$\therefore\ v = \sqrt{B^2 + 4A^2t^2B^4} = B\sqrt{1 + 4A^2t^2B^2}$

11. $\frac{v^2}{r} \not> a$ or $\frac{v^2}{a} \not> r$

or $r \not< \frac{v^2}{a}$, i.e. $r_{min} = \frac{v^2}{a}$

Since $v = 60$ km h^{-1}, $a = 1.5$ m s^{-2},

$\therefore\ r_{min} = 1.8 \times 10^2$ m

Terminal Questions

1. We have learnt from Sec. 1.5 that the terms 'rest' and 'motion' are relative. So, whenever I say that 'I am moving' or 'I am at rest', I am supposed to mention about the observer with respect to whom I am talking about my state. That is why the statement "I am moving" is meaningless.

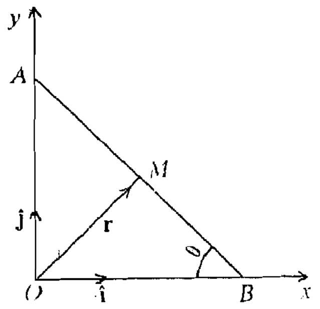


Fig. 1.29

2. Refer to Fig. 1.29. This is a modified form of Fig. 1.24 where the Cartesian x and y-axes are along OB and OA, respectively. (a) Let **r** be the position vector of the midpoint M of AB. Let the coordinates of B and A be $(x, 0)$ and $(0, y)$, respectively, at any time t.

$\mathbf{OB} = x\hat{\mathbf{i}}$, $\mathbf{OA} = y\hat{\mathbf{j}}$. Now, $\mathbf{OA} + \mathbf{AB} = \mathbf{OB}$, or $\mathbf{AB} = \mathbf{OB} - \mathbf{OA} = x\hat{\mathbf{i}} - y\hat{\mathbf{j}}$

and $\mathbf{r} = \mathbf{OM} = \mathbf{OA} + \mathbf{AM} = \mathbf{OA} + \frac{1}{2}\mathbf{AB} = y\hat{\mathbf{j}} + \frac{1}{2}(x\hat{\mathbf{i}} - y\hat{\mathbf{j}}) = \frac{1}{2}(x\hat{\mathbf{i}} + y\hat{\mathbf{j}})$.

The position vector of M is $\mathbf{r} = \frac{1}{2}(x\hat{\mathbf{i}} + y\hat{\mathbf{j}})$, where $x^2 + y^2 = L^2$ = a constant equal to the square of the length of the ladder.

Now, using Eq. 1.3b, we get $r = L/2$ ($\because x^2 + y^2 = L^2$). This means that the point M is always at a distance $L/2$ from O. In other words, it'describes a circle of radius $L/2$ with O as centre,

(b) The velocity of M $= \frac{d\mathbf{r}}{dt} = \frac{1}{2}\left(\frac{dx}{dt}\hat{\mathbf{i}} + \frac{dy}{dt}\hat{\mathbf{j}}\right)$.

Now $\frac{dx}{dt}$ = a constant $= v_0$ (given). Again as $x^2 + y^2 = L^2$, we have

$$x\frac{dx}{dt} + y\frac{dy}{dt} = 0 \text{ or } \frac{dy}{dt} = -\frac{x}{y}v_0 .$$

When B is at a distance b from O, we get $x = OB$ = b and $y = OA = \sqrt{L^2 - b^2}$.

Correspondingly, $\frac{dy}{dt} = -\frac{bv_0}{\sqrt{L^2-b^2}}$

So, the velocity of M at this instant is given by

$$\mathbf{v}_M = \frac{1}{2}v_0\hat{\mathbf{i}} - \frac{1}{2}\frac{bv_0}{\sqrt{L^2-b^2}})\hat{\mathbf{j}} = \frac{v_0}{2}\left[\hat{\mathbf{i}} - \frac{b}{\sqrt{L^2-b^2}}\hat{\mathbf{j}}\right]$$

And the speed $= v_M = \frac{v_0}{2}\left[1 + \frac{b^2}{L^2-b^2}\right]^{1/2} = \frac{Lv_0}{2\sqrt{L^2-b^2}}$

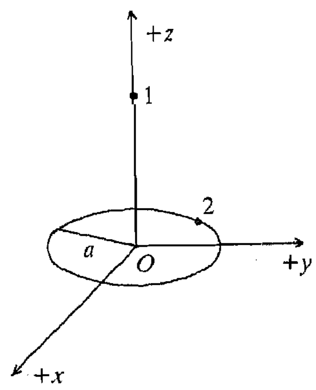


Fig. 1.30

3. Refer to Fig. 1.30. We first select a three-dimensional Cartesian coordinate system. Its origin is at the centre of the circle along which electron 2 executes uniform circular motion. Its z-axis is taken along the direction of the magnetic field, such that electron 1 moves along it. Let the position vectors of 1 and 2 at any time t be $\mathbf{r}_1$ and $\mathbf{r}_2$, respectively. Let the uniform angular speed of 2 be ω and the radius of the circle a. Then,

$\mathbf{r}_1 = v_1 t\hat{\mathbf{k}}$, $\mathbf{r}_2 = a(\cos\omega t\,\hat{\mathbf{i}} + \sin\omega t\,\hat{\mathbf{j}})$

The relative velocity of 2 with respect to 1 is given by

$\mathbf{v}_{12} = \frac{d}{dt}(\mathbf{r}_2 - \mathbf{r}_1) = \frac{d}{dt}\{a(\cos\omega t\,\hat{\mathbf{i}} + \sin\omega t\,\hat{\mathbf{j}}) - v_1 t\,\hat{\mathbf{k}})$

or $\mathbf{v}_{12} = -a\omega(\sin\omega t\,\hat{\mathbf{i}} - \cos\omega t\,\hat{\mathbf{j}}) - v_1\hat{\mathbf{k}}$, and the acceleration of 2 with respect to 1 is given by $\mathbf{a}_{12} = \frac{d\mathbf{v}_{12}}{dt} = -a\omega^2(\cos\omega t\,\hat{\mathbf{i}} + \sin\omega t\,\hat{\mathbf{j}})$

UNIT 2 FORCE AND MOMENTUM

Structure

2.1 INTRODUCTION

In Unit 1, we learnt how to describe the motion of a particle in terms of displacement, velocity and acceleration. We did not ask what caused the motion, In this unit we shall study the factors affecting motion. For this we shall recall Newton's laws of motion and apply them to a variety of situations. Using Newton's laws we shall also establish the condition for a particle's equilibrium, when it is acted on by several coplanar forces.

