIT 15 REFLECTION AND REFRACTION OF ELECTROMAGNETIC WAVES

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1 INTRODUCTION

In Unit 14, you have studied Maxwell's equations and derived equations for electromagnetic waves from them. You have learnt that light and several other forms of radiation, **viz.** radiowaves, infrared, X-rays and gamma-rays are **electromagnetic** radiations. You have also obtained plane wave and sinusoidal solutions of the electromagnetic wave equations in empty space devoid of free charges and currents, and in dielectrics.

In this unit we come to another interesting question related to electromagnetic waves in dielectric media. What happens when an electromagnetic wave passes from one dielectric medium to another? For example, you know what happens when light passes from air to glass or air to water. You get a reflected wave and a refracted wave, which we also call the transmitted wave. We shall derive the equations for the reflected and transmitted electromagnetic waves, when such waves are incident perpendicular to the boundary of the media. So far we have not talked about how electromagnetic waves are generated. In the last section of this unit, you will study how an oscillating electric dipole produces electromagnetic waves. Finally we shall briefly discuss the antenna — a device widely used to transmit and receive electromagnetic waves.

Objectives

After studying this unit you should be able to

- solve problems based on reflection and refraction of electromagnetic waves at the boundaries of dielectric media
- explain qualitatively the generation of electromagnetic waves from an oscillating electric dipole antenna.

15.2 REFLECTION AND REFRACTION AT A BOUNDARY BETWEEN TWO DIELECTRIC MEDIA

By the term linear, we mean that D and H are proportional to E and B. Also ε and μ are independent of position and direction. Thus, we are talking about linear, homogeneous and isotropic media.

Let us consider the situation in which uniform plane electromagnetic waves are incident on a boundary between two linear, dielectric media, **e.g.,** light passing from air to glass, or from oil to water. Let us assume that there are no free charges or currents in the materials.

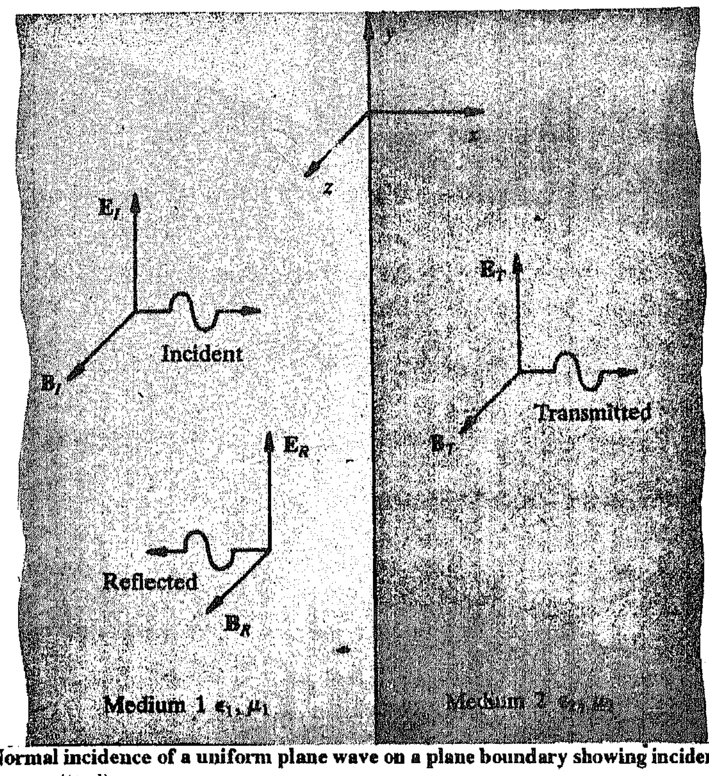


Fig.15.1: Normal incidence of a uniform plane wave on a plane boundary showing incident, reflected and refracted (transmitted) waves.

Fig. 15.1 shows a plane boundary between two dielectric media having different material properties: ε_1 , μ_1 for medium 1 and ε_2 , μ_2 for medium 2. A uniform plane wave travelling to the right in medium 1 is incident on the interface of the media, normal to the boundary. What happens to the wave? As in the case of waves on a string, and from our experience, we can expect to get a reflected wave propagating back into the medium and a transmitted (or refracted) wave travelling in the second medium. We would like to determine the expressions for the reflected and refracted waves in terms of the incident wave. We would also like to know what fraction of the incident energy is reflected and what transmitted? In order to do this we would need to know the **boundary conditions** satisfied by the waves at the interface of the media. ***These are the conditiotts we obtain when we stipulate that Maxwell's equations must be satisfied at the boundary between the media.*** So let us first obtain the appropriate boundary conditions.

15.2.1 Boundary Conditions

We will derive these boundary conditions from Maxwell's equations in a dielectric free of charge and currents. The relevant equations are:

over a closed surface S.

$$\text{(a)}\quad \varepsilon\int_S \mathbf{E}\cdot d\mathbf{S} = 0 \qquad (15.1a)$$

$$\text{(b)}\quad \int_S \mathbf{B}\cdot d\mathbf{S} = 0 \qquad (15.1b)$$

for any surface S bounded by the closed loop C.

$$\text{(c)}\quad \oint_C \mathbf{E}\cdot d\mathbf{l} = -\frac{d}{dt}\int_S \mathbf{B}\cdot d\mathbf{S} \qquad (15.1c)$$

$$\text{(d)}\quad \frac{1}{\mu}\oint_C \mathbf{B}\cdot d\mathbf{l} = \varepsilon\frac{d}{dt}\int_S \mathbf{E}\cdot d\mathbf{S} \qquad (15.16)$$

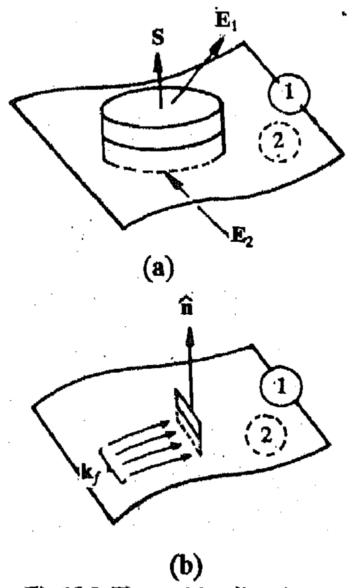


Fig.15.2: The positive direction for S and $E_\perp$ in (a) is from medium 2 towards 1.

