
UNIT 7 MICROSCOPIC PROPERTIES OF DIELECTRICS

Structure

- 7.1 Introduction
 - Objectives**
- 7.2 Microscopic Picture of a Dielectric in a **Uniform Electric** Field Review
 - Definition of Local **Field**
- 7.3 **Determination** of Local Field: Electric Field in Cavities of a Dielectric
- 7.4 Clausius-Mossotti Formula
 - Polarisation in a Gas**
 - Relation between Polarisability and Relative Permittivity**
- 7.5 Relation between the Polarisability and Refractive Index
- 7.6 **Behaviour** of Dielectric in Changing or Alternating Fields
- 7.7 Role of Dielectric in Practical Life
- 7.8 Summary
- 7.9 Terminal Questions
 - Solution/Answers**
 - SAQ's**
 - TQ's**

7.1 INTRODUCTION

In **Unit-5**, we have studied the **macroscopic (average)** behaviour of a dielectric in an electric field. We also **found** that the field is altered within the body of the dielectric, **This** can be accounted **for** by the charges appearing on the surface of the dielectric in the case of an isotropic material. In Unit 6, the macroscopic study of the dielectric behaviour was used to study the increase of capacitance in a condenser when a dielectric is placed between the plates of the condenser.

In the present unit, we will describe **microscopic** picture of a **dielectric** in which **we** will define the local field (E_L) and the average **macroscopic** field inside the dielectric (E_i). Further, we will derive the relationship between the local field and the macroscopic field. We will also study the effects of **polarisation** in nonpolar and polar molecules and derive the **famous** Clausius-Mossotti formula for **polarisation** of these molecules. Then we will derive Clausius-Mossotti equation for a **gas**. We will also study the relationship between **polarisability** and relative permittivity. After that, we will derive the relationship between **polarisability** and Refractive index. As you know that capacitors are used in alternating fields. So we will also study the effect of alternating field on a dielectric. In the last section of this unit we will study the role of dielectrics in our daily life.

In the **next** block, we will study the electric current produced by moving charges.

Objectives

After **going** through **this** unit, **you** will **be** able to:

- define the local field and relate it with **polarisation**,
find the macroscopic field within **the dielectric** and **relate** it to **polarisation**.
- relate the macroscopic **electric** field, the local **field** and the

- microscopic field **within the** dielectric,
- **write Clausius-Mossotti equation** for a liquid and a gas,
- establish a relationship between polarisability and Refractive index,
- discuss the role of dielectrics in daily life,

7.2 MICROSCOPIC PICTURE OF A DIELECTRIC IN A UNIFORM ELECTRIC FIELD—REVIEW

In Unit 5 you have studied the average (**macroscopic**) behaviour of dielectrics. In this section, we will study the **microscopic picture** of a dielectric in a uniform electric field. Let us consider a **dielectric** in a uniform electric field as shown in Fig. 7.1.

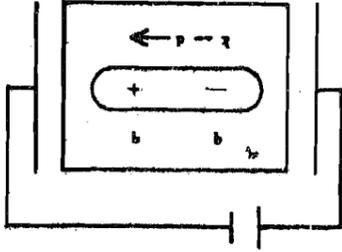


Fig. 7.1: A molecule in a dielectric medium.

In an electric field, **the electrons and** atomic nuclei of **the dielectric material** experience **forces** in opposite directions. We **know** that the electrons in a dielectric **cannot** move freely as in a conductor. Hence each **atom becomes a tiny dipole** with the positive and negative charge centres **slightly separated**. Taking the charge separation as **a**, the charge as **q** the dipole moment **p** in the **direction of field associated** with **the atom or molecule**

$$p = qa$$

Eq. (7.1) gives the dipole moment induced in **the atom/molecule** by the field. Hence we **call** it as induced **dipole moment**. If there are **n** such dipoles in an element of volume **V** of the **material**, we can define the **polarisation** vector **P** as the (dielectric) **dipole moment per unit volume as**

$$P = \frac{npV}{V}$$

Within the dielectric the charges **neutralise** each other, the negative charge of one **atom/molecule** is **neutralised** by the positive charge of its **neighbour**. Thus within the bulk of the **material, the electric field produces** on charge density but only a dipole moment density. However, at the **surface** this charge cancellation is not complete, and a polarisation charge densities of opposite signs **appears** at the **two surfaces** perpendicular to the **field**. **Now what** is the consequence of the appearance of polarisation charges?

