

UNIT 5 MACROSCOPIC PROPERTIES OF DIELECTRICS

Structure

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5.1 INTRODUCTION

In Unit 4 of Block 1 of the present course, you have learnt the concepts of **electric** field, electrostatic energy and the nature of the electrostatic force. **However**, for reasons of simplicity we confined our considerations of these concepts for charges that are placed in vacuum. For example, Coulomb's law of electrostatic force is the electric field due to a distribution of charges given in Unit 4; refer to **the** situation in which the surrounding medium is vacuum. Of equal importance is the situation in which the electrical phenomenon occurs in the presence of a material medium. Here we must distinguish between two different situations, **as** the physics of these **situations** is completely different. The **first** situation is when the medium consists of insulating materials **i.e.**, those materials which do not conduct electricity. **The** second situation corresponds to the case when the medium consists of conducting **materials**, **i.e.** materials like metals which are conductor of electricity. The conducting materials contain electrons which are free to move within the material. These electrons move under the action of an electric field and constitute current. We shall study conducting materials and electric fields in conducting **materials** at a later stage.

Dielectric substances are insulator (or non-conducting) substances as they do not allow conduction of electricity through them.

In the present unit, you will study the electric field in the presence of **an** insulator. In these materials there are practically no free electrons **or** number of such electrons is so **small** that the conduction is not possible. In 1837, **Faraday** experimentally found that when an insulating material, **also** called dielectric (such as mica, glass or **polyestrin etc.**) is introduced between two plates of a capacitor, it is found that the capacitance is increased by a factor which is greater than one. This factor is known as dielectric constant (**K**) of the material. It was also found that **this** capacitance is independent **of** the shape and size of the material but it varies from material to material. In the case of glass, the value of **the dielectric** constant is 6, while for water it is **80**. All the electrons in these materials are bound to their respective atoms or molecules.

When a potential difference is applied to the insulators no electric current flows; however, the study of their behaviour in the presence of an electric field gives us very useful information. **The** choice of a **proper** dielectric in a capacitor, the understanding of double **refraction** in quartz or calcite crystals are based on such **studies**. Natural materials, such as **wood**, cotton, natural rubber, mica are **some**

popular examples of electric insulators. A large number of varieties of plastics are also good **dielectrics**.

In this unit first of all we will study a simple model of dielectric material and **deduce** a relationship between applied field E and the dipole moment p of a **molecule/atom**. You will learn about electric **polarisation** in a dielectric material and define **polarisation** vector P . In Unit 2, you have studied Gauss's law in vacuum. You will **now** apply it to a dielectric medium. Here we will also introduce you to a new vector known as the electric displacement vector D . After that we will discuss the continuity of D and E at the interface between two dielectrics.

In recent years dielectric materials have become important specially due to their large **scale** use in electric and electronic **devices**. There are high demands for the improvement of operating reliability of these devices. Reliability of these devices is measured to a great extent by the quality of electrical insulation. In the last section you will study the dielectric strength and break down in dielectrics.

In the next unit you will study about the details of capacitors, specially the capacitance of a capacitor, energy stored in a capacitor, capacitor with a dielectric and different forms of the capacitors etc.

Objectives

After going through this unit, you will be able to

- explain the **behaviour** of a dielectric in an electric field,
- deduce Gauss's law for a dielectric medium,
- define dielectric polarisation and classify dielectrics as polar and nonpolar,
- explain Displacement Vector (D) and relate it to the electric field strength (E);
- define dielectric constant,
- **state** and derive the boundary conditions on E and D ,
- explain dielectric strength and **dielectric** breakdown.

5.2 SIMPLE MODEL OF THE DIELECTRIC MATERIAL

You must be aware that:

- every material is made up of a very large number of **atoms/molecules**,
- e an atom consists of a positively charged nucleus and negatively charged particles, with electrons revolving around it,
- a the total positive charge of the nucleus is balanced by the total negative charge of the electrons in the atom, so that the atom, as a whole, is electrically neutral **w.r.t.** any point present outside the atom,
- a molecule may be constituted by atom of the same kind, or of different kinds.

To understand the **polarisation** we shall consider a crude model of the atom. A simple crude model of an atom is shown in Fig. 5.1.

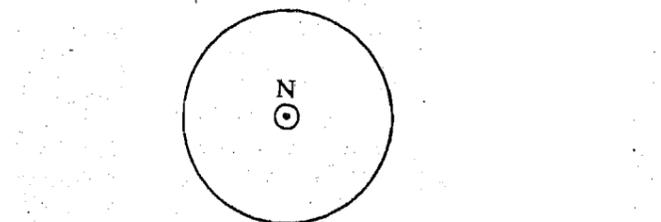


Fig. 5.1 : Model of an Atom.

The nucleus **is** at **the** centre and the various electrons revolving around it can be thought of as a spherically symmetric cloud of electrons. For **points** outside the atom this cloud of electrons can be regarded as concentrated at the centre of the atom as a point charge.

In most of the atoms and molecules the centres of positive and negative charges coincide with each other, whereas, in some molecules the centres of the two charges are located at different points. Such molecules are called polar molecules.

Further, we note that in dielectrics, all the **electrons** are firmly bound to their respective atoms and are unable to move about freely. In the absence of an electric field, the charges inside the **molecules/atoms** occupy their equilibrium positions. The arrangement of the molecules in a dielectric material is shown in Fig. 5.2.

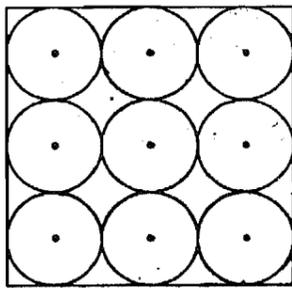


Fig. 5.2: The arrangement of the atoms in a dielectric material.

The charge centres are shown coincident at the centre of the sphere. Keeping this picture of a dielectric in mind we shall proceed to study its behaviour in an electric field in the next section,

5.3 BEHAVIOUR OF A DIELECTRIC IN AN ELECTRIC FIELD

You have seen in Section 5.2 that in a dielectric material, the centres of positive and negative charges of its atoms are found to coincide at the centre of the sphere. It is shown in Fig. 5.3.

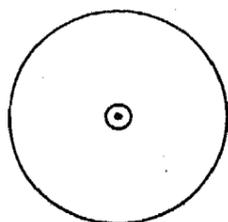


Fig. 5.3: Atoms in which the centres of charges are coincident with the centre of the spheres.