Let us apply Eq. (15.1a) to a tiny, thin Gaussian pill box extending just a little bit (hair-like) on either side of the boundary of the media (Fig. 15.2a). Eq. (15.1a) implies

$$\varepsilon_1 \mathbf{E}_1 . \mathbf{S} - \varepsilon_2 \mathbf{E}_2 . \mathbf{S} = 0$$

The edge of the wafer contributes nothing in the limit as the thickness goes to zero. Thus, the components of the electric fields perpendicular to the interface satisfy the condition

$$\varepsilon_1 E_{1\perp} - \varepsilon_2 E_{2\perp} = 0 \qquad (15.2a)$$

Using the same process, we obtain from Eq. (15.1b) the following boundary condition for the magnetic fields:

$$B_{1\perp} - B_{2\perp} = 0 \qquad (15.2b)$$

We now apply Eq. (15.1c) to a thin Ampérian loop across the surface (Fig. 15.2b) and obtain

$$\mathbf{E}_1 . \mathbf{l} - \mathbf{E}_2 . \mathbf{l} = -\frac{d}{dt}\int_S \mathbf{B} . d\mathbf{S}$$

Now in the limit as the width of the loop goes to zero, the flux vanishes and the contribution of the two ends to $\oint_C \mathbf{E} . d\mathbf{l}$ is zero. Therefore,

$$(\mathbf{E}_1 - \mathbf{E}_2) . \mathbf{l} = 0$$

which implies that

$$\mathbf{E}_{1\parallel} - \mathbf{E}_{2\parallel} = 0 \qquad (15.2c)$$

This means that the components of E parallel to the interface are continuous across the boundary. In the same way, from Eq. (15.1d) we can obtain the condition that

$$\frac{1}{\mu_1}\mathbf{B}_1 . \mathbf{l} - \frac{1}{\mu_2}\mathbf{B}_2 . \mathbf{l} = 0$$

which yields

$$\frac{1}{\mu_1}\mathbf{B}_{1\parallel} - \frac{1}{\mu_2}\mathbf{B}_{2\parallel} = 0 \qquad (15.2d)$$

Thus, we have derived the boundary conditions satisfied by the electric and magnetic fields at the interface of two linear dielectric media where there is no free charge or current. Let us put them together:

$$\varepsilon_1 E_{1\perp} = \varepsilon_2 E_{2\perp} \qquad (15.2a)$$

$$B_{1\perp} = B_{2\perp} \qquad (15.2b)$$

$$\mathbf{E}_{1\parallel} = \mathbf{E}_{2\parallel} \qquad (15.2c)$$

$$\frac{1}{\mu_1}\mathbf{B}_{1\parallel} = \frac{1}{\mu_2}\mathbf{B}_{2\parallel} \qquad (15.2d)$$

We shall now use these boundary conditions to study reflection and refraction (transmission) at normal incidence.

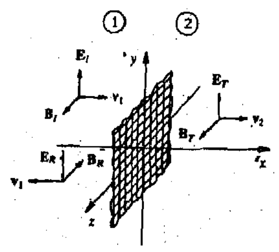


Fig.15.3: A sinusoidal plane electromagnetic wave incident normally at the boundary of two media.

15.2.2 Reflection and Refraction at Normal Incidence

Consider Fig. 15.3. Suppose the yz plane forms the boundary between the two linear dielectric media. Let a sinusoidal incident plane wave of frequency ω, travelling in the x direction approach the interface from the left. Suppose its electric field is along the y direction. The electric and magnetic fields of the incident wave are given by

$$\mathbf{E}_I(x,t) = E_{0I}\,\hat{\mathbf{j}}\exp[-i(\omega t - k_I x)] \qquad (15.3a)$$

$$\mathbf{B}_I(x,t) = \frac{E_{0I}}{v_1}\hat{\mathbf{k}}\exp[-i(\omega t - k_I x)] \qquad (15.3b)$$

At the interface of the media, the incident plane wave gives rise to a reflected wave and a refracted (or transmitted) wave. The reflected wave propagates back into the f i t medium and can be represented by the following E and B fields:

$$\mathbf{E}_R(x,t) = E_{0R}\,\hat{\mathbf{j}}\exp[-i(\omega t + k_I x)] \quad (15.4a)$$

$$\mathbf{B}_R(x,t) = -\frac{E_{0R}}{v_1}\hat{\mathbf{k}}\exp[-i(\omega t + k_I x)] \quad (15.4b)$$

Why have we put the minus sign in Eq. (15.4b)? This is because the direction of propagation is reversed and the fields of the wave must obey the relation

$$\mathbf{B}_R = \frac{1}{v_1}(\hat{\mathbf{k}}_I \times \mathbf{E}_R)$$

Will you now like to try writing down the E and B fields of the transmitted wave which travels to the right in medium 2? Fill in the following blanks.

$$\mathbf{E}_T(x,t) = \ldots\ldots\ldots\ldots\ldots \quad (15.5a)$$

$$\mathbf{B}_T(x,t) = \ldots\ldots\ldots\ldots\ldots \quad (15.5b)$$

These three electric and magnetic field vectors must satisfy the boundary conditions given by Eqs. (15.2) at every point on the plane interface at all times. Thus, at $x = 0$, the combined field to the left, viz. $\mathbf{E}_I + \mathbf{E}_R$ and $\mathbf{B}_I + \mathbf{B}_R$, must join the fields to the right, $\mathbf{E}_T$ and $\mathbf{B}_T$, according to the boundary conditions. In this case there are no field components perpendicular to the interface, since neither E nor B field is in the x direction. Thus, Eq. (15.2a and b) are trivial. The remaining Eqs. (15.2c and d) require that

$$E_{0I} + E_{0R} = E_{0T} \quad (15.6a)$$

$$\frac{1}{\mu_1}(B_{0I} + B_{0R}) = \frac{1}{\mu_2}B_{0T}$$

or

$$\frac{1}{\mu_1}\left(\frac{E_{0I}}{v_1} - \frac{E_{0R}}{v_1}\right) = \frac{1}{\mu_2}\frac{E_{0I}}{v_2}$$

or

$$E_{0I} - E_{0R} = \alpha E_{0T} \quad (15.6b)$$

where

$$\alpha = \frac{\mu_1 v_1}{\mu_2 v_2} = \sqrt{\frac{\mu_1 \varepsilon_1}{\mu_2 \varepsilon_2}} = \frac{\mu_1 n_2}{\mu_2 n_1} \quad (15.6c)$$

You may like to solve Eqs. (15.6a) and (15.6b) to obtain the reflected and transmitted amplitudes in terms of the incident amplitude. Try the following SAQ.

