UNIT 8 ELECTRIC CURRENT

Structure

8.1 INTRODUCTION

Water molecules flowing down a river constitute a water current. Analogously, electric charges flowing in a wire constitute an electric current. All the electrical appliances we use, such as the radio, electric heater and torch light, depend on the flow of electric charge. Such motion of charge usually occurs in **conductors** which contain free electrons; in the ionized gases of fluorescent lamps which contain charge **carriers of** both signs; and also **in** an evacuated region, for example, electrons in a TV **picture** tube.

In the last two Blocks we deait with electrostatics, in which charges are at **rest.** With this background, we now begin our study of electric charge that moves or flows from one point to another. You will find that an electric current results from charge motion due to an applied electric field whenever the charges are free to move. One of the reasons as to why we are studying electric current is that it forms a background for the trealment of electromagnetism in Blocks 3 and 4.

In Block 2 of this course, we discussed certain aspects of the **behaviour** of a substance under an applied electric field in **terms** of electric susceptibility of the material. In this unit, we discuss another important **property** of the material called electric conduction, which is also a response to an applied external electric field. The difference between the two cases is that in the **former** the charges are bound so that they undergo only small displacements, while in the latter case the charges are free and under the action of the field they flow and result in a current.

The concepts of electric current will be of much use to us. In the next unit, we will find that a new force field **viz.** magnetic field arises because of the motion of charge. We will also discuss the experimental relation between **current** and magnetic field and establish basic laws of magnetostatics, **viz.,** Gauss's law for magnetism and Ampere's law.

Objectives

After studying this unit you should be able to:

- explain the concept of electric **current** and obtain the expression for current density in terms of the **drift velocity,**
- explain the **conduction** mechanism microscopically,
- distinguish between ohmic and non-ohmic behaviour,
- use the continuity equation to discuss the behaviour of **current** in a diode.

8.2 ELECTRIC CURRENT AND CURRENT DENSITY

In Block 1, you learnt that when a charge is placed in an electric field it is acted on by a force and moves in the direction of lines of force. If the ends of a conductor, say, a copper wire are connected to a battery, an electric field **E** will be set up at every point within the conductor. Due to the presence of the field, the electrons present in the wire will move in the direction opposite to that of the field and give rise to an electric current in the wire. An electric current is caused whenever the charges move. (In the case of a copper wire the flow of electrons constitutes an electric current.) It is defined as the amount of the charge moving across a given cross-section of the wire per unit time. In Fig. 8.1, for a wire, it is defined as the rate at which charge passes through a plane perpendicular to the axis of the wire. For example, if charge q crosses the shown cross-section in Fig. 8.1 in time t then the average current I is given by

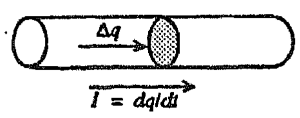


Fig. 8.1: The instantaneous current along a wire is defined as the net rate at which the charge passes through an area perpendicular to the axis of the wire.

$$I = \frac{\text{net charge transferred}}{\text{time taken}} = \frac{q}{t} \qquad (8.1)$$

When the current is not constant, i.e., the current varies with time, we define an instantaneous value of the current $I(t)$. If a net charge of Δq crosses the shaded area of Fig.8.1 in a time Δt, the instantaneous current is given by

$$I(t) = \lim_{\Delta t \to 0} \frac{\Delta q}{\Delta t} = \frac{dq}{dt} \qquad (8.2)$$

Eq. (8.1) or (8.2) shows that the unit of current is Coulombs per second (Cs^{-1}). In the SI system of units it has been given the name ampere (abbreviated A). In Unit 1 of Block 1 we have stated the definition of ampere.

Current is a scalar quantity, because both q and *t* are scalars. It is not a vector quantity as it does not obey the vector laws. Often, a current in a wire is represented by an arrow. Such arrows are not vectors, they only show a direction (or sense) of flow of charges along a conductor, not a direction in space.

SAQ 1

Name few other physical quantities, like current, that are scalars having a sense represented by an arrow in a diagram.

An electric current may consist of only one sign of charge in motion, or it may involve both positive and negative charges. By convention, the direction of current is defined as that direction in which the positive charge flows. If the moving charge is negative, as with electrons in a metal, then the current is opposite to the flow of the actual charges. When the current is due to both positive and negative charges it is determined by the net charge motion, that is, by the algebraic sum of the currents associated with both kinds of charges. For example, when salt (NaCl) is dissolved in water, it splits up into Na^+ ions and Cl^- ions. The sodium ion is positively charged and the chlorine ion is negatively charged. Under the influence of the electric field established between the two electrodes, these ions move through the liquid in opposite direction. Thus the motion of both positive and negative ions contributes to the current in the same direction.

As defined earlier, current is the total charge passing through the wire per unit time across any cross-section. Therefore, the current is determined by the total charge that flows through the wire, whether or not the charge passing through every element of the cross-section of the wire is the same. It is for this reason that current is a macroscopic quantity. If the charge passing through various elements of the cross-section of the wire is not the same, it is necessary to define a quantity at every point of the copductor. This is called the current density which is a microscopic quantity and denoted by **J**. It is defined as the charge flowing per unit time per unit area normal to the surface, and has a direction in which the positive charge moves.

Let us consider a simple system in which particles, each of charge q, are moving to the right as shown in Fig. 8.2.

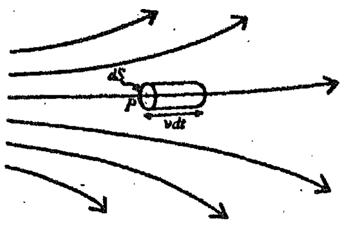


Fig. 8.2 : Calculation of current in terms of drift velocity.

Imagine a small area dS around **point** P so that all the particles crossing this area may be assumed to have the same speed $\boldsymbol{v}$. Let us further imagine a cylinder of length vdt as shown in Fig. **8.2.** Then all the particles within this cylinder of volume $dSvdt$ would cross the area dS in time dt. If n is the number of charged particle per unit volume, **then** the number of charged **particle** found in such a volume is $ndSvdt$. Therefore, the average rate at which the charge is passing through dS that is, the **current** through dS is given by

$$I = \frac{q\,(ndSvdt)}{dt} = ndSvq \tag{8.3}$$

Since **current** density is defined as the **current** per unit area held normal to the velocity of the current carriers, we have

$$J = \frac{I}{dS} = nqv \tag{8.4}$$

Since the direction of $\boldsymbol{J}$ **is** the direction of the actual flow of charges at that point, the above equation can be written in vector **form** as

$$\mathbf{J} = nq\,\mathbf{v} \tag{8.5}$$

Thus J is a vector quantity. In SI system of **units** J is expressed in amperes per square meter. When the **current** carriers are electrons, $q = -e$ and **Eq.** (8.5) takes the form