In Unit 1, you have studied that a charge experiences a force in the presence of an electric field. Therefore when a dielectric material is placed in an electric field, the positive charge of each atom experiences a force along the direction of the field and the negative charge in a direction opposite to it. This results in small displacement of charge centres of the atoms or molecules. This is also true of molecules whose charge centres do not coincide in the absence of an electric field. The separation of the charge centres due to an applied field E is shown in Fig. 5.4.

Electric dipole moment per unit volume is known as polarisation

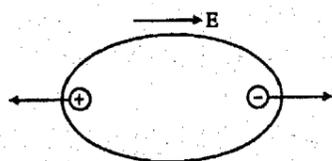


Fig. 5.4 : The separation of the charge centres due to an applied field E .

This phenomenon is called polarisation. Thus when an electrically neutral molecule is placed in an electric field, it gets **polarised**, with positive charges moving towards one end and negative charges towards the other. The otherwise **neutral** atom thus becomes a **dipole** with a dipole moment, which is proportional to electric field. The dipole and its dipole moment was discussed in Unit 3.

Now we consider another kind of molecule in which the charge centres do not coincide as shown in Fig. 5.5.

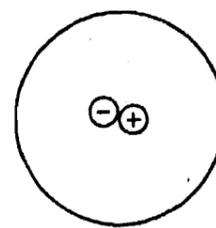


Fig. 5.5: A dielectric material in which charge centres do not coincide.

Due to this reason the molecule already possesses a dipole moment. Such materials are called **polar materials**. For such materials, let the initial orientation of the dipole axis be AOB as shown in Fig. 5.6.

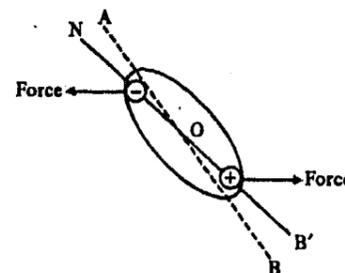


Fig. 5.6: Molecule Possessing a dipole moment.

Now an electric field E is applied, This field pulls the charge centres along lines **parallel** to its direction. Thus the electric field exerts a torque on the dipole causing it to **reorient** in the direction of the field. Recall our discussion of the torque on a dipole in Unit 3. In the absence of an electric field these polar materials do not have any resultant dipole moment, as the dipoles of the different molecules are oriented in random directions due to thermal agitation. When an electric field is applied, each of these molecules reorients itself **in the direction** of the field, and a net polarisation of the material results. The reorientation or **polarisation** of the medium is not perfect again due to thermal agitation. Thus polarisation depends both on field (linearly) and temperature.

SAQ 1

What are dielectrics? In what respects do they differ from conductor?

5.3.1. Non-polar and Polar Molecules

We have considered two types of molecules. One in which the centre of positive charges coincide with the centre of negative charges. The molecule as a whole has no **resultant charge**. Molecules of this type are called **Non-polar**. Examples of **Non-polar** molecules are air, hydrogen, benzene, carbon, tetrachloride **etc.** The second type is the one in which the centre of positive charges and the centre of negative charges do not coincide. In this case the molecule possesses a permanent dipole moment. This type of molecule is called a **Polar** Molecule. Examples of **polar molecules** are water, glass, **etc.**

Thus we see that, a **Non-polar** molecule acquires a Dipole **Moment only** in the presence of an electric field: **whereas** in a Polar Molecule the already **existing** dipole moment orients itself in the direction of the external electric field. **Even in polar** molecules, there is some induced dipole moment due to additional separation of

These are molecules in which there is electrical neutrality and the centres of positive and negative charges lie at one and the same point.

In such molecules the charge centres lie at different points and consequently there is an inherent dipole moment associated with the molecules.

charges, however this effect is comparatively much smaller than the reorientation effect and is thus ignored for polar molecules.

5.3.2 Polarisation Vector P

Let us study the effect of an electric field on a dielectric material by keeping a dielectric slab between two parallel plates as shown in Fig. 5.7. The electric field is

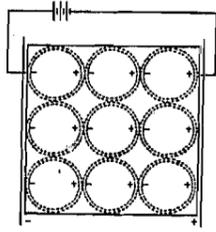


Fig. 5.7: Effect of an Electric field on a dielectric material by keeping a dielectric slab between two parallel plates.

set up by connecting the plates to a battery. We limit our discussion to a homogeneous and isotropic dielectric. A homogeneous and isotropic dielectric is one in which the electrical properties are the same at all points in all directions. The applied electric field displaces the charge centres of the constituent molecules of the dielectric. The separation of the charge centres are shown in Fig. 5.7. We find that the negative charges of one molecule faces the positive charges of its neighbour. Thus within the dielectric body, the charges neutralise. However, the charges appearing on the surface of the dielectric are not neutralised. These charges are known as Polarisation Surface Charges. The entire effect of the polarisation can be accounted for by the charges which appear on the ends of the specimen. The net surface charge, however, is bound and depends on the relative displacement of the charges. It is reasonable to expect that the relative displacement of positive and negative charges is proportional to the average field E inside the specimen.

From Fig. 5.7, we find that these polarisation charges appear only on those surfaces of the dielectric which are perpendicular to the direction of the field. No surface charges appear on faces parallel to the field. Such a situation occurs only in the special case of a rectangular block of dielectric kept between the plates of a parallel plate condenser. It is shown later in this section that surface density of bound charges depends on the shape of the dielectric material.

The polarisation of the material is quantitatively discussed in terms of dipole moment induced by the electric field. Recall that the moment of a dipole consisting of charges q and $-q$ separated by a displacement d is given by $P = -qd$. It is known from experiments that the induced dipole moment (p) of the molecule increases with the increase in the average field E . We can say that p is proportional to E

$$\text{or } p = \alpha E \quad (5.1)$$

where α is the constant of proportionality known as **Molecular/Atomic Polarisability**. Let us now define a new vector quantity which we shall represent by P and shall call it polarisation of the dielectric or just **polarisation**. **Polarisation P** is defined as the electric dipole moment per unit volume of the dielectric. It is important to note that the term **polarisation** is used in a general sense to describe what happens in a dielectric when the dielectric is subjected to an external electric field. It is also used in this specific sense to denote the dipole moment per unit volume.

Let us first consider a special case of n polarised molecules each with a dipole moment p present per unit volume of a dielectric and let all the dipole moments be parallel to each other. Then from the definition of P

$$P = np$$

From the above definition, units of P are

$$\text{Units of } P = \frac{\text{Coulomb m}}{\text{m}^3} = \frac{\text{Coulomb}}{\text{m}^2} = \text{C. m}^{-2}$$

These are the charges that appear on the face of a dielectric material when it is subjected to an external field. These charges are found on the faces that are perpendicular to the direction of the field.