SAQ 1

Spend 5 min

Show that

$$E_{0R} = \left(\frac{1-\alpha}{1+\alpha}\right)E_{0I} \quad (15.7a)$$

$$E_{0T} = \frac{2}{1+\alpha}E_{0I} \quad (15.7b)$$

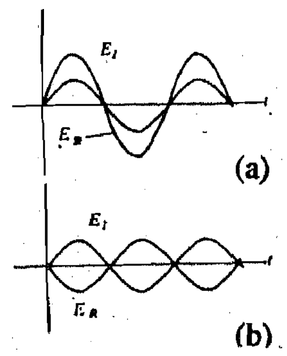


Fig. 15.4: (a) The reflected wave is in phase with the incident wave; (b) the reflected wave is out of phase with the incident wave.

For most dielectric media, the permittivities are close to their values in vacuum. In such cases $\alpha = \frac{v_1}{v_2}$ and we have

$$E_{0R} = \left(\frac{v_2 - v_1}{v_2 + v_1}\right)E_{0I},\; E_{0T} = \frac{2v_2}{v_2 + v_1}E_{0I} \quad (15.8)$$

The reflected wave is in phase with the incident wave if $v_2 > v_1$ and out of phase if $v_2 < v_1$ (See Fig. 15.4). In terms of the index of refraction $n = \frac{c}{v}$, we can write Eqs. (15.8) as

$$E_{0R} = \frac{n_1 - n_2}{n_1 + n_2} E_{0I}, E_{0T} = \frac{2\,n_1}{n_1 + n_2} E_{0I} \qquad (15.9)$$

when $n_1 < n_2$, **i.e.**, when the wave passes from a less dense medium to a more dense medium, the reflected wave is 180 "out of phase with the incident wave. This is well known in optics.

What fraction of incident energy is reflected and what fraction transmitted? You may like to work out this result yourself!

Spend 10 min

SAQ 2

Given that the intensity (average power per unit area) is $I = \frac{1}{2} \varepsilon\, v E_0^2$, and $\mu_1 = \mu_2 = \mu_0$, show that the ratio of reflected intensity to incident intensity is

$$R = \frac{I_R}{I_I} = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2 \qquad (15.10a)$$

and the ratio of the transmitted intensity to the incident intensity is

$$T = \frac{I_T}{I_I} = \frac{n_2}{n_1}\left(\frac{2\,n_1}{n_1 + n_2}\right)^2 \qquad (15.10b)$$

Show that $R + T = 1$. (15.10c)

R is called the **reflection coefficient** and T, the **transmission coefficient** of the surface. They measure the fraction of incident energy that is reflected and transmitted, respectively. You can now explain why most of the light is transmitted when it passes from air ($n_1 = 1$) to glass ($n_2 = 1.5$). You only need to calculate R and T to know the answer! Why don't you do so before studying further?

We will end this section by deriving the laws of reflection and refraction in a simple manner for the case of oblique incidence.

15.2.3 Laws of Reflection and Refraction

Consider Fig. 15.5. An incident plane wave in medium 1, at an angle θ_I, results in a reflected wave in 1 at an angle θ_R and a transmitted wave in medium 2.

We represent the waves by the following plane wave forms:

$$\mathbf{E}_I = \mathbf{E}_{0I}\, e^{-i(\omega_I t - \mathbf{k}_I \cdot \mathbf{r})}, \mathbf{B}_I = \frac{1}{v_1} \hat{\mathbf{k}}_I \times \mathbf{E}_I$$

$$\mathbf{E}_R = \mathbf{E}_{0R}\, e^{-i(\omega_R t - \mathbf{k}_R \cdot \mathbf{r})}, \mathbf{B}_R = \frac{1}{v_1} \hat{\mathbf{k}}_R \times \mathbf{E}_R$$

$$\mathbf{E}_T = \mathbf{E}_{0T}\, e^{-i(\omega_T t - \mathbf{k}_T \cdot \mathbf{r})}, \mathbf{B}_T = \frac{1}{v_2} \hat{\mathbf{k}}_T \times \mathbf{E}_T$$

Recall that in **Sec.** 15.2.2 we had said that the boundary conditions (Eqs. 15.2) **must** hold at every point on the interface at all times. If the boundary conditions hold at one point and at one time, they will hold at all points and all times only if the phases of the three waves are equal, **i.e.**

$$\omega_I t - \mathbf{k}_T \cdot \mathbf{r} = \omega_R t - \mathbf{k}_R \cdot \mathbf{r} = \omega_T t - \mathbf{k}_T \cdot \mathbf{r}$$

