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UNIT 6 MOTION UNDER CENTRAL CONSERVATIVE FORCES

Structure

6.1 INTRODUCTION

In Block 1 you have studied the basic concepts of mechanics, In Unit 5 of Block 1 we have discussed gravitation. You know that the planets move under the influence of the gravitational field of the sun. How do we solve the equation of motion of a planet ? In this unit we will try to answer this and similar questions.

In fact one of the most important problems of mechanics is to understand the motion of a particle moving under the influence of a force field: The force may be due to another particle or a system of particles, as in the Solar System or a system of fixed charged particles. It could even be due to an electromagnetic field. In this unit we will restrict ourselves to what we call central conservative forces. You will first learn what a central conservative force is. The motion of particles under the influence of such forces has special properties which simplify its description. So you will also study these properties.

There are many examples of such motion. We have mentioned the motion of planets around the sun. Other examples are the motion of satellites around the earth, of spacecrafts sent out to probe the universe and that of two charged particles with respect to each other. The forces associated with these systems, namely the gravitational and electrostatic, obey the inverse square law. We shall see that the inverse square central conservative forces are of special importance. So we shall concentrate chiefly on inverse squire central conservative forces. We shall solve the equation of motion of a particle moving under the influence of such forces, We shall then apply the results to determine the possible orbits of a body moving around the sun. This provides the theoretical basis for Kepler's empirical laws. We shall also determine the trajectory of an alpha particle approaching the nucleus. Such a calculation led to the nucle .r model of the atom.

So far we have studied single particle motion. In the next unit, we shall turn our attention to many-particle systems. In this unit we shall refer to the contents of Units 3.4 and 5 of Block 1 very often. So we suggest that you go through these units once again before studying this unit. You may also go through Appendix A on conic sections before studying Sec. 6.3. It is given after Unit 10.

Objectives

After studying this unit you should be able to

- identify a central conservative force
- solve problems by applying the properties of motion under a central conservative force
- determine the possible orbits under a given inverse square central conservative force.

6.2 CENTRAL CONSERVATIVE FORCE

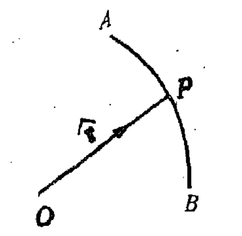


Fig. 6.1 : A particle moving under a central force.
O = Centre of force
P = Panicle
APB =Trajectory

In nature we come across many forces which are either directed towards or away from a fixed point. For example, the gravitational force experienced by a mass due to a fixed point mass is directed towards the point mass. Again the force experienced by a positive charge due to another fixed positive charge is directed away from the latter charge. The force on a particle of mass m attached to a string and moving in a circle in a horizontal plane is also directed toward the centre of the circle (see Fig. 4.17 of Unit 4). Such forces are examples of central forces. We define a central force as one that is everywhere directed towards or away from a fixed point. This fixed point is called the centre of force. Mathematically, we can express a central force acting on a particle as

$$\mathbf{F} = F\,\hat{\mathbf{r}} \tag{6.1}$$

where $\hat{\mathbf{r}}$ is a unit vector pointing from the centre of force to the particle (see Fig. *6.1).* For the above mentioned first three examples of central forces, F depends only on the separation between the centre of force and the particle. For such forces **Eq.** 6.1 can be written as

$$\mathbf{F} = f(r)\,\hat{\mathbf{r}}\,. \tag{6.2}$$

We can show that the central forces given by Eq. 6.2 are also conservative. For this recall the definition of a conservative force from Sec. **3.3** of Unit 3. Let us compute the work done by the force on particle P as it moves from point A to B (Fig. *6.2).*

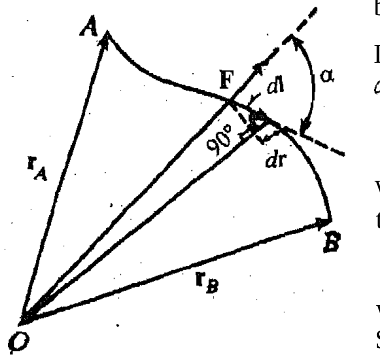


Fig. 6.2 : Work done on a particle moving from A to B. Since dl is infinitesimal, the angle indicated by double stripes can be considered alternate and hence equal to a.

Have you wondered about the use of the term 'conservative' force field? You know that for a conservative force, the work done in taking a system around a closed path is zero. If this were not so, we could find a closed path, traversing which would yield negative work, i.e. energy to us. Thus we could recover any amount of energy going around the loop. That this does not happen is related to the conservation of energy. Thus path independence of work in a 'conservative' force field is related to 'energy conservation'.

Let dW be the work done by the central force on the particle as it undergoes a displacement $d\mathbf{l}$ along the path, It is given by

$$dW = \mathbf{F}.d\mathbf{l} = f(r)\,\hat{\mathbf{r}}.d\mathbf{l} = f(r)dl\cos a, \tag{6.3}$$

where a is the angle between $\hat{\mathbf{r}}$ and dl . Since dl is infinitesimal, from Fig. 6.2 you can see that

$$dl\cos\alpha = dr\,,$$

where dr is the change in the particle's separation from O, as it undergoes displacement $d\mathbf{l}$. So Eq. **6.3** becomes

$$dW = f(r)dr.$$

The work done by the force on the particle as it moves from the **point** A to B is given as

$$W = \int_{r_A}^{r_B} f(r)dr. \tag{6.4}$$

Now, the value of this integral depends on its limits only. So the work done depends only on the end points and not on the path being followed by the particle. Thus, the central force given by Eq. *6.2* is conservative. We term the forces represented by Eq. *6.2* as central conservative **forces.**

You may like to use this concept to identify some central conservative forces in the following SAQ.

SAQ 1

Which forces among the following are central ? Also identify the central conservative force.

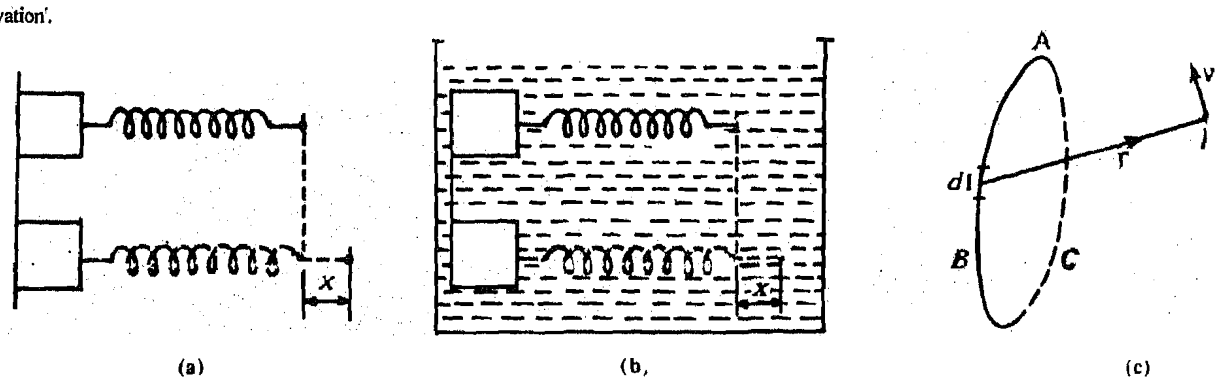


Fig. 6.3 : (a) Ideal spring-mass system; (b) real spring-mass system: (c) a current-carrying conductor.

a) The force acting on a particle of mass m in the spring-mass system shown in Fig. 6.3a for which $F = -kx$.

b) The force acting on a particle of a real spring-mass system kept inside water '(shown in Fig. 6.3b). Its vibration is subject to damping due to water. For such a system

$$\mathbf{F} = -k_2\mathbf{x} - k_3\mathbf{x}$$

where k_2 and k_3 are constants.

c) The force acting on a charge P due to an element $d\mathbf{l}$ of a current-carrying conductor shown in Fig. 6.3c for which

$$\mathbf{F} = \frac{k_1 (\mathbf{v} \cdot \hat{\mathbf{r}}) d\mathbf{l} - (\mathbf{v} \cdot d\mathbf{l}) \hat{\mathbf{r}}}{r^2}.$$

where v is the velocity of the charge and k_1 is a constant depending on the magnitude of the current and the nature of the medium.