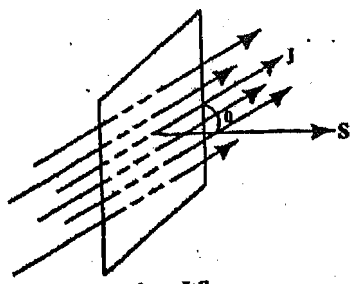
$$\mathbf{J} = -ne\,\mathbf{v} \tag{8.6}$$

The product nq in Eq. (8.5) represents the volume charge density ρ of the **current carriers.** Hence, in terms of ρ the current density is expressed as **follows**

$$\mathbf{J} = \rho\mathbf{v} \tag{8.7}$$

If current density is uniform over the cross section S of the wire, we can find the total **current** by multiplying the current density by the cross section of the wire. If the **current** density is not at right angles to the cross-sectional area, we consider only that component of J which is perpendicular to **it**. If we define a vector **S** the magnitude of which is the cross-sectional area S and the direction of which is along the perpendicular to the area, then a uniform **current** density **J** gives rise to a total current $I = \mathbf{J}.\mathbf{S}$ (Fig.8.3). When the current density **and/or** surface orientation vary with position; we can do the same thing for many small areas $d\mathbf{S}$, and then sum the results to get the **total current** (Fig, 8.4). The current through a small area $d\mathbf{S}$ is $\mathbf{J}.d\mathbf{S}$, so that the total current, **I,** through the entire surface is

$$I = \int_S \mathbf{J}.d\mathbf{S} \tag{8.8}$$



$I = \mathbf{J}.\mathbf{S}$

Fig. 8.3 : The current through a surface of area S is given by JS cos θ, or J . S where θ is the angle between the vectors S and J.

where the limits of the integral are chosen to cover the **entire** surface. Eq. 8.8 should remind you of the **definition** of the electric flux in Unit 2 of Block 1. (Compare Eq. 8.8 with **Eq. 2.4).** Indeed, the electric current through a surface is the flux of the current density through that surface. **Eq. 8.8** again shows that the current is a scalar because the integral $\mathbf{J}.d\mathbf{S}$, is a scalar.

In Fig. 8.4, **we** have taken the surface S to be open surface, In such situation the vector $d\mathbf{S}$, is taken to be positive in that direction along which the **current through** S is

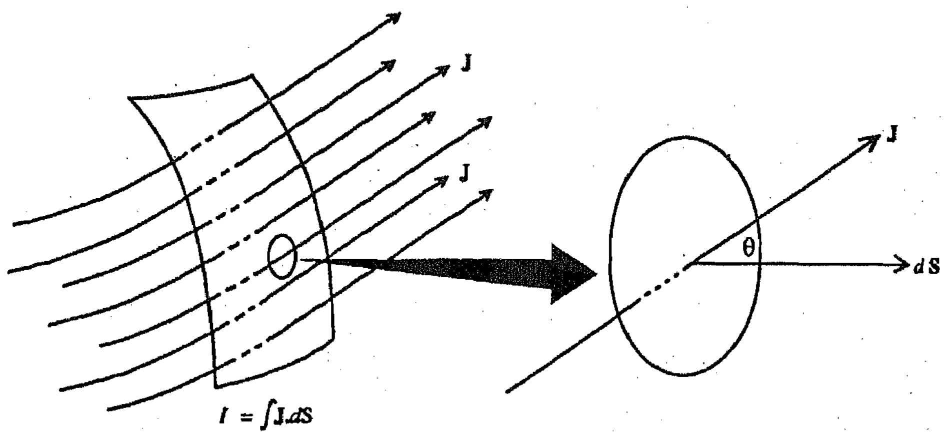


Fig. 8.4: When the current density and/or surface orientation vary with position, the total current may be written as $I = \int \mathbf{J}.d\mathbf{S}$.

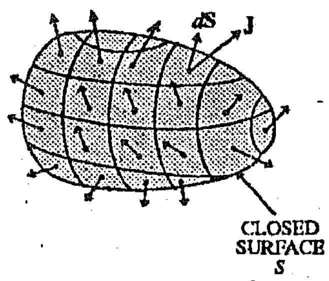


Fig. 8.5: The integral $\int \mathbf{J}\,.\,d\mathbf{S}$ over a closed surface is the rate of change of the total charge q inside.

required. When S is a closed surface, as shown in Fig. 8.5, the direction of every vector $d\mathbf{S}$ is taken along the outward normal to the surface. For such surfaces, the integral of J over S gives the rate at which the charge is going out of the volume enclosed by S. Now one of the basic laws of Physics is that an electric charge is indestructible; it is never lost or created. Electric charges can move from place to place but never appear from nowhere. We say that the charge is conserved.

Hence, if there is a net current out of a closed surface, it must be equal to the rate at which the total charge within the volume is depleting. The electric current I flowing out of the closed surface S enclosing the volume V, is given by

$$I = \int \mathbf{J}.d\mathbf{S} \tag{8.9}$$

We can, therefore, write the law of the conservation of charge as

$$\int_s \mathbf{J}.d\mathbf{S} = -\frac{d}{dt}\,(q_{inside}) \tag{8.10}$$

The charge within the volume can be written as a volume integral of the charge density p as follows

$$q_{inside} = \int_V \rho dV \tag{8.11}$$

where V is the volume enclosed by surface S.

Using Eq. (8.11) in Eq. (8.10) we get

$$\int_s \mathbf{J}.d\mathbf{S} = -\frac{d}{dt}\int_V \rho dV \tag{8.12}$$

Since we are dealing with a fixed volume V, the time derivative operates only on the function p. Since p is a function of spatial coordinates as well as time, the time derivative of ρ is written as the partial derivative with respect to time when it is moved inside the integral, Hence

$$\int_s \mathbf{J}.d\mathbf{S} = -\int_V \frac{\partial\rho}{\partial t}\,dV \tag{8.13}$$

The surface integral on the left hand side of the Eq. (8.13) can be converted into a volume integral through the divergence theorem (see Unit 2 of Block 1), leading to

$$\int_s \nabla \cdot \mathbf{J} dV \ \textit{mark=} -\int_V \frac{\partial \rho}{\partial t} dV$$

or
$$\int_s \left(\nabla \cdot \mathbf{J} + \frac{\partial \rho}{\partial t} \right) dV \ \textit{align=} \ 0 \qquad (8.14)$$

But V is completely arbitrary and Eq. (8.14) will hold for an arbitrary **volume element** only when the integrand is zero. Thus

$$\boxed{\nabla \cdot \mathbf{J} + \frac{\partial \rho}{\partial t} = 0} \qquad (8.15)$$

This differentialequation is **known** as the **continuity equation.** It expresses the conservationof charge in a differential form. Its meaning is clearer in Eq. **(8.14)**, according to which the change in the quantity of charge in any arbitrary volume **must** be accompanied by a net flow of charge inwards or outwards across its surface. **When** steady currents are involved we have

$$\frac{\partial \rho}{\partial t} = 0$$

This is because a steady current is one for which J is constant in time at every point. In other words, equal charges flow in and flow out of a section and, hence, there cannot be any accumulation of charge at any point of the system. Hence, in this case **the** continuity equation becomes

$$\nabla \cdot \mathbf{J} = 0 \qquad (8.16)$$

SAQ 2

Give an example of steady current system, and using the Eq. (8.16) list its **feature.**

The equation of continuity can be used to discuss the current distribution in a diode valve. But before discussing it **let** us find out why do the metals conduct electricity **and** what are the factors which influence conductivity of the metal.

