$$= \frac{1}{2}\left(\frac{5}{1000}\text{ kg}\right)\cdot(3\text{m})^2(2\text{ rad s}^{-1})^2$$
$$= 0.1 \text{ joule}$$

3. Since $\mathbf{r} = 6t^4\hat{\mathbf{i}} - 3t^2\hat{\mathbf{j}} + (4t^3 - 5)\hat{\mathbf{k}}$

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = 24t^3\hat{\mathbf{i}} - 6t\hat{\mathbf{j}} + 12t^2\hat{\mathbf{k}}$$

Angular momentum $\mathbf{L} = \mathbf{r} \times \mathbf{p} = m\mathbf{r} \times \mathbf{v}$

or $\mathbf{L} = m[6t^4\hat{\mathbf{i}} - 3t^2\hat{\mathbf{j}} + (4t^3 - 5)\hat{\mathbf{k}}] \times [24t^3\hat{\mathbf{i}} - 6t\hat{\mathbf{j}} + 12t^2\hat{\mathbf{k}}]$

$$= m[-36t^5\hat{\mathbf{k}} - 72t^6\hat{\mathbf{j}} + 72t^5\hat{\mathbf{k}} - 36t^4\hat{\mathbf{i}} + (96t^6 - 120t^3)\hat{\mathbf{j}} + (24t^4 - 30t)\hat{\mathbf{i}}]$$
$$= m[-(12t^4 + 30t)\hat{\mathbf{i}} + (24t^6 - 120t^3)\hat{\mathbf{j}} + 36t^5\hat{\mathbf{k}}]$$

b) Torque $\tau = \frac{d\mathbf{L}}{dt} = m[-(48t^3 + 30)\hat{\mathbf{i}} + (144t^5 - 360t^2)\hat{\mathbf{j}} + 180t^4\hat{\mathbf{k}}]$

c) Kinetic energy of rotation $= \frac{1}{2}m\mathbf{v}\cdot\mathbf{v}$

$$= \frac{1}{2}m[576t^6 + 36t^2 + 144t^4]$$
$$= 18mt^2[16t^4 + 4t^2 + 1].$$

4. a) Refer to Fig. 4.24. The total angular momentum of the system

$$= (0.02\text{ kg})(2\text{m s}^{-1})(0.5\text{ m}) + (0.03\text{ kg})(2\text{m s}^{-1})(0.5\text{m})$$
$$= 0.05\text{ kg m}^2\text{ s}^{-1}.$$

b) Since no external torque acts on the system, the angular momentum remains conserved. As the rod is light, we shall assume it to be massless. As the particles remain connected by the rod the magnitudes of their velocities must be same (=v,say). When the rod gets contracted to half its original length the radius of the circular path (shown dotted) becomes 0.25 m (Fig. 4.24). So, the total angular momentum

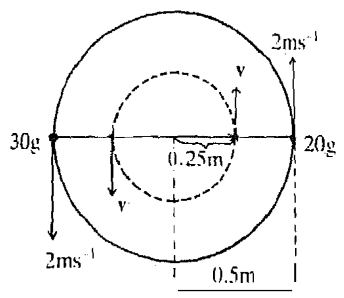


Fig. 4.24

$$= (0.02\text{ kg})(v)(0.25\text{m}) + (0.03\text{ kg})(v)(0.25\text{m})$$
$$= 0.05 \times 0.25\ v\text{ kg m}$$

From the principle of conservation of angular momentum, we get

$0.05\text{ kg m}^2\text{ s}^{-1} = 0.05 \times 0.25\ v\text{ kg m}$.

or $v = 4\text{m s}^{-1}$. So, speed of each particle becomes 4m s^{-1} i.e. double the original value.

UNIT 5 GRAVITATION

5.1 INTRODUCTION

In the previous four units you have studied linear as well as angular motion of a variety of objects. However, by and large we restricted our study to motion of objects on tile earth. We did discuss some examples of motion of heavenly bodies but they lacked in details for want of the knowledge of gravitation. Therefore, we shall study gravitation in this unit.

We shall start from the familiar Kepler's laws of planetary motion to arrive at the law of universal gravitation. We shall then develop the concept of gravitational field and potential and use them to revisit the ideas of earth's gravity, and escape velocity. Finally, we shall visualise the gravitational force as a fundamental force in nature. Alongwith that we shall discuss, in brief, the electroweak and strong forces which are the other basic forces in nature.

In Block 2, we shall apply the concepts of mechanics developed in this block to motion under central conservative forces, systems of many particles and rigid bodies. We shall also study motion in accelerating frames of reference.

Objectives

After studying this unit you should be able to:

- apply the law of gravitation
- infer that the law of gravitation is universally true
- compute gravitational intensity and potential
- solve problems related to the variation of acceleration due to gravity with the height, depth and latitude of a place
- derive expression for velocity of escape
- distinguish between the fundamental forces in nature.

5.2 LAW OF GRAVITATION

You must be aware that the 'Law of Gravitation' was formulated by Sir Isaac Newton. The popular story goes like this:

Newton was sitting under a tree from which an apple fell and struck him on his head. This gave him the necessary impetus to discover the law. There could have been another part in the story: Newton was stating at the moon when the apple hit him (Fig. 5.1)! Newton's stroke of genius was that he realised that ***the force which causes apples to fall to the ground is of tho same kind as the force which causes the moon to orbit the earth.*** In fact, the law of gravitation did not strike Newton in his first effort. He was looking for the answers to many questions related to wide-ranging topics from the 'Law of Falling Bodies' due to Galileo to Kepler's 'Laws of Planetary Motion'. Let us first arrive at the law of gravitation using Kepler's laws (Fig. 5.2). We shall then examine its universality

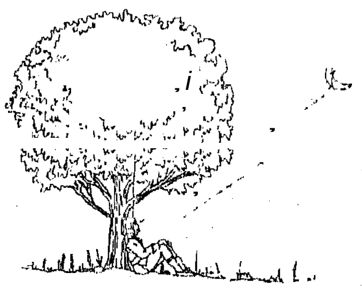


Fig. 5.1: Newton realised that all objects in the universe whether on earth or in heavens move under the influence of the same force of gravity.

through the discussion of the motion of the falling apple and that of the moon around the earth.

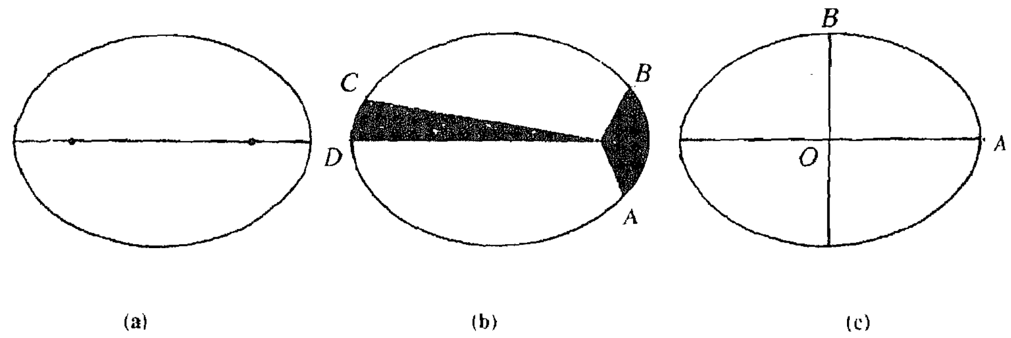


Fig. 5.2: Kepler's laws: (a) All planets move around the sun in elliptic orbits with the sun at one focus; (b) the equal area law: the line joining a planet and the sun sweeps out equal areas in equal intervals of time; (c) the square of the time period of revolution of a planet around the sun is directly proportional to the cube of the semi-major axis of the elliptic orbit. Here OA = semi-major axis, OB = semi-minor axis.

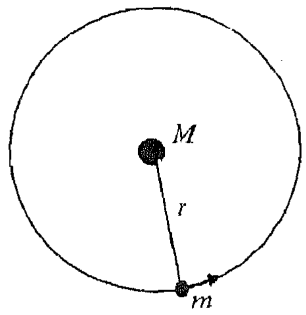


Fig. 5.3: Planet of mass m moving around the sun of mass M.

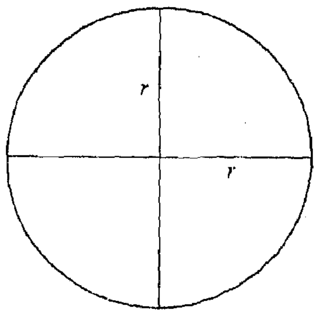


Fig. 5.4: A circle is a special case of an ellipse.