Now that you know what a central conservative force is, let us find out the equation of motion for a particle of mass m moving under its influence. From Newton's second law it is given as

$$m\mathbf{a} = f(r)\,\hat{\mathbf{r}} \qquad (6.5)$$

We find that the study of motion under central conservative forces is much simplified because it has certain general properties. Let us first discuss these properties.

6.2.1 Properties of Motion under Central Conservative Forces

The central force is directed along $\hat{\mathbf{r}}$. So, the torque on the particle about the centre of force is

$$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F} = \mathbf{r} \times F\,\hat{\mathbf{r}} = 0.$$

Angular momentum is constant

You know from Unit 4 that $\boldsymbol{\tau} = \frac{d\mathbf{L}}{dt}$ So for zero net torque, L is a constant. This means that *for motion under central force, the magnitude and direction of angular.momentum is constant.* We shall now see that another interesting property arises only from the fact that the direction of angular momentum is constant.

Motion is restricted to a **plane**

We know from Unit 4 that $\mathbf{L} = \mathbf{r} \times \mathbf{p} = \mathbf{r} \times m\mathbf{v}$. So $\mathbf{L}$ is a vector perpendicular to $\mathbf{r}$. In other words, the vector r always remains in a plane perpendicular to $\mathbf{L}$. Since the direction of $\mathbf{L}$ is fixed, this plane is also fixed (Fig. 6.4).

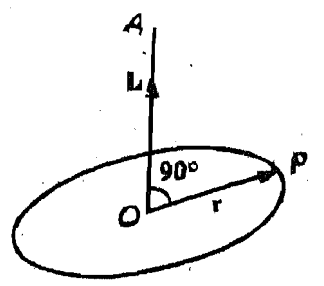


Fig. 6.4 : A particle having constant angular momentum L moves on a fixed plane perpendicular to L.

Since the motion is restricted to a plane, we can use a two-dimensional coordinate system to describe the particle's motion. Since r will be occurring very often in the mathematical treatment, it will be convenient to use plane polar coordinates which you have studied in Unit 4.

We have already determined the magnitude of $\mathbf{L}$ in Unit 4. From Eq. 4.25

$$L = mr^2\,\theta \qquad (6.6)$$

which is constant for a central force.

The property that angular momentum is constant for central force motion gives rise to the following law.

Law of equal areas

Refer to Fig. 6.5. Let $\mathbf{r}$ be the radius vector of a particle at a time t, executing central force

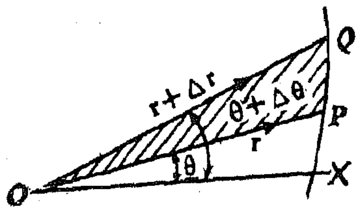


Fig. 6.5 : Area swept out by the radius vector. OX – polar axis, OP = r , OQ = r + Δr.

The area of a triangle can be expressed as half of the product of the length of any two sides and the sine of the angle contained between them.

motion. Let its radius vector be $\mathbf{r} + \Delta\mathbf{r}$ at time $t + \Delta t$. The polar coordinates of the particle at t and $t + \Delta t$ are (r, θ) and $(r + \Delta r, \theta + \Delta\theta)$, respectively. The area ΔA swept out by the radius vector during the time interval Δt is shown shaded in the figure. For small values of $\Delta\theta$, the area ΔA is approximately equal to the area of the triangle OPQ, i.e.

$$\Delta A = \frac{1}{2} r (r + \Delta r) \sin \Delta\theta \simeq \frac{r}{2} (r + \Delta r) \Delta\theta, \; (\because \sin \Delta\theta \simeq \Delta\theta, \text{ for small } \Delta\theta)$$

Ignoring the term $\Delta r \Delta\theta$, we get $\frac{\Delta A}{\Delta t} = \frac{1}{2} r^2 \frac{\Delta\theta}{\Delta t}$

Therefore, the rate at which area is swept out is given by

$$\frac{dA}{dt} = \lim_{\Delta t \to 0} \frac{\Delta A}{\Delta t} = \lim_{\Delta t \to 0} \left(\frac{1}{2} r^2 \frac{\Delta\theta}{\Delta t} \right) = \frac{1}{2} r^2 \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{1}{2} r^2 \dot{\theta}.$$

From Eq. 6.6. we understand that $r^2\dot{\theta}$ = a constant for a particle of given mass m. So $\frac{dA}{dt}$ is a constant, which gives the **law of equal areas.** It states that ***for any central force the radius vector of a particle sweeps out equal areas in equal times.*** Kepler's second law of planetary motion is precisely this law applied to the central force of gravitation. You will understand the physical meaning of this law better after deriving Kepler's fist law.

The property that angular momentum is a constant vector holds for all central forces. Motion under central conservative forces has another property that the total mechanical energy is constant.

Total mechanical energy is constant

From **Eq.** 3.21 of Unit **3,** you know that the total mechanical energy E for a conservative force is constant, i.e.,

$$E = \frac{1}{2} m v^2 + U(r) = \text{constant.} \tag{6.7a}$$

The potential energy $U(r)$ is given by

$$U(r) - U(r_0) = - \int_{r_0} f(r)\, dr \tag{6.7b}$$

where r_0 is some arbitrary reference position. Both these equations 6.7a and 6.7b apply to those central forces which are conservative.

Let us now apply the concepts that angular momentum and total mechanical energy of a particle moving under a central conservative force are constants of motion.

Example 1

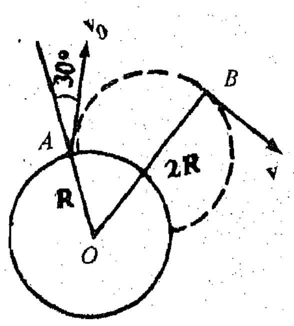


Fig. 6.6

A spacecraft is launched from the point A of the surface of a spherical planet of mass M having no atmosphere with a speed v_0 at an angle of 30° from the radial direction. It goes into an orbit, where its maximum distance OB from the centre of the planet is twice its radius R. Find v_0 in terms of G, R and M.

Refer to Fig. 6.6. Let the mass of the spacecraft be m. When the spacecraft is at position $\mathbf{r}$ with respect to the centre of the planet then the force of gravitation on it is $\mathbf{F} = -\frac{GMm}{r^2}\hat{\mathbf{r}}$.