8.3 CONDUCTION MECHANISM

In this **Unit,** we are **concentrating** on the currents **flowing** in metal wires. In a metal, **the** metal ions are fixed in a **regular** array, known as lattice, **making** them relatively immobile. The metal **ions** are positively charged because the atoms **forming the metal** lose one or more electrons which become free in the **sense** that these electrons wander through the ion lattice as shown in Fig. 8.6. It is the motion of these negatively **charged** electrons that gives metals their conducting properties.

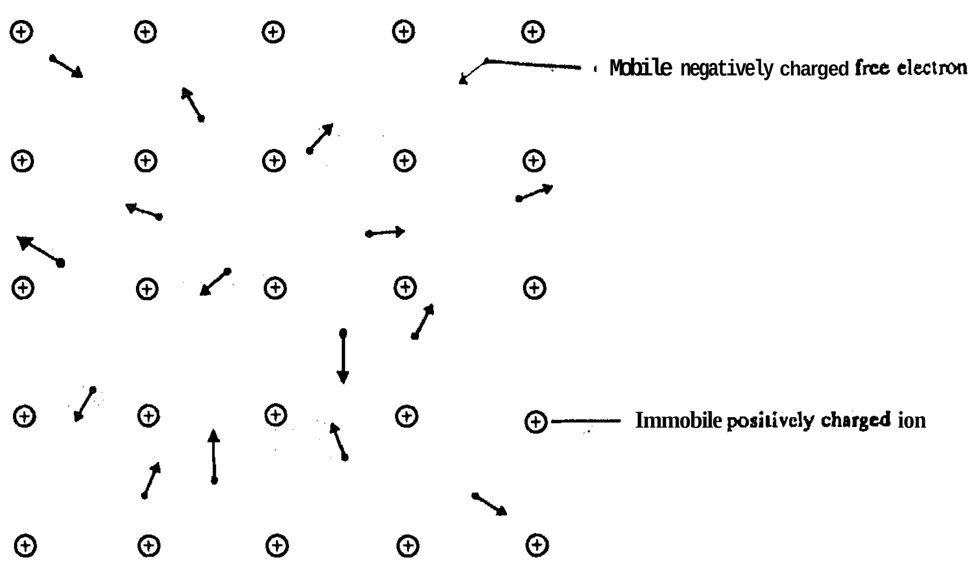


Fig. 8.6: A schematic view **of the crystal structure of** a **metal.** The **positive metal ions exist** on a **rigid lattice. Each atom, on forming** an ion, **gives** up m e or **more electrons, which are then free** to wander through **the crystal.**

When a battery is connected between the ends of a metallic wire MN as shown in Fig. 8.7,

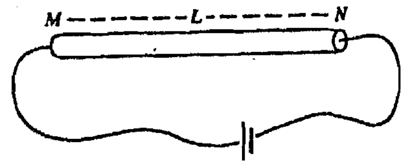


Fig. 8.7: A battery (source of emf) can maintain an electric field within a conducting wire.

we find that the current flows **through** it from M to N (current flowing in the wire can be detected by putting an ammeter inseries). Let us find out why and how the current starts flowing in a particular direction by taking a microscopic view of the situation.

8.3.1 Drift Velocity and Ohm's Law

When the metallic conductor is not connected to the battery, the free **electrons** present in the metal are in constant motion because of their thermal energy, the motion being random in velocity as shown in Fig. **8.8(a).**

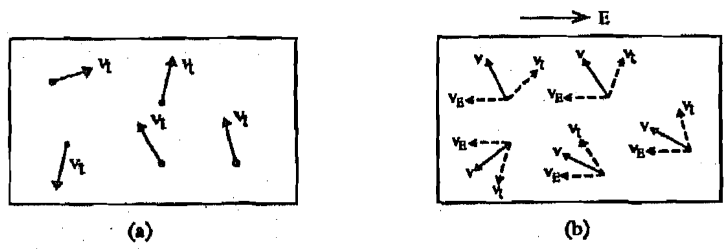


Fig. 88: Motion of some **free electrons** (a) **in** the **absence of an external field** a d (b) **in the presence of** an external field (**as** shown by **the** solid **lines**). v_t **represents** thermal **velocity**, v_E is the **velocity** only in the presence of electric **field** and v is the net **velocity.**

In this state, the free electrons encounter frequent collisions with positive ions and impurity atoms (if any). At each collision, the velocity changes both in magnitude and direction; Since the motion is completely random, at any instant, the **average velocity** along any direction in the bulk of the conductor is zero. Hence no current. But remember that **average** speed of these free electrons at any instant is not zero. **Its** value is of the order of 10^5 ms^{-1}.

When a battery is connected between the ends of the metallic **wire**, it maintains a uniform electric field E at each point in the wire. The electrons experience a force in a direction opposite to that of the applied electric field. Due to this force, besides having a thermal velocity v_t, an electron also experiences a constant acceleration $a = eE/m_e$, where m_e is the mass of the electron. You may now wonder and ask if the electron velocity, v_E, which is in the presence of the electric field, increases continuously as it moves in the wire. Experiments show that this does not happen. **As** an electron picks up speed under the action of the field, it collides with the oscillating ion or the impurity atom within the metal. The result of this collision is that the electron loses all its velocity acquired due to acceleration in the field. In other words, at each collision the velocity of the electron is **randomised,** and it begins at fresh acceleration in the direction of the field. If u is the velocity of an **electron** just after a collision, its velocity $\mathbf{v}_E$ just before the next collision will be

$$\mathbf{v}_E = \mathbf{u} + \frac{e\mathbf{E}}{m_e}t \qquad (8.17)$$

where t is the time of travel between the two **collisions.** The **average of the** velocities of all electrons before collision can be written as

$$\langle \mathbf{v}_E \rangle = \langle \mathbf{u} \rangle + \frac{e\mathbf{E}}{m_e}\langle t \rangle \qquad (8.18)$$

where the sign < > denotes the average value of the parameter.