It is the ratio of the induced dipole moment of the molecule to the applied electric field.

In **general**, \mathbf{P} is a point function depending upon the coordinates. In such cases, where the ideal situation mentioned above is not satisfied, we would consider an **infinitesimal** volume V throughout which **all** the \mathbf{p} 's can be expected to be parallel and write the **equation**

$$\mathbf{P} = \lim_{\Delta V \rightarrow 0} \sum_{i=1}^N \frac{\mathbf{p}_i}{V} \quad (N \text{ is the number of dipoles in volume } V) \quad (5.1a)$$

Here V is large compared to the molecular volume but small compared to ordinary volumes, **thus**, although \mathbf{p} is a point function, it is a space average of \mathbf{p} . The direction of \mathbf{p} will of course, be parallel to the vector sum of the dipole moment of the **molecules** within V . In such a **case** where the \mathbf{p} 's are not parallel, as in a dielectric that has polar molecules, Eq. (5.1a) still **holds** as the **defining** equation for \mathbf{p} ,

SAQ 2

Show **that** the dipole moment of a molecule \mathbf{p} and the dipole moment per unit volume are related by

$$\mathbf{p} = n\mathbf{p}$$

where n is the number of molecules per unit volume of the dielectric.

To understand the **physical** meaning of \mathbf{P} , we consider the **special case** of a rectangular block of a dielectric material of length L and cross-sectional area A . Fig. 5.8 represents such a block.

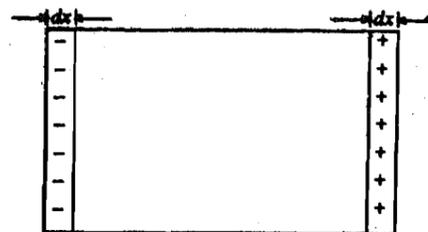


Fig. 5.8: Surface polarisation charges on a rectangular block of dielectric.

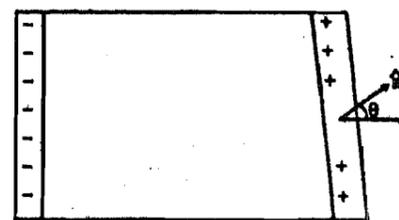


Fig. 5.8a: Surface polarisation charges. Actual displacement of charge on right is $dx \cos \theta$.

Let ρ be the **surface** density of **polarisation** charges, **viz.**, the number of charges on a unit area or **charge/unit** area on the surface. The total number of **polarisation** charges appearing on the surface = $A\rho$

$$\text{Induced dipole moment} = A\rho L \dots \quad (5.2)$$

$$\text{Volume of the slab} = AL$$

By definition dipole moment per unit volume = \mathbf{P}

$$\text{Induced dipole moment} = \mathbf{P}AL \dots \dots \quad (5.3)$$

Now we can compare the magnitudes of **Eqs.** (5.2) and (5.3) to obtain the magnitude ρ of the **polarisation** vector to be

$$\mathbf{P} = \rho \mathbf{p} \quad (5.4)$$

Thus, the surface density of charges appearing on the faces perpendicular to the field is a measure of \mathbf{P} , the polarisation vector. Eq. (5.4) is true for a special geometry when the dielectric material is a rectangular block. For a block shown in Fig. 5.8a the surface on right is not perpendicular to \mathbf{P} . The normal unit vector (\mathbf{n}) to the surface makes an angle θ with \mathbf{P} . If the charges are displaced by a distance dx the effective displacement is $dx \cos\theta$ for the surface on the right. If n is the number of charged particle and q is the charge on each particle, then the surface charge density σ is given by

$$\sigma_p = n q dx \cos \theta = \mathbf{P} \cdot \mathbf{n} = P_n \quad (5.5)$$

where q is the positive charge on each atom/molecule and P_n is the component of \mathbf{P} normal to the surface on the right. This also shows why no charges appear on the surfaces parallel to the applied field ($\theta = 90^\circ$) and on the left side of the block the angle between \mathbf{P} and \mathbf{n} , unit vector normal to the surface is 180° the surface charge density is negative.

For an ideal, homogeneous and isotropic dielectric, the polarisation \mathbf{P} is proportional to the average field \mathbf{E} , i.e.,

$$\mathbf{P} = \chi \epsilon_0 \mathbf{E} \quad (5.6)$$

Where $\chi = \mathbf{P}/\epsilon_0 \mathbf{E}$ and is known as electrical susceptibility. This relation is related to Eq. (5.1), Eq. (5.1) refers to one molecule, whereas Eq. (5.6) refers to the material. Thus the latter is a macroscopic version of Eq. (5.1). The constant ϵ_0 is included for the purpose of simplifying the later relationships.

The relation (5.6) requires that \mathbf{P} is linearly related to the average (microscopic) field. This average field would be the external applied field as modified by the polarisation surface charges. The susceptibility is a characteristic of the material and gives the measure of the ease with which it can be polarised, it is simply related to σ for the nonpolar materials.

From SAQ1 $p = nq$ using Eq. (5.4), we get

$$\sigma_p = np$$

The dipole moment per atom in this case $p = q \cdot dx \cos \theta$

5.4 GAUSS' LAW IN A DIELECTRIC.

In Unit 2, you have studied Gauss law in vacuum. Here, we shall modify and generalise it for dielectric material. Consider two metallic plate as shown in Fig. 5.9. Let \mathbf{E}_0 be the electric field between these two plates. Now, we introduce a dielectric material between these two plates. When the dielectric is introduced, there is a reduction in the electric field, which implies a reduction in the charge per unit area, since, no charge has leaked off from the plates, such a reduction can be only due to the induced charge appearing on the two surfaces of the dielectric. Due to this reason, the dielectric surface adjacent to the positive plate must have an induced negative charge, and the surface adjacent to the negative plate must have an induced positive charge of equal magnitude. It is shown in Fig. 5.9.

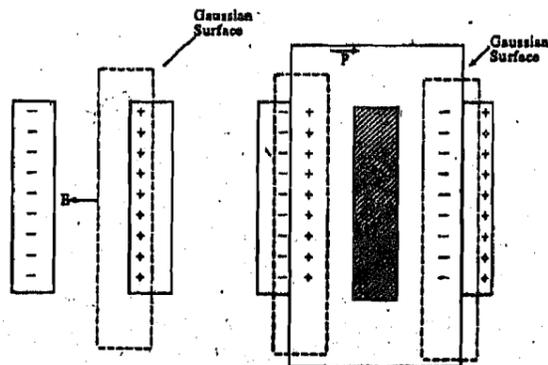


Fig. 5.9 : Induced charges on the faces of a dielectric in an external field.