On comparing with Eq.6.2 we realise that this is a central conservative force. This means that the spacecraft is moving under the influence of a central conservative force. Hence, its angular momentum and the total mechanical energy E are constant. We know that E = K.E + P.E. From Eq. 5.16 we also know that the P.E. of a mass m at a point at a distance r from the centre of a spherical mass M is $-\frac{GMm}{r}$. Therefore, the total mechanical energy of the spacecraft at point A on the surface of the planet is

$$E_A = \frac{1}{2} m v_0^2 - \frac{GMm}{R}.$$

The total mechanical energy of the spacecraft at point B corresponding to the maximum distance $2R$ is

$$E_B = \frac{1}{2}mv^2 - \frac{GMm}{2R}.$$

We know that $E_A = E_B$, since the total mechanical energy is constant. The conservation of angular momentum gives us another relation. Recalling that $\mathbf{L} = m\,\mathbf{r} \times \mathbf{v}$, we get for the magnitudes of angular momentum at points A and B,

$$L_A = mRv_0 \sin 30^\circ = \frac{mRv_0}{2},$$

$$L_B = mv\,(2R)\sin 90^\circ = 2mRv.$$

Since $L_A = L_B$, we get

$$v = \frac{v_0}{4}$$

Setting $E_A = E_B$ and putting $v = \frac{v_0}{4}$ in the equation, we get

$$\frac{1}{2}mv_0^2 - \frac{GMm}{R} = \frac{1}{2}m\left[\frac{v_0}{4}\right]^2 - \frac{GmM}{2R}.$$

After simplification, we get

$$v_0 = 4\sqrt{\frac{GM}{15R}}.$$

So far we have studied some general properties of motion under central conservative forces. We shall now use these properties to determine the path of a particle moving under inverse square central conservative forces. Examples of such forces are the familiar gravitational and electrostatic forces.

6.3 INVERSE SQUARE CENTRAL CONSERVATIVE FORCES

For any general inverse square central conservative force, Eq. *6.2* is expressed as

$$\mathbf{F} = \frac{k}{r^2}\hat{\mathbf{r}}, \tag{6.8}$$

If k is positive, then the force is repulsive and if it is negative, the force is attractive. For example, you know that the force between two like charges is repulsive and that between two unlike charges is attractive. Similarly, gravitation is an attractive inverse square force. Let us now solve the equation of motion to determine the orbit of a body moving under the influence of gravitational force of the sun. We will regard the sun to be stationary. In order to determine the orbit, we need to know r (t) and θ (t), or r as a function of θ. We will now use a simple method to obtain $r(\theta)$.

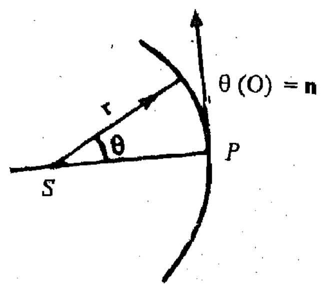


Fig. 6.7 : Motion of a body moving under the gravitational force of the sun (S). P is the position of the body at $t = 0$.

Refer to Fig. *6.7*. As has been pointed out in Sec. 6.2.1, we shall be using plane polar coordinates. Let the sun be at the origin located at the centre of force. The equation of motion of the body under the influence of the gravitational attraction of the sun is given by

$$m\frac{d\mathbf{v}}{dt} = -\frac{GMm}{r^2}\hat{\mathbf{r}}, \tag{6.9a}$$

where m and M are the masses of the body and the sun, respectively.

O-

$$\therefore \quad \frac{d\mathbf{v}}{dt} = -\frac{GM}{r^2}\,\hat{\mathbf{r}}$$

Let us first solve this equation to obtain v. Then we will use Eq. **4.13a** to obtain r from the expression of $\mathbf{v}$.

Since the force is central, we have from Eq. 6.6 that $L = mr^2\,\dot{\theta} = a$ constant.

We also know from Eq. 4.10 that $\frac{d\hat{\boldsymbol{\theta}}}{dt} = -\dot{\theta}\,\hat{\mathbf{r}}$. Using **Eqs.** 4.10 and 6.6 we can write Eq. 6.9b as

$$\frac{d\mathbf{v}}{dt} = \frac{GM}{r^2\dot{\theta}}\frac{d\hat{\boldsymbol{\theta}}}{dt} = \frac{GMm}{L}\frac{d\hat{\boldsymbol{\theta}}}{dt} = \frac{A}{L}\frac{d\hat{\boldsymbol{\theta}}}{dt},$$

where $A = GMm$ = a constant.

$$\text{or} \quad \frac{L}{A}\,d\mathbf{v} = d\hat{\boldsymbol{\theta}}.$$

On integrating, we get

$$\frac{L}{A}\mathbf{v} = \hat{\boldsymbol{\theta}} + \mathbf{C}, \tag{6.10a}$$

where C is a constant vector of integration. We shall use the initial conditions to determine C. Let us choose the origin of time (t = 0) at the instant when the body is closest to the sun, i.e., r is a minimum. Thus $\frac{dr}{dt} = 0$ at $t = 0$. Again v (0) (i.e. v at t = 0) is in the same diition as $\hat{\boldsymbol{\theta}}$ (0) (i.e. $\hat{\boldsymbol{\theta}}$ at t = 0). Let $\hat{\boldsymbol{\theta}}(0) = \hat{\mathbf{n}}$. Hence, from Eq. **6.10a** we get

$$\frac{L}{A}v(0)\,\hat{\mathbf{n}} = \hat{\mathbf{n}} + \mathbf{C},$$

$$\therefore \quad \mathbf{C} = \left(\frac{L}{A}\,v(0) - 1\right)\hat{\mathbf{n}} = e\hat{\mathbf{n}}, \text{ say}$$

where

$$e = \frac{L}{A}\,v(0) - 1 = \text{a constant}. \tag{6.10b}$$

Hence, from Eqs. **6.10a** and b, we get

$$\frac{L}{A}\mathbf{v} = \hat{\boldsymbol{\theta}} + e\hat{\mathbf{n}}. \tag{6.10c}$$

Now that we have obtained an expression for v, we can find r as a function of θ in a simple manner. Taking the scalar product of **Eq.6.10c** with $\hat{\boldsymbol{\theta}}$, we get

$$\frac{L}{A}\mathbf{v}.\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\theta}}.\hat{\boldsymbol{\theta}} + e\hat{\mathbf{n}}.\hat{\boldsymbol{\theta}} = 1 + e\cos\theta. \tag{6.11}$$

We know from **Eq.4.13a** that $\mathbf{v} = \dot{r}\,\hat{\mathbf{r}} + r\,\dot{\theta}\,\hat{\boldsymbol{\theta}}$.