5.2.1 Arriving at the Law

Let us make an approximatibn and consider the orbit of a planet to be circular rather than elliptic. Let a planet of mass m revolve round the sun of mass M in a circular orbit of radius r with a linear speed v (Fig. 5.3). Let us further assume that m and M are point masses as their sizes are much smaller compared to the distance between their centres. The planet for being in the orbit demands a centripetal force (as discussed in Unit 4) whose magnitude is given by $F = \frac{mv^2}{r}$. Since, the time-period of revolution of the planet is $T = \frac{2\pi r}{v}$, we get

$$F = \frac{4\pi^2 mr}{T^2}. \qquad (5.1)$$

Now, a circle is a special case of an ellipse (Fig. 5.4) whose semi-major axis is equal to its radius. According to Kepler's third law $T^2 = Cr^3$, where C is a constant. So from

Eq. 5.1 we get $F = \frac{4\pi^2 m}{Cr^2} = \frac{km}{r^2}$, where $k = \frac{4\pi^2}{C}$ = a constant. (5.2)

Eq. 5.2 gives an expression of F, the centripetal force necessary for the planet for being in the circular orbit. F is a force experienced by the planet towards the centre of circular orbit, i.e. towards the sun. With these ideas in mind you may like to try an SAQ.

SAQ 1

Define the radius vector **r** as originating from the sun and ending at the planet and write down the expression for the vector F.

We have not yet arrived at the law of gravitation. We may rewrite Eq. 5.2 as

$$F_{planet} = \frac{k_{sun} m}{r^2} \qquad (5.2a)$$

This indicates that the force is on the planet due to sun. At this stage let us recall the good old third law of motion. We know that the sun experiences the same farce due to the planet as the planet due to the sun. So following Eq. 5.2a we have

$$F_{planet} = F_{sun} = \frac{k_{sun} m}{r^2} = \frac{k_{planet} M}{r^2}, \qquad (5.2b)$$

$$\text{or } k_{sun} m = k_{planet} M, \text{ i. e. } \frac{k_{sun}}{M} = \frac{k_{planet}}{m}. \qquad (5.3)$$

So we get the nature of dependence of k on the mass of the respective celestial object, k is directly proportional to its mass and the constant of proportionality is called the **'Universal Gravitational Constant'** and is denoted by G. We have not yet explained why we call the constant '*universal*'. We shall take that up soon. Going back to Eq. 5.3 we get,

k_{sun} =MG and hence from **Eq.** 5.2b we have

$$F_{planet} = F_{sun} = \frac{GMm}{r^2}. \qquad (5.4)$$

So the force between a planet and the sun is one of mutual attraction and is proportional to the product of their masses and inversely proportional to the square of the distance between them.

If you go back and read the paragraph before Eq. 5.1 you will remember that m and M were considered as point masses. Keeping this in view we consider earth and apple **A** to be point masses, having masses M_e and M_a, respectively. Let the distance between them be r, i.e. $r = R_e + h$, (see Fig. 5.5), where

R_e = Radius of earth

h = Height of the point above the earth's surface from which the apple falls.

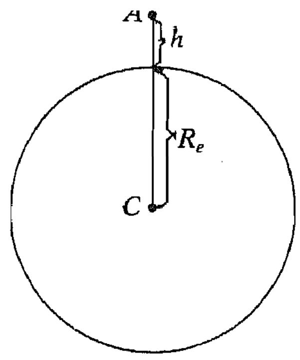


Fig. 5.5: An apple (A) at a distance $(R_e + h)$ from the centre of the earth (C).

Here, the apple experiences a force of attraction due to the earth. Its magnitude (according to Eq. 5.4) is given by

$$F = \frac{GM_eM_a}{r^2}. \qquad (5.5)$$

Now $r^2 = (R_e + h)^2 \simeq R_e^2$, as h is much smaller in comparison to R_e. So $F = \frac{GM_eM_a}{R_e^2}$,

Thus, the acceleration of the apple towards the earth is

$$a = \frac{F}{M_a} = \frac{GM_e}{R_e^2}. \qquad (5.6)$$

Study Eq. 5.6 carefully. Everything on its right-hand side is a constant on the earth and the left-hand side stands for the acceleration of an object falling near the surface of the earth. Now, as you are very well aware of the **'Law of Falling Bodies'** due to Galileo try the following SAQ.

SAQ 2

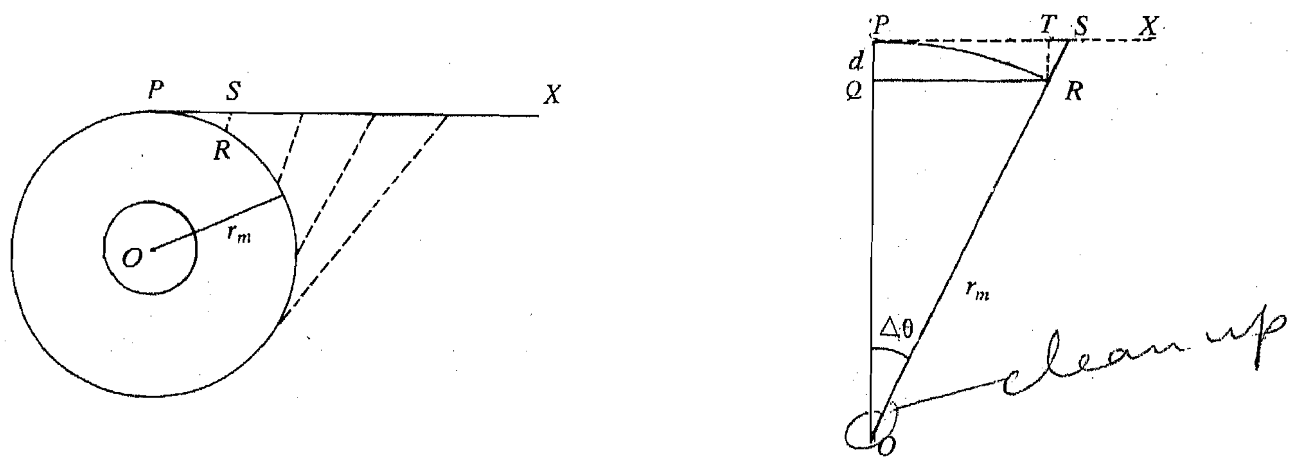
Show that Eq. 5.6 agrees with the 'Law of Falling Bodies'. [*Hint*: The acceleration of a falling body near the surface of earth is a constant irrespective of the body.]

Let us now apply Eq. 5.6 to analyse the fact that the moon falls towards the earth very much as the apple does.

5.2.2 Moon's Rotation about the Earth

Let the moon be at any position P in its orbit (Fig. 5.6a).

If no force were acting on the moon, it would have travelled along a straight line PX tangent to the orbit at **P.** Instead it follows the circular path of radius r_m about the centre of the earth.



(a) (b)

Fig. 5.6 (a) In absence of gravity the moon would have followed the straight path PX. However, in its circular orbit it can be regarded as falling away from the straight path (shown by dashed lines). Inner circle represents earth and the outer circle represents the orbit of the moon around the earth; (b) $\Delta\theta = \omega\ \Delta t$, $\omega = \frac{2\pi}{T}$ = the angular velocity of moon's rotation about the earth where T is the time-period of rotation.

Now let the moon travel from P to R in a very short time Δt. Let its linear velocity be **v**. In the absence of any force it would have travelled a distance $v\Delta t$ (= PS, say) along PX. Its motion along the circular arc **PR** can, therefore, be considered as a fall towards the earth through a distance SR. ST and RQ are drawn perpendicular to PX and OP, respectively. Since PR is infinitesimal, SR = TR = PQ = d, say. So, effectively the fall in the time interval Δt is d.

Now, $d = r_m(1-\cos\Delta\theta) = 2r_m \sin^2\left(\frac{\Delta\theta}{2}\right) \simeq \frac{r_m}{2}(\Delta\theta)^2$, as $\sin\frac{\Delta\theta}{2} \simeq \frac{\Delta\theta}{2}$ for small $\Delta\theta$.

Since $\Delta\theta = \omega(\Delta t) = \frac{2\pi}{T}(\Delta t)$,

$$d = \frac{r_m}{2}\left(\frac{2\pi}{T}\Delta t\right)^2 = \frac{2\pi^2 r_m}{T^2}(\Delta t)^2. \qquad (5.7)$$

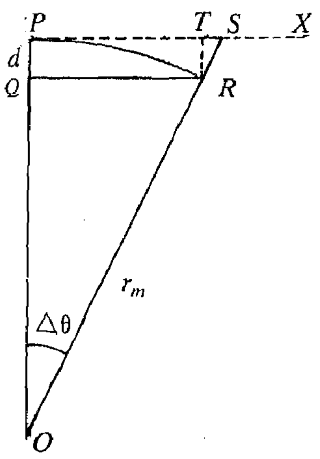


Fig. 5.6b repeated for ready reference

Now, r_m and T are known to have the values 3.85×10^8 m and 2.4×10^6 s, respectively. If we put $\Delta t = 1$s, d is found to be 1.3×10^{-3} m. This means that while moon turns for 1s around the earth, the distance by which it falls towards the earth is a little over 1 mm. If the acceleration of the moon towards the earth is a_m in ms^{-2}, then the distance through which the moon falls in 1 s is given by

$$d = \frac{1}{2}a_m(1s)^2 = 1.3\times10^{-3}\,m, \qquad (5.8)$$

or $a_m = 2.6\times10^{-3}\,m\,s^{-2}$.