For the sake of simplicity, you consider the charge on the surface of dielectric material as shown in Fig. 5.9a. Now we apply Gauss' flux theorem to a region which is wholly within the dielectric such as the Gaussian volume at region 1 of Fig. 5.9a.

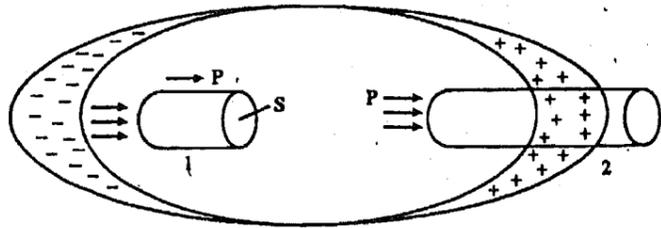


Fig. 5.9a: Gaussian volumes at 1 and 2 inside a dielectric. The displacement of charges at the surfaces perpendicular to the applied field are shown

The net charge inside this volume is zero even though this material is polarised. The positive charges and negative charges are equal. For this volume the flux of field through the surface is zero. We can write

$$\int_{\text{surface at 1}} \mathbf{E} \cdot d\mathbf{S} = \int_{S_1} \epsilon_0 \chi \mathbf{P} \cdot d\mathbf{S} = 0 \quad (5.7)$$

This shows that "lines" of \mathbf{P} are just like lines of \mathbf{E} except for a constant (ϵ_0). Instead of this Gaussian volume suppose we take another one at region 2. In this Gaussian volume one surface is inside the dielectric and the other is outside it. The curved surface is parallel to the lines of field (\mathbf{E} or \mathbf{P}). For the surface of this Gaussian volume outside the material \mathbf{P} is nonexistent. However, lines of \mathbf{P} must terminate inside the Gaussian volume. Hence the net flux of \mathbf{P} is finite and negative as shown in Fig.5.9a since the component of \mathbf{P} normal to the surface, i.e. P_n and σ_p the surface charge density are equal to each other in magnitude, the surface integral

$$\begin{aligned} \mathbf{P} \cdot d\mathbf{S} &= P_n dS = -\sigma_p dS \\ &= -q_p \end{aligned} \quad (5.8)$$

Where q_p is the charge inside the Gaussian volume. Thus the flux of \mathbf{P} is equal to the negative of the charge included in the Gaussian volume. Notice the difference in the flux of \mathbf{P} and flux of \mathbf{E} .

Note : See Kip's book for a good account of the generalised Gauss' Law.

Now we can generalise Gauss' flux theorem. Since the effects of polarised matter can be accounted for by the polarisation surface charges, the electric field in any region can be related to the sum of both free and polarisation charges. Thus in general

$$\int_{\text{closed surface}} \mathbf{E} \cdot d\mathbf{S} = \frac{1}{\epsilon_0} (q_f + q_p) \quad (5.9)$$

where q_f represents free charges and q_p the polarisation charges.

SAQ 3

Two parallel plates of area of a cross section of 100m^2 are given equal and opposite charge of $1.0 \times 10^{-7} \text{ C}$. The space between the plates is filled with a dielectric material, and the electric field within the dielectric is $3.3 \times 10^5 \text{ V/m}$. What is the dielectric constant of the dielectric and the surface charge density on the plate?

Using Gauss' theorem for vectors this surface integral can be converted into a volume integral. Thus the above equation becomes

$$\int_V (\nabla \cdot \mathbf{E}) dV = \frac{1}{\epsilon_0} (\rho_f + \rho_p) dV \quad (5.10)$$

where ρ_f and ρ_p are respectively the free and bound charge densities. As this is true for any volume, the integrands can be equated. Thus

$$\epsilon \nabla \cdot \mathbf{E} = \rho_f + \rho_p \quad (5.11)$$

The flux of \mathbf{p} through the closed surface is given by (See equation 5.8)

$$\int \mathbf{P} \cdot d\mathbf{S} = -q_p = -\int \rho_p dV$$

which can be written using Gauss' flux theorem

$$\begin{aligned} \nabla \cdot \mathbf{P} &= \rho_p \\ \epsilon_0 \nabla \cdot \mathbf{E} &= \rho_f - \nabla \cdot \mathbf{P} \\ \therefore \epsilon_0 \nabla \cdot \mathbf{E} + \nabla \cdot \mathbf{P} &= \rho_f \\ \nabla \cdot (\epsilon_0 \mathbf{E} + \mathbf{P}) &= \rho_f \\ \nabla \cdot \mathbf{D} &= \rho_f \end{aligned} \quad (5.12)$$

where $\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P}$

is known as **electric displacement vector**.

(Note that 5.12 is already Gauss's Law.)

SAQ 4

Show that Eq. (5.12) reduces to Eq. (5.11) when $\mathbf{P} = 0$.

The dimension of \mathbf{D} is the same as that of \mathbf{P} .

The units of \mathbf{D} are $C \cdot m^{-2}$.

From Eqs. (5.12) and (5.10) we observe that the source of \mathbf{D} is the free charge density ρ_f , whereas the source of \mathbf{E} is the total charge density $\rho_f + \rho_p$.

When we write $\mathbf{P} = \epsilon_0 \chi \mathbf{E}$ (see Eq. 5.5)

$$\begin{aligned} \text{We have } \mathbf{D} &= (1 + \chi) \epsilon_0 \mathbf{E} \\ &= \epsilon_r \epsilon_0 \mathbf{E} \end{aligned} \quad (5.13)$$

Where $\epsilon_r = (1 + \chi)$ is known as the relative permittivity. Another usual form of **electric displacement vector** \mathbf{D} is given by

$$\mathbf{D} = \epsilon \mathbf{E} \quad (5.14)$$

where $\epsilon = \epsilon_r \epsilon_0$

Eq. (5.14) provides the relation between Electric displacement \mathbf{D} and electric field \mathbf{E}

SAQ 5

Consider two rectangular plates of ~~area~~ a cross section of $6.45 \times 10^{-4} m^2$ each are kept parallel to each other. The **separation** between them is $2 \times 10^{-3} m$, and a voltage of $10V$ is applied across these plates. If a material of dielectric constant 6.0 is **introduced** within the region between the **two** plates, calculate:

- 1) Capacitance
- 2) The magnitude of the charge stored on each plate.
- 3) The dielectric displacement \mathbf{D}
- 4) The polarisation.