Since $\hat{\mathbf{r}}.\hat{\boldsymbol{\theta}} = 0$ **and** $\hat{\boldsymbol{\theta}}.\hat{\boldsymbol{\theta}} = 1$, **we get** $\mathbf{v}.\hat{\boldsymbol{\theta}} = r\dot{\theta} = \frac{1}{r}r^2\dot{\theta} = \frac{L}{mr}$ **from Eq. 6.6.**

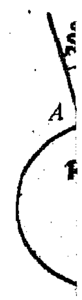
So we get from **Eq.** 6.11,

$$\frac{L^2}{Am}\frac{1}{r} = 1 + e\cos\theta. \tag{6.12}$$

Comparing Eq. 6.12 and Eq. A.3 of Appendix A, we get

$$p = \frac{L^2}{Am}. \tag{6.13}$$

So we can say that the orbit of the body is a conic with its pole inside. e is **called the** eccentricity, of the conic. Now this conic can be either a parabola, a hyperbola or an ellipse depending on whether e is equal to, greater than or less than 1.



Fig

8

In the special case when $e = 0$, the conic is a circle.

The solution that we have obtained for the path of a body moving under the sun's gravitational field is based on some simplifying assumptions. We have assumed that the sun is stationary and that the only force acting on the body is the gravitational attraction of the sun. We know that both these assumptions are not exactly true in the real Solar System. The sun is not stationary and all other members of the Solar System also exert gravitational forces on the body. However, these forces are negligible in comparison with the gravitational attraction of the massive sun. For our Solar System containing one huge sun and a small number of little planets (called a Keplerian system) these assumptions are reasonable.

Let us now analyse what kinds of orbits (elliptical, parabolic or hyperbolic) are followed by the various bodies in the Solar System. For this we shall relate the eccentricity e to the total mechanical energy of the moving body.

Energy and eccentricity

We know that E = K.E. + P.E. (6.14a)

$$\text{K.E.} = \frac{1}{2} m v^2 = \frac{1}{2} m \mathbf{v} \cdot \mathbf{v}$$

To calculate K.E. in polar coordinates we use Eq.6.10c to get

$$\text{K.E.} = \frac{m}{2} \frac{A^2}{L^2} (\hat{\boldsymbol{\theta}} + e\, \hat{\mathbf{n}}) \cdot (\hat{\boldsymbol{\theta}} + e\, \hat{\mathbf{n}}) = \frac{A^2 m}{2L^2} (1 + 2e \cos\theta + e^2). \quad (6.14b)$$

Similarly, from Eq.5.16 we know that P.E. $= -\frac{GMm}{r} = -\frac{A}{r}$.

From Eq. 6.12. P.E. $= -\frac{A^2 m}{L^2} (1 + e \cos\theta)$. (6.14c)

From Eqs.6.14a, 6.14b and 6.14c we get

$$E = \frac{A^2 m}{2L^2} (e^2 - 1), \quad (6.15a)$$

$$\text{or } e = \sqrt{1 + \frac{2L^2 E}{A^2 m}} \quad (6.15b)$$

Eqs. 6.13 and 6.15b give the values of p and e, which together determine the orbit of the body moving under the sun's gravitation. These can be calculated if we know the values of E, L and A. Although we have determined the orbit of a body moving under the sun's gravitation, these results can be applied more generally. These equations hold for every particle of mass m moving under the influence of an attractive inverse square force given by

$$\mathbf{F} = -\frac{a}{r^2} \hat{\mathbf{r}}.$$

Before proceeding further you may like to solve an SAQ to get some practice on Eqs. 6.12 to 6.15.

SAQ 2

The elliptical orbit of a 2000 kg satellite about the earth is given by the equation

$$r = \frac{8000 \text{ km}}{1 + 0.5 \cos\theta}$$

Find the (a) eccentricity of the orbit; (b) angular momentum and (c) total mechanical energy of the satellite. (Note that for this problem m and M are the masses of the satellite and the earth, respectively.)

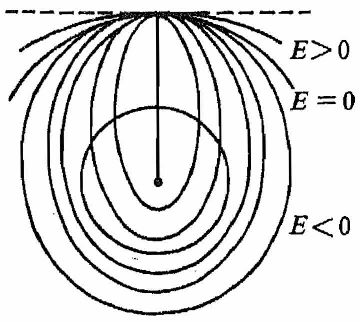


Fig. 6.8: Possible orbits under an inverse square attractive central conservative force.

Let us now consider the various kinds of orbits corresponding to different values of E.

Case 1: $E > 0$. For this $e > 1$ and the orbit is **hyperbolic**. This means that the object starts its motion at an infinite distance from the sun and slowly falls towards the sun. Its loss in P.E. appears as a gain in K.E. It passes by the sun at some minimum separation and goes away along the hyperbola, never to return (see Fig. 6.8). Some comets have been seen with hyperbolic orbits.

Case 2: $E = 0$. For zero energy, $e = 1$ and the object moves along a **parabola**. It too passes the sun once and moves away, without ever returning. Parabolic orbits are highly unlikely because $E = 0$ means that a perfect balance is required between the negative P.E. and the positive K.E.

Case 3: $E < 0$. In addition, we have to put another condition on E, namely $E \geq \frac{-A^2m}{2L^2}$.

If $E < \frac{-A^2m}{2L^2}$ the number under the square root in Eq. 6.15b becomes negative and no orbit is possible. For $\frac{-A^2m}{2L^2} \leq E < 0$, we have $0 \leq e < 1$. For $0 < e < 1$, the orbit is an **ellipse** and for $e = 0$, it is a **circle**. Negative total energy means that the gravitational P.E. is always greater in magnitude than the positive K.E. The object never gains enough K.E. to escape. So it remains bound to the sun or to the centre of force forever in a closed elliptical orbit. Such is the case for all planets and the asteroids of the Solar System. Let us consider these orbits in some detail.

Orbits of planets and comets

You have already read about Kepler's laws of planetary motion in Unit 5 of Block 1. As you know Kepler had arrived at these laws on the basis of the detailed observations made by Tycho Brahe. Here we have applied Newton's laws of motion to show that a planet, comet, meteor or any heavenly body that orbits the sun must move along a conic section given by Eq. 6.12. The shape of the orbit is determined by Eqs. 6.13 and 6.15.

Table 6.1

Planet	e
Mercury	0.2056
Venus	0.0068
Earth	0.0167
Mars	0.0934
Jupiter	0.0483
Saturn	0.0560
Uranus	0.0461
Neptune	0.0100
Pluto	0.2484

In fact, Case 3 corresponds to Kepler's first law. Let us recall the law of equal areas derived in Sec. 6.2.1. When applied to planetary motion, this is Kepler's second law. Kepler observed that a planet did not orbit the sun with a constant angular speed $\dot{\theta}$. For constant $\dot{\theta}$ the law of equal areas demands that r should remain constant, i.e. the orbit should be circular. Since $\dot{\theta}$ varied, Kepler conjectured that the planetary orbits were not circular but elliptical. This turned out to be consistent with the observations. However, the eccentricities (e) of most of the planetary orbits are very small and they are very nearly circular (Table 6.1). For example, the earth's distance from the sun varies by only 3% throughout the year. From Table 6.1, you can see that the approximation that a planet's orbit is circular, made in Sec. 5.2.1, is quite justifiable.

Let us complete the discussion of planetary orbits by deriving Kepler's third law from his first and second laws.