Now, the acceleration a of a freely falling object near the surface of the earth has an experimentally determined approximate value of 9.8 m s^{-2}. This gives a value for the ratio of the two accelerations:

$$a_m/a = 2.6\times10^{-4}. \qquad (5.9)$$

Note that in the estimation of this ratio, the law of gravitation has not been used at all. Now using Eq. 5.6, we get

$$\frac{a_m}{a} = \frac{GM_e/r_m^2}{GM_e/R_e^2} = \left(\frac{R_e}{r_m}\right)^2; \qquad (5.10)$$

Now, putting the values of R_e and r_m we have $a_m/a = 2.7\times10^{-4}$ which agrees reasonably well with Eq. 5.9.

Newton argued that this could not be a coincidence. There is a force of attraction between two objects that is proportional to the product of their masses and inversely proportional to the square of the distance between the two. The law is indeed universally applicable to all objects in the universe, be these dust particles or stars and galaxies. Hence the constant G is universal, Its value is 6.673×10^{-11} $Nm^2\,kg^{-2}$ and is same for all pairs of particles. So we are now in a position to state the ***'Law of Universal Gravitation':***

Newton's theory of gravitation was the culmination of two centuries of scientific revolution that began in 1543 through Copernicus. After that the works of Tycho Brahe, Kepler and in particular Galileo provided the necessary launching pad for Newton's law.

The force between any two particles having masses m_1 and m_2 separated by a distance r is attractive, acting along the line joining the particles and has the magnitude,

$$F = G\frac{m_1 m_2}{r^2} \qquad (5.11)$$

But force as we know is a vector quantity. Hence, we must take care of the direction of **F**. So we shall rewrite Eq. 5.11 vectorially. Refer to Fig. 5.7. Let $\mathbf{r}_{12}$ be the position vector of m_2 with respect to m_1, i.e it points from m_1 to m_2. The gravitational force $\mathbf{F}_{12}$, exerted by m_1 on m_2 is given by

$$\mathbf{F}_{12} = -G\frac{m_1 m_2}{r_{12}^2}\hat{\mathbf{r}}_{12}. \qquad (5.12)$$

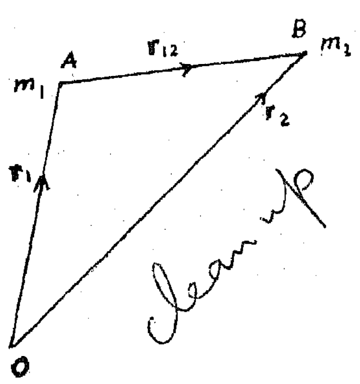


Fig. 5.7 : $\mathbf{r}_{12} = \mathbf{r}_2 - \mathbf{r}_1$

$\hat{\mathbf{r}}_{12}$ is the unit vector from m_1 to m_2. The minus sign indicates that the force on m_2 due to m_1, is directed opposite to $\mathbf{r}_{12}$, it being a force of attraction. Likewise $\mathbf{F}_{21}$, the force experienced by m_1 due to m_2 will be directed along $\mathbf{r}_{12}$. We know from Newton's third law of motion that, $\mathbf{F}_{21} = -\mathbf{F}_{12}$. So we have from Eq. 5.12

$$\mathbf{F}_{12} = -\mathbf{F}_{21} = -G\frac{m_1 m_2}{r_{12}^2}\hat{\mathbf{r}}_{12}. \qquad (5.12a)$$

However, did you note one point? Eq. 5.12 was stated for two point masses. For dealing with situations like attraction between earth and moon we considered them as point masses as the distance between them was much greater compared to their sizes. But otherwise we may have to calculate the force between a sphere and a point mass, say for example the force between earth and a particle. To tackle such a problem we need to know the 'Principle of Superposition'.

5.3 PRINCIPLE OF SUPERPOSITION

Eq. 5.12 gives us the force between two point masses. If there are several masses, like m_1, m_2 and m_3 as shown in Fig. 5.8, how would we calculate the gravitational force on one of them, say m_1? If only m_1 and m_2 were present, the force on m_1 due to m_2 would be

$$\mathbf{F}_{21} = -G\frac{m_1 m_2}{r_{21}^2}\hat{\mathbf{r}}_{21}.$$

Similarly if only m_1 and m_3 were present, the force on m_1 due to m_3 would be

$$\mathbf{F}_{31} = -G\frac{m_1 m_3}{r_{31}^2}\hat{\mathbf{r}}_{31}.$$

Now, if both m_2 and m_3 are attracting m_1, the total force on m_1 is the vector sum of $\mathbf{F}_{21}$ and $\mathbf{F}_{31}$, i.e.

$$\mathbf{F}_1 = \mathbf{F}_{21} + \mathbf{F}_{31},$$

$$\text{or } \mathbf{F}_1 = -G\frac{m_1 m_2}{r_{21}^2}\hat{\mathbf{r}}_{21} - G\frac{m_1 m_3}{r_{31}^2}\hat{\mathbf{r}}_{31}. \quad (5.13)$$

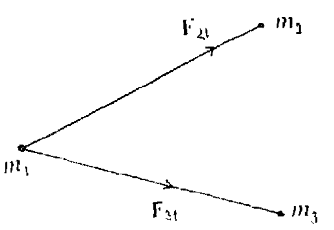


Fig. 5.8 : $\mathbf{F}_1 = \mathbf{F}_{21} + \mathbf{F}_{31}$

This is the superposition **principle** according to which *the resultant force on a mass is the vector sum of the individual forces acting on it.* We shall apply this principle to determine the gravitational force due to an extended body. But before we go into that discussion, you may like to **try an SAQ.**

SAQ **3**

Show that the location of the point between two fixed masses m_1 and m_2 at which a mass m does not feel any resultant gravitational force due to than is independent of m.

If the force between a point mass O and an extended body (Fig. 5.9) is required, we can apply the principle of superposition. But the problem will be complicated as the number of particles is very large. Effectively we shall have to perform integration. In order to get rid of this difficulty we take resort to the concept of Gravitational Potential, which we shall discuss next.

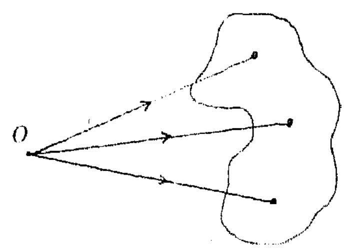


Fig. 5.9 : Force between a point mass and an extended body.

5.4 GRAVITATIONAL FIELD AND POTENTIAL

Let us consider a particle of mass m_1 placed at some point. We place another particle of mass m_2 at a distance r from it. So each particle experiences a force of attraction due to the other. If the distance between the particles is changed then also there will be a force. In other words, however large the value of r might be, there will be some force of attraction. We say that m_1 modifies the space around it in some way and sets up a field of influence, called the gravitational **field.**

The strength of a gravitational field is given by **its** intensity. The intensity of the gravitational field due to a mass M at a distance r from it is given by the force experienced by a unit mass placed at that point. Hence, the intensity at point P due to mass M , say at O, (see Fig. 5.10) will be given by

$$\mathbf{E} = -\frac{GM}{r^2}\hat{\mathbf{r}}, \quad (5.14)$$

where $\hat{\mathbf{r}}$ is the unit vector along OP, and $OP = r$. The force F experienced by a mass m at P due to M at O is given by

$$\mathbf{F} = -\frac{GMm}{r^2}\hat{\mathbf{r}}. \quad (5.14a)$$

Thus, from Eq. 5.14 we get $\mathbf{F} = m\mathbf{E}$. (5.14b)

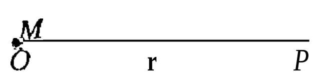


Fig. 5.10: Gravitational intensity at P due to M at O.

If mass m is now removed from this point and placed at a larger distance then work has to **be** done against this attractive force. In Unit **3,** we have dealt with a similar case. Look up Sec. 3.2.2 and try the following **SAQ.**

SAQ 4

Show that the work done in bringing a mass m from a point Q to a point P in the gravitational field of a mass M placed at O (Fig. 5.11) is given by

$$W = -GMm\left(\frac{1}{R} - \frac{1}{r}\right), \text{ where } OQ = R, OP = r. \quad (5.15)$$

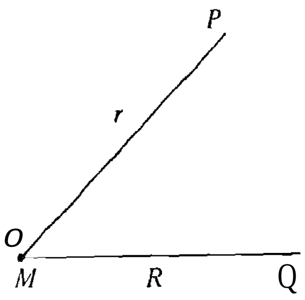


Fig. 5.11

Since the above work has to be performed, the mass m acquires a P.E. called the gravitational potential energy. This P.E. is mutual to the masses m and M. By convention, the gravitational potential energy U of a mass m in the field of mass M at a distance r is measured as the negative of the work done in bringing m from infinity to the said point. So we can get U by putting R = ∞ in the expression of $-W$ of Eq. 5.15, i.e.

$$U = -\frac{GMm}{r}. \quad (5.16)$$

We shall introduce another term called *gravitational potential* of a mass M at a distance r. It is defined as the negative of the work done in bringing a unit mass from infinity to that point.