5.5 DISPLACEMENT VECTOR D

In Section 5.4, we introduced a new vector \mathbf{D} and called it **Displacement Vector** (or **Electric Displacement**).

It is one of the basic vectors for an electric field that depends only on the magnitude of free charge and its distribution.

We found (see Sec. 5.4) that the electric displacement is defined by $\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P}$; Gauss' law in dielectric is given by $\mathbf{D} \cdot d\mathbf{S} = q_f dV$. For an isolated charge q , kept at the centre of a dielectric sphere of radius r , we find that the Gauss' flux theorem gives (being a case of spherical symmetry)

$$(4\pi r^2) (D) = q$$

Which gives

$$\mathbf{D} = q\mathbf{r} / 4\pi r^2 \quad (5.15)$$

$$\therefore \mathbf{D} = \epsilon \mathbf{E} \text{ we get } \mathbf{E} = q\mathbf{r} / 4\pi \epsilon r^2 \quad (5.16)$$

From (5.16) it follows that the force \mathbf{F} , between two charges q_1 , and q_2 , kept at a distance r in a dielectric medium is given by

$$\mathbf{F} = \frac{q_1 q_2}{4\pi \epsilon r^2} \mathbf{r} \quad (5.17)$$

and the expression for the potential ϕ at a distance r from q is

$$\phi = q / 4\pi \epsilon r \quad (5.18)$$

When we compare Eq. 5.16 with the corresponding expression for \mathbf{E} in free space, 4.5.17 and 5.18 shows similar expressions for Coulomb force and potentials (see Unit 3). We may find that in all these expressions, ϵ_0 has been replaced by ϵ in a dielectric medium.

SAQ 6

Two large metal plates each of area 1 sq. metre face each other at a distance. (One metre apart they carry equal and opposite charge on their surface.) If the electric intensity between the plates is 50 newton per coulomb, calculate the charge on the plates.

With this background, we may wrongly conclude that \mathbf{D} for a dielectric medium is same as \mathbf{E} for free space. It is therefore important to clearly distinguish between these two vector quantities:

- \mathbf{E} is defined as the force acting on unit charge, irrespective of whether a dielectric medium is present or not. It is to be calculated taking into account the free or external charges as well as the induced charges of the medium. On the other hand \mathbf{D} is defined by Eq. (5.10), viz., $\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P}$, and it is a vector like electric field, but is determined only by free or external charges. Note from Eqs. (5.15) and (5.16) that the value of \mathbf{D} does not depend upon the dielectric constant while the value of \mathbf{E} as well as the force between the charges involve ϵ .
- The quantity $\int \mathbf{D} \cdot d\mathbf{S}$ is usually referred to as the electric flux through the element of area $d\mathbf{S}$. For this reason \mathbf{D} is also known as electric flux density. From the integral form of Gauss' law in dielectrics, we find that the total flux is q , through an area surrounding a charge q , and this flux is unaltered by the presence of a dielectric medium. This is not true in the case of total flux of electric intensity, since

$$\int_s \mathbf{E} \cdot d\mathbf{S} = (q/\epsilon)$$

- Since \mathbf{D} is a vector, we may draw lines of displacement in the same way as we draw the lines of force. The number of lines of displacement passing through

unit area is proportional to (D) . These lines of **displacement begin** and end only on free charges, since the origin of D is the conduction **charges/charge density** (see Section 5.4).

Again by using Gauss' law it can be shown easily that the lines of displacement are continuous in space containing no free charges. In other words, at the boundary of two dielectrics, if there are no free lines of charges D are continuous, while the **lines** of E are not continuous because lines of electric force can end on **both** free and polarisation charges. This behaviour of D and E is dealt with in greater **detail** in the next section. These rules are contained in two **Boundary Conditions** at the interface between two **dielectric media**.

5.6 BOUNDARY CONDITIONS ON D AND E

We wish to **determine** the relationships that E and D must **satisfy** at the interface between two dielectrics. Here, we will assume that **there** are only **polarisation** charges at the interface **i.e.**, since the **dielectrics** are ideal they have no free electrons, and thus there is no conduction charge **at** the interface. **Later**, these boundary conditions will **be** useful for proving laws of reflection and refraction of **electromagnetic** waves. Now we will determine the boundary condition for vector D .

Boundary Condition for D:

We apply the Gauss' law for dielectrics to a small cylinder in the shape of a pill box which intersects **the** boundary between two **dielectric** media and whose **axis** is normal to the boundary.

Fig. 5.10 shows the cylinder, let the height of **the pill box** be **very** small compared to its cross **sectional** area. The contribution to $\int \mathbf{D} \cdot d\mathbf{S}$ comes from **the components** of D normal to the boundary. That is

$$\int_s \mathbf{D} \cdot d\mathbf{S} = D_{n2} dS - D_{n1} dS \quad (5.19)$$

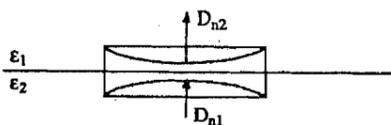


Fig. 5.10

Fig. 5.10: Boundary condition for D between two dielectric media.

where D_{n1}, D_{n2} are the normal **components** of D in media 1 and 2 respectively.

D_{n1} is opposite to the direction of the normal to dS in the medium (ϵ_1)

Further $\int \mathbf{D} \cdot d\mathbf{S} = 0$ since there are no free charges on the **boundary** surface,

$$\therefore D_{n1} = D_{n2} \quad (5.20)$$

Thus the normal components of electrical displacement vectors **are** continuous across the boundary (having no free charges).

Boundary condition for E

We shall make use of **the conservative** nature of the **electric** field in this case. To **obtain** the boundary condition for \mathbf{E} , we calculate the **workdone** in taking a unit **charge** around a rectangular **loop** ABCDA, Fig. 5.11 shows such a loop. The sides BC

Boundary conditions give the way in which the **basic vectors** change when they are incident on the **surface of discontinuity** in dielectric behaviour.