The consequence **of** this is that **the electric** field inside the dielectric is **less** than the **electric** field causing the **polarisation**. The **polarisation** charges give rise to an electric field in the opposite direction. This **field** opposes the **electric** field causing **polarisation**. It is shown in Fig. 7.2.

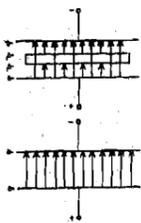


Fig. 7.2: Field inside a dielectric.

Hence we conclude that inside the dielectric, the average electric field is less than the electric field causing polarisation. However, the macroscopic or average field is not a satisfactory measure of the local field responsible for the polarisation of each atom.

Let us denote the field at the site or location of the atom or molecule as the **local** field. In next section, we will calculate the local field inside a dielectric.

7.21 Definition of Local field

In this section we will define the local field in a dielectric material. This is the field on a unit positive charge kept at a location or site from which an atom or molecule has been removed provided the other charges remain unaffected. Fig.7.3 shows a site in a uniformly polarised medium from which a molecule/atom is removed when all other charges are kept intact at their positions.

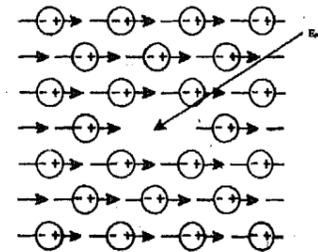


Fig. 7.3: A site in a uniformly polarised medium.

The extent of the charge separation depends on the magnitude of the local field. Hence we conclude that the induced dipole moment, p , is directly proportional to the local field, E_{loc} . Thus we have,

$$P = a E_{loc}$$

Where a is the constant of proportionality and is known as **atomic/molecular polarisability** and E_{loc} the **local** field.

To use Eq. (7.3) we require the value of E_{loc}

7.3 DETERMINATION OF LOCAL FIELD: ELECTRIC FIELDS IN CAVITIES OF A DIELECTRIC

The **polarisation** of dense materials such as liquids and many solids changes the electric **field inside** the material. The field experienced by an individual **atom/molecule** depends on the **polarisation** of atoms in its immediate vicinity. The actual value of the field varies rapidly from point to point. Very close to the nucleus it is very high and it is relatively small in between the **atoms/molecules**. By taking the **mean** of the fields over a space containing a very large number of atoms one gets the average value of the field.

SAQ1

Show that the field at the centre of a spherical cavity (filled with air) is zero.

The field experienced by an individual **atom/molecule** may be **called** the local field which is different **from** the average field. The local field is the one which causes the **polarisation** of the atom. The **average field** can be expressed as V/d where V is the potential difference between two points of a **dielectric**, distant d apart (just as one obtains the field between the plates of a parallel plate condenser). The estimation of local field is not so **easy**. Let us consider **three** different cavities to find the local field in a **dense** dielectric which has been **uniformly polarised**. See Fig. 7.4.

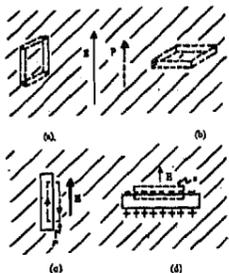


Fig. 7.4: The field in a slot cut in a dielectric depends on the shape and orientation of the slot. E shown is the average field.

The directions of electric (average) field E and P are shown in Fig. 7.4. Suppose we cut a rectangular slot ABCDEFGH as in (a) of Fig. (7.4). The field E and the polarisation P are parallel to the faces ABCD, EFGH. The field inside this slot can be found out by evaluating the line integral of E around the curve C shown in Fig. 7.4(b). Since $\oint E \cdot dl$ has to be zero for the closed curve C the field inside this slot has to be the same as the field outside the slot. Therefore the field inside a thin slot cut parallel to the field is equal to the average field E .

Now consider a thin rectangular slot with faces perpendicular to the average field E cut from the dielectric as shown in (c) A'B'C'D'E'F'G'H' of Fig. 7.4. To find the field inside this slot we use the Gauss' flux theorem on a surface S with one face outside the slot and one face inside the slot. See Fig.7.4(d). The flux of E through faces parallel to E is zero. Instead of the flux of E let us consider the flux of electric displacement D . Let E_{loc} be the field inside the slot; then D_1 inside the slot is $\epsilon_0 E_{loc}$. The D vector outside the slot is $\epsilon_0 E + P$. Now, as the flux of D through the closed surface S has to be zero (no free or external charges inside the Gaussian surface), we must have

$$\begin{aligned} \epsilon_0 E_{loc} &= \epsilon_0 E + P \\ \therefore E_{loc} &= E + P / \epsilon_0 \end{aligned} \quad (7.4)$$

The field inside the slot in this case is different from the field outside by P/ϵ_0 because of the surface polarisation charges appearing on A'B'C'D' shown in Fig. 7.4(c).