SAQ 5

Using Eqs. 5.14a and 5.16, verify that

$$\mathbf{F} = -\frac{dU}{dr}\hat{\mathbf{r}} \quad (5.17)$$

The meaning of Eq. 5.17 is that the *gravitational force of attraction between two masses can be obtained as the negative space-rate of change of theb gravitational P.E.* If you look back at Eq. 3.18 you will find that we have discussed this aspect about a conservative force field. And the gravitational force is indeed a conservative one.

At this stage we shall recall the problem of determination of force between a point mass and an extended body. We were worried over the complication involved in performing the vector sum of the individual forces. Eq. 5.17 provides us with a way out. We can determine the overall gravitational P.E. by taking the sum of individual P.E.s. This sum can be obtained quite conveniently as potential energy is a scalar quantity. We can then use Eq. 5.17 to determine the force.

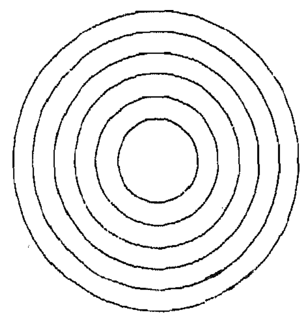


Fig. 5.12

We shall apply whatever we have learnt so far to the study of the force of gravity. For that we shall consider earth to be a sphere, neglecting its equatorial bulge. So let us take up the problem of determining the force of attraction between a sphere and a point mass. In the course of this discussion we shall have to do quite a bit of mathematical calculations. But do not let that put you off.

You might have noticed that when an onion is peeled off, thin layers come out one after another till we reach its central part when almost nothing remains. Similarly, we may consider a solid sphere as an aggregate of several concentric thin spherical shells as shown in Fig. 5.12. We shall first find out the gravitational potential energy of a point mass due to a spherical shell and then go over to the study of spheres.

5.4.1 Gravitational P.E. due to a Spherical Shell

Let a point mass m be placed at a distance r from the centre of a spherical shell of mass M_s and radius R_s. Let us calculate the gravitational potential energy of the mass due to the shell when i) $r > R_s$ ii) $r < R_s$.

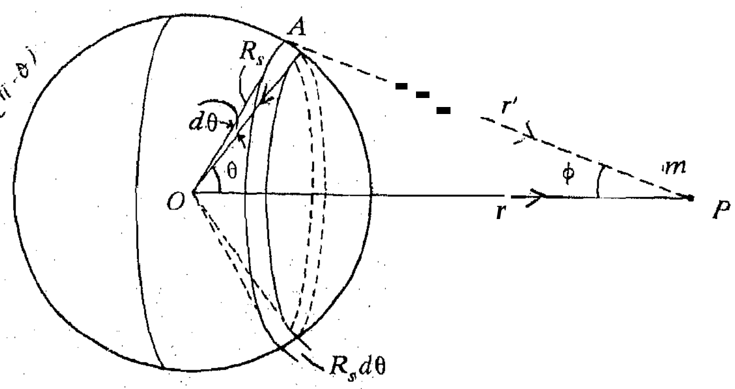


Fig. 5.13: Gravitational P.E. due to a circular ring of a thin spherical shell

Refer to Fig. 5.13. We first consider a ring-like portion of the shell contained between the directions θ and $\theta + d\theta$ with respect to the axis OP. Let it be of infinitesimal width so that every point on it is at the same distance, say r', from P. The angular width of the ring is $d\theta$, its width is $R_s d\theta$, and its radius is $R_s \sin\theta$. What is its mass? The mass per unit area of the shell is $\sigma = M_s / 4\pi R_s^2$. The ring's mass is then $M_{ring} = \sigma dA$, dA being the surface area of the ring. The circumference of the ring is $2\pi R_s \sin\theta$. Its area is, therefore, given by

$$dA = (2\pi R_s \sin\theta)\ R_s d\theta, \qquad (5.18)$$

or $$M_{ring} = \frac{M_s}{4\pi R_s^2}(2\pi R_s \sin\theta)\ R_s d\theta = \frac{M_s}{2}\sin\theta d\theta. \qquad (5.19)$$

We shall now determine the gravitational P.E. at P due to this ring. The ring is made up of a large number of point masses each having mass equal to, say δM.

The gravitational P.E. at P due to one such point mass is $-\frac{Gm\delta M}{r'}$. So the gravitational P.E. at P due to the ring will be $dU_{ring} = \Sigma\left[-\frac{Gm\delta M}{r'}\right]$, where the summation (Σ) extends over all the points on the ring, Here G is a constant. Again as every point on the ring is at the same distance r' from P, r' is also a constant for the points on the ring. So

$$dU_{ring} = \Sigma\left[-\frac{Gm\delta M}{r'}\right] = -\frac{Gm}{r'}\Sigma\delta M = -\frac{Gm}{r'}M_{ring}.$$

On using Eq. 5.19 we get

$$dU_{ring} = -\frac{GmM_s \sin\theta\, d\theta}{2r'} \qquad (5.20)$$

The shell can be imagined to be made up of such rings having a common axis OP. Since P.E. is a scalar quantity we shall integrate Eq. 5.20 to get the gravitational P.E. U of the shell. On the right hand-side of Eq. 5.20 we now have two variables θ and r'. It would be convenient if we can express it in terms of a single variable. For this we shall consider the relation between r', r and R_s. From triangle OAP we have,

$$r'^2 = r^2 + R_s^2 - 2rR_s \cos\theta. \qquad (5.21)$$

On differentiating with respect to θ we get

$$2r'\frac{dr'}{d\theta} = 2rR_s \sin\theta,$$

or $$\frac{dr'}{rR_s} = \frac{\sin\theta\ d\theta}{r'} \qquad (5.22)$$

Hence, from Eq. 5.20, we get

$$dU_{ring} = -\frac{GmM_s}{2rR_s}dr' \qquad (5.23)$$

And on integrating Eq. 5.23, we get for the entire spherical shell

$$U = -\frac{GM_s m}{2rR_s}\int_{r_1'}^{r_2'} dr', \qquad (5.24)$$

where r_1' and r_2' are, respectively, the minimum and maximum values of r'. Now study Fig. 5.14. For a point P outside the shell, i.e. for $r > R_s$,

$$r_1' = r - R_s \quad r_2' = r + R_s \qquad (5.25a)$$

and for a point P inside the shell, i.e. for $r < R_s$

$$r_1 = R_s - r,\ r_2 = R_s + r. \qquad (5.25b)$$

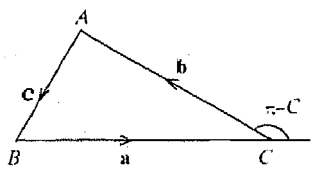
So the gravitational potential energy

$$U = -\frac{GmM_s}{2rR_s}(r_2' - r_1') = -\frac{GmM_s}{2rR_s}2R_s, \quad r > R_s \ (\text{from } 5.25a)$$

or $$U = -\frac{GmM_s}{r},\ r > R_s. \qquad (5.26)$$

The force on mass m is given by Eq. 5.17 as

$$\mathbf{F} = -\left(\frac{dU}{dr}\right)\hat{\mathbf{r}} = -\frac{GmM_s}{r^2}\hat{\mathbf{r}}, \qquad (5.27)$$



In the above figure
a + **b** + **c** = 0
i.e. **a** + **b** = − **c**
or (**a** + **b**). (**a** + **b**) = c^2
or $a^2 + b^2 + 2\mathbf{a}.\mathbf{b} = c^2$
or $a^2 + b^2 + 2ab\cos(\pi - C) = c^2$
or $c^2 = a^2 + b^2 - 2ab\cos C$

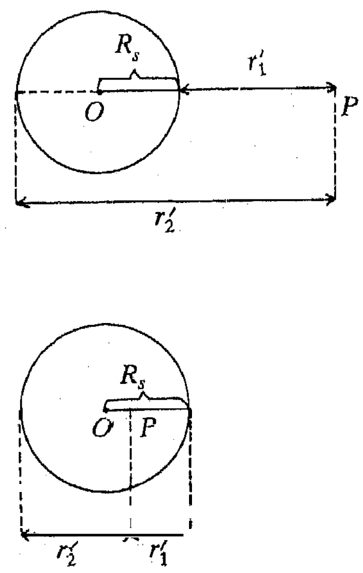


Fig. 5.14

The negative **sign** indicates that it is a force of attraction. **The** negative sign in the expression of U in Eq. 5.26 also indicates that it is attractive. Comparing Eqs. 5.12 and 5.27, it can be said that the shell behaves as a point mass having the same mass **as** that of the shell and located at its centre.