We will use the familiar concept of linear momentum to study the motion of systems having more than one particle. In this process we shall establish the principle of conservation of linear momentum and apply it to solve problems in which a knowledge of the forces acting on the system is not needed. Finally, we shall recall the concept of impulse and use it to study the motion of variable mass systems. Any change of motion of an object is accompanied by perfoimance of work and expenditure of energy. Therefore, in the next unit we shall study the concepts of work and energy.

Objectives

After studying this unit you should be able to:

- apply Newton's laws of motion
- solve problems using conditions for equilibrium of forces
- apply the law of conservation of linear momentum
- solve problems concerning impulse and variable mass systems.

2.2 CAUSES OF MOTION

What makes things move? An answer to this question was suggested by Aristotle, way back in the fourth century B.C. For nearly 2,000 years following the work of Aristotle, most people believed in his answer, that a force — a push or a pull — was needed to keep something moving. And the motion ceased when the force was removed. This idea made a lot of common sense. When an ox stopped pulling an ox-cart, the cart quickly came to a stop.

But these ideas were first critically examined by Galileo who carried out a series of experiments to show that no cause or force is needed to maintain the motion of an object. Study Fig. 2.1 carefully to understand this.

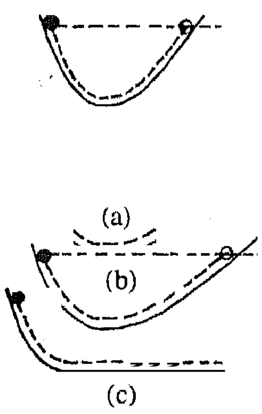


Fig. 2.1: (a) The ball rolling down a frictionless incline will rise approximately to its starting height on a second frictionless incline; (b) making the second incline more gradual would result in the ball's travelling further in the horizontal direction to attain the same height; (c) what happens in this case?

What do you think actually happens in the case of *(c)?*

The ball does stop on the flat surface after some time. But it is seen that the smoother the surface, the longer it takes for the ball to come to rest. Moreover, if the surface is reasonably smooth and flat, the ball moves more or less in a straight line. So if the element of friction can be completely removed, the ball would move indefinitely with a constant velocity as it would never be able to reach the starting height. Galileo concluded that ***any object in motion, if not obstructed will continue to move with a constant speed along a horizontal line.*** So, there would be no change in the motion of an object, unless an external agent acted on it to cause the change, That was Galileo's version of inertia. Inertia resists changes, not only from the state of rest, but also from motion with a constant speed along a straight line. So the interest shifted from the *causes of motion* to the *causes for changes in motion.*

Galilee's work set the stage for centuries of progress in mechanics, beginning with the achievements of Isaac Newton. Newton's laws of motion are the basis of mechanics. We will now briefly discuss these laws.

2.2.1 Newton's Laws of Motion

Galileo's version of inertia was formalised by Newton in a form that has come to be known as Newton's first law of motion.

Newton's first **law** of motion
Stated in Newton's words, the first law of motion is:

> *"Every body continues in its state of rest or of uniform motion in a straight line unless it is compelled to change that state by forces impressed upon it."*

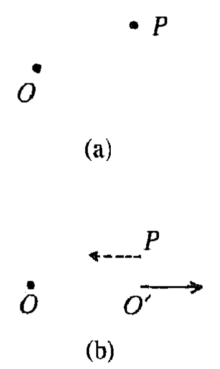


Fig. 2.2: (a) The observer O and the object P are at rest with respect to each other; (b) O' is accelerating with respect to O.

Newton's first law is also known as ***law of inertia*** and the motion of a body not subject to the action of other forces is said to be inertial motion. With the help of this law *we can dejine force as an external cause which changes or tends to change the state of rest or of uniform motion of a body.*

Have you noted that the first law does not tell you anything about the observer? But we know from Sec. 1.5 that the description of motion depends very much on the observer. So it would be worthwhile to know: For what kind of observer does Newton's first law of motion hold?

Suppose that an object P is at rest with respect to an observer O who is also at rest (Fig. 2.2a). Let another observer O' be accelerating with respect to O. P will appear to O' to be accelerating in a direction opposite to the acceleration of O' (Fig. 2.2b). According to Newton's first law the cause of the acceleration is some force. So O' will infer that P is being acted upon by a force. But O knows that no force is acting on P. It only appears to be accelerated to O'. Hence, the first law does not hold good for O'. It holds good for 0 .

An observer like O who is at rest or is moving with a constant velocity is called an inertial observer and the one like O', a non-inertial observer.

But hbw do we know whether an observer is inertial or not. For this, we need to measure the observer's velocity with respect to some standard. It is a common practice to consider the earth as a standard. Now the place where one is performing one's experiment has an acceleration (as discussed in Sec. 1.5) towards the polar axis due to the daily rotation of the earth. Again the centre of the earth has an acceleration towards the sun owing to its yearly motion around the sun. The sun also has an acceleration towards the centre of the Galaxy, and so on. Hence, the search for an absolute inertial frame is unending.

So we modify the definition of an inertial observer. We say *two observers are inertial with respect to one another when they are either at rest or in uniform motion with respect to one another. If an observer has an acceleration with respect to another then they are non-inertial with respect to one another.* Thus, a car moving with a constant velocity and a man standing on a road are inertial with respect to one another while a car in the process of gathering speed, and the man, are non-inertial with respect to each other.

The first law tells you how to detect the presence or absence of force on a body. In a sense, it tells you what a force does — it produces acceleration (either positive or negative) in a body. But the first law does not give a quantitative, measurable definition of force. This is what the second law does.