Another possible slot is a spherical hole which is the most likely way an atom finds itself in most liquids and solids. We would expect that an atom finds itself, on the average, surrounded by other atoms in what would be a good approximation to a spherical hole. What is the local field in a spherical hole? Suppose we cut a spherical hole after "freezing" the state of polarisation from a uniformly polarised material. If we call E_{loc} as the field inside the spherical hole at its centre and E_p as the field produced by the uniformly polarised dielectric spherical plug at its centre, then by adding E_{loc} and E_p , we should get the average field E inside the dielectric, See Fig. 7.5. This should be true because of the superposition principle. Thus

$$E = E_{loc} + E_p \quad (7.5)$$

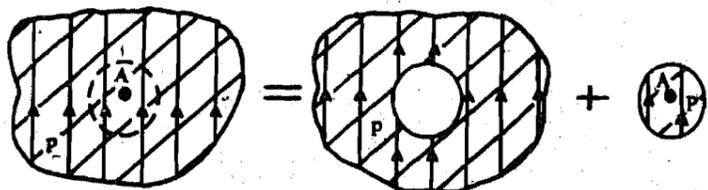


Fig. 7.5: The field at any point A in a dielectric can be considered as a sum of the field in a spherical hole plus the field due to the spherical plug.

and the required field

$$E_{loc} = E - E_p \quad (7.6)$$

One can calculate E_p (the field produced by the uniform polarised dielectric) as follows:

The field E_p arises from bound charges of density $= \sigma$. $n = P \cos\theta$. Hence the field due to the charges over an area dS is given

$$dE_p = \frac{1}{4\pi\epsilon_0} \frac{(\cos\theta) dS}{r^2} r$$

where r is the unit vector from the surface to the centres of the sphere where the field is to be calculated.

Resolving dE_p into components parallel and perpendicular to P , it is clear from the symmetry of the situation that only the components parallel to the direction of P will contribute to the total field E_p . Thus

$$E_p = dE_p \cdot \cos\theta$$

It should be noted that the direction of E_p is parallel to that of P , we then have

$$E_p = \frac{1}{4\pi\epsilon_0} P \frac{\cos^2\theta}{r^2} dS$$

$$\text{Now } dS = r^2 \sin\theta d\theta d\phi$$

and the limits of θ are from 0 to π and that of ϕ from 0 to 2π .

Hence

$$\begin{aligned} E_p &= \frac{P}{4\pi\epsilon_0} \int_0^{2\pi} \int_0^\pi \cos^2\theta \sin\theta d\theta d\phi \\ &= \frac{P}{4\pi\epsilon_0} \int_0^{2\pi} d\phi \int_0^\pi \frac{\cos^2\theta}{2} d\theta \\ &= \frac{2}{3} \frac{P}{4\pi\epsilon_0} \\ &= \frac{P}{3\epsilon_0} \end{aligned} \quad (7.7)$$

Then the field experienced by an atom in a spherical hole is

$$E_i = E - \frac{P}{3\epsilon_0} \quad (7.8)$$

To determine the field E_p at an arbitrary point r inside the dielectric sphere, we consider the polarised sphere as a superposition of slightly displaced spheres of positive and negative charges. See Fig. 7.6. Further note that the field at point r , is entirely determined by the charge contained in the sphere of radius r , interior to point r .

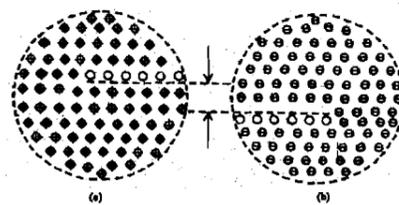


Fig. 7.6: Superposition of slightly displaced sphere of positive and negative charges.