For $r < R_s$, we use Eq. 5.25b to get

$$U = -\frac{GmM_s}{2rR_s}(r_2' - r_1') = -\frac{GmM_s}{2rR_s}2r$$

$$\text{or } U = -\frac{GmM_s}{R_s} = \text{a constant} = U_0, \text{ say. } r < R_s. \qquad \textbf{(5.28)}$$

From Eq. 5.17, we get $\mathbf{F} = -\frac{dU}{dr}\hat{\mathbf{r}} = 0.$ (5.29)

So P.E. of a mass placed at any point within the shell remains constant and the gravitational force on it **is** zero.

You can now apply the concepts you have learnt in working out the following SAQ.

SAQ 6

Draw a graph of U vs. r for the spherical shell. Take the range of r as $r = 0$ to $r = 2R_s$. Explain using physical argument whether U should be continuous at $r = R_s$ or not. Does your graph agree with your argument'?

So **far** we have determined the gravitational P.E. of a point mass due to a spherical shell. Let us now **extend** these ideas to the case of a solid sphere.

5.4.2 Gravitational B.E. due to a Solid Sphere

We have seen earlier that a solid sphere is an aggregate of concentric spherical shells. The determination of a gravitational P.E. due to a solid sphere at an external point is a straightforward application of the ideas of **Sec.** 5.4.1 So you may like to work it out yourself.

SAQ 7

Refer to Fig. **5.15a.** A solid sphere of mass M and radius a has been shown. Show that the gravitational **potential energy** of a mass m at A $(OA = r)$ due to the sphere is given by

$$U_A = -\frac{GMm}{r}. \qquad (5.30)$$

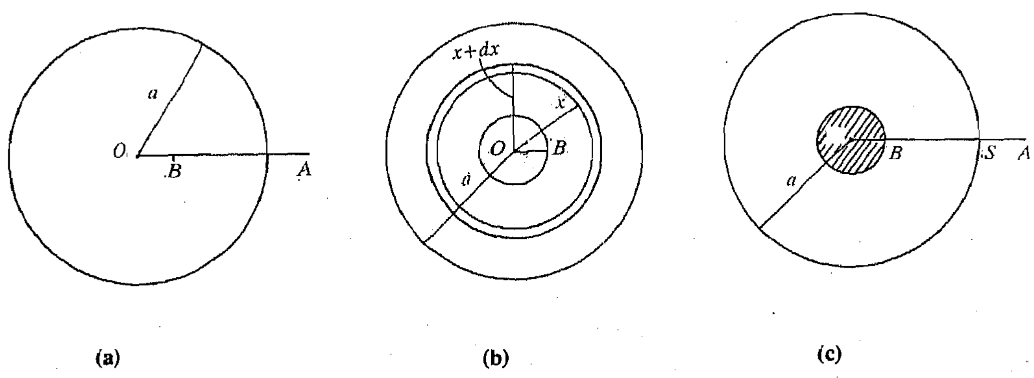


Fig. 5.15

We took up **the** problem of the gravitational force on a mass due to a sphere with a view to studying the variation of earth's gravity. For this we not only need to know about the force experienced by a mass placed external to a sphere but **also** by a mass placed inside **it.** So let us work out **the** following example.

Example 1

Refer to Fig. **5.15a.** Show that the gravitational P.E. of a mass m at B (OB = r) due to the solid sphere of mass M and radius a is given by

$$U_B = -\frac{GMm}{2a^3}(3a^2 - r^2). \qquad (5.31)$$

See Fig. 5.15b. The point B is on the surface of a solid sphere of radius r and on the inner surface of a thick spherical shell included between radii r and a. These two will contribute to the P.E. of m at B. Let us name the contributions as U_{B1} and U_{B2}. From the result of SAQ 7, we have $U_{B1} = -\frac{GM_1 m}{r}$, where M, is the mass of the inner sphere of radius r:

$$M_1 = \frac{M}{\frac{4}{3}\pi a^3}\left(\frac{4}{3}\pi r^3\right) = \frac{Mr^3}{a^3}. \tag{5.32}$$

Hence, from Eq. 5.32, $U_{B1} = -\frac{GMmr^2}{a^3}$. (5.33)

For determining U_{B2} we consider the concentric shell included between radii x and $x + dx$. The volume of this shell is

$4\pi x^2 dx$ and its mass $= \frac{M}{\frac{4}{3}\pi a^3} 4\pi x^2 dx = \frac{3M}{a^3} x^2 dx.$

Since dx is infinitesimal, we can consider this thin shell equivalent to a spherical shell of radius x. Hence, from Eq. 5.28 we have the P.E. of m due to this shell at B as

$$dU_{B2} = -\frac{Gm\left(\frac{3M}{a^3}x^2 dx\right)}{x} = -\frac{3GMm}{a^3} x dx. \tag{5.34}$$

But the thick shell is made up of a number of such thin shells with radii ranging from r to a. So in order to get U_{B2}, we shall have to integrate Eq. 5.34. Thus,

$$U_{B2} = -\int_r^a \frac{3GMm}{a^3} x dx = -\frac{3GMm}{2a^3}(a^2 - r^2). \tag{5.35}$$

Now, $U_B = U_{B1} + U_{B2}$.

From Eqs. 5.33 and 5.35, we get

$$U_B = -\frac{GMm}{a^3}\left[r^2 + \frac{3}{2}(a^2 - r^2)\right] = -\frac{GMm}{2a^3}(3a^2 - r^2) \text{ which is Eq. 5.31.}$$

Now, we can calculate the force of attraction on the mass m due to the solid sphere of mass M and radius a. Refer to Fig. 5.15c. When m is placed external to the sphere at A (i.e. $r = OA$) we have from Eqs. 5.17 and 5.30 that the force is

$$\mathbf{F}_A = -\frac{d}{dr}\left(-\frac{GMm}{r}\right)\hat{\mathbf{r}} = -\frac{GMm}{r^2}\hat{\mathbf{r}}. \tag{5.36}$$

This is the same as Eq. 5.14a. Eq. 5.36 signifies that the force of attraction due to a solid sphere on a mass m placed external to it is *the same as that due to a point mass having the same mass as that of the sphere and located at its centre.*

When m is placed inside the sphere at B (i.e. r = OB) we have from Eqs. 5.17 and 5.31 that the force is

$$\mathbf{F}_B = -\frac{d}{dr}\left\{-\frac{GMm}{2a^3}(3a^2 - r^2)\right\}\hat{\mathbf{r}} = -\frac{GMm}{a^3} r\hat{\mathbf{r}}. \tag{5.37}$$

Now, using Eq. 5.32 we may write Eq. 5.37 as

$$\mathbf{F}_B = -\frac{GM_1 m}{r^2}\hat{\mathbf{r}}. \tag{5.38}$$

Now, refer to Fig. 5.15c. Eq. 5.38 signifies that the force experienced by m at B is the same as that due to a point mass at O having mass M_1, which is, incidentally, the mass of the sphere with radius OB. So we can infer that when the point mass m is placed *inside a solid sphere it experiences a force of attraction only due to the spherical mass shown shaded in Fig. 5.15c. The thick spherical shell (shown unshaded in Fig. 5.15c) does not contribute to the force of attraction.* Now, putting $r = a$ in each of Eqs. 5.36 and 5.37, we get the force of attraction when m is placed at S on the surface of the sphere as

$$\mathbf{F}_S = -\frac{GMm}{a^2}\hat{\mathbf{r}}. \tag{5.39}$$

We shall now use the results of Eqs. 5.36, 5.37 and 5.39 to study the variation of earth's gravity.

5.4.3 Gravity and its Variation

The phenomenon of attraction between the earth and any other body is called **gravity.** Due to such an attraction a body experiences an acceleration towards the centre of the earth. This is known as the *acceleration* due *to gravity*, and is denoted by **g.** We shall study how **g** at a place varies with altitude and depth.