$\mathbf{D} \cdot d\mathbf{S} = D \cdot \mathbf{n} \, dS$ where \mathbf{n} is the unit vector along the outward drawn normal to the area dS . This representation gives the boundary condition as

$$\mathbf{n} \cdot \mathbf{D}_1 = \mathbf{n} \cdot \mathbf{D}_2$$

which gives Eq. (5.20). Otherwise the boundary conditions becomes

$$D_1 \cos\theta_1 = D_2 \cos\theta_2$$

where θ_1 and θ_2 are the angles between \mathbf{n} and \mathbf{D}_1 and \mathbf{D}_2 respectively.

and AC of the loop are very small. As the work done in taking a unit charge round a closed path is zero (conservative force)

$$\oint_{\text{ABCD}} \mathbf{E} \cdot d\mathbf{l} = 0 \quad (5.21)$$

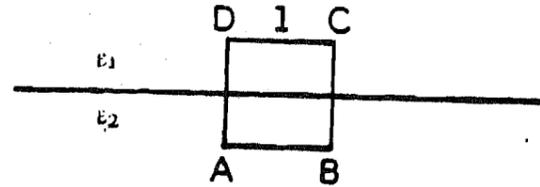


Fig. 5.11 : Boundary condition for \mathbf{E} between two dielectric media.

Let E_{t1} and E_{t2} be the tangential components of \mathbf{E} in the media 1 and 2 respectively as shown in Fig. 5.11. Then

$$\begin{aligned} \int_{\text{ABCD}} \mathbf{E} \cdot d\mathbf{l} &= \int_{\text{AB}} E_{t1} dl - \int_{\text{CD}} E_{t2} dl \\ &= E_{t1} l - E_{t2} l \end{aligned} \quad (5.22)$$

where $l = AB = CD$.

Using Eq. 5.21 in Eq. 5.22 we get

$$E_{t1} = E_{t2} \quad (5.23)$$

Eq. 5.23 states that tangential component of electric field is continuous along the boundary. Note that to calculate work done, we need force which is related to the electric field.

The boundary condition contained in Eq. (5.23) may be written in the vector form as

$$\mathbf{n} \times \mathbf{E}_1 = \mathbf{n} \times \mathbf{E}_2 \quad (5.23a)$$

where \mathbf{E}_1 , \mathbf{E}_2 are the corresponding electric fields and \mathbf{n} is the unit vector normal to the boundary.

SAQ 7

Prove Eq. 5.23a using equation 5.23. Using the vector identity.

$$\oint_{\text{Surface}} \mathbf{E} \cdot d\mathbf{l} = \int_{\text{Surface}} (\mathbf{V} \times \mathbf{E}) \cdot \mathbf{n} dS = - \int \mathbf{V} (\mathbf{n} \times \mathbf{E}) dS$$

Note on Eq. 5.23a

We write Eq. (5.23a) as

$$E_1 \sin \theta_1 = E_2 \sin \theta_2 \quad (5.23b)$$

where θ_1 and θ_2 are angles between \mathbf{n} and \mathbf{E}_1 and \mathbf{n} and \mathbf{E}_2 respectively in the media 1 and 2.

This is yet another form of the boundary condition. We write Eq. 5.23b as

$$\begin{aligned} \frac{D_1}{\epsilon_1} \sin \theta_1 &= \frac{D_2}{\epsilon_2} \sin \theta_2 \\ \text{or } \frac{D_1 \sin \theta_1}{D_2 \sin \theta_2} &= \frac{\epsilon_1}{\epsilon_2} \end{aligned} \quad (5.23c)$$

Eq. (5.23c) implies that the tangential component of \mathbf{D} is not continuous across the boundary.

SAQ 8

Show that the normal component of \mathbf{E} is discontinuous across a dielectric boundary.

5.7 DIELECTRIC STRENGTH AND BREAKDOWN

We have seen that under the influence of an external electric field, polarisation results due to displacement of the charge centres. In our discussion, we have treated the phenomenon as an elastic process. A question that arises in our minds is, "what would happen if the applied field is increased considerably? One thing that is certain is that the charge centres will experience a considerable pulling force. If the pulling force is less than the binding force between the charge centres, the material will retain the dielectric property and on removing the field the charge centres will return to their equilibrium positions. If the pulling force just balances the binding force, the charges will just be able to overcome the strain of the separation and any slight imbalance will loosen the bonds between the electrons and the nucleus. A further increase of the applied field will result in the separation of the charges. Once this happens the electrons will be accelerated. The fast moving electrons will collide with the other atoms and multiply in number. This will result in the flow of conduction current. The minimum potential that causes the charge separation is known as the BREAKDOWN POTENTIAL and the process is known as the DIELECTRIC BREAKDOWN.

Breakdown potential varies from substance to substance. It also depends on the thickness of the dielectric (thickness measured along the direction of the field). The field strength at which the dielectric is about to break down is known as the **Dielectric Strength**. It is measured in kilo volts per metre. Knowledge of the breakdown potential is very important for practical situations, as in the use of capacitors in electrical circuits.

When a dielectric is subjected to a gradually increasing electric potential a stage will reach when the electron of the constituent molecule is torn away from the nucleus. Now the dielectric breakdown, viz., loses its dielectric properties, and begins to conduct electricity.

It is the applied potential difference per unit thickness of the dielectric when the dielectric just breakdown.

5.8 SUMMARY

- o When an electric field is applied to an insulating material, it gets **polarised**. This means that a **dipole moment** is created in the material. This dipole moment is also exhibited as a surface charge density.
- Electric dipole moment per unit volume is known as **polarisation**.
- At atomic level **polarisation** of the medium takes place in two ways, as there are two kinds of molecules **polar** and nonpolar. In nonpolar molecules the **centres** of positive and negative charges lie at one point and their inherent **dipole moment** is **zero**.
- In polar molecules the positive and negative charge centres lie at different points and consequently there is an inherent dipole moment associated **with** the molecules, though the net charge of the molecule is zero.
- For a dielectric medium, it is convenient to introduce another vector related to \mathbf{E} and \mathbf{P} . This is called the displacement vector \mathbf{D} defined in

$$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P}$$

- o For the analysis of dielectric behaviour, the relation between the **polarisation** vector \mathbf{P} and the total electric field \mathbf{E} is important. For an ideal, homogeneous and isotropic dielectric, the relation is expressed as

$$\mathbf{P} = \epsilon_0 \chi_e \mathbf{E}$$

The constant χ_e is known as the electric susceptibility of the medium.