The sphere of positive charge can be regarded as a point charge at its centre and if P is the volume charge density then the positive charged sphere is equivalent to charge at its centre equal to $\frac{4\pi}{3} r^3$. Similarly the negative charged sphere is equivalent to a

point charge at its centre. The magnitude of this point charge is same as $\frac{4\pi}{3} r^3$. If 'a' is the separation of the positive and negative charges in an atom then the uniformly polarised dielectric is equivalent to a dipole of moment $\frac{4\pi}{3} r^3 a$. If there are n dipoles per unit volume, q is the charge on each dipole then $\sigma_p = qn$. [The number of positive or negative charges per unit volume is also equal to n in the spheres considered above]. Then the dipole moment of the sphere is given by

$$\frac{4\pi}{3} r^3 nqa = \frac{4\pi}{3} r^3 P$$

and the polarised sphere is equivalent to a dipole of moment $\frac{4\pi}{3} r^3 P$ kept at its centre. The potential due to this dipole at the point r on the surface is given by

$$\frac{4\pi}{3} \frac{r^3 P \cos \theta}{4\pi \epsilon_0 r^2} = \frac{P \cos \theta r}{3\epsilon_0}$$

where p, r, θ are as shown in the Fig. 7.7.

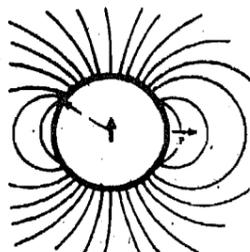


Fig. 7.7: Field outside a uniformly polarised sphere.

The polarisation is in the direction of E and if we take this to be the z-direction with the origin at the centre then the potential at T is

$$\phi = \frac{Pz}{3\epsilon_0}$$

This shows that the potential at a point depends only on its z coordinate. Hence the electric field is along z direction and is given by

$$E_p = \frac{-\phi}{z} = -\frac{P}{3\epsilon_0}$$

This shows that the electric field inside the dielectric sphere is uniform and in the direction of the polarisation vector. Hence the field experienced by an atom in a spherical hole is

$$E_{loc} = E + \frac{P}{3\epsilon_0} \tag{7.8}$$

The field in a spherical hole is greater than the average field by $P/3\epsilon_p$.

SAQ 2

Show that the field inside a uniform spherically symmetric charge distribution with as charge density is equal to $\frac{r}{3\epsilon_0}$ where r is the position vector of the point with origin at the centre.

7.4 CLAUSIUS-MOSSOTTI EQUATION

In a liquid we would expect an individual atom to be **polarised** by a field obtained in a spherical cavity rather than by the average (macroscopic) **field**. Thus using Eq. 7.8 and Eq. 7.3 we have

$$\mathbf{P} = n\alpha \mathbf{E}_{\text{loc}}$$

$$\mathbf{P} = n\alpha \mathbf{E} + \frac{\mathbf{P}}{3\epsilon_0} \quad (7.9)$$

This can be rewritten as

$$\mathbf{P} = \frac{n\alpha}{1 - \frac{n\alpha}{3\epsilon_0}} \mathbf{E} \quad (7.10)$$

The susceptibility χ was defined in Unit 5 by the equation

$$\mathbf{P} = \epsilon_0 \chi \mathbf{E}$$

Hence

$$\chi = \frac{n\alpha / \epsilon_0}{1 - n\alpha / 3\epsilon_0} \quad (7.11)$$

Eq. 7.11 gives the relation between susceptibility and **atomic/molecular polarisability**. This is one form of Clausius-Mossotti Equation.

7.4.1 Polarisation in a Gas.

Unlike the **atoms/molecules** of a liquid or solid it is possible to consider the **atoms/molecules** of a gas **as far apart** and independent. We can neglect the field due to the dipoles on the immediate **neighbourhood** of an individual molecule. Hence the local field causing **polarisation** is the average or macroscopic **field E**. Therefore we can write

$$\mathbf{P} = \epsilon_0 \chi \mathbf{E} = n\mathbf{p}$$

where n is the number of molecules per unit volume. If we consider only an individual **atom/molecule** and write the dipole moment \mathbf{p} as

$$\mathbf{p} = \epsilon_0 \alpha \mathbf{E} \quad (7.12)$$

where α is known as **atomic polarisability**. Therefore α has the dimensions of volume and roughly equals the volume of an atom.