Using the result $r^2\dot{\theta} = \frac{L}{m}$, Kepler's second law can be written as

$$\frac{dA}{dt} = \frac{1}{2}r^2\dot{\theta} = \frac{L}{2m}. \tag{6.16}$$

If the time taken to complete one elliptical orbit is T, then integrating Eq. 6.16 from $t = 0$ to $t = T$, we find

$$\int_0^T \frac{dA}{dt}dt = \frac{L}{2m}\int_0^T dt = \frac{LT}{2m}$$

The quantity on the left side of the above equation is the area of the region enclosed by the ellipse. Now, the area of an ellipse = πab, where a is the semi-major axis and b the semi-minor axis of the ellipse. So we have,

$$\pi ab = \frac{L}{2m}T$$

$$\text{or } T^2 = \left[\frac{2m\pi}{L}\right]^2 a^2 b^2$$

We know that for an ellipse (see Eqs. A.5 to A.8 of Appendix A),

$b^2 = a^2 (1-e^2)$ and $p = a\,(1-e^2)$. Hence, from Eq. 6.13,

$$a\,(1-e^2) = \frac{L^2}{A\,m}.$$

So, we get

$$T^2 = \left(\frac{2m\pi}{L}\right)^2 a^4\,(1-e^2) = \left(\frac{2m\pi}{L}\right)^2 \frac{L^2}{A\,m}\,a^3$$

$$\text{or } T^2 = \frac{4\pi^2\,ma^3}{A}.$$

Since $A = GMm$, we have Kepler's third law :

$$T^2 = \frac{4\pi^2 a^3}{GM} = ka^3, \tag{6.17}$$

where $k = \left(\dfrac{4\pi^2}{GM}\right)$ depends only on the mass of the sun, and is the same for all the planets.

Kepler's third law holds not only for planetary orbits but also for elliptical orbits of the satellites of planets. For the motion of satellites, M, in Eq. 6.17, is the mass of the planet.

So we have studied the laws of planetary motion. We shall now study comets very briefly.

The motion of comets remained an enigma for a long time even after Kepler formulated the three laws. In fact, it was Isaac Newton who observed a comet in 1682 and was the first to explain its trajectory. He could see that the orbit of the comet was governed by the same principles of dynamics that applied to the motion of the planets. He realised that some comets could move past the sun in parabolic and hyperbolic orbits and so would never return. But other comets should move along the elliptical path like the planets. Only the eccentricity would be much higher. Newton's insight revealed that comets are members of the Solar System. You must be familiar with Halley's comet which returns every 76 years. It has a highly elliptical orbit with $e = 0.967$.

We have seen earlier that **L** and E are constants of motion. We must also be able to determine these constants provided the geometrical features of the orbits are known. We can use the results $p = a\,(1-e^2) = \dfrac{L^2}{A\,m}$ and $A = GMm$ in Eq. 6.15a to obtain the following relation between E and a.

$$E = -\frac{GMm}{2a} \tag{6.18}$$

You may now like to apply these results (Eq. 6.15 to 6.18) to some actual situations. So how about trying the following SAQ.

SAQ 3

a) Given that $e = 0.0167$ and $a = 1.5 \times 10^8$ km for the earth's orbit, calculate its energy and angular momentum about the sun.

b) The absolute magnitude of energy of a meteor approaching the sun is given by $|E| = \dfrac{G^2M^2m^3}{L^2}$ where M and m are the masses of the earth and the meteor, respectively. **L** is the angular momentum of the meteor. What are the types of the possible orbits?

So, we have taken care of the factors that determine the orbit of a planet, a comet or a satellite. Let us now try to see how the orbit of a body can be determined if some initial conditions are known.

Example 2: Calculating the orbit from initial conditions

Let us consider the example of a satellite of mass m launched from a space shuttle at a distance r_0 from the centre (C) of the earth (see Fig. 6.9). The initial velocity $\mathbf{v}_0$ of the satellite with respect to the earth and the angle of launch ϕ are given as initial conditions. What will the orbit of the satellite be?

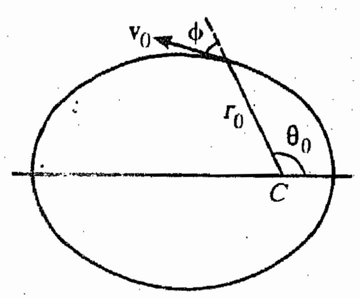


Fig. 6.9: Orbit of a satellite.

Let us first determine the energy of the satellite which is a constant. As you know the value of E will give us the shape of the orbit. It is given by

$$E = \frac{1}{2} m v_0^2 - \frac{GMm}{r_0},$$

where M is the mass of the earth. If the orbit of the satellite is to be a closed one, then $E < 0$, i.e.

$$\frac{1}{2} m v_0^2 < \frac{GMm}{r_0},$$

$$\text{or } v_0^2 < \frac{2GM}{r_0} \quad \text{or} \quad v_0 < \sqrt{\frac{2GM}{r_0}}$$

Thus, for the satellite to be in an elliptical or a circular orbit v_0 must satisfy the above condition. The size of the orbit will be determined from the length of the major axis which is given from Eq. 6.18 as

$$2a = -\frac{GMm}{E}.$$

Similarly, the angular momentum of the satellite remains a constant equal to its initial value given by

$$L = m v_0 r_0 \sin \phi. \qquad (6.19)$$

The point on a planetary orbit where the planet is nearest from the sun is called the *perihelion* and where it is farthest is called the *aphelion*.

The eccentricity of the orbit can then be found from Eq. 6.15b. Then we can find the point where the satellite is nearest from the earth (called the **perigee**) and where it is farthest (called the **apogee**). From Eq. A.6 of the Appendix A, these points are, respectively, given as

$$r_p = a\,(1 - e\,),\; r_a = a\,(1 + e\,). \qquad (6.20)$$

This is how the shape of the orbit given by 'e', and its size given by 'a' can be found from the initial conditions. To completely specify the satellite's orbit we also need to know its orientation in space. It is specified by the line joining the focus to the perigee. We can find this line by determining the angle θ_0 between this line and the known vector $\mathbf{r}_0$ as shown in Fig. 6.9. The angle θ_0 can be found from the polar equation of the ellipse by putting the values of r_0, e, m and L, i.e.

$$r = \frac{L^2}{Am\,(1 + e\cos\theta)}$$

However, from the equation only $\cos\theta_0$ is determined. It does not tell us the sign of θ_0 which can be positive or negative depending on whether we are moving away from the perigee or approaching it. This information is obtained by considering the angle ϕ in Fig. 6.9. You can see that $\phi < 90^\circ$ when moving away from the perigee and $\phi > 90^\circ$ when approaching it. You can now apply these results to determine the orbit of an actual satellite.

SAQ 4

A satellite of mass 5,000 kg is launched in space with an initial speed of 4,000 m s^{-1} at a distance 3.6×10^7 m from the centre of the earth. It is projected at an angle of 30° with respect to the radial direction. Calculate (a) the lengths of semi-major and semi-minor axes, (b) the angular momentum, and (c) the apogee and perigee distances of the orbit.

So far we have determined the possible orbits of a body moving under an attractive inverse square force. As you know the electrostatic force between two positively charged particles is a repulsive inverse square central conservative force. What will the path of a particle acted upon by such a force be?

Orbits under a repulsive inverse square force

We can follow the same procedure that we adopted for determining the planetary orbits. But we have to replace the right hand side of Eq. 6.9a by $\frac{k}{r^2}\hat{\mathbf{r}}$, where k is a positive constant. You can now determine such an orbit by solving the following SAQ.