Since the effect of each collision is to reduce the velocity to zero and to restore the random thermal motion, we can write $\langle u \rangle$ as $\langle v_t \rangle$ which is zero, as explained earlier. If $\langle t \rangle$ is represented by τ, then we get

$$\langle v_E \rangle = \frac{e\,E}{m_e}\tau \qquad (8.19)$$

For this reason v_E does not increase continuously with time, but will rather have an average value $\langle v_E \rangle$ as given by Eq. (8.19). Here τ denotes the average time between successive collisions,—i.e., the time over which the electron accelerated freely under the action of the electric field. This is called mean free time. The thermal motion of the free electrons is, therefore, modified as shown in Fig. 8.8b. It is clear from the figure that at any instant, the resultant velocity is $v_t + v_E$ and for each electron it is different. The average resultant velocity of all the electrons can be expressed as

$$\langle v \rangle = \langle v_t + v_E \rangle = \langle v_t \rangle + \langle v_E \rangle$$

As already stated, v_t is zero, but v_E is not zero because of the fact that the v_E for all the free electrons is in the same direction. Therefore, $\langle v \rangle = \langle v_E \rangle$. Hence, the free electrons in a metallic wire have an average velocity which is caused only by the applied electric field. This velocity is called the drift velocity of the electrons denoted by $\langle v_d \rangle$. That is,

$$\langle v_d \rangle = \frac{e\,E}{m_e}\tau \qquad (8.20)$$

It is this velocity that appeared in Eq. (8.5). Thus, the current density in a conductor can be written as

$$J = nq\,v_d \qquad (8.21)$$

You must be knowing that a freely falling body in vacuum has a velocity $v = gt$ which increases continuously with time, but if the body falls through a viscous fluid, the motion becomes uniform with a constant limiting velocity. By analogy, the effect of the crystal lattice can be represented by a viscous force, acting on the conduction electrons when their natural motion is disturbed by the applied electric field.

In most substances and over a wide range of electric field strengths, it has been experimentally found that the current density is proportional to the strength of the electric field that causes it. The relation may be written as

$$\boxed{J = \sigma E} \qquad (8.22)$$

where σ is the proportionality constant and is known as the **conductivity** of the material. Eq. (8.22) is a statement of Ohm's law. It is an empirical law, a generalization derived from experiment for some materials under certain conditions. It is not a theorem that must be universally obeyed. The value of σ is very large for metallic conductors and extremely small for good insulators. It may also depend on the physical state of the material, for instance, on its temperature, about which you will study in next section. But for many common conductors, for given conditions, it does not depend on the magnitude of E. Such materials are called ohmic or linear and for such materials Eq. (8.22) implies that the direction of J is always the same as the direction of E. Eq. (8.22) shows that the units of conductivity are $(Am^{-2})/(Vm^{-1})$ or $AV^{-1}m^{-1}$. But one VA^{-1} is given the name ohm (symbol Ω). Therefore, the SI unit of conductivity is $(\Omega\ m)^{-1}$. Instead of the conductivity we can use its reciprocal, called resistivity ρ in stating the relation between current density and electric field as follows:

Eq. 8.22: holds only for isotropic materials—those materials in which the electric properties are the same in all directions.

$$\boxed{E = \rho J} \qquad (8.23)$$

The units of resistivity are Ω m. Since both E and J are microscopic parameters ρ also defines a microscopic property of the conductor. In Fig. 8.7, the electric field along the wire is in the direction MN and its value is $E = V/L$ everywhere. Here V is an applied potential difference between the ends of the wire. Thus

$$J = \sigma\frac{V}{L}$$

and the total current is $I = JS = \frac{\sigma SV}{L}$,

where S is the cross-sectional area of the wire. This gives,

$$\frac{V}{I} = \frac{L}{\sigma S} = \frac{\rho L}{S} \qquad (8.24)$$

It is customary to use ρ as the symbol for resistivity and σ as the symbol for conductivity inspite of their use in some of our other units for volume charge density and surface charge density respectively. In the rest of the units, ρ will denote resistivity and σ conductivity if not stated otherwise.

The ratio V/I is called the **resistance** R of the wire and the Eq. (8.24) is written in the **form**

$$\boxed{V = IR} \qquad (8.25)$$

with

$$R = \rho L/S \qquad \textbf{(8.26)}$$

This gives **another more** familiar expression of Ohm's law. It implies that resistance R of the conductor is independent of the applied potential difference V. Therefore, for the linear conductors (those conductors which obey **Ohm's** law) a graph between $V - I$ is a straight line as shown in Fig. 8.9.

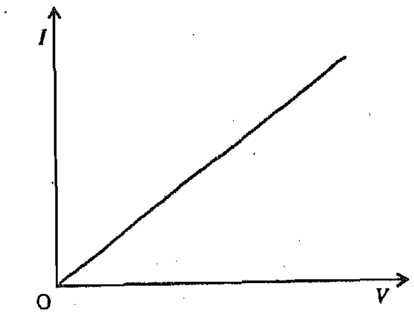


Fig. 8.9 : $V - I$ characteristic of a linear conductor.

The resistivity p depends on the nature of the conducting material whereas the resistance depends not only on the nature of the **medium** but also on its physical dimensions. Resistivity p is of basic significance to those who wish to study the **behaviour** of the conductor from the atomic viewpoint. If we apply Eq. **(8.20)** to Eq. **(8.21),** we find

$$\mathbf{J} = ne\,\mathbf{v}_d = \frac{ne^2\tau}{m_e}\mathbf{E} \text{ (Here we have replaced q by e)}$$

By comparison with Eq. (8.22) we **find the expression** for conductivity as follows:

$$\sigma = \frac{ne^2\tau}{m_e} \qquad \textbf{(8.27)}$$

and for the resistivity

$$\rho = \frac{m_e}{ne^2\tau} \qquad \textbf{(8.28)}$$

This equation shows that the resistivity of a metal depends on the density of the free electrons, their mass and charge and on mean free time. The cause of the dependence of resistivity on temperature is the variation **of τ** with temperature. In the next sub section, we shall explain how resistivity depends upon **temperature.**

SAQ 3

Eq. (8.22) and Eq. (8.25) are mathematical expressions of **Ohm's** law. Derive Eq. (8.22) from Eq. (8.25).

8.3.2 Temperature Dependence of Resistivity

Till now we have been considering the simple classical model in which electrons are considered as a "gas" of charged particles getting accelerated under the influence of an external electric field. On this basis, we arrived at the expression of resistivity. But if we wish to go further in order to understand the dependence of ρ on temperature or, in other words, to **understand** the dependence of τ on **temperature, we** have to look for other model. Fortunately, this can be **explained** on the basis of quantum mechanical model. In this model, we should not now think of the electron as a tiny charged particle. But we should **think** electron to be behaving **more** like a wave interacting with a larger region of metal.

If the temperature of metal is very low, say zero, then all the ions are rigidly fixed **at** their regular lattice positions. This makes the classical collision between the **electron and** ion unlikely to occur. It means that, in such situations, the time between collisions is very large or infinite. The hindrance which interrupts the progress of an electron wave is not the regular array of ions but an irregularity in the array. On increasing the temperature, the ions vibrate, and it causes the solid to look less regularly spaced then it would if the ions were at rest. The effect is that the time between collisions is shortened. Hence the mean free time τ decreases with increase of temperature. This leads to increase of resistivity with temperature.