Refer to Fig. 5.16. We consider the positions of a particle of mass m at A and B, respectively, where SA = h = the altitude of A and SB = d = the depth of B. S is a point on the surface of the earth. $OS = R_e$ = the radius of earth. Let the mass of earth be M_e. Let the forces of attraction experienced by m at A, B and S be denoted by $\mathbf{F}_A$, $\mathbf{F}_B$ and $\mathbf{F}_S$, respectively. In each of Eqs. 5.36, 5.37 and 5.39, we put $M = M_e$, $a = R_e$. Then we put $r = R_e + h, R_e - d$ in the Eqs. 5.36 and 5.37, respectively, to get the magnitudes of the force as

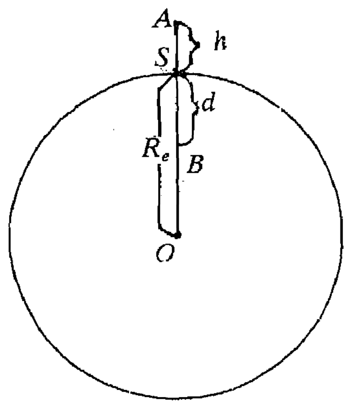


Fig. 5.16

$$F = \frac{GM_e m}{R_e^2},\ F_A = \frac{GM_e m}{(R_e + h)^2},\ F_B = \frac{GM_e m}{R_e^3}(R_e - d). \tag{5.40}$$

Let the magnitudes of acceleration due to gravity on the surface of earth, and at points A and B be denoted by g_S, g_A and g_B. Then

$$g_S = \frac{F_S}{m} = \frac{GM_e}{R_e^2} = g_0,\ \text{say}, \tag{5.41}$$

$$g_A = \frac{F_A}{m} = \frac{GM_e}{(R_e + h)^2}, \tag{5.42}$$

$$g_B = \frac{F_B}{m} = \frac{GM_e(R_e - d)}{R_e^3} \tag{5.43}$$

From Eqs. 5.41 and 5.42, we get

$$g_A = \frac{g_0 R_e^2}{(R_e + h)^2}, \tag{5.42a}$$

$$\text{or}\quad g_A = g_0\left(1 + \frac{h}{R_e}\right)^{-2} \approx g_0\left(1 - \frac{2h}{R_e}\right),\ \text{for } (h \ll R_e). \tag{5.42b}$$

From Eqs. 5.41 and 5.43, we get

$$g_B = \frac{g_0}{R_e}(R_e - d). \tag{5.43a}$$

From Eqs. 5.42a and 5.43a we get that the acceleration due to gravity varies inversely as the square of the distance from the centre of earth for points above the surface of earth, and directly as the distance from the centre of earth for points below the surface of earth. Now you can work out an SAQ on Eqs. 5.42a and 5.43a.

SAQ 8

a) Plot a graph of g vs. r with r ranging from 0 to $2R_e$.

b) By what percentage of its value at sea-level does g increase or decrease when one goes to i) an altitude of 2500 km and ii) Kolar Gold Field at a depth of 3000 m.

We have discussed the variation of g with altitude and depth. It also varies with latitude due to the rotation of the earth about its axis. We are stating the formula for this variation without proof, which will be given in Unit 10.

$$g(\lambda) = g_e + \omega^2 R \sin^2 \lambda, \tag{5.44}$$

where $g(\lambda)$ = Value of g on the surface of earth at a place having latitude λ

g_e = Value of g on equator = 9.7805 m s^{-2}

ω = Angular speed of rotation of earth.

We have discussed how g is affected due to several factors. We understand from Eqs. 5.42 and 5.42a that at any finite distance from the surface of earth, g is non-zero. *So* the effect of gravity can be felt at any point irrespective of its distance from the centre of the earth. But we shall see that at any position **a** particle may be made to escape from the bounds of earth's attraction if it is provided with a certain minimum velocity. **This** is called the **velocity of escape.** This concept applies to any spherical celestial object. We shall now derive an expression for it.

5.4.4 Velocity of Escape

Let us consider a particle of mass m at a distance r from the centre of a huge spherical body of mass M (see Fig. 5.17). Its gravitational P.E. at this position is $U = -GMm/r$. Read the paragraph after Eq. 5.27 and you will realise the significance of the negative sign of U. It indicates that the mass m is bound by the attraction of the body of mass M.

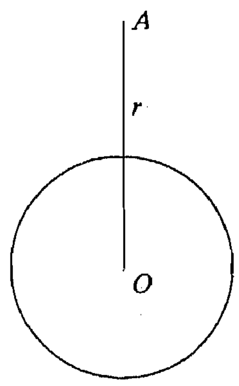


Fig. 5.17

Now, if the particle is to become free from the bounds of the gravitational attraction of the body then it must be provided with an external energy E ($\geq U$). Thereby, its total energy $(E + U)$ becomes non-negative. Thus, the particle ceases to remain bound and escapes from the attraction of M. If E is provided in the form of K.E. then $E = \frac{1}{2} mv^2$, where v is the velocity given to the particle. Accordingly the condition becomes

$$\frac{1}{2} mv^2 + U \text{ is not negative}$$

$$\text{or } \frac{1}{2} mv^2 + \left(-\frac{GMm}{r}\right) \geq 0, \quad \text{i.e. } v^2 \geq \frac{2GM}{r} \quad \text{or } v \nless \sqrt{\frac{2GM}{r}}.$$

Hence, $\sqrt{\frac{2GM}{r}}$ is the required minimum velocity and it is the expression for the *velocity of escape* (v_e) which we can see is independent of m, the mass of the particle. Thus.

$$v_e = \sqrt{\frac{2GM}{r}}. \qquad (5.45)$$

If the particle were originally on the surface of earth, then $r = R_e$, $M = M_e$ and from Eqs. 5.45 and 5.41, we get

$$v_e = \sqrt{\frac{2GM_e}{R_e}} = \sqrt{2 g_0 R_e}. \qquad (5.46)$$

Now, taking $g_0 = 9.8 \text{ m s}^{-2}$, we get $v_e = 1.1 \times 10^4 \text{ m s}^{-1} = 11 \text{ km s}^{-1}$ — a velocity that will take you from Srinagar to Kanyakumari in about five minutes! So, now you can work out a simple SAQ.

SAQ 9

Find the velocity of escape on the surface of moon.

So far we have dealt with the phenomenon of gravitation and some of its applications. Newton's law of gravitation was the fountainhead of all the discussion. But now we raise the question — 'why at all there is a force of attraction between any two material bodies?' Does Newton's law provide an answer? It cannot because the gravitational force between two bodies exists naturally. Such a force is called a *'Fundamental Force* in *Nature'*. There are three different kinds of fundamental forces in nature. We shall now discuss briefly about them.

5.5 FUNDAMENTAL FORCES IN NATURE

The three kinds of fundamental forces are (i) *gravitational* (ii) *electroweak* and (iii) *strong*. You have read in detail about (i) which acts on all matter as you have seen so far. It varies inversely as the square of the distance but its range is infinite. This force is responsible for holding together the planets and stars and in overall organisation of solar system and galaxies.

The second kind is the electroweak force. It includes the forces of electromagnetism and the so-called weak nuclear force. We shall discuss about the latter towards the end of this section, But let us first identify the electromagnetic forces. The force between two charged particles at rest (electrostatics) or in motion (electrodynamics) comes under the purview of electromagnetic forces. The electrostatic force between two charges obeys the inverse square law like the gravitational force between two masses. However, there is an important dissimilarity, Charges can be of two kinds — positive and negative. If the charges are of opposite kind the force between them is *attractive* and if they are of the same kind, then the force between them is *repulsive*. It can be shown that the gravitational force between an electron and a proton in a hydrogen atom is 10^{39} times weaker than the electrostatic force between them. Thus, we get a comparative estimate of the strengths of gravitational and electrostatic force.

Now, let us come to the case of moving charges. We know that charges in motion give rise to electric current. You have also studied Oersted's experiment in your school science courses. From this experiment we understand that a current carrying conductor is equivalent to a magnet. This is the meeting point of electricity and magnetism and hence the word 'electromagnetic' got associated with this field of force. The forces that one comes across in daily life, like friction, tension, **etc.** can be explained from the standpoint of the electromagnetic force field.

Now, if we make an estimate of the relative strengths of the repulsive electrostatic and the attractive gravitational force between two protons in a nucleus we shall **find** that the former is 10^{36} imes **larger** than the latter. So how is it that the protons in an atomic nucleus, stay **together** instead of flying away? The answer lies in the third kind of fundamental force known as the **strong** (nuclear) force that exists between the protons inside the nucleus which is strongly attractive, much stronger than the electrostatic force between them. As the nucleus **also** contains neutrons, which are as tightly bound as the protons, this force must also exist between two neutrons as well as between neutrons and protons. Unlike the gravitational and the electromagnetic forces, the nuclear force acts only when the nucleons (protons and neutrons) are very close to each other (10^{-15} m or less). The nuclear forces decrease very rapidly with distance, so rapidly that a nucleon only interacts with its closest neighbours. You will study in detail about the nuclear forces in the Nuclear Physics course.

times

The strong nuclear force as we have seen just now accounts for the binding of atomic nuclei. But this cannot account for processes like radioactive beta decay about which once again you will read in the Nuclear Physics course. This can be explained from the point of view of the so-called ***weak*** nuclear ***force.*** It is much weaker than the electromagnetic force at nuclear distance but still greater by a factor of 10^{34} than the gravitational force. Just a few years ago, this weak force was listed separately from the electromagnetic force. However a theory was proposed which led to the unification of the weak forces and the electromagnetic forces and hence the name 'electroweak' forces.

We shall now give the different characteristics of the fundamental forces in nature in Table 5.1.

Table 5.1: Some Characteristics of the Three Fundamental Forces

Force		Relative strength	Range	Importance
Strong nuclear		1	10^{-13} cm	Holds nucleons together
Electroweak	Electromagnetic	10^{-2}	Infinite	Controls everyday phenomena— friction, tension etc.
	Weak nuclear	10^{-5}	10^{-15} cm	Nuclear transmutation
Gravitational		10^{-39}	Infinite	Organises large-scale phenomena and universe

Now let us sum up what you have learnt in this unit.