SAQ 5

Show that the path followed by an alpha particle approaching an atomic nucleus is a hyperbola.

Let us now summarise what we have studied in this unit.

6.4 SUMMARY

- A central force is one which, everywhere, is directed towards or away from a fixed point called the centre of force. It is represented as

$$\mathbf{F} = F\,\hat{\mathbf{r}}.$$

- A central force whose magnitude depends only on r is also conservative. A central conservative force can be represented as :

$$\mathbf{F} = f(r)\,\hat{\mathbf{r}}.$$

- For motion under central conservative forces, angular momentum $\mathbf{L}$ and total mechanical energy E are constant. The law of equal areas holds for such a motion which is restricted to a plane.

- The equation of an orbit for an inverse square central conservative force $\mathbf{F} = \pm\frac{k}{r^2}\hat{\mathbf{r}}$ is a conic given by

$$\frac{1}{r} = \frac{\mp 1 + e\cos\theta}{p}.$$

For a repulsive force, the orbit will be a hyperbola. For an attractive force, its shape would depend on the value of e. Eccentricity depends on E and is given by

$$e = \sqrt{1 + \frac{2L^2E}{k^2m}}.$$

6.5 TERMINAL QUESTIONS

1. Indicate which of the following central force fields are attractive and which are repulsive.

$$\text{(i)}\ \mathbf{F} = -4\,r^3\hat{\mathbf{r}}, \quad \text{(ii)}\ \mathbf{F} = \frac{\hat{\mathbf{r}}}{\sqrt{r}}, \quad \text{(iii)}\ \mathbf{F} = \frac{(r-1)}{r^2+1}\,\hat{\mathbf{r}}.$$

2. a) Had the force of gravitational attraction been inverse cube instead of being inverse square, which one of the three Kepler's laws would still be true?

   b) Justify the statement: The angular speed of a planet in its orbit is minimum at aphelion and maximum at the perihelion.

3. A rocket is fired from Thumba with an initial speed

$$v_0 = \frac{3}{4}\sqrt{\frac{2GM_E}{R_E}},$$

where R_E and M_E are the radius and mass of the earth, respectively. Ignore air resistance and the earth's rotation. Consider conservation of energy and angular momentum and calculate the farthest distance it reaches from the centre of the earth if it is fired off (a) radially and (b) tangentially.

4. It is given that the eccentricity of the orbit of Halley's comet is 0.967 and the length of its semi-major axis is approximately 2.7×10^{12}m. Calculate the following for the comet:

   a) the perihelion and the aphelion distances,

   b) the period.

6.6 ANSWERS

SAQs

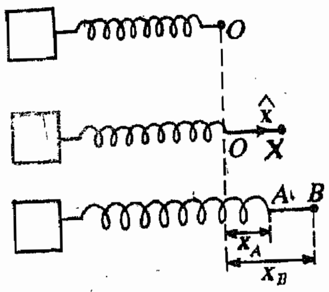


Fig. 6.10: Diagram for SAQ1a.

1. a) Since $\mathbf{F} = -k\mathbf{x}$ where $\mathbf{x} = \mathbf{OX}$ (Fig. 6.10), the force is directed towards the fixed point O. So it is central.

   The work done in stretching it from A $(x = x_A)$ to B $(x = x_B)$ is given by

$$W = \int_A^B \mathbf{F}.d\mathbf{x} = \int_{x_A}^{x_B} -kx\,\hat{\mathbf{x}}.(-dx\,\hat{\mathbf{x}})$$

$$\text{or } \quad W = \int_{x_A}^{x_B} kx\,dx = \frac{k}{2}(x_B^2 - x_A^2)$$

   So the work done is dependent only on the initial and final positions. Hence, the force is also conservative.

   b) This force is again directed towards a fixed point and hence it is central. However, from the working of part (a), it is evident that the work done in taking the particle from $x = x_A$ to $x = x_B$ will not only depend on the initial and final positions of the path, it would also depend on the velocity. So the force is not conservative.

   c) The given force can be expressed as $\mathbf{F} = m\,d\mathbf{l} + n\,\hat{\mathbf{r}}$ where m and n are scalars ($\because$ $\mathbf{v.r}$ and $\mathbf{v}.d\mathbf{l}$ are scalar quantities). It is evident from the expression of $\mathbf{F}$ that it is not along $\hat{\mathbf{r}}$ as $m \neq 0$ in general. Hence, it is not central.

2. On comparing the given equation with Eq. 6.12, we get

   a) $e = 0.5$ and

   b) $\dfrac{L^2}{Am} = 8000 \times 1000\text{m} = 8 \times 10^6\text{m}$

   But $A = GMm$

$$= (6.673 \times 10^{-11}\text{Nm}^2\,\text{kg}^{-2}) \times (5.97 \times 10^{24}\,\text{kg}) \times (2000\text{kg})$$

$$= 7.97 \times 10^{17}\,\text{Nm}^2$$

   or $L^2 = (7.97 \times 10^{17}\text{Nm}^2) \times (2000\text{kg}) \times (8 \times 10^6\,\text{m})$

$$= 7.97 \times 16 \times 10^{26}\,\text{kg}^2\,\text{m}^4\,\text{s}^{-2}.$$

   or $L = 1.13 \times 10^{14}\,\text{kg m}^2\,\text{s}^{-1}$

   c) From Eq. 6.15 a, $E = \dfrac{A^2m}{2L^2}(e^2 - 1)$

$$= \frac{(7.97 \times 10^{17})^2\,\text{N}^2\,\text{m}^4 \times (2000\text{kg})}{2 \times (1.13 \times 10^{14})^2\,\text{kg}^2\,\text{m}^4\,\text{s}^{-2}} \times \left(-\frac{3}{4}\right) = -3.73 \times 10^{10}\text{J}.$$

3. a) From Eq. 6.18, $E = -\dfrac{GMm}{2a}$

$$= -\frac{(6.673 \times 10^{-11}\text{Nm}^2\,\text{kg}^{-2}) \times (1.99 \times 10^{30}\,\text{kg}) \times (5.97 \times 10^{24}\,\text{kg})}{2 \times 1.5 \times 10^{11}\text{m}}$$

$$= -2.6 \times 10^{33}\text{J}$$

Since from Eq. 6.13 $p = \frac{L^2}{Am}$ and also $p = a(1 - e^2)$ (See Eq. A.5 of Appendix A.)

we get

$$\frac{L^2}{Am} = a(1-e^2), \quad \text{and as } A = GMm$$

$$L^2 = GMm^2 a(1-e^2).$$

For the earth's orbit $e = 0.0167$

$$\therefore L^2 = (6.673 \times 10^{-11} \text{Nm}^2 \text{kg}^{-2}) \times (1.99 \times 10^{30} \text{kg}) \times (5.97 \times 10^{24} \text{ kg})^2$$

$$\times (1.5 \times 10^{11} \text{m}) \times (0.9997) = 7.1 \times 10^{80} \text{ kg}^2 \text{ m}^4 \text{ s}^{-2}$$

$$\therefore L = 2.7 \times 10^{40} \text{kg m}^2 \text{s}^{-1}$$

b) From Eq.6.15b, $e = \sqrt{1 + \frac{2L^2E}{A^2m}}$.