8.3.3 Breakdown of Ohm's Law

You may think that the metals always behave as linear conductors. But it is not so. Under certain conditions metals do not behave as linear conductors. In this analysis, we have been making an unstated assumption that the electric field applied to the metal is so small that it does not disturb the electron velocity pattern in the metal in a major way. Immediately, you would be tempted to ask, what will happen if the field is increased to high values. Let us first see what is the time between collisions when a small field is applied. In such situation, if the average distance travelled by an electron before it encounters the next collision is denoted by λ called the mean free path of the electron, the average time τ between two collisions is given by

$$\tau = \frac{\lambda}{\langle | \mathbf{v}_t + \mathbf{v}_d | \rangle} \qquad (8.29)$$

where $\langle | \mathbf{v}_t + \mathbf{v}_d | \rangle$ is the average speed and not the average velocity between two collisions. For the electric fields normally used in the laboratory, v_d ($\approx 10^{-2}$ cm s^{-1}) is very small compared to $\mathbf{v}_t$ ($\approx 10^8$ cm s^{-1}). Therefore, Eq. (8.29) can be written as

$$\tau = \frac{\lambda}{\langle | \mathbf{v}_t | \rangle} \qquad (8.30)$$

When we apply a very large field, the drift speed of the electrons, i.e., v_d becomes comparable to v_t. Then the time between collisions will be shorter than it was before the field was applied. This is an effect, which is not included in our theory. In this case, the expression for ρ would also contain parameter v_d, which is strongly field-dependent. Therefore, metals under these circumstances would not obey Ohm's law.

At very large fields, yet another thing happens, the free electrons are accelerated so much that they gain sufficient energy. With these energies, they can strike the atom hard enough to knock another electron out of its grip. Thus, extra electrons are freed and get accelerated. These accelerated, extra electrons release more charges when they collide with other atoms. This process, thus, causes an avalanche of free charge carriers. Under these conditions, it produces a rapidly increasing current and unless the avalanche is limited in some way, the process may destroy the material. This is a complete breakdown of Ohm's law.

We end this unit with a short discussion of a vacuum tube. We will apply the current electricity ideas, i.e., whatever you have learnt just now to vacuum tube and discover that the vacuum tube does not obey Ohm's law.

8.4 CURRENT-VOLTAGE RELATIONSHIP FOR DIODE (NON-OHMIC CONDUCTOR)

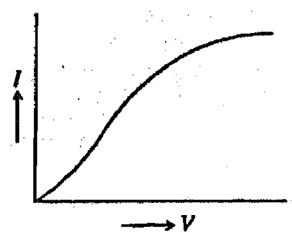


Fig. 8.10: Current-voltage characteristic of a vacuum diode.

There are many conductors which do not obey Ohm's law. Such conductors are called non-linear conductors. The V – I characteristic of such conductors is not a straight line as shown in Fig. 8.9. You must be knowing that examples of such non-linear conductors are: vacuum tube diodes and electrolytes. For vacuum diode, the V – I characteristic have the form shown in Fig. 8.10. In this section we will find out the relation between current and voltage for a vacuum diode using the continuity equation.

In its simplest form, a diode can be assumed to wnsist of two electrodes. One electrode, called cathode, is coated with a material that emits electrons copiously when heated. The other electrode, known as anode, is simply a metal plate. By means of a battery the anode is maintained at a positive potential with respect to the cathode. Both these electrodes are enclosed usually in a glass tube the inside of which is evacuated to a very low pressure (of the order of 10^{-5} cm of Hg). Electrons emerge from this hot cathode

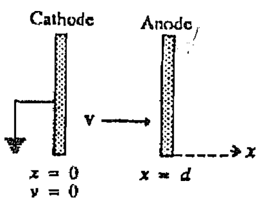


Fig. 8.11: A vacuum diode with plane parallel cathode and anode.

with very low velocities and then, being negatively charged, are accelerated towards the positive anode due to the presence of electric field between the cathode and anode. In the space between the cathode and anode the electric current consists of these moving electrons.

In this diode, the local charge density ρ (here and in this section ρ denotes charge density), is simply equal to nq, where n is the local density of the electrons. According to Eq. (8.4), the local current density $\mathbf{J}$ is $\rho \mathbf{v}$, where v is the velocity of electrons in the region concerned. (This is because in this case the drift velocity is the actual velocity.) If it is assumed that $\mathbf{J}$ has no y or z components as shown in Fig. 8.11 and if conditions are steady, then div $\mathbf{J}$ = 0. i.e., $\frac{\partial J_x}{\partial x} = 0$. This means if we have a steady stream of electrons moving in the x direction only, the same number per second have to cross any intermediate plane between cathode and anode. We conclude that ρv is constant. But observe that v is not constant; it varies with x, because the electrons are accelerated by the field. Hence ρ is not constant either. Instead, the negative charge density is high near the cathode, low near the anode, just as the density of vehicles on a highway is high near a trafficsignal and low where traffic is moving at high speed. This is because the electrons coming out of the cathode form a space charge which prevents further emission of electrons for low anode voltages (low acceleration of electrons between the two electrodes).

Using this conclusion let us find the diode current. To make the mathematics simple, we assume that

i) the potential at the cathode is zero while the polential at the anode is V_a;

ii) the distance between the cathode and anode is 'a', which is small so that the field can be assumed uniform and normal to the surfaces of the electrodes;

iii) the velocity (v) of the electrons soon after their emission at the cathode is zero and anywhere in between the cathode and anode, say at a distance x from the cathode, the velocity is denoted by v_x and potential by $V(x)$.

Differential form of Gauss's law is given by

$$\text{div}\,\mathbf{E} = \frac{\rho}{\varepsilon_0}$$

where ρ is the charge density.

In rectangular co-ordinates it can be written as

$$\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = \frac{\rho}{\varepsilon_0}$$

Since $E_x = -\frac{\partial V(x)}{\partial x}$, etc, as shown in Unit 3, by substitution we find

$$\frac{\partial^2 V(x)}{\partial x^2} + \frac{\partial^2 V(y)}{\partial y^2} + \frac{\partial^2 V(z)}{\partial z^2} = -\frac{\rho}{\varepsilon_0}$$

This is called Poisson's equation.

In the given example of diode since we have assumed that the electric field acts only in the x-direction so that $\mathbf{J}$ has no y- or z-component, and $E_y = E_z = 0$. Therefore, potential $V(x)$ at a distance x from the cathode must satisfy the following equation

$$\frac{\partial^2 V(x)}{\partial x^2} = -\frac{\rho}{\varepsilon_0} \qquad (8.31)$$

The potential $V(x)$ at a distance x from the cathode is given by:

$$\frac{d^2 V(x)}{dx^2} = -\frac{\rho}{\varepsilon_0} \qquad (8.31)$$

where ρ is the charge density at the point x.

or

$$\frac{d^2 V(x)}{dx^2} = -\frac{J}{\varepsilon_0 v_x}, \qquad (8.32)$$

since $J = \rho v$ and $v = v_x$ at x.