We can relate α or χ to the natural frequency of oscillation of electrons in the **atom/molecule**. If the atom is placed in an oscillating field \mathbf{E} the centre of charge of electrons obeys the equation

$$m \frac{d^2 x}{dt^2} + m\omega_0^2 x = qE$$

where m is the mass of electron of charge q , $m\omega_0^2 x$ is the restoring force term and qE the force from outside field—this equation is the same as the equation of forced oscillation. If the **electric** field varies with angular frequency ω then

$$x = \frac{qE}{m(\omega_0^2 - \omega^2)}$$

For our purposes in the **electrostatic** case $\omega = 0$ which means that

$$x = \frac{q E}{m\omega_0^2}$$

and the dipole moment p is

$$P = q x = \frac{q^2 E}{m\omega_0^2}$$

From Eq. (7.12) we can write the atomic polarisability as

$$\alpha = \frac{q^2}{\epsilon_0 m \omega^2} \quad (7.13)$$

and

$$\frac{P}{E} = \epsilon_0 = \epsilon \chi (\epsilon_0 - 1) = \epsilon_0 n \alpha$$

$$\epsilon_r - 1 = n \alpha = \frac{n q^2}{\epsilon_0 m \omega^2}$$

For hydrogen gas we can get a rough estimate of ω_0 . The energy needed to ionise the hydrogen atom is equal to 13.6 eV. Equating this a $\frac{h\omega_0}{2\pi}$ where h is Planck's constant we get

$$\begin{aligned} & \approx \frac{13.6 \times 1.6 \times 10^{-19} \times 2\pi}{6.62 \times 10^{-34}} \\ \omega_0 & \approx 20.65 \times 10^{15} \end{aligned}$$

Substituting this in the equation 7.13 (a) we get

$$\epsilon_r \approx 1 + \frac{n q^2}{\epsilon_0 m_0} \approx 1.00020$$

The experimentally observed value is $\epsilon_r = 1.00026$.

7.4.2 Relation between Polarizability and Relative Permittivity

In Unit 5, you have studied that one can write P as

$$P = \epsilon_0 (\epsilon_r - 1) E \quad (7.14)$$

where ϵ_r is the relative permittivity.

Using Eq. 7.14 in Eq. 7.8 we get

$$E_0 = E + \frac{P}{3\epsilon_0} = (\epsilon_r + 2)E / 3 \quad (7.15)$$

Using Eqs. 7.14 and 7.15 one can rewrite Eq. 7.9 as

$$P = \epsilon_0 (\epsilon_r - 1) E = n \alpha \frac{(\epsilon_r + 2)}{3} E$$

which yields

$$\alpha = \frac{3\epsilon_0}{n} \frac{(\epsilon_r - 1)}{(\epsilon_r + 2)} \quad (7.16)$$

Eq. 7.16 gives us the relation between atomic/molecular polarisability and the relative permittivity. Eq. (7.16) is another form of the Clausius-Mossotti equation.

Obtain Eq. 7.15 from Eq. 7.14.

Proofs of this relation will be given in the unit on propagation of electromagnetic waves.

7.5 RELATION BETWEEN THE POLARISABILITY AND REFRACTIVE INDEX

For a dielectric, the refractive index μ defined as the ratio of the speed of light in vacuum to the speed in the dielectric medium, can be shown to be equal to $\sqrt{\epsilon_r}$.

$$\mu^2 = \epsilon_r$$

Using Eq. 7.15 in Eq. 7.14 we get

$$\alpha = \frac{3\epsilon_0}{n} \frac{(\mu^2 - 1)}{(\mu^2 + 2)} \quad (7.17)$$

Eq. 7.17 gives the relation between polarisability and refractive index. This relation is known as the Lorentz-Lorenz formula.

In all the equations discussed above n represents the number density of atoms or molecules which is equal to $N_A d/W$ where N_A is the Avogadro number, d the mass density and W the molecular weight. For gases, we have the gas equation relating pressure, P volume V and absolute temperature T given by

$$PV = RT = N_A kT$$

where q is the mole number.

$$\text{and } P = qN_A kT / V = nkT$$

Therefore, $n = p' / kT$.

Thus if we determine ϵ_r at different pressures for a gas, we can calculate the atomic/molecular polarisability of gas. For this we write Eq. 7.16 as

$$\alpha = \frac{3\epsilon_0 kT}{p'} \frac{(\epsilon_r - 1)}{(\epsilon_r + 2)}$$

or

$$\frac{(\epsilon_r - 1)}{(\epsilon_r + 2)} = \frac{\alpha}{3\epsilon_0 k} (p' / T) \quad (7.18)$$

Eq. 7.18 represents the linear relation between $(\epsilon_r - 1) / (\epsilon_r + 2)$ and (p' / T) . If now a graph is drawn with $(\epsilon_r - 1) / (\epsilon_r + 2)$ on the y-axis and (p' / T) on the x-axis, we get a straightline the slope of which gives $(\alpha / 3\epsilon_0 k)$.