Since, the absolute magnitude of E is given, it can be positive as well as negative.

If $E > 0$, $e = \sqrt{1 + \frac{2L^2}{A^2m}\left(\frac{G^2M^2 m^3}{L^2}\right)} = \sqrt{3}$, so the orbit is a hyperbola.

If $E < 0$, $e = \sqrt{1-2} = \sqrt{-1}$, so orbit is not possible.

Using Eq. 6.18, we get

$$a = -\frac{GMm}{2E} \tag{6.21}$$

Now, we know that the initial energy is

$$E = \frac{1}{2} m v_0^2 - \frac{GMm}{r_0},$$

where $r_0 = 3.6 \times 10^7$m, $v_0 = 4000 \text{m s}^{-1}$, $m = 5000$ kg and M is the mass of the earth. Putting these values alongwith those of G and M, we get $E = -1.5 \times 10^{10}$J.

Putting this value in Eq. 6.21, we get $a = 6.6 \times 10^7$m.

The angular momentum $L = m v_0 r_0 \sin\phi$ (where $\phi = 30°$). Putting the values of m, v_0, r_0 and ϕ, we get $L = 3.6 \times 10^{14} \text{ kg m}^2 \text{s}^{-1}$.

Once we know both E and L, we can use Eq. 6.15b to calculate e.

$$e = \sqrt{1 + \frac{2L^2E}{A^2m}}, \text{ where } A = GMm.$$

Putting the values of L, E, G, M and m, we get $e = 0.9$. $\therefore b = 2.9 \times 10^7$m

The apogee and perigee distances are given by

$$r_a = a(1+e) = 1.3 \times 10^8 \text{m},$$

$$r_p = a(1-e) = 6.6 \times 10^6 \text{m}.$$

The orbit is shown in Fig. 6.11.

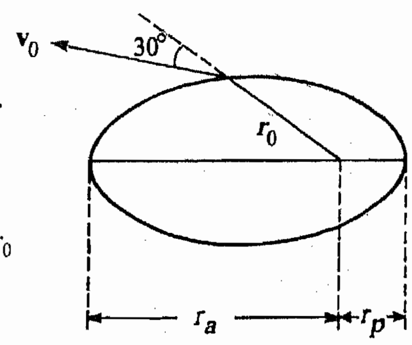


Fig. 6.11: Orbit of the satellite of SAQ 4.

5. Refer to Fig. 6.12. Let the charges on the positively charged nucleus N and the alpha-particle A be q_1 and q_2, respectively. Let the mass of alpha-particle be m. So, as we have obtained Eq. 6.9a, we can get here,

$$m \frac{d\mathbf{v}}{dt} = C \frac{q_1 q_2}{r^2} \hat{\mathbf{r}}, \text{ where } C \text{ is a constant dependent on the}$$

nature of the medium.

$$\text{or} \quad \frac{d\mathbf{v}}{dt} = \frac{k}{r^2} \hat{\mathbf{r}}, \tag{6.22}$$

where $\quad k = \frac{C}{m} q_1 q_2$ = a positive constant.

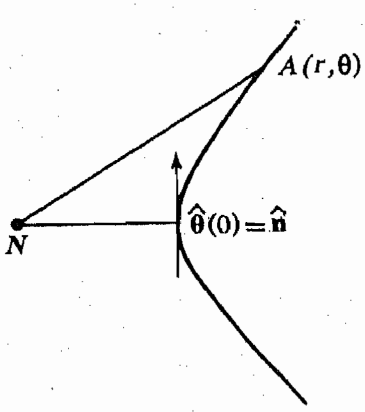


Fig. 6.12 : An alpha particle approaching a nucleus.

You may recall from Eq. 4.10 of Unit 4 that $\frac{d\hat{\boldsymbol{\theta}}}{dt} = -\dot{\theta}\hat{\mathbf{r}}$, or $\hat{\mathbf{r}} = -\frac{1}{\dot{\theta}}\frac{d\hat{\boldsymbol{\theta}}}{dt}$

Thus, $$\frac{d\mathbf{v}}{dt} = -\frac{k}{r^2\dot{\theta}}\frac{d\hat{\boldsymbol{\theta}}}{dt} = -\frac{km}{L}\frac{d\hat{\boldsymbol{\theta}}}{dt} \quad (\because L = mr^2\dot{\theta})$$

$$\text{or } d\mathbf{v} = -\frac{km}{L}d\hat{\boldsymbol{\theta}}$$

$$\mathbf{v} = -\frac{km}{L}\hat{\boldsymbol{\theta}} + \mathbf{C}_1, \quad (6.23)$$

where $\mathbf{C}_1$ is a constant vector of integration. We shall now determine the constant $\mathbf{C}_1$. For this we shall follow the same procedure as we did for determining $\mathbf{C}$ in the problem of planetary orbits. So looking back at the few steps worked out after Eq. 6.10a you will realise that $\mathbf{v}(0)$ and $\hat{\boldsymbol{\theta}}(0)$ are in the same direction. Let $\hat{\boldsymbol{\theta}}(0) = \hat{\mathbf{n}}$ then $\mathbf{v}(0) = v(0)\,\hat{\mathbf{n}}$.

Hence, from Eq. 6.23, we get

$$v(0)\,\hat{\mathbf{n}} = -\frac{km}{L}\hat{\mathbf{n}} + \mathbf{C}_1,$$

$$\mathbf{C}_1 = \left[v(0) + \frac{km}{L}\right]\hat{\mathbf{n}} = k_1\hat{\mathbf{n}}, \text{ where } k_1 \text{ is a positive constant.}$$

$$\therefore \quad \mathbf{v} = -\frac{km}{L}\hat{\boldsymbol{\theta}} + k_1\hat{\mathbf{n}}.$$

Taking dot product with $\hat{\boldsymbol{\theta}}$ on both sides and using $\mathbf{v}\cdot\hat{\boldsymbol{\theta}} = r\dot{\theta}$, $\hat{\boldsymbol{\theta}}\cdot\hat{\boldsymbol{\theta}} = 1$ and $\hat{\mathbf{n}}\cdot\hat{\boldsymbol{\theta}} = \cos\theta$, we get

$$r\dot{\theta} = -\frac{km}{L} + k_1\cos\theta$$

$$\text{or } \frac{L}{mr} = -\frac{km}{L} + k_1\cos\theta \quad [\because L = mr^2\dot{\theta}]$$

$$\text{or } \frac{1}{r} = -\frac{km^2}{L^2} + \frac{mk_1}{L}\cos\theta$$

$$\text{or } \frac{1}{r} = \frac{km^2}{L^2}\left[\frac{Lk_1}{mk}\cos\theta - 1\right]. \quad (6.24)$$

Eq. 6.24 can be compared with Eq . A.9 of Appendix A

$$\frac{1}{r} = \frac{e\cos\theta - 1}{p}, \text{ where } p = \frac{L^2}{km^2}, e = \frac{Lk_1}{mk}$$

which is the equation of a conic with pole outside.

Such a conic can only be a hyperbola. Hence, the orbit is hyperbolic, with the nucleus being the pole, i.e. the focus.

Terminal questions

1. (i) The negative sign indicates that the force is directed towards the centre of force and hence it is attractive.