Assume the initial velocity of the electron as zero. Then its velocity v at any point is related with potential V through which the electron has traversed by the following relation :

$$\frac{1}{2} m v_x^2 = qV(x) \qquad (8.33)$$

Using Eq. (8.33) in Eq. (8.32) we get

$$\frac{d^2 V(x)}{dx^2} = \frac{J}{\varepsilon_0}\left(\frac{m}{2q}\right)^{1/2} V(x)^{-1/2}$$

Multiplying both sides by $2\frac{dV(x)}{dx}$ we get

$$2\frac{dV(x)}{dx}\frac{d^2V(x)}{dx^2} = 2\frac{J}{\varepsilon_0}\left(\frac{m}{2q}\right)^{1/2} V(x)^{-1/2}\frac{dV(x)}{dx}$$

or

$$\frac{d}{dx}\left(\frac{dV(x)}{dx}\right)^2 = \frac{4J}{\varepsilon_0}\left(\frac{m}{2q}\right)^{1/2}\frac{d}{dx}\left(V(x)\right)^{1/2}$$

Integrating above with respect to x we get

$$\left(\frac{dV(x)}{dx}\right)^2 = \frac{4J}{\varepsilon_0}\left(\frac{m}{2q}\right)^{1/2} V^{1/2}(x) + C_1$$

where C_1 is constant of integration. At $x = 0$, $V(x)$ and $\frac{dV(x)}{dx}$ arc both zero, so $C_1 = 0$.

$$\therefore \quad \frac{dV(x)}{dx} = 2\left[\frac{J}{\varepsilon_0}\left(\frac{m}{2q}\right)^{1/2}\right]^{1/2} V^{1/4}(x)$$

or $$V^{-1/4}(x)\, dV(x) = 2\left[\frac{J}{\varepsilon_0}\left(\frac{m}{2q}\right)^{1/2}\right]^{1/2} dx$$

On integrating again with respect to x, we have

$$\frac{4}{3} V^{3/4}(x) = 2\left[\frac{J}{\varepsilon_0}\left(\frac{m}{2q}\right)^{1/2}\right]^{1/2} x + C_2$$

Again constant of integration $C_2 = 0$ as at $x = 0$, $V(x) = 0$. On squaring the above equation we get

$$V^{3/2}(x) = \frac{9}{4}\frac{J}{\varepsilon_0}\left(\frac{m}{2q}\right)^{1/2} x^2 \qquad (8.34)$$

From Eq. (8.34), at the anode, $x = a$, we get

$$V_a^{3/2} = \frac{9}{4}\frac{J}{\varepsilon_0}\left(\frac{m}{2q}\right)^{1/2} a^2 \qquad (8.35)$$

or

$$J = \frac{4}{9}\varepsilon_0\left(\frac{2q}{m}\right)^{1/2}\frac{V_a^{3/2}}{a^2} \qquad (8.36)$$

This is the famous ***Child - Langmuir or three halves power law,*** It suggests that the current density **J** or current **I** in a diode is proportional to the **three-halves** power of the applied potential difference between the anode and cathode.

SAQ 4

From Eq. (8.31) obtain the following **relation** :

$$\rho(x) = -\frac{4\varepsilon_0}{9}\frac{V_a}{a^{4/3}} x^{-2/3} \qquad (8.37)$$

Eq. (8.37) gives variation of charge density with distance from the cathode. This variation is represented in Fig. 8.12.

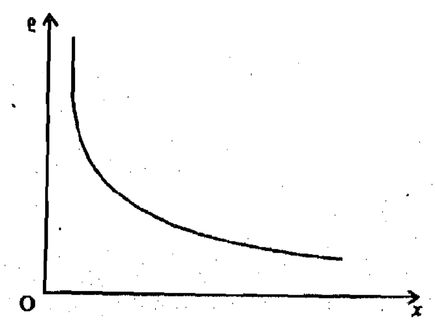


Fig. 8.121 Variation of charge density in a diode as a function of the distance from cathode.

SAQ 5

You know that the electric field inside a conductor is zero. Hence, if a charge is placed inside a conductor, it will move to the surface and distribute itself in such a way that the zero field exists within the surface. How fast this happens is of **importance, and**

interestingly the continuity equation helps in evaluating this time. **Determine** the characteristic time for the decay of charge inside a conductor. (Assume that ρ_0 is the initial charge density.)

8.5 SUMMARY

Current is the flow of **charge.** The unit of current is the ampere. Current is defined as the amount of charge per unit time passing a given **point.**

$$I = \frac{dq}{dt}$$

- Current density J is a vector specifying the current per unit area. The direction of J at any point is that in which a positive charge-carrier would move if placed at that point.

$$\mathbf{J} = nq\,\mathbf{v}_d$$

- The total current through a surface is the flux of the current density over that surface:

$$I = \int_s \mathbf{J}.d\mathbf{S}$$

where $d\mathbf{S}$ is an element **of** area and the integral is taken over the surface.

- **The** total charge **crossing** a surface S **in unit** time **is** $\int \mathbf{J}.d\mathbf{S}$. If S is a closed surface enclosing a volume V, the rate of loss of charge through S must be the same as the rate of depletion of charge contained in V, **i.e.**

$$\int \mathbf{J} \,.\, d\mathbf{S} = -\frac{\partial}{\partial t}\int \rho dV$$

The differential form of the above equation is the continuity **equation** :

$$\nabla . \mathbf{J} + \frac{\partial \rho}{\partial t} = 0$$

Both these statements express the law of conservation of charge.

- Conductivity a is a property of a material which is equal to the ratio of current density to electric field in the material

$$\mathbf{J} = \sigma \mathbf{E}$$

Resistivity ρ is the inverse of conductivity.

- Resistance is a property of a particular **piece** of material. It is defined as the ratio of voltage V across the material to the **current** I through the material:

$$R = \frac{V}{I}$$

The resistance of a piece of material depends on **its resistivity** and physical dimensions. For a material of length l, uniform cross-sectional area S and resistivity p, the **resistance** is

$$R = \rho \frac{l}{S}$$

- The conduction in metals is due to the presence of free electmns. In metals, the combined effects of acceleration of free electrons in an applied electric field and collisions between electrons and metal ions and impurities result in a drift velocity. On this basis, the expression **for** electrical resistivity is

$$\rho = \frac{m_e}{ne^2\tau}$$

Thus, resistivity depends on the number of free electrons per unit volume, their mass and charge, and mean free time.

8.6 TERMINAL QUESTIONS

1) TV set shoots out a beam of electrons. The beam current is **10 μA.** How many electrons strikes the TV screen each second? How much charge strike the screen in a minute?

2) In the **Bohr** Model, the electron of a hydrogen atom moves in a circular orbit of radius 5.3 x 10^{-11} m with a speed of 2.2×10^6 ms^{-1}. Determine its frequency f and the current I in the orbit.