5.6 SUMMARY

- Newton's law of universal gravitation states that any two particles in the universe exert an attractive force on each other, given by

$$\mathbf{F}_{12} = -G\frac{m_1 m_2}{r_{12}^2}\hat{\mathbf{r}}_{12} = -\mathbf{F}_{21},$$

where $\mathbf{F}_{12}$ is the force exerted by m_1 on m_2 and $\hat{\mathbf{r}}_{12}$ is the unit vector directed from m_1 to m_2, along the line joining the two masses.

- **Any** mass creates about itself a field of influence called the gravitational field. The intensity E and the potential U of a **gravitational** field due to a point mass M at a point are given by

$$\mathbf{E} = -\frac{GM}{r^2}\hat{\mathbf{r}} \text{ and } U = -\frac{GM}{r},$$

where $\hat{\mathbf{r}}$ is the unit vector along the line joining the mass M to the point

- The gravitational force of attraction due to a solid sphere experienced by a point mass placed external to it is the same as that due to a point mass placed at the centre of the sphere and whose mass is equal to that of the sphere.

  When the point mass is placed inside the sphere, it experiences force of attraction only due to a concentric spherical mass on whose surface it lies. The matter contained in the shells external to this point mass does not contribute at all to the force of attraction.
- The value of acceleration due to gravity at the points above and below the surface of earth varies, respectively, as the inverse square of and directly as the distance of the point from the centre of earth.
- The minimum velocity that an object of mass m at a distance r from the centre of a spherical body of mass M must have so that it can escape from the bounds of the gravitational attraction of M is called its escape velocity. Its value is $\sqrt{2GM/r}$.
- Gravitational force is fundamental force in nature. There are two other kinds of fundamental forces — the electroweak force and the strong nuclear force.

5.7 TERMINAL QUESTIONS

1. The weight of a body on the surface of the earth is 900N. What will be its weight on the surface of Mars whose mass is 1/9 and radius 1/2 that of the earth'?
2. The Gravitational P.E. of an object of mass m at a height h above the surface of earth is equal to $-GM_e m/(R_e + h)$ according to Eq. 5.30. Show that it is consistent with the expression 'mg_0h' for Gravitational P.E., where g_0 is the value of acceleration due to gravity on the surface of earth.
3. Three bodies A, B, C of masses 5 x 10^6 kg each are arranged in space at the vertices of an equilateral triangle of side 2 km (see Fig. 5.18). How much work should be done to separate them to infinite distance apart?

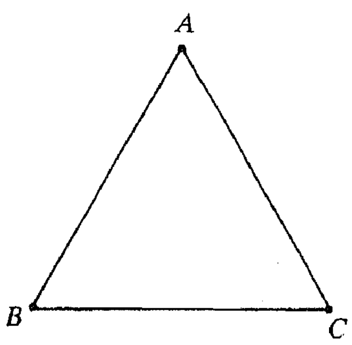


Fig. 5.18

5.8 ANSWERS

SAQs

1 Refer to Fig. 5.19. S and P refer to the positions of the sun and the planet, respectively.

$$\mathbf{F} = -\frac{km}{r^2}\hat{\mathbf{r}}$$

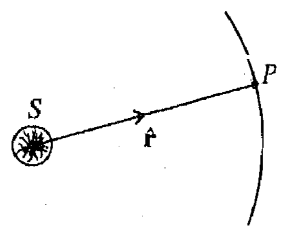


Fig. 5.19

2. Since a is constant near the surface of earth, we have for any object falling freely from rest for t s the following relations:

$$v = at\ ,\ s = \tfrac{1}{2}\ at^2,$$

or v is proportional to t and s is proportional to t^2 which are consistent with the law of falling bodies.

3. Refer to Fig. 5.20. Let the distance between m_1 and m_2 be a. Let m be at a distance x from m_1 when the resultant gravitational force on m due to m_1 and m_2 is zero. Then in this situation the magnitudes of forces of attraction between m, m_1 and m, m_2 must be same. Hence

$$\frac{Gm_1m}{x^2} = \frac{Gm_2m}{(a-x)^2} \quad \text{or} \quad \frac{a-x}{x} = \sqrt{\frac{m_2}{m_1}} = b, \text{ say.}$$

Here b is the value of the positive square root of m_2/m_1 as $x < a$.

$$\therefore x = \frac{a}{b+1} = \text{a constant independent of } m.$$

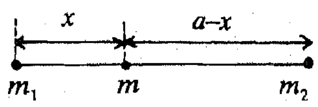
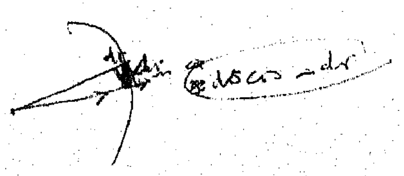


Fig 5.20

4. Required work done $W = \int_Q^P \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F} = -\frac{GMm}{r^2}\hat{\mathbf{r}}$

$$\mathbf{F} \cdot d\mathbf{r} = -\frac{GMm}{r^2}\hat{\mathbf{r}} \cdot d\mathbf{r} = -\frac{GMm}{r^2}dr \quad \text{(as explained in Sec. 3.2.2).}$$

$$\therefore W = -\int_R^r \frac{GMm}{r^2}dr = GMm\left[\frac{1}{r}\right]_R^r = -GMm\left(\frac{1}{R} - \frac{1}{r}\right).$$



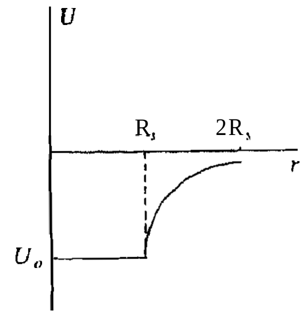


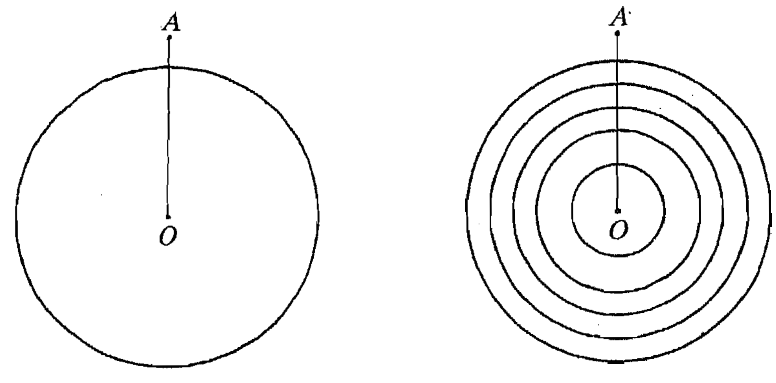
Fig 5.21 : U vs r graph; $U_0 = -\frac{GmM}{R}$

5. From Eq. 5.16, $U = -\frac{GMm}{r}$, ∴ $\frac{dU}{dr} = \frac{GMm}{r^2}$.

   Again, from Eq. 5.14a, $\mathbf{F} = -\frac{GMm}{r^2}\hat{\mathbf{r}}$ ∴ $\mathbf{F} = -\frac{dU}{dr}\hat{\mathbf{r}}$

6. U vs. r graph is shown in Fig. 5.21. If U is discontinuous at $r = R_s$, then $\frac{dU}{dr}$ becomes infinite at that point. This indicates that in accordance with Eq. 5.17 the gravitational force of attraction has to be infinite at the boundary of the spherical shell. But that is absurd. So U vs. r must be continuous everywhere as shown by the graph.

7. Refer to Fig. 5.22a. The P.E. has to be determined at **A**. See Fig. 5.22b.



Pig. 5.22

The solid sphere as we have seen earlier can be considered as an aggregate of a number of concentric spherical shells of masses, say $m_1, m_2, m_3, ...$ where

$$m_1 + m_2 + m_3 + ... = M \qquad (5.47)$$

The point A is at a distance r from the centre of each shell. From Eq. 5.26, the P.E. due to the shells will be given by

$$U_1 = -\frac{Gm_1m}{r},\ U_2 = -\frac{Gm_2m}{r},\ U_3 = -\frac{Gm_3m}{r} \text{ and so on}$$

Hence, the P.E. due to the sphere at A is

$$U_A = U_1 + U_2 + U_3 + \ldots$$

$$= -\frac{Gm}{r}(m_1 + m_2 + m_3 + \ldots)$$

∴ From Eq. 5.47, $U_A = -\frac{GMm}{r}$.