Newton's second **law of motion**
If you are struck by a very fast moving cricket ball you get injured but if you are hit by a flower moving with the same velocity as that of the ball you do not at all feel perturbed. However, if you are struck by a slower ball the injury is less serious. This indicates that any kind of impact made by an object depends on two things — its mass and velocity. Hence, Newton felt the necessity of defining the product of mass and velocity which later came to be known as linear momentum. Mathematically speaking, linear momentum

$$\mathbf{p} = m\mathbf{v}. \tag{2.1}$$

Thus, p is a vector quantity in the direction of velocity. The introduction of the above quantity paved the way for stating the second law, which in the words of Newton is as follows:

> ***'The change of motion of an object is proportional to the force impressed; and is made in the direction of the straight line in which the force is impressed."***

By "change of motion", Newton meant the rate of change of momentum with time. So mathematically we have

$$\mathbf{F} \propto \frac{d}{dt}(\mathbf{p}),$$

$$\text{or } \mathbf{F} = k\frac{d}{dt}(\mathbf{p}), \tag{2.2}$$

where F is the impressed force and k is a constant of proportionality. The differential operator $\frac{d}{dt}$ indicates the rate of change with time. Now, if the mass of the body remains constant (i.e. neither the body is gaining in mass like a conveyer belt nor is it disintegrating like a rocket), then

$$\frac{d\mathbf{p}}{dt} = \frac{d}{dt}(m\mathbf{v}) = m\,\frac{d\mathbf{v}}{dt} = m\mathbf{a},$$

where $\mathbf{a} = \frac{d\mathbf{v}}{dt}$ = the acceleration of the body. Thus, from Eq. 2.2, we get

$$\mathbf{F} = k\,m\,\mathbf{a}, \text{ and} \tag{2.3a}$$

$$F = kmn, \tag{2.3b}$$

We had seen earlier that the need for a second law was fell in order to provide a quantitative definition of force. So something must be done with the constant ***k***. We have realised that the task of a force F acting on a body of mass ***m*** is to produce in it an acceleration **a.** Hence, anything appearing in the expression for force other than m and **a** must be a pure number, i.e. k is a pure number. So we can afford to make a choice for its numerical value.

We define unit force as one which produces unit acceleration in its direction when it acts on a unit mass. So, we obtain from **Eq. 2.3b** that **1** = k. 1.1 or k = 1. Thus, **Eqs.** 2.2 and **2.3** take the form

$$F = \frac{d\mathbf{p}}{dt}, \text{ and} \tag{2.4a}$$

$$\mathbf{F} = m\mathbf{a}, \text{ for constant mass.} \tag{2.4b}$$

Now we know from Sec. 1.3 that if the position vector of a particle is **r** at a time ***t*** then its velocity v and acceleration a are given by **Eqs. 1.24a** and **1.26a.** Substituting for a and v in Eq. 2.4, we get

$$\mathbf{F} = m\frac{d\mathbf{v}}{dt} = m\,\frac{d}{dt}\left(\frac{d\mathbf{r}}{dt}\right), \text{ or } \mathbf{F} = m\,\frac{d^2\mathbf{r}}{dt^2}. \tag{2.5}$$

Eq. 2.5 is a second order differential equation in r. If we know the force F acting on a body of mass m, we can integrate **Eq.** 2.5 to determine **r** as a function of *t*. The function **r** (*t*) would give us the path of the particle. Since **Eq.** 2.5 is of second order we shall come across two constants of integration. So we require two initial conditions to work out a solution of this equation. Conversely, if we know the path or trajectory of an accelerating particle, we can use **Eq.** 2.5 to determine the force acting on the body. **Eq.** 2.5 also enables us to determine unknown masses from measured forces and accelerations.

So far, we have considered only one force acting on the body. But often several forces act on the same body. For example, the force of gravity, the force of air on the wings and body of the plane and the force associated with engine thrust act on a flying jet (Fig. 2.3).

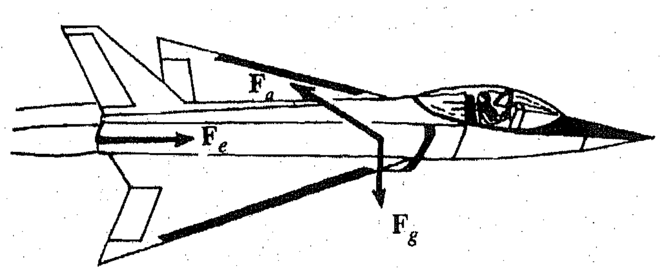


Fig. 2.3: Forces on a jet: F_e, the thrust of the engine, F_a, the force of the air provides both lift and drag, F_g the force of gravity.

In such cases, we add the individual forces vectorially, to find the *net force* acting on the object. The object's mass and acceleration are related to this net force by Newton's second law. You may now like to apply Newton's second law to a simple situation.

SAQ 1

Astronauts on the Skylab mission of the 1970s found their masses by using a chair on which a known force was exerted by a spring. With an astronaut strapped in the chair, the 15 kg chair underwent an acceleration of 2.04×10^{-2} m s^{-2} when the spring force was 2.07 N. What was the astronaut's mass?

Newton's third law **of motion**

So far we have been trying to understand how and why a single body moves. We have identified force as the cause of change in the motion of a body. But how does one exert a force on this body? Inevitably, there is an agent that makes this possible. Very often, your hands or feet are the agents. In football, your feet bring the ball into motion. Thus, forces arise from interactions between systems. This fact is made clear in Newton's third law of motion. To put it in his own words:

"To every uction there is an equal and opposite reaction."

Here the words 'action' and 'reaction' mean forces as defined by the first and second laws. If a body A exerts a force $\mathbf{F}_{AB}$ on **a** body B, then the body B in turn exerts a force $\mathbf{F}_{BA}$ on **A**, such that

$$\mathbf{F}_{AB} = -\mathbf{F}_{BA}.$$

So, we have $\mathbf{F}_{AB} + \mathbf{F}_{BA} = \mathbf{0}.$ (2.6)

Notice that Newton's third law deals with *two* forces, each acting on a different body. You may now like to work out an SAQ based on the third law.

SAQ 2

a) When a footballer kicks the ball, the ball and the man experience forces of the same magnitude but in opposite directions according to the third law. The ball moves but the. man does not move. Why'?

b) The earth attracts an apple with a force of magnitude F. What is the magnitude of the force with which the apple attracts the earth? The apple moves towards the earth. Why does not the reverse happen?

Newton's laws of motion give us the means to understand most aspects of motion. Let us now apply them to a variety of physical situations involving objects in motion.

2.2.2 Applications of Newton's Laws

To apply Newton's laws, we must identify the body whose motion interests us. Then we should identify all the forces acting *on* the body, draw them on a vector diagram and find the ***net*** force acting *on* the body. Newton's second law can then be used to determine the body's acceleration. We will now use this basic method to solve a few examples.