- The constant a , corresponding to the susceptibility χ_e , is known as the atomic (or molecular) polarisability when we consider the polarisation of a single atom (or molecule).
- In a polarised piece of a dielectric, volume charge density $\rho_p (= -\text{div } \mathbf{P})$ and surface charge density σ_p are given by $\mathbf{P} \cdot \mathbf{n}$ or P_n .
- The presence of dielectric leads to the modification of the Gauss' law. It's modification is

$$\oint \epsilon_0 \mathbf{D} \cdot \mathbf{n} \, dS = q$$

where q is the total unit free or external charge

$$\text{or } \text{div } \mathbf{D} = \rho$$

where \mathbf{D} , depends only on the magnitude of free charge and distribution.

- The general relation between the vectors \mathbf{D} , \mathbf{E} and \mathbf{P} can be used to define the dielectric constant K and permittivity ϵ_r of dielectric medium. Using the permittivity ϵ , the relation between \mathbf{D} , \mathbf{P} and \mathbf{E} can be expressed in the linear form

$$\mathbf{D} = \epsilon \mathbf{E}$$

$$\mathbf{P} = \epsilon (k - 1) \mathbf{E} = (\epsilon - \epsilon_0) \mathbf{E}$$

- The vectors \mathbf{E} and \mathbf{D} satisfy certain boundary conditions on the interface between two dielectric media. These conditions are:
 - i. the tangential component of \mathbf{E} is the same on each side of the boundary, i.e., $E_{t1} = E_{t2}$ and
 - ii. the normal component of \mathbf{D} is same on each side of the boundary, i.e., $D_{n1} = D_{n2}$
- Dielectric strength is the applied potential difference per unit thickness of the dielectric when the dielectric just breaks down.

5.8 TERMINAL QUESTIONS

- 1) Calculate the relative displacement of the nucleus of the molecule, modelled in Fig. 5.12 (spherically symmetric molecule) when it is subjected to an external electric field and hence its polarisability.
- 2) Suppose two metallic conducting plates are kept as shown in Fig. 5.13.

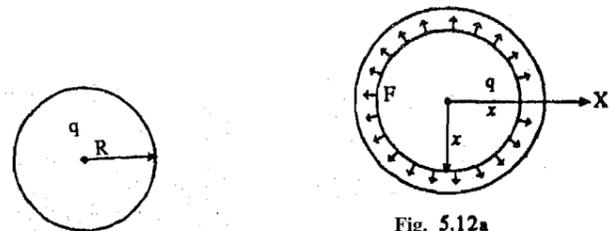


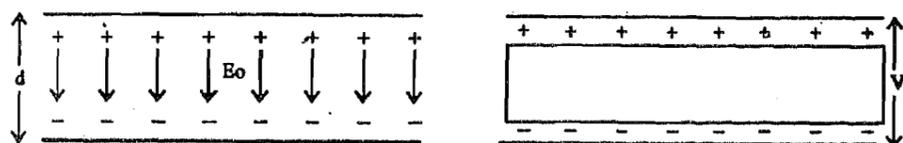
Fig. 5.12a

Fig. 5.12: Model of atom.

The area of cross section of each plate is 2.0 m^2 and are separated 10^{-2} apart, The potential difference between them in vacuum, V_0 is 3000 volts, and it decreases to

1000 volts when a sheet of dielectric 1 cm thick is inserted between the plates. Calculate the followings:

- The relative permittivity K of the dielectric
- its permittivity, ϵ ,
- its susceptibility χ
- the electric intensity between the plates in vacuum (here it is given that Intensity = Voltage / Area of Cross section)
- the resultant electric intensity in the dielectric
- the electric intensity set up by the bounded charges



(a) Two conducting plates without dielectric (b) Two conducting plates with dielectric

Fig. 5.13: Two metallic conducting plates (a) and (b) with dielectric material.

- Consider two isotropic dielectric medium 1 and 2 separated by a charge free boundary as shown in Fig. 5.14

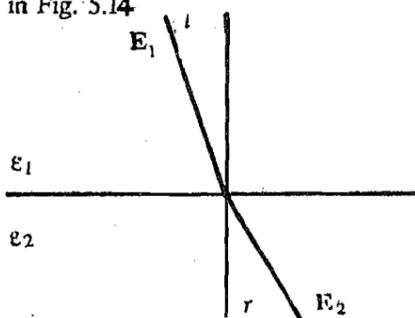


Fig. 5.14 Line of force across the boundary between two dielectrics.

Now, an electric vector E_1 goes from medium 1 and enters into the medium 2. If i is the angle of incidence and r is the angle of reflection, prove that

$$\frac{\tan i}{\tan r} = \frac{\epsilon_1}{\epsilon_2}$$

- Show that the polarisation Charge density at the interface between two dielectrics is

$$p^1 = \epsilon_0 \frac{\epsilon_1 - \epsilon_2}{\epsilon_1} n \cdot E_1$$

5.9 SOLUTIONS & ANSWERS

SAQs

SAQ 1) Please see text.

SAQ 2) The dipole moment per molecule = p
The number of molecules per unit volume = n

$$\oint \mathbf{E} \cdot d\mathbf{l} = 0$$

From vector **analysis** we have

$$\oint \mathbf{E} \cdot d\mathbf{l} = \int_{\text{surface}} (\nabla \times \mathbf{E}) \cdot \mathbf{n} dS = -\int \nabla \cdot (\mathbf{n} \times \mathbf{E}) dS = 0.$$

For $\int \nabla \cdot (\mathbf{n} \times \mathbf{E}) dS$ to be zero, the integrand $\nabla \cdot (\mathbf{n} \times \mathbf{E})$ has to be zero.

Again, in as much as $\nabla \cdot (\mathbf{n} \times \mathbf{E})$ represents a space derivative operation we can set $(\mathbf{n} \times \mathbf{E})$ to be either a constant or zero. If we set $\mathbf{n} \times \mathbf{E} = 0$ then a trivial result follows. So it is better to choose

$$\mathbf{n} \times \mathbf{E} = \text{a constant}$$

Applying this to Fig. 5.11, we get

$$\mathbf{n} \times \mathbf{E}_1 = \mathbf{n} \times \mathbf{E}_2$$

which is Eq. (5.23a).