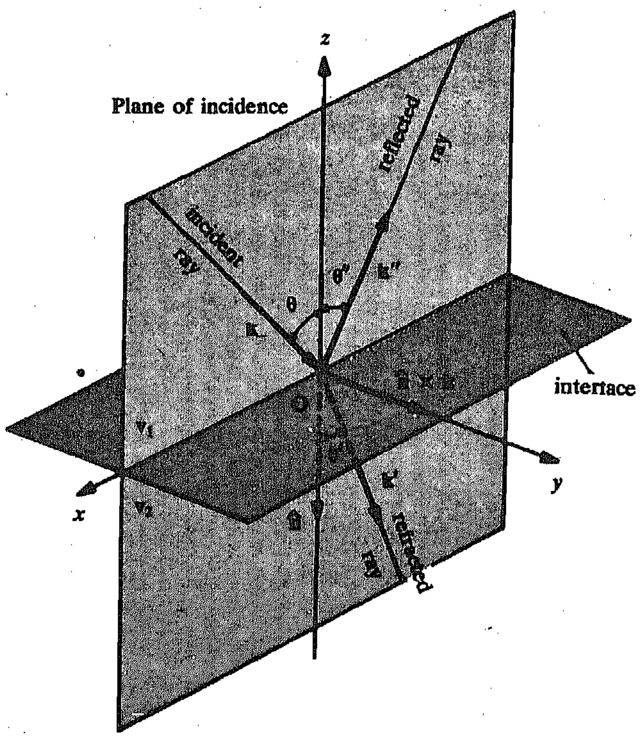


Fig.15.5: A plane wave, represented by propagation vector k, encounters an interface between two media (ε_1, μ_1 and ε_2, μ_2) **. The** normal *to the interface,*pointing *into* medium 2, **is** *the unit* **vector** $\hat{\mathbf{n}}$**.**

The equality of phases at all times requires that the three angular frequencies must be the same:

$$\omega_I = \omega_R = \omega_T = \omega \qquad (15.11a)$$

Since $k = \frac{\omega}{c} n$, we have

$$k_I = k_R \text{ and } \frac{k_I}{n_1} = \frac{k_T}{n_2} \qquad (15.11b)$$

Equality of phases for all points on the interface requires that

$$\mathbf{k}_I \cdot \mathbf{r} = \mathbf{k}_R \cdot \mathbf{r} = \mathbf{k}_T \cdot \mathbf{r} \qquad (15.12)$$

for all **r** on the interface. These equations yield

$$(\mathbf{k}_I - \mathbf{k}_R) \cdot \mathbf{r} = 0 \text{ and } (\mathbf{k}_I - \mathbf{k}_T) \cdot \mathbf{r} = 0$$

Since $\mathbf{r} \neq 0$, the above equation tell us that either

i) $\mathbf{k}_I = \mathbf{k}_R$ and $\mathbf{k}_I = \mathbf{k}_T$

or

ii) $(\mathbf{k}_I - \mathbf{k}_R)$ and $(\mathbf{k}_I - \mathbf{k}_T)$ are perpendicular to **r**, for all r on the surface.

Case(i) is the trivial case of no reflection and no refraction. So we consider only case (ii). Since **r** is any vector in the plane interface, condition (ii) will be satisfied only if $(\mathbf{k}_I - \mathbf{k}_R)$ and $(\mathbf{k}_I - \mathbf{k}_T)$ are along the normal to the plane interface. If $\hat{\mathbf{n}}$ represents the unit vector normal to the plane, it will be parallel to $(\mathbf{k}_I - \mathbf{k}_R)$ and $(\mathbf{k}_I - \mathbf{k}_T)$. So we can write that

$$\hat{\mathbf{n}} \times (\mathbf{k}_I - \mathbf{k}_R) = 0$$

and $$\hat{\mathbf{n}} \times (\mathbf{k}_I - \mathbf{k}_T) = 0$$

This gives us that

$$\hat{\mathbf{n}} \times \mathbf{k}_I = \hat{\mathbf{n}} \times \mathbf{k}_R = \hat{\mathbf{n}} \times \mathbf{k}_T \qquad (15.13)$$

Eq. (15.13) gives us the following informations.

1. $\hat{\mathbf{n}} \times \mathbf{k}_I = \hat{\mathbf{n}} \times \mathbf{k}_R$ says that the plane **defined** by $\hat{\mathbf{n}}$, $\mathbf{k}_I$ (the **plane** of **incidence)** coincides with the plane defined by $\hat{\mathbf{n}}$, $\mathbf{k}_T$ (the plane of refraction).

2. The equality of magnitudes

   $|\hat{\mathbf{n}} \times \mathbf{k}_I| = |\hat{\mathbf{n}} \times \mathbf{k}_T|$ gives the condition that

   $$k_I \sin\theta_I = k_T \sin\theta_T \qquad (15.14)$$

3. Similarly the relation

   $$\hat{\mathbf{n}} \times \mathbf{k}_I = \hat{\mathbf{n}} \times \mathbf{k}_R$$

   tells us that the plane of incidence coincides with the plane formed by $\hat{\mathbf{n}}$ and $\mathbf{k}_R$, the plane of reflection and

   $$k_I \sin\theta_I = k_R \sin\theta_R \qquad (15.15)$$

   Since $k_I = k_R$ from Eq. **(15.11b),** Eq. (15.15) yields

   $$\theta_R = \theta_I \qquad \text{(Law of reflection)} \qquad (15.16)$$

   Substituting Eq. (15.14) in Eq. **(15.11b)** we get

   $$\frac{\sin\theta_I}{\sin\theta_T} = \frac{n_2}{n_1} \qquad \text{(Snell's Law of refraction)} \qquad (15.17)$$

Thus, we have arrived at the well known laws of reflection and refraction in optics.

We end this section with an SAQ for you.

Spend 10 min

SAQ3

(a) A uniform plane wave whose electric field is given by

$$\mathbf{E}_I = 100\cos(\omega t - 6\pi x)\,\hat{\mathbf{z}}\ \mathrm{Vm^{-1}}$$

is incident from a region having $\varepsilon_1 = 4\varepsilon_0$, $\mu_1 = \mu_0$ normal to the plane surface of a material having $\varepsilon_2 = 9\varepsilon_0$, $\mu_2 = 4\mu_0$.

Write **complete** expressions for the incident, reflected and transmitted electric and magnetic fields.

(b) A **plane** electromagnetic wave propagates from one dielectric to another at normal incidence. Find the ratio of the indices of refraction of the two dielectrics for which the reflection and transmission coefficients are both equal to 0.5.