7.6 BEHAVIOUR OF DIELECTRIC IN CHANGING OR ALTERNATING FIELDS

So far we have considered only electrostatic fields in matter. Now we would like to look at the effects of electric fields that vary with time, like the field in the dielectric of a capacitor used in an alternating current circuit.

Will the changes in polarisation keep up with the changes in the field? Will the polarisability, the ratio of P to E , at any instant be the same as in a static electric field?

For very slow changes or small frequencies we do not expect any difference. However, for high frequencies or faster process we have to look at the response time for the polarisation. We have to separately consider two polarisation processes viz., induced polarisation and the orientation of permanent dipoles. we know that the

induced **polarisation occurs** by the distortion of the electronic structure. In the distortion mass involved is that of electron and the distortion is very **small**, which means the **structure** is very stiff. From our knowledge of oscillatory motion (see the course on oscillations and waves), its natural frequencies of vibration are extremely high. Alternatively, the motions of electrons in atoms and molecules are **characterised** by periods of the order of the period of a visible light wave (10^{-16} seconds). Thus the readjustment of the electronic structure **i.e.** the polarisation response is very rapid, **occurring** at the time scale of 10^{-14} sec. For this reason we **find** that nonpolar substances behave the same way from dc **upto** frequencies close to those of visible light.

We **shall** examine the situation in the light of Eq. 7.15, where we have expressed the **Clausius-Mossotti** formula in terms of the refractive index. We know that the refractive index is dependent on the wavelength or frequency. Thus, in a way 7.13 implies the variation of the polarisability with frequency.

Experimentally, **d.c.** values of ϵ_r can be found. The refractive index of the same substance can be determined by optical methods, using a spectrometer. A fairly good agreement is found between the refractive index and ϵ_r values **for non-polar** substances. However for polar substances, ϵ_r varies with frequency; it decreases with increase in frequency. The drop in the value of ϵ_r at high frequencies is due to the fact that the permanent dipoles are not able to follow the rapid alternation of the field. In other words the polarisation response of polar molecules is much slower. However, in the frequency range of visible light the refractive index and ϵ_r values shows a fairly good agreement as indicated by nonpolar substances.

7.7 ROLE OF DIELECTRIC CAPACITOR IN OUR PRACTICAL LIFE

Dielectrics have several applications. Dielectrics are used very widely in capacitors. Although the actual requirements vary depending on the application, there are certain characteristics which are desirable for their use in capacitors, A capacitor should be **small**, have high resistance, be capable of being used at high temperatures and have long life. From a commercial point of view it should also be cheap. Specially prepared thin **kraft** paper, free from holes and **conducting** particles, is used in power capacitors where withstanding high voltage stresses is more important than incurring dielectric losses. In addition, the **kraft** paper is impregnated with a suitable liquid such as chlorinated diphenyl. This increases the dielectric constant and thus reduces the size of the capacitor. In addition the breakdown strength is increased.

In addition to paper capacitors for general purpose, other types of capacitors are used. In the film capacitors, thin film of **teflon**, mylar or polythene are used. These not only reduce the size of the capacitor but also have high resistivity. Teflon is used at high frequencies as it has low loss. In **electric** capacitors, an electrolyte is deposited on the impregnating paper. The size of such a capacitor is small as the film is very thin. Polarity and the maximum operating voltage are important specifications for these capacitors,

Some ceramics can be used as **temperature** compensators in electronic circuits. High **dielectric** constant materials, where small variations in dielectric constant with temperature can be tolerated, help **miniaturise** capacitors. Barium titanate and its modifications are the best examples of such materials.

7.8 SUMMARY

- **Inside** a dielectric the average electric field is **less** than the electric field which causes the **polarisation**.
- In a **dielectric** material, the induced dipole moment p , is **directly proportional** to the local field and mathematically given by

$$P = \alpha E_{loc}$$

where the symbols have their usual meanings.

- The field inside a spherical hole is given by

$$\mathbf{E}_{loc} = \mathbf{E} + \mathbf{P} / 3\epsilon_0$$

which shows that the field in a **spherical** hole is greater **than** the average field.