Example 1: Projectile **Motion**

The motion of a shot fired by a gun and that of a ball thrown by a fieldsman to another are all examples of projectiles. Let us consider such a projectile of mass m (Fig. 2.4).It is thrown from a point *O* with a velocity **v**, along OA making an angle θ with the horizontal. Let the particle be at a point P (OP = r) at time *t*. If we neglect air resistance, then the only force acting on the particle is a constant force, $\mathbf{F} = m\mathbf{g}$, due to gravity. Let us determine the particle's path. Eq. 2.5 gives

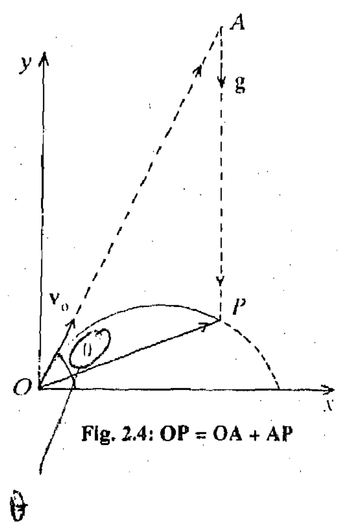


Fig. 2.4: OP = OA + AP

$$m\frac{d^2r}{dt^2} = m\mathbf{g}, \quad (2.7)$$

$$\text{or } \frac{d^2r}{dt^2} = \mathbf{g},$$

$$\text{or } \frac{d}{dt}\left(\frac{d\mathbf{r}}{dt}\right) = \mathbf{g}.$$

On integrating with respect to *t* we get

$$\frac{d\mathbf{r}}{dt} = \mathbf{g}t + \mathbf{A}, \quad (2.8a)$$

where **A** is a constant of integration. As the other two factors in Eq. **2.8a** are vectors **having** dimensions of velocity, A must also be a vector having the dimension of velocity. To determine A, we use the initial condition that velocity $= \frac{dr}{dt} = \mathbf{v}_0$ when $t = 0$.
So $\mathbf{A} = \mathbf{v}_0$. Hence,

$$\frac{d\mathbf{r}}{dt} = \mathbf{g}t + \mathbf{v}_0 . \qquad (2.8b)$$

On integrating with respect to t again, we get

$$\mathbf{r} = \mathbf{v}_0 t + \tfrac{1}{2}\mathbf{g}t^2 + \mathbf{B}, \qquad (2.9)$$

where **B**, like **A** in Eq. **2.8a**, is a constant vector of integration, but it has the dimension of length. To determine **B**, we need another initial condition. Letting $\mathbf{r} = \mathbf{0}$ at $t = 0$, we get B =0. Hence,

$$\mathbf{r} = \mathbf{v}_0 t + \tfrac{1}{2}\mathbf{g}t^2 \qquad \textbf{(2.10)}$$

We have essentially used two initial conditions: $\frac{d\mathbf{r}}{dt} = \mathbf{v}_0$ and $\mathbf{r} = \mathbf{0}$ at t = 0. Since $\mathbf{v}_0$ is along OA and t is scalar, we understand that $\mathbf{v}_0 t$ is along OA. Again $\mathbf{g}$ is directed vertically downwards and $\frac{1}{2}t^2$ is a scalar, so $\frac{1}{2}\mathbf{g}t^2$ is directed vertically downwards, **i.e.** along AP (Fig. 2.4).

We use the law of vector addition to get

$$\mathbf{OP} = \mathbf{OA} + \mathbf{AP}. \qquad (2.11)$$

Thus, we get the location of the particle. As time advances OA is lengthened and so is AP, and we get the location of the particle by adding **OA** and AP.

Example 2: **Friction**

A heavy-block is kept on a rough floor. You apply a force by pulling on a rope attached to it, but it still does not move. Is it a contradiction of Newton's laws? Discuss the motion of the block.

Refer to Fig. 2.5a. Let us first find out all the forces that act on the heavy block. There is a force of gravity $m\mathbf{g}$ acting downwards. The block exerts this force on the floor. Therefore, the floor exerts an equal and opposite normal force of reaction N on the block. N is normal to the surface of the floor. The third force results from your pull on the rope. Let $\mathbf{F}_1$ be the force that you exert on the rope. The rope exerts a force of reaction $\mathbf{F}_1'$ on you and a force of action, say $\mathbf{F}_2$ on the block. Let $\mathbf{F}_2'$ be the force that the block exerts on the rope. Then according to Newton's third law of motion

$$\mathbf{F}_1 = -\mathbf{F}_1'; \quad \mathbf{F}_2 = -\mathbf{F}_2'. \qquad (2.12)$$

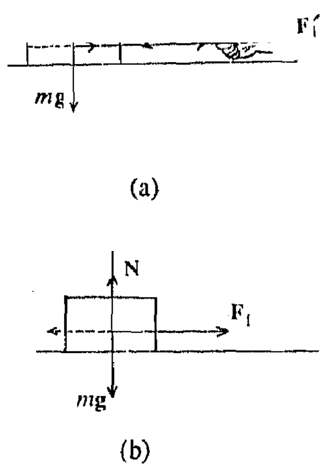


Fig. 2.5

Let us assume that the rope is massless. Then, from Newton's second law, the net force acting on the rope is zero and we have,

$$\mathbf{F}_1 + \mathbf{F}_2' = \mathbf{0},$$

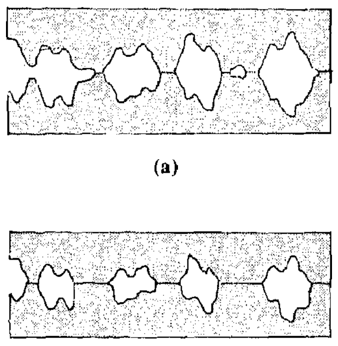
$$\text{or } \mathbf{F}_2' = -\mathbf{F}_1,$$

$$\text{or } \mathbf{F}_2 = \mathbf{F}_1, \text{ from Eq. 2.12.}$$

So a massless rope transmits the force you exert on it to the block without any change. The three forces $m\mathbf{g}$, N and $\mathbf{F}_1$ acting on the block do not add up to zero. N and $m\mathbf{g}$ cancel each other, leaving a net force $\mathbf{F}_1$ shown in Fig. **2.5b**. Since the block remains at rest, the net force acting on it must be zero according to the first law. There must, therefore, be another force which acts on the block. This force must also be horizontal, directed opposite to $\mathbf{F}_1$ and equal in magnitude. Actually, there is such a force which is the contact force between the floor and the block, known as the force of friction. It is shown in Fig. **2.5b** by a dotted line.