15.3 GENERATION OF ELECTROMAGNETIC WAVES

So far you have studied **about** the existence of electromagnetic waves, their properties in free space and in **dielectrics,** and their reflection and refraction at the boundaries of dielectric media. But how are these electromagnetic waves generated? This is the question we are going to answer in the present section.

From what you have studied about the nature of electromagnetic waves, you **can** guess that to generate such a wave, we must create a changing electric or magnetic field. Once we do so, the self-regenerating process described by **Ampére**-Maxwell's Law and Faraday's law occurs, and the wave propagates on its own. So how do we create a changing electric or magnetic field?

We create changing fields when we **alter** the motion of an electric charge. This can be done by accelerating an electric charge. It is also possible to create such fields when **an** electric dipole oscillates. Let us now study qualitatively how electromagnetic waves are generated from oscillating electric dipoles.

15.3.1 Radiation from an Oscillatory Electric Dipole

Fig. 15.6 shows an experimental arrangement depicting an oscillating electric dipole. Since an ac source of current is connected to the wires, charges will move back and forth along the vertical wires. This will produce an electric dipole, alternately pointed up and then down. Currents also flow in the wires, producing a magnetic field B around the wires. The B field also reverses direction periodically. Let us now examine the electric and magnetic fields produced near the oscillating dipole. When the charges near the ends of the wire are at a maximum, the current will be zero. Consider a point P near the wire. The E field at P will be a maximum when B is zero, and vice-versa. Thus, near the wire, E and B are 90° out of phase in time. Hence, the direction of energy flow which is determined by the Poynting vector $\mathbf{S} = \frac{1}{\mu_0}(\mathbf{E} \times \mathbf{B})$ alternates between outward and inward directions. So the net energy flow at P is zero. Thus, the near field of an oscillating dipole moves energy in and out equally and does not result in a radiated wave.

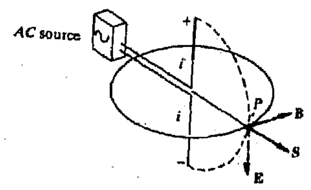


Fig.15.6: An oscillating electric **dipole. The E** and **B fields resulting from currents and charges** on **the wire** are **shown** along with **the Poynting vector S.** At **the instant shown, the Poynting** vector **is directed** outward at the **point *P*.** At **greater distances there** is a **net flow of energy, produced** by **the local time-varying E and** B **fields.**

What happens at distances far away from the oscillating dipole? Recall that time-varying E and B fields can produce B and E fields, respectively. **The time**-varying electric and magnetic fields generated by the oscillating dipole continue moving outward, because the local time-varying E fields produce in-phase B fields and in-phase E fields are produced by the local time-varying B fields (in accordance with the two Maxwell equations for induced electric and magnetic fields). Thus, the total E and B fields at P are the vector sums of the out-of-phase components generated by the oscillating electric dipole and in- phase fields induced locally by the local time-varying fields. The energy density of out-of-phase components falls off rapidly with distance as $\frac{1}{r^3}$. **The** in-phase components provide the net transfer of energy' and become the dominant fields at large distances from the dipole. We **can** see that the time-varying fields near the dipole serve to launch the electromagnetic waves at large distances.

An interesting device based on this simple process of generating electromagnetic waves is the **antenna.** Can you think of any modem **communication** network without an antenna? An antenna is used for the **generation** (transmission) as well as for the reception of electromagnetic waves. In the last subsection of this **unit** we are briefly going to discuss the antenna.

15.3.2 Antenna

The antenna based on the mechanism of an oscillating electric dipole is termed the **oscillating** electric dipole **antenna.** It is, perhaps, the simplest of antenna systems. Such an antenna consists of two conducting rods connected to an ac generator as shown in Fig. 15.6. Fig. 15.7 shows the **electromagnetic** wave generated by this kind of antenna. The field pattern shown in the figure holds for all radial distances r from the antenna such that $r > \lambda$, where A is the wavelength of the electromagnetic wave. Remember that certain electromagnetic radiations such as X-rays, gamma rays and **light** come from atomic and nuclear sources.

In this case, we have restricted outselves to the region of the spectrum ($\lambda \approx$ 1mm to 1m) in which the source of radiation is both macroscopic and of manageable dimensions. Essentially we are speaking of radio wave aisd microwave generation.

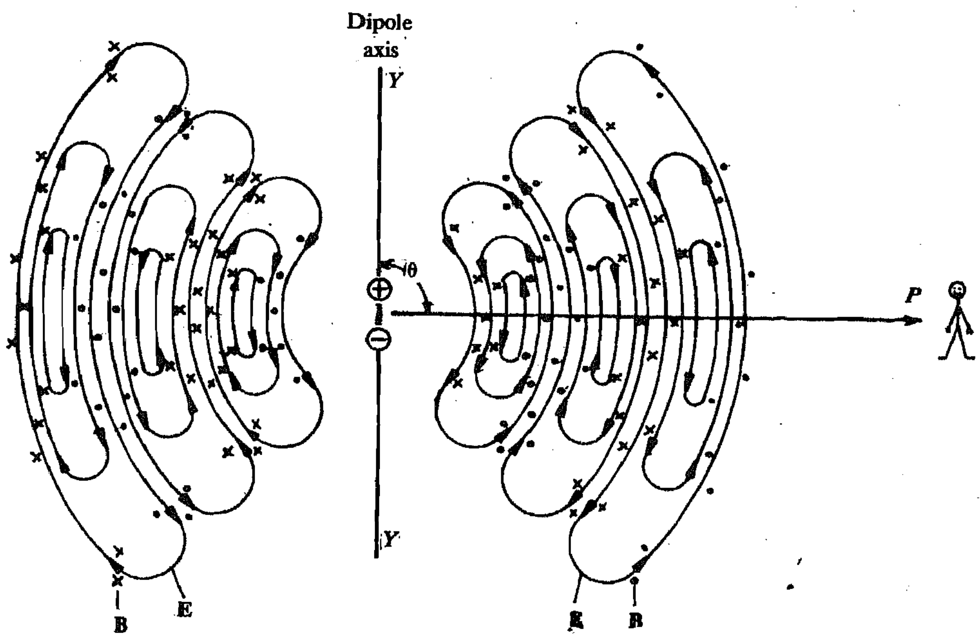
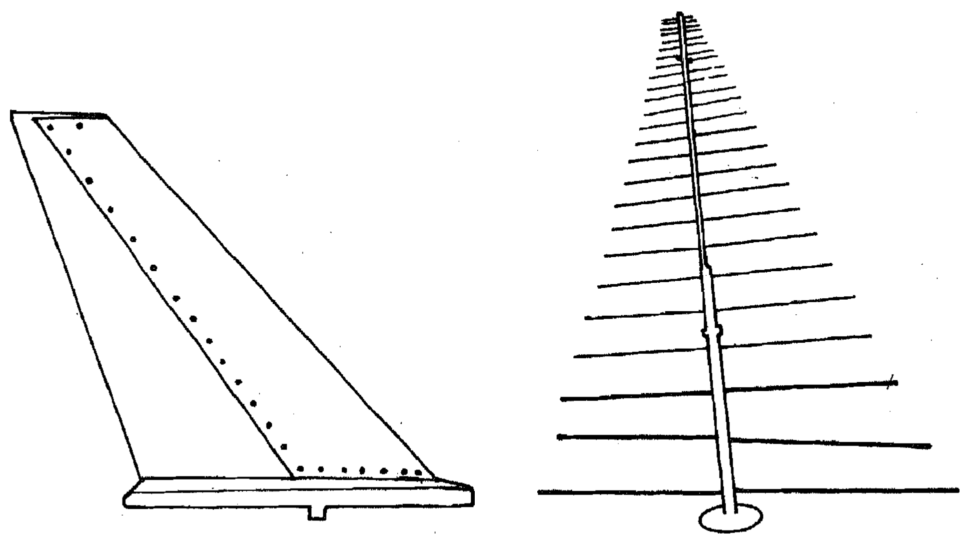