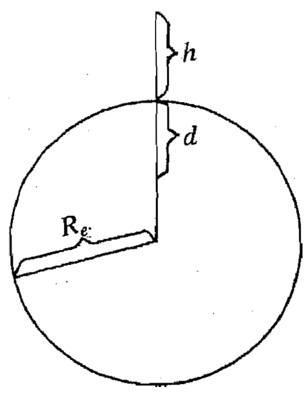


Fig. 5.23

8. a) Refer to Fig. 5.23. For $r > R_e$, $r = R_e + h$ and for $r < R_e$, $r = R_e - d$. From Eqs. 5.43 and 5.42,.

$$g = g_0\frac{r}{R_e} \quad \text{for } r < R_e$$

and

$$g = g_0\frac{R_e^2}{r^2} \quad \text{for } r > R_e$$

See Fig. 5.24 for variation of g with r.

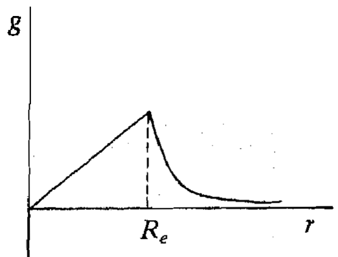


Fig. 5.24

b) (i) $g = g_0\left(\frac{R_e}{R_e + h}\right)^2$. R_e = 6,370 km, h = 2,500 km.

$$\therefore \left(\frac{R_e}{R_e + h}\right)^2 = \left(\frac{6370}{8870}\right) = 0.5157$$

$$g = g_0\,(0.5157)$$

Percentage decrease = $\frac{g_0 - g}{g_0} \times 100 = 48.4$

(ii) $g = \frac{g_0}{R_e}(R_e - d), d = 3$ km. $R_e - d = 6367$ km.

$\therefore g = g_0 \frac{6367}{6370} = g_0\,(0.9995)$

$\therefore$ Percentage decrease = **0.05**.

9. The velocity of escape on moon $v_{em} = \sqrt{\frac{2GM_m}{R_m}}$

where M_m = mass of the moon, R_m = radius of the moon. Putting these values of G. M_m and R_m, we get $v_{em} = 2.37 \times 10^3$ m s^{-1}.

Terminal Questions

1. Mass of earth = M_e, Radius of earth = R_e, Mass of body = m. Newton's law of gravitation gives

$$F = G\frac{M_e m}{R_e^2} = 900 \text{ N}$$

Mass of Mars = $\frac{M_e}{9}$, Radius of Mars = $\frac{R_e}{2}$.

Suppose weight of the body on the surface of Mars = x

$$\frac{G\frac{M_e}{9}m}{\left(\frac{R_e}{2}\right)^2} = \frac{4}{9}\frac{GM_e m}{R_e^2} = \frac{4}{9} \times 900 \text{ N} = 400 \text{ N}$$

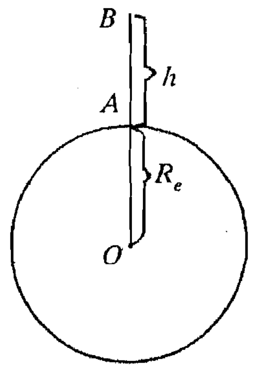


Fig. 5.25

2. Refer to Fig. **5.25.** A is a point on the surface of earth and AB = h. For the point A, $h = 0$. So using the result given in the question we have the values of P.E.s at A and B as

$$U_A = -\frac{GM_e m}{R_e},\ U_B = -\frac{GM_e m}{R_e + h}, \text{ respectively.}$$

So the P.E. of the object with respect to the surface of earth is

$$U_{BA} = U_B - U_A = -GM_e m\left(\frac{1}{R_e + h} - \frac{1}{R_e}\right) = \frac{GM_e mh}{(R_e + h)R_e}$$

But usually $h \ll R_e$ $\therefore (R_e + h)R_e \approx R_e^2$. Hence, $U_{BA} = \frac{GM_e}{R_e^2}mh$.

From Eq. 5.41, $U_{BA} = mg_0 h$

3. The gravitational P.E. of a mass m_1 in the field of m_2 when they are separated by a distance r is $\left(-\frac{Gm_1m_2}{r}\right)$. Refer to Fig. **5.26.** Let the masses kept at the corners of the equilateral triangle be m and its side a. So the overall gravitational P.E. of the system is given by

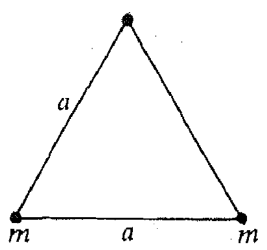


Fig.5.26

$$U = \left(-\frac{Gmm}{a}\right) + \left(-\frac{Gmm}{a}\right) + \left(-\frac{Gmm}{a}\right)$$

or $U = -\frac{3Gm^2}{a}$, where $m = 5 \times 10^6$ kg, $a = 2 \times 10^3$ m

$$U = -\frac{3 \times (6.673 \times 10^{-11}\text{Nm}^2\text{kg}^{-2}) \times (25 \times 10^{12}\text{kg}^2)}{2 \times 10^3\text{m}} = -2.5 \text{ J}$$

Since, the energy is negative the system is bound with an energy of 2.5J. So in order to take them infinite distance apart an external energy **2.5** J is required.

FURTHER READING

1. ***Mechanics, Berkeley Physics Course*** — Volume *I;* C. Kittel, **W. D.** Knight, M.A. Ruderman, A.C. Helmholz, B.J. Moyer; Asian Student Edition, McGraw-Hill International Book Company, 1981.
2. **An *Introduction to Mechanics;*** **D.** Kleppner, **R.J.** Kolenkow; International Student Edition, McGraw-Hill International Book Company, 1984.
3. *Introduction to Classical Mechanics*; ***A.P.*** French, **M. G.** Ebison; Van Nostrand Reinhold (UK) Co Ltd, 1986.

4. ***Physics Part I***; Robert Resnick and David Halliday; Wiley Eastern Ltd, 1988.

5. ***The Mechanical Universe,*** Mechanics and Heat, Advanced Edition; S.C. Frautschi, R.P. Olenick, T.M. Apostol, D.L.Goodstein; Cambridge University Press, 1986.

6. ***Physics Volume I; R.*** Wolfson, J.M. Pasachoff; Little, Brown and Company, 1987.

Acceleration		$m\ s^{-2}$	$[LT^{-2}]$
Angular displacement	radian	rad	
Angular velocity		$rad\ s^{-1}$	$[T^{-1}]$
Angular acceleration		$rad\ s^{-2}$	$[T^{-2}]$
Angular momentum		$kg\ m^2 s^{-1}$	$[ML^2T^{-1}]$
Force	newton	N	$[MLT^{-2}]$
Work, Energy	joule	J	$[ML^2T^{-2}]$
Power	watt	W	$[ML^2T^{-3}]$
Gravitational potential		$J\ kg^{-1}$	$[L^2T^{-2}]$
Gravitational Intensity		$N\ kg^{-1}$	$[LT^{-2}]$
Momentum, Impulse		$kg\ m\ s^{-1}$	$[MLT^{-1}]$
Period		s	$[T]$
Moment of inertia		$kg\ m^2$	$[ML^2]$
Area		m^2	$[L^2]$
Volume		m^3	$[L^3]$
Density		$kg\ m^{-3}$	$[ML^{-3}]$
Torque		N m	$[ML^2T^{-2}]$
Temperature	kelvin	K	$[K]$
Electric charge	coulomb	C	$[Q]$
Electric current	ampere	A	$[T^{-1}Q]$

Table of Constants

Physical Constants

Symbol	Quantity	Value
c	speed of light in vacuum	2.998×10^{8} m s^{-1}
μ_0	permeability of free space	1.257×10^{-6} N A^{-2}
ε_0	permittivity of free space	8.854×10^{-12} C^{2}N^{-1}m^{-2}
$1/4\pi\varepsilon_0$		X 988 x 10^{9} N m^{2}C^{-2}
e	charge of the proton	1.602×10^{-19}C
$-e$	charge of the electron	-1.602×10^{-19}C
h	Planck's constant	6.626×10^{-34}J s
$\hbar$	$h/2\pi$	1.055×10^{-34}J s
m_e	electron rest mass	9.109×10^{-31}kg
$-e/m_e$	electron charge to mass ratio	-1.759×10^{11}C kg^{-1}
	proton rest mass	1.673%10^{-27}kg
m_n	neutron rest mass	1.675×10^{-27}kg
R	Rydberg constant	1.097×10^{7}m^{-1}
	Bohr radius	5.292×10^{-11}m
N_A	Avogadro constant	6.022×10^{23}mol^{-1}
R	Universal gas constant	8.314 J K^{-1}mol^{-1}
k_B	Boltzmann constant	1.381×10^{-23}J K^{-1}
G	Universal gravitational constant	6.673×10^{-11}N m^{2}kg^{-2}

Astrophysical Data

Celestial Body	Mass (kg)	Mean Radius (m)	Mean distance from the centre of Earth (m)
Sun	1.99×10^{30}	6.96×10^{8}	1.50×10^{11}
Moon	7.35×10^{22}	1.74 x 106	3.85×10^{8}
Earth	5.97×10^{24}	6.37 x 106	0