Friction is a force that acts between two surfaces to oppose their relative motion (see Fig. 2.6). The **force of** static **friction** $\mathbf{f}_s$ acts between surfaces at rest with respect to each other. The maximum force of static friction $\mathbf{f}_{sm}$ is the same as the smallest force necessary to start motion. Once motion has started, the force of friction usually decreases, so that a smaller force is required to maintain a uniform motion.

The force acting between surfaces in relative motion is called the **force** of kinetic **friction** $\mathbf{f}_k$. $\mathbf{f}_k$ is less than $\mathbf{f}_{sm}$. The ratio of the magnitude of maximum force of static friction f_{sm} to the



(a)

(b)

Fig. 2.6: Friction acts between two surfaces to oppose their relative motion. Even the smoothest surface is actually rough on a microscopic scale. (a) When two surfaces are in contact, their irregularities adhere because of electrical forces between the molecules. This gives rise to a force that opposes their relative motion; (b) when the normal force between the surfaces increases, the irregularities are crushed together and the contact area between the surfaces increases. This increases the force of friction.

magnitude of normal force of reaction N between the two surfaces is called the coefficient of static friction μ_s, i.e.

$$f_{sm} = \mu_s N.$$

Similarly,

$$f_k = \mu_k N$$

where μ_k is the coefficient of kinetic friction,

The discussion on friction brings us to an important class of problems in which an object undergoes motion against resistive forces. Another example of resistive force is air resistance to projectile motion. The motion of raindrops, or cars is also affected by air resistance. So let us discuss an example on motion where resistive forces are present.

Example 3: Motion against resistive forces

Suppose an object moves under the influence of a constant force $\mathbf{F}_0$, with a resistive force R opposing its motion. Let R always act in a direction opposite to the object's instantaneous velocity. In general the resistive force is a function of speed, so that Newton's second law becomes:

$$\mathbf{F}_0 - \mathbf{R}(v) = m\frac{d\mathbf{v}}{dt} \qquad (2.13)$$

The resistive force of dry friction (Fig. 2.7a) is almost independent of v, so that

$$R(v) = F_1 = \text{constant}.$$

In this case Eq. 2.13 reduces to the simple case of acceleration under a constant net force.

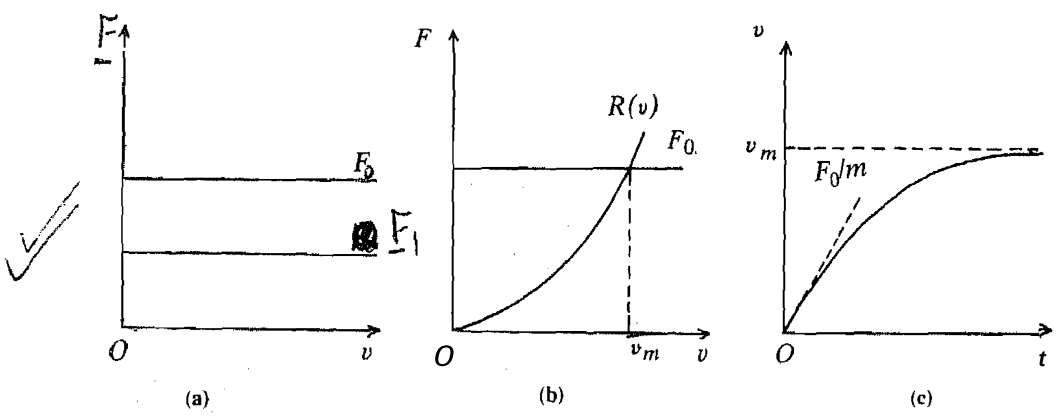


Fig. 2.7: (a) Resistive force for an object resisted by dry friction and (b) by fluid friction; (c) terminal speed v_m of an object in a fluid resistive medium. F_0/m is the slope of the curve at O.

In the case of air resistance or fluid resistance, $R(v)$ increases with v (Fig. 2.7b). It is usually described by the relation

$$R(v) = Av + Bv^2. \qquad (2.14)$$

For the sake of simplicity, let us consider only one-dimensional motion under the resistive force of Eq. 2.14. So we can use the scalar form of Eq. 2.13, which is

$$m\frac{dv}{dt} = F_0 - Av - Bv^2. \qquad (2.15)$$

Eq. 2.15 is not very easy to solve and we do not intend to go into its formal mathematical solution. Let us, however, consider some qualitative features of the possible solution.

Let the object start moving under a constant force F_0. Its initial acceleration will have almost a constant value $\frac{F_0}{m}$, since v is very small. Thus, v will be a linear function of t (Fig. 2.7 c). As v increases, R (v) will increase and the net driving force is reduced to a value below F_0, giving a steadily decreasing slope in the graph of $v(t)$. When $R(v)$ approaches F_0, the net force acting on the body tends to be zero. Then the object's velocity acquires a limiting constant magnitude v_m. The value of v_m is the positive solution of the quadratic equation

$$Bv^2 + Av - F_0 = 0.$$

In such a situation, the body moves with zero acceleration under zero net force. ***It is not the unaccelerated motion of objects moving under no force at all.*** So every time we see a car moving along a straight road at a steady speed, a jet plane flying through the air at a constant speed, or raindrops falling with a uniform terminal velocity, we see bodies moving under ***zero net force.*** Their motion at a constant speed does ***not*** mean that no force is acting on them. Now you may like to work out an SAQ based on this concept.

SAQ 3

A box of mass m is being pulled across a rough floor by means of a massless rope that makes an angle θ with the horizontal (Fig. 2.8). The coefficient of kinetic friction between the box and the floor is μ_k. What is the tension in the rope when the box moves at a constant velocity?

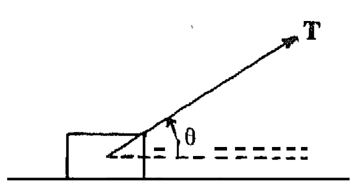


Fig. 2.8

One simple but important application of Newton's laws is the study of bodies in equilibrium. A large number of situations may be reduced to problems concerning the equilibrium of forces on a particle. For example, the construction of buildings and suspension bridges, design of aircrafts and ships, loading or unloading operations, involve forces in equilibrium. So let us now study equilibrium of forces acting on a particle.