- The relation between susceptibility and atomic/molecular polarisability is given by

$$= \frac{n \alpha / \epsilon_0}{1 - n \alpha / 3\epsilon_0}$$

7.9 TERMINAL QUESTIONS

- i) A sphere of Linear dielectric material is placed in a **uniform** electric field E_0 (see Fig. TQ1). Find the field inside the sphere and polarisation in terms of external field E_{loc}

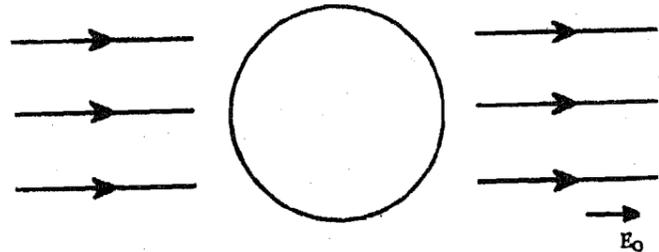


Fig. TQ1 : A linear dielectric material placed in a uniform magnetic field.

- 2) The electric field inside a **polarised** sphere is uniform and equal to $-P/3\epsilon_0$. Prove this by superposing the internal fields of two spheres of charge whose centres are separated.
- 3) Show that ϵ_0 times the force on a unit charge placed in a disc shaped cavity will measure the electric displacement (D) in a solid dielectric.
- 4) A dielectric consists of a cubical array of atoms (or molecules) with spacing d between each atom along the (x,y,z) axis. It is influenced by a field E_{loc} applied along the direction of z -axis. evaluate the average field produced by all the dipoles.

Solutions/Answers

SAQ's

- 1) We identify pairs of dipoles equidistant from the centre. **from** Unit 3, we **know** that the dipole field falls off with distance as $1/r^2$. Since **equidistant** pairs have directions of p opposite to one another, the overall field **at** the centre due to the pair is zero. This is the case for every other pair. Hence $E_{in} = 0$.
- 2) According to integral **form** of Gauss's law

$$\mathbf{E} \cdot \mathbf{n} \, ds = \frac{1}{\epsilon_0} \, dV$$

Therefore,

$$\mathbf{E} 4\pi r^2 = \frac{1}{\epsilon_0} \frac{4}{3} \pi r^3$$

or

$$E = \frac{1}{3\epsilon_0} \mathbf{r}$$

in the vector form

$$\mathbf{E}(\mathbf{r}) = (\mathbf{r})\mathbf{r} / 3\epsilon_0$$

$$\begin{aligned} 3) \quad E_{loc} &= \mathbf{E} + \frac{\mathbf{P}}{3\epsilon_0} = \mathbf{E} + \frac{\epsilon_0(\epsilon_r - 1)\mathbf{E}}{3\epsilon_0} \\ &= \frac{3\mathbf{E} + (\epsilon_r - 1)\mathbf{E}}{3} \\ &= (\epsilon_r + 2)\mathbf{E} / 3 \end{aligned}$$

Terminal Answers

- 1) The resultant field is no longer uniform in the **neighbourhood** of the sphere because of the **polarisation** of the sphere. Let \mathbf{P} be the dipole moment per unit volume inside the sphere. We would expect \mathbf{P} to be uniform as the **dielectric** material is linear. Then \mathbf{P} is proportional to the electric field, \mathbf{E}_{in} , inside the sphere. If χ is the susceptibility of the **material** then we can write \mathbf{P} as

$$\mathbf{P} = \epsilon_0 \chi \mathbf{E}_{in}$$

This polarisation \mathbf{P} produces a field inside the sphere which is given by $-\mathbf{P}/3\epsilon_0$.

The electric field inside the sphere, \mathbf{E}_{in} , can be regarded as a superposition of the uniform field \mathbf{E}_0 and the field due to **polarised** (dipoles) charges. Thus.

$$\mathbf{E}_{in} = \mathbf{E}_0 - \frac{\mathbf{P}}{3\epsilon_0}$$

Substituting for \mathbf{P} in terms of \mathbf{E}_{in}

$$\mathbf{E}_{in} = \mathbf{E}_0 - \frac{\epsilon_0 \chi \mathbf{E}_{in}}{3\epsilon_0}$$

Thus

$$\mathbf{E}_{in} = \frac{3\mathbf{E}_0}{(3 + \chi)} = \frac{3}{(\epsilon_r + 2)} \mathbf{E}_0$$

$$\therefore (1 + \chi) = \epsilon_r$$

$$\text{and } \mathbf{P} = \epsilon_0 (\epsilon_r - 1) \mathbf{E}_{in}$$

$$= \frac{3\epsilon_0 (\epsilon_r - 1)}{(\epsilon_r + 2)} \mathbf{E}_0$$

The assumption of uniform **polarisation** is now seen to be self-consistent.