Fig.15.7: The electric and magnetic field lines associated with the electromagnetic wave radiated by an oscillating dipole antenna. The dots and crosses represent field lines emerging from and entering into the plane of the figure. The field pattern close to the antenna is not shown.

Antennas are made of metals as well as dielectrics and come in different shapes. In fact, any metallic or dielectric structure which is designed so as to launch (or radiate) waves efficiently into space and to focus (or concentrate) these waves in a particular direction is referred to as an antenna. Two common antennas in use today for various purposes are shown in Fig. 15.8.



(a) (b)

Fig.15.8: (a) Monopole (blade) antenna used on aircraft; (b) log-periodic antenna.

Let us now summarise what you have studied in this unit.

15.4 SUMMARY

- The boundary conditions satisfied by electromagnetic waves at the interface of two charge-free and current-free dielectric media characterised by ϵ_1, μ_1 (medium 1) and ϵ_2, μ_2 (medium 2) are given as follows:

$$\varepsilon_1 E_{1\perp} = \varepsilon_2 E_{2\perp}$$

$$B_{1\perp} = B_{2\perp}$$

$$\mathbf{E}_{1\parallel} = \mathbf{E}_{2\parallel}$$

$$\frac{1}{\mu_1}\mathbf{B}_{1\parallel} = \frac{1}{\mu_2}\mathbf{B}_{2\parallel}$$

The boundary conditions are consequences of Maxwell's equations.

- The **incident, reflected** and **transmitted** waves at the interface of two dielectric media can be represented as follows:

$$\mathbf{E}_I = \mathbf{E}_{0I} \exp\left[-i(\omega_I t - \mathbf{k}_I \cdot \mathbf{r})\right]$$

$$\mathbf{B}_I = \frac{1}{v_1}\hat{\mathbf{k}}_I \times \mathbf{E}_I$$

$$\mathbf{E}_R = \mathbf{E}_{0R} \exp\left[-i(\omega_R t + \mathbf{k}_I \cdot \mathbf{r})\right]$$

$$\mathbf{B}_R = \frac{1}{v_1}\hat{\mathbf{k}}_I \times \mathbf{E}_R$$

$$\mathbf{E}_T = \mathbf{E}_{0T} \exp\left[-i(\omega_T t - \mathbf{k}_T \cdot \mathbf{r})\right]$$

$$\mathbf{B}_T = \frac{1}{v_2}\hat{\mathbf{k}}_T \times \mathbf{E}_T$$

- The **amplitudes** of the electric fields of the **reflected** and **transmitted waves** when the incident wave is normal to the interface of the dielectric media are:

$$E_{0R} = \left(\frac{1-\alpha}{1+\alpha}\right)E_{0I}$$

$$E_{0T} = \frac{2}{1+\alpha}E_{0I}$$

where

$$\alpha = \frac{\mu_1 v_1}{\mu_2 v_2}$$

- The **reflection and transmission coefficients** are defined as the ratio of reflected intensity to incident intensity, and that of transmitted intensity to incident intensity, respectively. For the dielectric media for which $\mu_1 = \mu_2 = \mu_0$, these are given as

$$R = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2$$

$$T = \frac{n_2}{n_1}\left(\frac{2 n_2}{n_1 + n_2}\right)^2$$

- The **law** of **reflection** and **Snell's law** of **refraction** for oblique incidence are derivable from **boundary** conditions in electromagnetic fields.
- An **oscillating electric dipole** generates electromagnetic waves. The **antenna,** a device used for radiating and receiving electromagnetic waves in the radio and micro frequency region is based on this principle.

15.5 TERMINAL QUESTIONS

Spend 20 min

1. A uniform plane wave has a wavelength of 3 cm in free space and 2 cm in a dielectric for which $\mu = 4.7 \times 10^{-7}$ NA^{-2}. Determine the dielectric constant of the dielectric.

2. A uniform plane wave of 200 MHz travelling in free space strikes a large block of a material having $\varepsilon = 4\varepsilon_0$, $\mu = 9\mu_0$ and $\sigma = 0$ normal to the surface. If the incident magnetic field vector is given by

$$\mathbf{B} = 10^{-8}\cos(\omega t - \beta y)\,\hat{\mathbf{z}}\ \text{tesla}$$

write complete expressions for the incident, reflected, and transmitted field vectors.