2.2.3 Equilibrium of Forces

We say that ***a particle*** is ***in equilibrium, when the resultant of all the forces acting on it is zero.*** It then follows from Newton's first law of motion that a particle in equilibrium is either at rest or is moving in a straight line with constant speed. It is found that for a large number of problems, we have to deal with equilibrium of forces lying in a plane. Therefore, we shall restrict our discussion to the case when a particle is in equilibrium under the influence of a number of coplanar forces, $\mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_3, \ldots$. The required condition is given by

$$\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \ldots = \mathbf{0}. \qquad (2.16a)$$

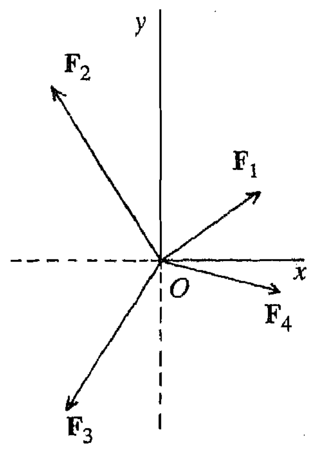


Fig. 2.9

Since the forces are coplanar, we can resolve them along two mutually perpendicular directions of x and y-axes (Fig. 2.9), O being the particle. So Eq. 2.16a can be rewritten as

$$(F_{1x}\hat{\mathbf{i}} + F_{1y}\hat{\mathbf{j}}) + (F_{2x}\hat{\mathbf{i}} + F_{2y}\hat{\mathbf{j}}) + \cdots = \mathbf{0},$$

$$\text{or } (F_{1x} + F_{2x} + \cdots)\hat{\mathbf{i}} + (F_{1y} + F_{2y} + \cdots)\hat{\mathbf{j}} = 0,$$

$$\text{or } \quad F_{1x} + F_{2x} + \ldots = 0,$$

$$\text{and } F_{1y} + F_{2y} + \ldots = 0. \qquad (2.16b)$$

Eqs. 2.16b can be expressed in a concise form as

$$\Sigma F_x = 0, \ \Sigma F_y = 0. \qquad (2.16c)$$

where Σ denotes summation of the x- or y-components of the forces. We shall now apply Eq. 2.16c to work out an example.

Example 4

A particle of mass m is hung by two light strings as shown in Fig. 2.10a. The ends A and B are held by hands. The strings OA and OB make angles θ with the vertical. Find the values of T and T' in terms of m and θ.

Through O, we consider two mutually perpendicular directions of x and y-axes, the latter being along the vertical.

From Eq. 2.16c, we have

$$-T\cos(90^\circ - \theta) + T'\cos(90^\circ - \theta) = 0, \qquad (2.17)$$

$$\text{and} \quad T\cos\theta + T'\cos\theta - mg = 0 \qquad (2.18)$$

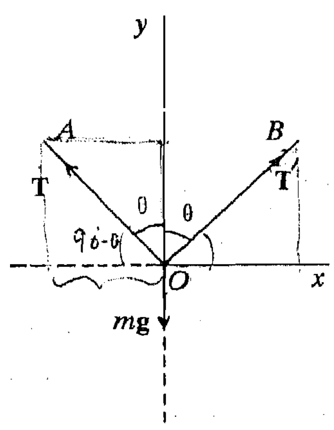
Hence, from Eq. 2.17, we get

$$T' = T, \ (\because \theta \neq 0^\circ).$$

Thus, from Eq. 2.18, we have

$$2T\cos\theta = mg,$$

$$\text{or } T = T' = \frac{mg}{2\cos\theta}. \qquad (2.19)$$



(a)

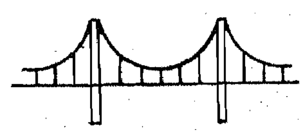

(b)

Fig. 2.10

If θ is increased, $\cos\theta$ will decrease, thereby increasing the tension. This may lead to the breaking of the string. It is for this reason that the main cable supporting the suspension of a bridge must be hung with a substantial curvature as shown in Fig. 2.10b. If the cable were stretched straight across, the tension would be so large that it may break.

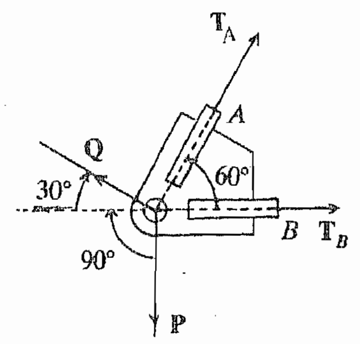


Fig. 2.11

Now that you have studied equilibrium of forces, you can work out the following SAQ.

SAQ 4

A connection used for joining different parts of a machine is maintained in equilibrium by applying two forces **P** and Q of magnitude $P = 3000$ N and $Q = 4000$ N as shown in Fig. 2.11. Determine the tension in rods *A* and *B*.

So far we have applied Newton's laws to a single particle or a single body which could be treated as a particle. We will now extend our study to the motion of a system of particles. One example of such a system is the sun and the planets. These bodies are so far apart compared to their diameters that together they can be treated as a system of particles. We find that linear momentum (recall Eq. 2.1) plays a vital role in describing the motion of such systems. It is also significant because of the principle of conservation of linear momentum. So let us now study linear momentum in some detail.

2.3 LINEAR MOMENTUM

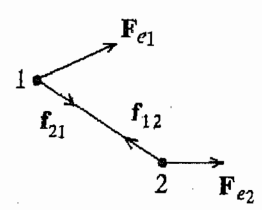


Fig. 2.12

Let us first study a system of two interacting particles '1' and '2' having masses m_1 and m_2 (Fig. 2.12). Let $\mathbf{p}_1$ and $\mathbf{p}_2$ be their linear momenta. The total linear momentum **p** of this system is simply the vector sum of the linear momenta of these two particles.

$$\mathbf{p} = \mathbf{p}_1 + \mathbf{p}_2. \tag{2.20}$$

From Newton's second law, the rate of change of $\mathbf{p}_1$ is the vector sum of all the forces acting on 1, i.e. the total external force $\mathbf{F}_{e1}$ on it and the internal force $\mathbf{f}_{21}$ due to 2:

$$\mathbf{F}_{e1} + \mathbf{f}_{21} = \frac{d\mathbf{p}_1}{dt}. \tag{2.21a}$$

Similarly, for particle 2:

$$\mathbf{F}_{e2} + \mathbf{f}_{12} = \frac{d\mathbf{p}_2}{dt}. \tag{2.21b}$$

From Newton's third law, we know that $\mathbf{f}_{12} = -\mathbf{f}_{21}$. Therefore, on adding Eqs. 2.21a and 2.21b, we get

$$\mathbf{F}_{e1} + \mathbf{F}_{e2} = \frac{d\mathbf{p}_1}{dt} + \frac{d\mathbf{p}_2}{dt},$$ which may be written as

$\mathbf{F}_e = \frac{d}{dt}(\mathbf{p}_1 + \mathbf{p}_2)$, where $\mathbf{F}_e$ is the net external force on the system. Therefore, from Eq. 2.20,

$$\mathbf{F}_e = \frac{d\mathbf{p}}{dt}. \tag{2.22}$$

Thus, in a system of interacting particles, it is the net external force which produces acceleration and not the internal forces. Now, we shall see how Eq. 2.22 leads to the principle of conservation of linear momentum.