2)

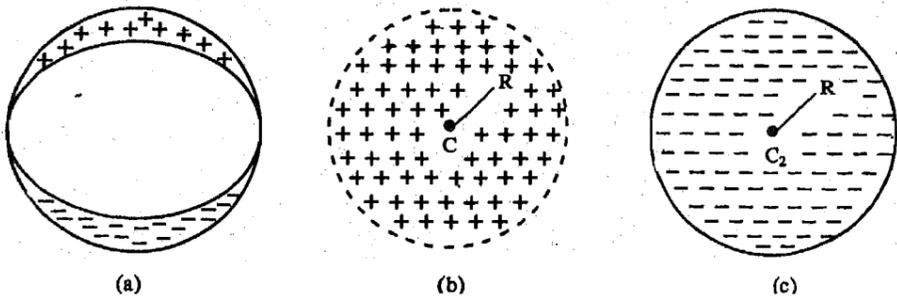


Fig. TQ2 : A sphere of lined-up molecular dipoles

The polarised dielectric sphere (a) of Fig. TQ2 can be regarded as a superposition of two spheres charged uniformly one with positive charge as in (b) and another with negative charge as in (c) of Fig. TQ2. The two spheres have centres at C_1 and C_2 which are separated by a distance a , say along the z -axis. This means that P is along the z direction. The field at R due to the positively charged sphere is given by

$$E_+ = \frac{4\pi\rho}{3} r^3 \frac{\mathbf{r}}{4\pi\epsilon_0 r^2} = \rho \frac{\mathbf{r}}{3\epsilon_0}$$

where $\mathbf{r} = C_1 R$ and ρ is the charge density. Similarly the field at R due to the negatively charged sphere is

$$\mathbf{E} = -\rho \frac{\mathbf{r}'}{3\epsilon_0}$$

where $\mathbf{r}' = C_2 R$. Now ρ is along the z -direction and $C_2 C_1 = a$ is also along the z -direction. Adding E_+ and E_- vectorially (See Fig. TQ) we get

$$\begin{aligned} \mathbf{E}_+ + \mathbf{E}_- &= \frac{\rho}{3\epsilon_0} (\mathbf{r} - \mathbf{r}') = -\frac{\rho a}{3\epsilon_0} \\ &= -a / 3\epsilon_0 \end{aligned}$$

as a is equal to the no. of either positive or negative charges per unit volume.

- 3) We assume that the radius of the cavity to be greater than its thickness, measured parallel to the field. By this assumption, the fields near A and B is the same as E . the field E_0 at the centre of the cavity is therefore parallel to E_A , the field near A. This is in accordance with the boundary condition for the normal component of $D (= \epsilon_0 E)$, viz.,

$$\epsilon_0 E_A = E_C = (D)_n$$

Here the normal component measures the complete vector D . Therefore we can conclude that ϵ_0 times the force on unit charge, viz., E , placed in a disc shaped cavity measures the electric displacement (D).

- 4) The scalar potential due to a dipole of moment P at (x', y', z') is

$$\phi(x, y, z) = \frac{p \cos \theta}{4\pi\epsilon_0 R^2} = \frac{p(z-z')}{4\pi\epsilon_0 R^3}$$

since $(z-z') = R \cos \theta$ and

$$R^2 = (x-x')^2 + (y-y')^2 + (z-z')^2$$

The z -component of the field

$$\begin{aligned} E &= -\frac{\partial \phi}{\partial z} = -\frac{p}{4\pi\epsilon_0 R^2} + \frac{3p(z-z')}{4\pi\epsilon_0 R^3} \\ &= \frac{p}{4\pi\epsilon_0} \frac{2(z-z')^2 - (x-x')^2 - (y-y')^2}{[(x-x')^2 + (y-y')^2 + (z-z')^2]^{5/2}} \end{aligned}$$

We assume the dipole to be present at the origin. To find the average value of the field, we integrate the field. The integration is taken over by an Octant of the unit cell. The unit volume of the unit cell is d^3 . Thus

$$\begin{aligned} E &= \frac{8p}{4\pi\epsilon_0 d^2} \int_0^{d/2} \int_0^{d/2} \int_0^{d/2} \frac{2z^2 - x^2 - y^2}{(x^2 + y^2 + z^2)^{3/2}} dx dy dz \\ &= \frac{p}{3\epsilon_0 d^3} \langle E \rangle = \frac{P}{3\epsilon_0} \therefore P = \frac{p}{d^3} \end{aligned}$$

[Kindly see PHE-04 & PHE-05 Course]