15.6 SOLUTIONS AND ANSWERS

Self-Assessment Questions (SAQs)

1. Summing Eqs. (15.6a and b) we get

$$2E_{0I} = (1+\alpha)E_{0T}$$

or

$$E_{0T} = \frac{2}{1+\alpha}E_{0I}$$

Substituting E_{0T} in Eq. (15.6a)

$$E_{0I} + E_{0R} = \frac{2}{1+\alpha}E_{0I}$$

or

$$E_{0R} = \left(\frac{2}{1+\alpha} - 1\right)E_{0I} = \left(\frac{1-\alpha}{1+\alpha}\right)E_{0I}$$

2. $$R = \frac{I_R}{I_I} = \frac{\varepsilon_1 v_1 E_{0R}^2}{\varepsilon_1 v_1 E_{0I}^2} = \left(\frac{E_{0R}}{E_{0I}}\right)^2 = \left(\frac{1-\alpha}{1+\alpha}\right)^2$$

where $\alpha = \dfrac{\mu_1 n_2}{\mu_2 n_1} = \dfrac{n_2}{n_1}$ in this case, since $\mu_1 \simeq \mu_2$

$$\therefore R = \left(\frac{1 - n_2/n_1}{1 + n_2/n_1}\right)^2 = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2$$

$$T = \frac{I_T}{I_I} = \frac{\varepsilon_2 v_2 E_{0T}^2}{\varepsilon_1 v_1 E_{0I}^2} = \frac{\varepsilon_2 v_2^2}{\varepsilon_1 v_1^2}\cdot\frac{v_1}{v_2}\left(\frac{2}{1 + n_2/n_1}\right)^2$$

$$= \frac{n_2}{n_1}\left(\frac{2n_1}{n_1 + n_2}\right)^2$$

since $n_1 = c/v_1$, $n_2 = c/v_2$, $v_1^2 = \dfrac{1}{\mu_1\varepsilon_1}$, $v_2^2 = \dfrac{1}{\mu_2\varepsilon_2}$

3.a) $\mathbf{E}_I = 100\cos(\omega t - 6\pi x)\,\hat{\mathbf{z}}$ Vm^{-1}

Since $\hat{\mathbf{k}}$ is in the x-direction, $\mathbf{B}_I$ will be in the y-direction. Its magnitude is given by the relation

$$\frac{|\mathbf{E}_I|}{|\mathbf{B}_I|} = v_1,\ \text{where}\ v_1 = \frac{1}{\sqrt{\varepsilon_1\mu_1}}$$

It is given that $\varepsilon_1 = 4\varepsilon_0$, $\mu_1 = \mu_0$

$$\therefore v_1 = \frac{c}{\sqrt{4}} = 0.5\, cm\, s^{-1}$$

Thus $\mathbf{B}_I = \frac{200}{c} \cos(\omega t - 6\pi x)\,\hat{\mathbf{y}}$ tesla.

The reflected wave travels back in the negativex- direction in medium 1. The E and B fields of the reflected wave are

$$\mathbf{E}_R = E_{0R} \cos(\omega t + 6\pi x)\,\hat{\mathbf{z}}$$

where $E_{0R} = \left(\frac{1-\alpha}{1+\alpha}\right) E_{0I}$

$$\alpha = \frac{\mu_1 v_1}{\mu_2 v_2} = \frac{c}{4} \times \frac{0.5\text{ms}^{-1}}{v_2}$$

where $v_2 = \frac{1}{\sqrt{\varepsilon_2 v_2}} = \frac{c}{\sqrt{9 \times 4}} = \frac{c}{6}\text{ms}^{-1}$

$$\therefore \alpha = 0.75$$

and $E_{0R} = \frac{0.25}{1.75} E_{0I} = \frac{1}{7} E_{0I}$

Thus,

$$\mathbf{E}_R = \frac{100}{7} \cos(\omega t + 6\pi x)\,\hat{\mathbf{z}}\ \text{Vm}^{-1}$$

and

$$\mathbf{B}_R = -\frac{200}{7c} \cos(\omega t + 6\pi x)\,\hat{\mathbf{y}}\ \text{tesla}$$

The E and B fields of the transmitted wave are

$$\mathbf{E}_T = E_{0T} \cos(\omega t - k_2 x)\,\hat{\mathbf{z}}$$

where

$$E_{0T} = \left(\frac{2}{1+\alpha}\right) E_{0I} = \frac{2}{1.75} E_{0I}$$

$$= \frac{8}{7} E_{0I}$$

and $k_2 = \frac{\omega}{v_2} = 6\frac{\omega}{c}.$

$$\therefore \mathbf{E}_T = \frac{800}{7} \cos\left(\omega t - \frac{6\omega x}{c}\right)\hat{\mathbf{z}}$$

and

$$\mathbf{B}_T = \frac{4800}{7c} \cos\left(\omega t - \frac{6\omega x}{c}\right)\hat{\mathbf{y}}$$

$$\left[\because |\mathbf{B}_T| = \left|\frac{\mathbf{E}_T}{\mathbf{v}_2}\right|\right]$$

b) It is given that R = T = 0.5

Let

Then we have that

$$\left(\frac{1-\beta}{1+\beta}\right)^2 = \beta\left(\frac{2}{1+\beta}\right)^2$$

or $$(1-\beta)^2 = 4\beta$$

or $$1-\beta+\beta^2 = 0$$

whence $$\beta = 5.83.$$

Terminal Questions

1. The dielectric constant is given by $\kappa = \frac{\varepsilon}{\varepsilon_0}$. It is given that the plane wave has wavelength 3 cm in free space and 2 cm in the dielectric. Hence, the speed of the wave in the dielectric is

$$v = \frac{\lambda_2 c}{\lambda_1} = \frac{2c}{3}$$

Thus

$$\frac{v}{c} = \frac{2}{3} = \sqrt{\frac{\mu_0\varepsilon_0}{\mu\varepsilon}}$$

or $$\frac{4}{9} = \frac{\mu_0\varepsilon_0}{\mu\varepsilon}$$

$$\therefore \kappa = \frac{\varepsilon}{\varepsilon_0} = \frac{9}{4}\frac{\mu_0}{\mu} = \frac{9\times 1.257\times 10^{-6}\ \mathrm{NA^{-2}}}{4\times 4.7\times 10^{-7}\ \mathrm{NA^{-2}}}$$

$$= 6$$

2. The incident magnetic field is given by

$$\mathbf{B}_I = 10^{-8}\cos(\omega t - \beta y)\,\hat{\mathbf{z}}\ \text{tesla}$$

where $\omega = 200$ MHz and $\beta = \frac{\Omega}{c} = \frac{2}{3}\mathrm{m}^{-1}$.