2.3.1 Conservation of Linear Momentum

In the special case when the net external force $\mathbf{F}_e$ is zero, Eq. 2.22 gives

$$\frac{d\mathbf{p}}{dt} = \mathbf{0}, \tag{2.23}$$

so that $\mathbf{p} = \mathbf{p}_1 + \mathbf{p}_2$ = a constant vector.

This is the principle of conservation of linear momentum for a two-particle system. It is equally valid for a system of any number of particles. Its formal proof for a many-particle system will be given in Unit 7 of Block 2. It states that:

> *"If the net external force acting on a system is zero, then its total linear momentum is conserved."*

Let us now apply this principle.

Example 5

A vessel at rest explodes, breaking into three pieces. Two pieces having equal mass fly off

perpendicular to one another with the same speed of 30 ms^{-1}. Show that immediately after the explosion the third piece moves in the plane of the other two pieces. If the third piece has three times the mass of either of the other piece, what is the magnitude of its velocity immediately after the explosion?

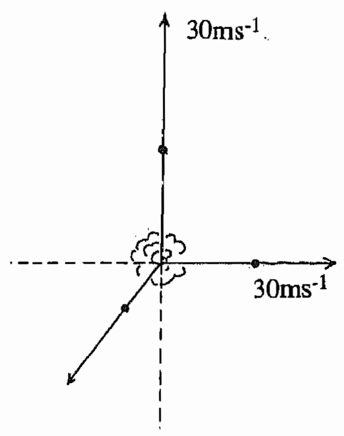


Fig. 2.13

The process is explained in the schematic diagram (Fig. 2.13). The vessel was at rest prior to the explosion. So its linear momentum was zero. Since no net external force acts on the system, its total linear momentum is conserved. Therefore, the final linear momentum is also zero, i.e.

$$\mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3 = \mathbf{0}, \qquad (2.24a)$$

$$\text{or } \mathbf{p}_1 + \mathbf{p}_2 = -\mathbf{p}_3. \qquad (2.24b)$$

$(\mathbf{p}_1 + \mathbf{p}_2)$ lies in the plane contained by $\mathbf{p}_1$ and $\mathbf{p}_2$. So in accordance with Eq. 2.24b, $-\mathbf{p}_3$ must also lie in that plane. Hence, $\mathbf{p}_3$ lies in the same plane as $\mathbf{p}_1$ and $\mathbf{p}_2$. Now, from Eq. 2.24b,

$$(\mathbf{p}_1 + \mathbf{p}_2) \cdot (\mathbf{p}_1 + \mathbf{p}_2) = (-\mathbf{p}_3) \cdot (-\mathbf{p}_3),$$

$$\text{or } p_1^2 + p_2^2 + 2\mathbf{p}_1 \cdot \mathbf{p}_2 = p_3^2$$

But $\mathbf{p}_1 \cdot \mathbf{p}_2 = 0$ ($\because$ $\mathbf{p}_1$ is perpendicular to $\mathbf{p}_2$).

$$\text{So, } p_3^2 = p_1^2 + p_2^2, \qquad (2.24c)$$

$$\text{or } (3m\,v)^2 = (mu)^2 + (mu)^2,$$

$$\text{or } 9m^2v^2 = 2m^2u^2, \text{ or } v = \frac{\sqrt{2}}{3}u.$$

According to the problem $u = 30$ ms^{-1}. $\therefore$ $v = 10\sqrt{2}$ ms^{-1}

There is another method of finding the magnitude of the velocity. We can express Eq. 2.24b in terms of the components of $\mathbf{p}_1$, $\mathbf{p}_2$ and $\mathbf{p}_3$ in two mutually perpendicular directions of x and y-axes. Let $\mathbf{p}_1$ be along x-axis, $\mathbf{p}_2$ along y-axis and let $\mathbf{p}_3$ make an angle θ with x-axis. Then Eq. 2.24b gives

$$p_1\hat{\mathbf{i}} + p_2\hat{\mathbf{j}} = -(p_3\cos\theta\hat{\mathbf{i}} + p_3\sin\theta\hat{\mathbf{j}}). \qquad (2.25a)$$

This equation is satisfied iff (see Eq. 1.6)

$$-p_3\cos\theta = p_1, \quad -p_3\sin\theta = p_2. \qquad (2.25b)$$

$$\text{or } p_3^2 = p_1^2 + p_2^2, \text{ which is the same as Eq. 2.24c.}$$

SAQ 5

Find the direction of $\mathbf{v}$ in Example 5.

From the above example and the way we obtained the principle of conservation of momentum, it may appear that the principle is limited in its application. This is because we have assumed that no net external force acts on the system of particles. However, the scope of the principle is much broader.

There are many cases in which an external force, such as gravity, is very weak compared to the internal forces. The explosion of a rocket in mid air is an example. Since the explosion lasts for a very brief time, the external force can be neglected in this case. In examples of this type, linear momentum is conserved to a very good approximation.

Again, if a force is applied to a system by an external agent, then the system exerts an equal and opposite force on the agent. Now if we consider the agent and the system to be a part of a new, larger system, then the momentum of this new system is conserved. Since there is no larger system containing the universe, its total linear momentum is conserved.

We have seen that whenever we have a system of particles on which no net external force acts, we can apply the law of conservation of linear momentum to analyse their motion. In fact, the advantage is that this law enables us to describe their motion without knowing the details of the forces involved. Let us now see what happens to each individual particle in the system. In such a case, the particle experiences a net force, and its linear momentum changes. This change depends on the magnitude of the force and also the time for which it acts. Further, the force itself may vary with time. Let us now study the relationship of force, its duration and the resulting change in momentum.