SAQ 8) The integral form of Gauss' law in dielectrics is

$$\int_{\text{surface}} \mathbf{D} \cdot d\mathbf{S} = \text{charge enclosed}$$

(Refer to the Fig. 10).

$$(\mathbf{n} \cdot \mathbf{D}_{n2} - \mathbf{n} \cdot \mathbf{D}_{n1}) ds = \sigma_f ds$$

where σ_f is the surface charge density on the interface between the dielectrics and \mathbf{n} the unit vector along the outward drawn normal to the surface. \mathbf{D}_{n2} , \mathbf{D}_{n1} are the normal components of displacement vectors in media 2 and 1 respectively.

when $\sigma_f = 0$, we get $\mathbf{n} \cdot \mathbf{D}_{n1} = \mathbf{n} \cdot \mathbf{D}_{n2}$

Now $\mathbf{D}_{n1} = \epsilon_1 \mathbf{E}_{n1}$ and $\mathbf{D}_{n2} = \epsilon_2 \mathbf{E}_{n2}$

$$\therefore \mathbf{D} = \epsilon \mathbf{E}$$

$$\therefore \epsilon_1 \mathbf{n} \cdot \mathbf{E}_{n1} = \epsilon_2 \mathbf{n} \cdot \mathbf{E}_{n2}$$

or

$$\frac{\mathbf{n} \cdot \mathbf{E}_{n1}}{\epsilon_1} = \frac{\mathbf{n} \cdot \mathbf{E}_{n2}}{\epsilon_2}$$

Thus we find that the normal component of \mathbf{E} is discontinuous.

TQs

Answers

- 1) Let the applied electric field be \mathbf{E} , the relative displacement of the nucleus be x , the radius of the electron cloud be R and the charge of the nucleus be q . The electron cloud is equivalent to a uniform sphere of charge with the charge density given by

$$\text{The density of this charge} = \frac{-q}{(4/3)\pi R^3}$$

the total charge of the electron cloud = $-q$

we find the field at a distance x from the centre of the sphere using Gauss' law (see Unit 2). This gives

$$\epsilon_0 4\pi x^2 E_0 = \frac{4}{3} \pi x^2 \frac{-q}{\frac{4}{3} \pi R^2}$$

$$E_0 = \frac{-qx}{4\pi\epsilon_0 R^2}$$

Force on the nucleus, $F = qE_2$, when it is displaced by an amount x . (F is the coulomb restoring force on the nucleus). Now

$$F = -\frac{q^2 x}{4\pi\epsilon_0 R^2}$$

External force on the nucleus = $E q$

This balances the coulomb restoring force

$$\therefore E q = -F$$

or

$$E q = \frac{q^2 x}{4\pi\epsilon_0 R^2}$$

and

$$x = \frac{4\pi\epsilon_0 R^2}{q} E$$

Resulting dipole moment per molecule

$$\begin{aligned} (\mathbf{p}), P &= qx \\ &= 4\pi\epsilon_0 R^2 E \end{aligned}$$

The dipole moment is proportional to E

The molecular polarisability $(a) = 4\pi\epsilon_0 R^2$

2) a) The dielectric constant K is given by

$$K = \frac{E}{\epsilon} = \frac{V}{V_0} = \frac{3000 \text{ V}}{1000 \text{ V}} = 3$$

$$b) \epsilon = k\epsilon_0 = 3\epsilon_0$$

$$c) \epsilon = \epsilon - \epsilon_0 = 2\epsilon_0$$

$$d) \epsilon = \frac{V_0}{A} = \frac{3 \times 10^3 \text{ V}}{10^{-2} \text{ m}} = 3 \times 10^5 \text{ V/m}$$

$$e) \epsilon = \frac{V}{A} = \frac{1 \times 10^5 \text{ V}}{10^{-2} \text{ m}} = 10^5 \text{ V/m}$$

The bound charges of the dielectric set up which opposes the electric field ϵ_0 due to the plate charges. The new field E is the resultant of the two

$$e) E_b = \epsilon_0 - \epsilon$$

$$= 2 \times 10^5 \text{ V/M}$$

3) The tangential component of E at the boundary is continuous. Thus $E_1 \sin i = E_2 \sin r$.

The **normal** component of D is continuous. Here we will use $D = \epsilon E$ and write

$$\epsilon_1 E_1 \cos i = \epsilon_2 E_2 \cos r$$

$$\therefore \frac{\tan i}{\epsilon_1} = \frac{\tan r}{\epsilon_2}$$

$$\text{or } \frac{\tan i}{\tan r} = \frac{\epsilon_1}{\epsilon_2}$$

- 4) The **polarisation** charges appear on the surfaces of the dielectric, perpendicular to the direction of the electric field. We write Eq. (5.4), viz., $\sigma_p = P$ in the vector form as

$$\sigma_p = \mathbf{n} \cdot \mathbf{P}$$

where \mathbf{n} is the unit vector normal to the face on which **polarisation** charges appear and \mathbf{P} the **Polarisation** vector. Let \mathbf{P}_1 and \mathbf{P}_2 be the polarisation vector in the two media. At the interface between the two dielectrics, the surface density of **polarisation** charge σ_p is

$$\sigma_p = \mathbf{n} \cdot (\mathbf{P}_2 - \mathbf{P}_1) \quad (\text{i})$$

From the Boundary condition for D we have

$$\mathbf{n} \cdot \mathbf{D}_1 = \mathbf{n} \cdot \mathbf{D}_2 \quad (\text{ii})$$

or

$$\epsilon_1 \mathbf{n} \cdot \mathbf{E}_1 = \epsilon_2 \mathbf{n} \cdot \mathbf{E}_2 \quad (\text{iii})$$

$$\because \mathbf{D} = \epsilon_0 \mathbf{E} - \mathbf{P}, \text{ we have } \mathbf{P} = \epsilon_0 \mathbf{E} - \mathbf{D}$$

$$\because \mathbf{P}_1 = \epsilon_0 \mathbf{E}_1 - \mathbf{D}_1 \text{ and } \mathbf{P}_2 = \epsilon_0 \mathbf{E}_2 - \mathbf{D}_2 \quad (\text{iv})$$

using (iv) in (i) we get

$$\begin{aligned} \sigma_p &= \mathbf{n} \cdot (\epsilon_0 \mathbf{E}_2 - \mathbf{D}_2) - (\epsilon_0 \mathbf{E}_1 - \mathbf{D}_1) \\ &= \epsilon_0 (\mathbf{n} \cdot \mathbf{E}_2 - \mathbf{n} \cdot \mathbf{E}_1) \text{ in view of (ii)} \\ &= \epsilon_0 (\epsilon_1/\epsilon_2 \mathbf{n} \cdot \mathbf{E}_1 - \mathbf{n} \cdot \mathbf{E}_1) = \epsilon_0 (\epsilon_1/\epsilon_2 - 1) \mathbf{n} \cdot \mathbf{E}_1 \\ &= \epsilon_0 [(\epsilon_1 - \epsilon_2)/\epsilon_2] \mathbf{n} \cdot \mathbf{E} \end{aligned}$$