Titus, the wave is travelling in the y-direction. So the electric field is in the *x*-**direction**. The magnitude of the electric field can be obtained from

$$|\mathbf{E}_I| = c\,|\mathbf{B}_I| = 3\times 10^8\times 10^{-8}\ \mathrm{Vm^{-1}}$$

$$= 3\ \mathrm{Vm^{-1}}$$

$$\therefore \mathbf{E}_I = 3\cos(\omega t - \beta y)\,\hat{\mathbf{x}}\ \mathrm{Vm^{-1}}$$

The reflected wave travels in the negative x-direction with amplitude

$$E_{0R} = \left(\frac{1-\alpha}{1+\alpha}\right)E_{0I}$$

where

$$\alpha = \frac{\mu_1 v_1}{\mu_2 v_2} = \frac{c\,\mu_0\sqrt{4\,\varepsilon_0\times 9\,\mu_0}}{9\,\mu_0}$$

$$= \frac{6}{9} = \frac{2}{3}$$

$$\therefore E_{0R} = 0.2E_{0I}$$

$$\therefore \ \mathbf{E}_R = 0.6 \cos(\omega t + \beta y)\,\hat{\mathbf{x}}\ \text{Vm}^{-1}$$

and

$$\mathbf{B}_R = -2 \times 10^{-9} \cos(\omega t + \beta y)\,\hat{\mathbf{z}}\ \text{tesla}$$

The E and B fields of the transmitted wave are

$$\mathbf{E}_T = E_{0T} \cos(\omega t - \beta_2 y)\,\hat{\mathbf{x}}\ \text{Vm}^{-1}$$

where

$$E_{0T} = \frac{2E_{0I}}{1+\alpha} = \frac{2 \times 3 E_{0I}}{5} = 1.2 E_{0I}$$

and

$$\beta_2 = \frac{\omega}{v_2} = \frac{200\ \text{MHz} \times 6}{3 \times 10^8\ \text{ms}^{-1}} = 4\ \text{m}^{-1}$$

$$\therefore \ \mathbf{E}_T = 3.6 \cos(\omega t - 4y)\,\hat{\mathbf{x}}\ \text{Vm}^{-1}$$

and

$$\mathbf{B}_T = \frac{3.6}{v_2} \cos(\omega t - 4y)\,\hat{\mathbf{z}}\ \text{tesla}$$

$$= 7.2 \times 10^{-8} \cos(\omega t - 4y)\,\hat{\mathbf{z}}\ \text{tesla}$$

FURTHER READING

1. *Electricity and Magnetism* - Volume 2; Edward M. Purcell; International Student Edition, McGraw-Hill Book Company; 1985.
2. *Introduction to Electrodynamics* - David J. Griffiths; Prentice Hall of India Private Limited, 1984
3. *Fundamentals of Electricity and Magnetism* - Arthur F. Kip; International Student Edition; McGraw-Hill International Book Company; 1984

Table of Constants

Symbol	Quantity	Value
c	speed of light in vacuum	2.998×10^{8} m s^{-1}
μ_0	permeability of free space	1.257×10^{-6} N A^{-2}
ε_0	permittivity of free space	8.854×10^{-12} $C^2N^{-1}m^{-2}$
$1/4\pi\varepsilon_0$		8.988×10^{9} N m^2C^{-2}
e	charge of the proton	1.602×10^{-19}C
$-e$	charge of the electron	-1.602×10^{19}C
h	Planck's constant	6.626×10^{-34}J s
$\hbar$	$h/2\pi$	1.055×10^{-34}J s
m_e	electron rest mass	9.109×10^{-31}kg
$-e/m_e$	electron charge to mass ratio	-1.759×10^{11}C kg^{-1}
m_p	proton rest mass	1.673×10^{-27}kg
m_n	neutron rest mass	1.675×10^{-27}kg
R	Rydberg constant	$1.097 \times 10^{7}m^{-1}$
a_0	Bohr radius	5.292×10^{-11}m
N_A	Avogadro constant	$6.022 \times 10^{23}mol^{-1}$
R	Universal gas constant	8.314 J K^{-1} mol^{-1}
k_B	Boltzmann constant	1.381×10^{-23}J K^{-1}
G	Universal gravitational constant	6.673×10^{-11}N m^2kg^{-2}

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33	0909	New Progressive Education Trust, Mehsana-384 002, **Gujarat**
34	0922 (R)	Shree Gattu Vidyalaya,Plot No 910, GIDC Estate,Ankleshwar,**Gujarat**
35	0928(R)	National Institute for Management and Information Technology(NIMIT) C/o Parag Ad, Jansatta Press, Rajkot-5
36	2901	Govt Arts College, Daman and Diu (U.T.)-396 310
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38	1005	Chhotu Ram College of Education, Rohtak-124 001, **Haryana** (All India Jat Heroes Memorial Collcgc. Rohtak, **Haryana**).
33	1008 **Haryana**	Govt. College (Girls Wing), Sector-14, Railway Road, Karnal-132 001,
40	1009	Govt. P.G. College, Hissar-125 001, **Haryana.**
41	1012	Markanda National College, Shahabad, Kurukshetra, **Haryana**
42	1013	Government P.G. College, Jind-126 102, **Haryana**
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59	1s;?	Shri Narayan College, Kannur-670 007
60	1412	St. Alberts College, Ernakulam-682 018, Kerala.