3) A current of 2.00 A flows in a copper wire of 1.00 mm^2 cross section. What is the drift velocity of electrons in that wire? How long does it take **an electron** to travel 10.0 cm (about the length of an incandescent bulb filament)in this wire under these circumstances? Assume that the number of conduction **electron** per cubic meter is 8.43×10^{28}.

4) A potential difference V is applied to a copper wire of diameter d and **length** L. What is the effect on the electron drift speed on (a) doubling V, (b) **doubling** L, **and** (c) doubling d ?

5) i) See Fig. **8.13.** What is the electric field in a copper conductor of resistivity $\rho = 1.72 \times 10^{-8}$ **ohm** meter having a current density J = 2.54×10^6 amp m^{-2}?

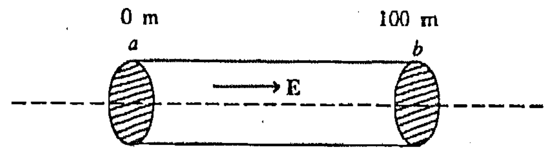


Fig. 8.13

ii) What is the potential difference between two points of a copper wire 100 m apart?

6) As shown in Fig. **8.14,** a metal rod of radius r_1 is concentric wih a metal cylindrical shell of radius r_2 and **length** L. The space between the rod and cylinder is tightly packed with a high-resistance material of resistivity ρ. A battery, having a terminal voltage V_t, is connected as shown. Neglecting **resistances** of the rod and the cylinder, derive expressions for (a) the total current **I,** (b) the current **density J** and the electric field E at any point P between the rod and the cylinder, **and** *(c)* the -resistance R between rod and cylinder.

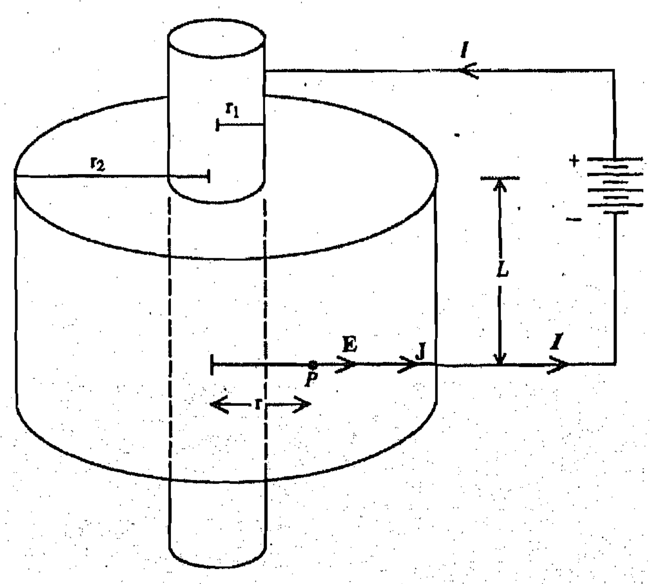


Fig. 8.14

8.7 SOLUTIONS AND ANSWERS

SAQs

1) The other physical quantities like ,current are

i) Thermal current in heat conduction,

ii) Flow current of water or any incompressible liquid in stream line motion.

A source or sink of a current is one by which the current may be injected into or withdrawn from a conducting region. For example, an electrode.

2) A wire carrying a constant current is an example of steady current. As we have noted a steady current system is one for which J is a constant in time at every point. In such a system, there cannot be any accumulation of charge at any point. This means that in any region of current flow there is no source or sink of current.

3) From Eq. 8.25, we have $\frac{V}{I} = R$

Multiplying both sides by (A/L) we get

$$\frac{V}{I} \times \frac{A}{L} = R\frac{A}{L}$$

Rearranging the terms, we write

$$\frac{V}{L}\frac{1}{I/A} = \frac{AR}{L} = \rho = \frac{1}{\sigma}$$

Since

$$\frac{V}{L} = E \text{ and } \frac{I}{A} = J$$

$$\therefore E\,.\frac{1}{J} = \frac{1}{\sigma} \text{ or } \mathbf{J} = \sigma E \qquad \text{which is Eq. (8.22)}$$

4) We have $\frac{d^2V(x)}{dx^2} = -\frac{\rho}{\varepsilon_0}$ Eq. (8.31)

From above equation we have already derived Eq. (8.34), i.e.,

$$V^{3/2}(x) = \frac{9}{4}\frac{J}{\varepsilon_0}\left(\frac{m}{2q}\right)^{1/2} x^2$$

and also Eq. (8.35), i.e.,

$$V_a^{3/2} = \frac{9}{4}\frac{J}{\varepsilon_0}\left(\frac{m}{2q}\right)^{1/2} a^2$$

Diyiding one by the other and rearranging we get

$$V^{3/2}(x) = \left(\frac{V_a^{3/2}}{a^2}\right)x^2 \qquad \text{(I)}$$

$$V(x) = V_a\left(\frac{x}{a}\right)^{4/3}$$

$$\frac{dV(x)}{dx} = \frac{V_a}{a^{4/3}}\frac{4}{3}x^{1/3}$$

$$\frac{d^2V(x)}{dx^2} = \frac{V_a}{a^{4/3}}\frac{4}{9}x^{-2/3} = -\frac{\rho(x)}{\varepsilon_0}$$

$$\therefore \rho(x) = -\frac{4\varepsilon_0}{9}\frac{V_a}{a^{4/3}}x^{-2/3}$$

3) From continuity equation we have

$$\nabla.\mathbf{J} = -\frac{d\rho}{dt}$$

within a conductor $\mathbf{J} = \sigma \mathbf{E}$ where σ is the conductivity. According to Gauss's law $\nabla.\mathbf{E} = \rho/\varepsilon$ where ε is the permittivity of the conductor. Thus

$$\nabla.\mathbf{J} = \nabla.\sigma\mathbf{E} = \sigma\nabla.\mathbf{E} = \frac{\sigma\rho}{\varepsilon} = -\frac{d\rho}{dt}$$

On rearranging we get

$$\frac{d\rho}{\rho} = -\frac{\sigma}{\varepsilon}dt$$

Integrating, $\int_{\rho_0}^{\rho}\frac{d\rho'}{\rho'} = -\int_0^t \frac{\sigma}{\varepsilon}dt$

or $\ln\frac{\rho}{\rho_0} = -\frac{\sigma t}{\varepsilon}$

Then $\rho = \rho_0 e^{-\frac{\sigma}{\varepsilon}t}$

where ρ_0 is the initial charge density and ρ is the charge density after time t.

The quantity $\frac{\varepsilon}{\sigma} = \tau$ is called the characteristic time.

For extremely good conductors τ will be very small and for insulators τ will be large. (τ is usually referred to as the **Relaxation** time.)

Terminal Questions

1) Let n be the number of electrons per second.