   (ii) The positive sign on the right-hand side indicates that the force is directed away from the centre and hence it is repulsive.

   (iii) The force is attractive for $0 < r < 1$ and repulsive for $r > 1$. It vanishes at $r = 1$.

2. a) So long as the force is central, we have from Sec. 6.2.1 that

$$\boldsymbol{\tau} = \mathbf{0} \text{ or, } \frac{d\mathbf{L}}{dt} = \mathbf{0}, \text{ i.e. } \mathbf{L} = \text{a constant vector.}$$

   We have seen that the law of equal areas follows from the constancy of angular momentum vector. Kepler's second law is precisely the law of equal areas. So Kepler's second law will still be true.

b) From the law of equal areas we know that $r^2\dot{\theta}$ = a constant for all the planets. At aphelion r is maximum, so $\dot{\theta}$ is minimum. And at perihelion r is minimum, so $\dot{\theta}$ is maximum. So a planet moves faster as it approaches the sun and slower when it moves far away from the sun.

3. (a) Refer to Fig. 6.13. The total mechanical energy of the rocket at point P on the surface of earth is

$$E_P = \frac{1}{2} mv_0^2 - \frac{GM_E m}{R_E} = -\frac{7}{16}\frac{GM_E m}{R_E}.$$

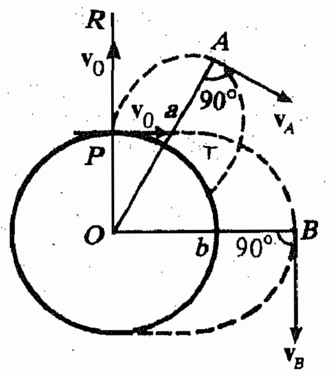


Fig. 6.13: Trajectories of the rocket. PR and PT show, respectively, the radial and tangential directions of firing the rocket.

The total mechanical energy of the rocket at point A corresponding to the maximum distance a is

$$E_A = \frac{1}{2} mv_A^2 - \frac{GM_E m}{a},$$

where v_A = the velocity of the rocket at A.

We know from the principle of conservation of energy that $E_P = E_A$,

$$\text{i.e. } \frac{1}{2} mv_A^2 - \frac{GM_E m}{a} = -\frac{7}{16}\frac{GM_E m}{R_E}. \quad (6.25)$$

We know that $\mathbf{L} = \mathbf{r} \times m\mathbf{v}$. At P, $\mathbf{r} = \mathbf{OP}$ which is parallel to $\mathbf{v}_0$. So the magnitude of angular momentum at P is zero. Hence $L_P = 0$. Again $L_A = mav_A$ ($\because \mathbf{v}_A$ is perpendicular to the radial direction **OA**). Now since, $L_P = L_A$, $v_A = 0$.

Hence, we get from Eq. 6.25 that $a = \frac{16}{7} R_E$.

b) The total mechanical energy of the rocket at point B corresponding to the maximum distance b is

$$E_B = \frac{1}{2} mv_B^2 - \frac{GM_E m}{b},$$

where $\mathbf{v}_B$ = the velocity of the rocket at B. Now $E_P = E_B$.

$$\frac{1}{2} mv_B^2 - \frac{GM_E m}{b} = -\frac{7}{16}\frac{GM_E m}{R_E}. \quad (6.26)$$

Now, at P, $\mathbf{r}$ is perpendicular to $\mathbf{v}_0$. $\therefore L_P = mR_E v_0$.

Again $L_B = mbv_B$. Since $L_P = L_B$, we get $v_B = \frac{R_E v_0}{b}$.

$$v_B^2 = \frac{R_E^2 v_0^2}{b^2} = \frac{R_E^2}{b^2}\frac{9}{16}\frac{2GM_E}{R_E} = \frac{9}{8}\frac{GM_E R_E}{b^2}.$$

So, we get from Eq. 6.26, that

$$\frac{9}{16}\frac{GM_E m R_E}{b^2} - \frac{GM_E m}{b} = -\frac{7}{16}\frac{GM_E m}{R_E}$$

or $\quad \frac{9R_E}{16b^2} - \frac{1}{b} = -\frac{7}{16R_E}$

or $\quad 9R_E^2 - 16bR_E = -7b^2$

or $\quad 7b^2 - 16bR_E + 9R_E^2 = 0$

or $\quad (7b - 9R_E)(b - R_E) = 0$

But $b \neq R_E$, so $b = \frac{9R_E}{7}$

4. a) The perihelion and aphelion distances are given by

$$r_p = a(1-e) \text{ and } r_a = a(1+e).$$

For Halley's comet $a = 2.7 \times 10^{12}$m and $e = 0.967$. Hence we get

$$r_p = 8.9 \times 10^{10}\text{m}, r_a = 5.3 \times 10^{12}\text{m}.$$

b) From Eq. 6.17, we get

$$T = 2\pi\sqrt{\frac{a^3}{GM}}.$$

where M is the mass of the sun. So on putting the values of a, G and M, we get

$$T = 2\pi\sqrt{\frac{(2.7\times10^{12})^3\ \mathrm{m}^3}{(6.673\times10^{-11}\ \mathrm{Nm^2\ kg^{-2}})\times(1.99\times10^{30}\ \mathrm{kg})}}$$

$= 2.42\times10^9\mathrm{s} = 76.7$ years.

UNIT 7 MANY-PARTICLE SYSTEMS

Structure

7.1 INTRODUCTION

So far you have studied the motion of single particles. In Unit 6 we did take up the example of a planet moving in the sun's gravitational field. However, we assumed that the sun was at rest. You may have wondered as to why only the planet moves due to their mutual gravitational attraction. Should not the sun also move ? Indeed, as we shall find in this unit the sun also has a motion. Then why did we neglect it in Unit 6 ? We can answer this question if we analyse the motion of the two-body system of the sun and the planet.

In this unit we shall first study the motion of two bodies moving under the influence of their mutual interaction force. We shall, of course, be applying the basic concepts of mechanics to this system. In addition, you will learn the concepts of the motion of centre-of-mass and the relative coordinates and apply them to two-body systems. We shall then determine the other dynamical variables like the linear and the angular momenta and the K.E. of each system.

We shall next extend these concepts to study the motion of many-particle systems. The Solar System made up of planets and their satellites, asteroids and comets is one such system. Gas filled in a cylinder is also a many-particle system if its molecules can be regarded as point masses in a given problem. Objects such as exploding stars, an acrobat, a javelin thrown in air, a cup of tea, a planet, a car, a ball are all systems composed of many particles. In some systems, e.g. a solid metallic sphere the distances between the particles remain fixed. We shall study the motion of such systems in Unit 9. In other systems the constituent particles move with respect to one another. In this unit you will learn the basic concepts needed to understand these more complex and realistic systems. However, predicting the motion of even more complicated many-particle systems, such as air masses that determine earth's weather, is still very difficult. We need supercomputers to apply these coneepts to such systems.

In the next unit we shall use the concepts of mechanics to study the phenomenon of scattering.

Objectives

After studying this unit you should be able to

- define the centre-of-mass and relative coordinates, and reduced mass
- solve problems involving motion of two-body systems
- derive and explain the physical significance of the expressions of linear and angular momenta and K.E. of a many-particle system.