Then $n = \frac{I}{e} = \frac{10 \times 10^{-6}\ \mathrm{Cs}^{-1}}{1.6 \times 10^{-19}\ \mathrm{C}} = 6.3 \times 10^{13}$ electrons per second. The charges Q striking the screen is given by

$$Q = It = (10\ \mu\mathrm{Cs}^{-1})(60\ \mathrm{s}) = 600\ \mu\mathrm{C}.$$

Since the charges are electrons, the actual charge is

$$Q = -600\ \mu\mathrm{C}.$$

2) $f = \frac{v}{2\pi r} = \frac{2.2 \times 10^6\ \mathrm{ms}^{-1}}{2\pi(5.3 \times 10^{-11}\ \mathrm{m})} = 6.6 \times 10^{15}\ \mathrm{rev\ s}^{-1}$

Each time the electron goes around the orbit, it carries a charge q around the loop. The charge passing a point on the loop each second, i.e., current is given as follows:

Current $\mathbf{I} = ef = (1.6 \times 10^{-19}\ \mathrm{C})(6.6 \times 10^{-15}\ \mathrm{s}^{-1})$

$= 1.06\ \mu\mathrm{A}.$

Note that the current flows in the direction opposilc to the electron, which is negatively charged.

3) Solving Eq. (8.21) for v_d, we obtain

$$v_d = \frac{I}{nqS} \quad (\text{because } \mathbf{I} = \mathrm{JS})$$

$$v_d = \frac{2.00\ \mathrm{A}}{(8.43 \times 10^{28})(-1.6 \times 10^{-19}\ \mathrm{C})(10^{-6}\ \mathrm{m}^2)}$$

$$= -1.48 \times 10^{-4}\ \mathrm{ms}^{-1}.$$

(The negative sign appears because the electronic charge is negative, and v_d is therefore directed opposite to $\mathbf{I}$.)

The time required to traverse 10.0 cm at this speed is

$$t = \frac{0.100\ \mathrm{m}}{1.48 \times 10^{-4}\ \mathrm{ms}^{-1}}$$

$$= 0.674 \times 10^3\ \mathrm{s} = 11\ \mathrm{min}\ 14\ \mathrm{s}$$

That is a long time. Yet we know that as soon as we close the proper switch, the charge flows through a circuit and lamps light up. We need not wait several minutes, not even seconds, to witness the effect of the current in a circuit, and there appears to be no observable dependence on the distance between the wall switch and the light fixture, a distance generally considerably greater than 10 cm.

The point is that one does not have to wait until a particular electron at the battery terminal reaches the lamp for the lamp filament to respond to the current. When the switch is closed, the entire charge distribution within the conductor is set in motion almost instantaneously, much as water starts to flow in a long pipe as soon as we open a tap.

4) a) drift velocity will be doubled,

b) drift velocity will be halved,

c) drift velocity will remain unchanged.

5) i) By definition, E, the electric field, is related to the current density J, through the relationship

$$E = \frac{J}{\sigma}$$

But $\sigma = \frac{1}{\rho}$, and therefore,

$$|\mathbf{E}| = \rho\,|\mathbf{J}|$$

$$= (1.72 \times 10^{-8}\ \text{ohm m})(2.54 \times 10^{6}\ \text{amp m}^{-2})$$

$$= 4.37 \times 10^{-2}\ \text{volt m}^{-1}$$

ii) E is related to V by

$$V_b - V_a = \int_a^b \mathbf{E}.d\mathbf{l} \qquad (1)$$

From the Fig. 8.13, E is parallel to the axis of the cylindrical wire. If we evaluate (1) along a line in the direction of E and parallel to the cylinder axis, we obtain

$$V_b - V_a = E\,(a - b)$$

$$V_b - V_a = (4.37 \times 10^{2}\ \text{volt})(0 - 100\ \text{m})$$

$$V_b - V_a = -4.37\ \text{volts.}$$

Therefore, V_b is at a lower potential than V_a.

6) a) Assuming radial flow of charge between rod and cylinder, we have at P

$J = \frac{I}{2\pi rL}$ and $E = \rho J = \frac{\rho I}{2\pi rL}$, where ρ is resistivity, with both J and E in the direction of r. Then, by definition of the potential,

$$dV = -\mathbf{E}.d\mathbf{l} = -E dr = -\frac{\rho I}{2\pi L}\frac{dr}{r}$$

and so, noting the polarity of V_t, we get

$$-V_t = \int_{r_1}^{r_2} dV = \frac{\rho I}{2\pi L}\int_{r_1}^{r_2}\frac{dr}{r} = -\frac{\rho I}{2\pi L}\ln\frac{r_2}{r_1}$$

solving for I,

$$I = \frac{2\pi L V_t}{\rho \ln(r_2/r_1)}$$

b) From (a), $J = \frac{I}{2\pi rL} = \frac{V_t}{\rho r \ln(r_2/r_1)}$ and $E = \rho J = \frac{V_t}{r \ln(r_2/r_1)}$

c) From Ohm's law, $R = \frac{V_t}{I} = \frac{\rho \ln(r_2/r_1)}{2\pi L}$.

UNIT 9 MAGNETIC FIELD

Structure

9.1 INTRODUCTION

In Block 1 of this course, you were introduced to the concept of an electric charge and studied some properties of charges at rest. You learnt that a static distribution of charge produces a static electric field. Similarly, steady flow of charge (**i.e.,** a **steady** current) produces a static magnetic field, which is, **infact,** the topic of this unit. However, there are some major differences between the two fields which you will discover in this unit.

In the science laboratory, during your school days, you must **have** been fascinated with magnets. Recall, when you tried to push two magnets together in a way they didn't want to go, you felt a mysterious force! In fact, magnetic fields or the effect of such fields have been known since ancient times when the effect of the naturally **occurring** permanent magnet **(Fe_3O_4)** was first observed. The north and south seeking properties of such materials played a large role in early navigation and exploration. Except for this application, magnetism was a little known phenomenon until the 19th century, when Oersted discovered that an electric current in a wire deflects a compass needle. **This** discovery showed that electric **current** has something to do with the magnetic field because a compass needle gels deflected and finally points **in the north-south** direction only when placed in a magnetic field.

In this Unit, we shall consider in detail the production of the magnetic fields due to steady currents, and the forces they exert on circuits carrying steady **currents** and on isolated moving charge.

A good way of gaining a better understanding of the nature of fields **is** to **know** how they affect the charged particles on which they act. Hence, in the next unit, you will study the **behaviour** of charged particles in both electric and magnetic fields.

Objectives

After studying this unit you should be able to :

- understand what is meant by the magnetic field, the right hand rule, Biot-Savart law, right hand method, Ampere's law,
- define the magnetic field at a point in **terms** of the **force** on a steady **current** element and also on a moving charged particle,
- use the formula for the force on a steady current element or on charged particle due to a magnetic field to calculate **the** force on a certain simple current **carrying** circuits, and solve simple **problems,**