2.3.2 Impulse

In cricket, when a batsman strikes a ball with his bat, the bat is in contact with the ball for a very short but finite time. At the two instants, when the ball is just about to make contact with the bat and when it just leaves the bat, it experiences no force. In between, it experiences a large varying force. The variation of the magnitude of such a force $\mathbf{F}(t)$ can be as shown in Fig.2.14. We generally assume that the force has a constant direction. We can find the change in the linear momentum of an object on which such a force acts by integrating Eq. 2.4a over the time interval from t_1 to t_2:

$$\int_{t_1}^{t_2} \mathbf{F}(t)dt = \int_{t_1}^{t_2} \frac{d\mathbf{p}}{dt} dt = \mathbf{p}(t_2) - \mathbf{p}(t_1) \equiv \Delta\mathbf{p}. \quad (2.26)$$

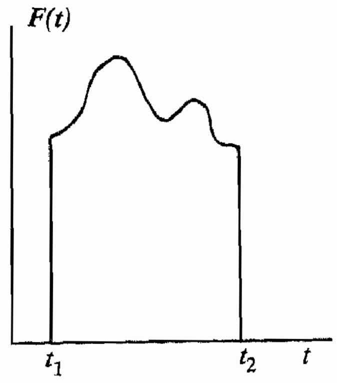


Fig. 2.14

The integral of force over time is called the **impulse** of the force and is given by

$$\mathbf{J} = \int_{t_1}^{t_2} \mathbf{F}(t)dt. \quad (2.27)$$

Thus, according to Eq.2.26, impulse of a force is equal to the change in linear momentum. If $\mathbf{F}$ acts during a time interval Δt but is variable, then to calculate impulse we would need to know the function $\mathbf{F}(t)$ explicitly. However, this is usually not known. A way out is to define the **average force** $\overline{\mathbf{F}}$ by the equation

$$\overline{\mathbf{F}} = \frac{1}{\Delta t}\int_{t_1}^{t_2} \mathbf{F}(t)dt, \text{ where } \Delta t = t_2 - t_1. \quad (2.28)$$

From Eqs. 2.27 and 2.28, we get

$$\mathbf{J} = \overline{\mathbf{F}}\Delta t = \Delta\mathbf{p}. \quad (2.29)$$

There are many examples which illustrate the relationship between the average force, its duration and change of linear momentum. A tennis player hits the ball while serving with a great force to impart linear momentum to the ball. To impart maximum possible momentum, the player 'follows through' with the serve. This action prolongs the time of contact between the ball and the racquet. Therefore, to bring about the maximum possible change in the linear momentum, we should apply as large a force as possible over as long a time interval as possible. You may now like to apply these ideas to solve a problem.

SAQ 6

a) A ball of mass 0.25 kg moving horizontally with a velocity 20 m s^{-1} is struck by a bat.The duration of contact is 10^{-2} s. After leaving the bat, the speed of the ball is 40 m s^{-1} in a direction opposite to its original direction of motion. Calculate the average force exerted by the bat.

b) Give an example in which a weak force acts for a long time to generate a substantial impulse.

So far we have dealt with examples which do not involve variation of mass of objects in motion. We shall now take up such cases and apply the concepts of impulse and momentum.

2.3.3 Motion with Variable Mass

If the mass of a system varies with time, we can express Newton's second law of motion as

$$\mathbf{F} = \frac{d\mathbf{p}}{dt} = \frac{d(m\mathbf{v})}{dt} = m\frac{d\mathbf{v}}{dt} + \mathbf{v}\frac{dm}{dt}. \quad (2.30)$$

Under the special case when $\mathbf{v}$ is constant, Eq. 2.30 becomes

$$\mathbf{F} = \mathbf{v}\frac{dm}{dt}. \quad (2.31)$$

Let us study an example of this special type.

Example 6

Sand falls on to a conveyer belt B (Fig.2.15) at the constant rate of 0.2 kg s^{-1}. Find the force required to maintain a constant velocity of 10 m s^{-1} of the belt.

Here, we shall apply Eq. 2.31, as velocity remains constant. Since the mass is increasing, $\frac{dm}{dt}$ is positive. The direction of $\mathbf{F}$, therefore, is same as that of $\mathbf{v}$, i.e. the direction of motion of the conveyer belt.

Thus, using Eq. 2.31, we get

$$F = (10 \text{ m s}^{-1}) \times (0.2 \text{ kg s}^{-1}) = 2 \text{ kg m s}^{-2} = 2\text{N}.$$

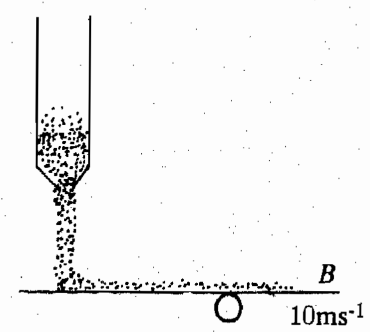


Fig. 2.15

Another example of a varying mass system is the rocket. In a rocket (Fig. 2.16) a stream of gas produced at a very high temperature and pressure escapes at a very high velocity through an exhaust nozzle. Thus, the rocket loses mass and $\frac{dm}{dt}$ is negative. So the main body of the rocket experiences a huge force in a direction opposite to that of the exhaust causing it to move. This is a very simplified way of dealing with the motion of a rocket. We shall next analyse the motion of a rocket with a little more rigour using the idea of impulse.

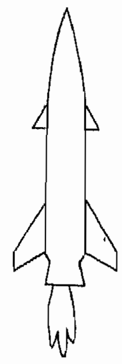


Fig. 2.16

Motion of a rocket

Let us assume that the rocket has a total mass M at a time t. It moves with a velocity $\mathbf{v}$ and ejects a mass ΔM during a time interval Δt. The situation is explained schematically in Figs. 2.17a and 2.17b.

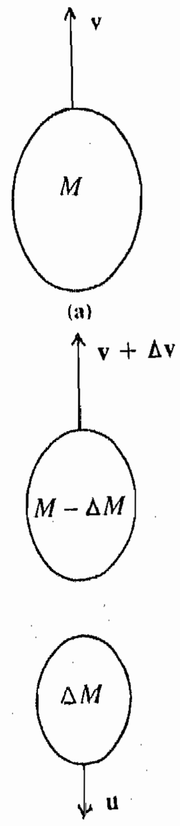


Fig 2.17

At time t the total initial momentum of the system = $M\mathbf{v}$ (Fig. 2.17a).
At time $t + \Delta t$ the total final momentum of the system = $(M - \Delta M)(\mathbf{v} + \Delta\mathbf{v}) + (\Delta M)\,\mathbf{u}$ (Fig. 2.17b).

Notice that we have used the positive sign for $\mathbf{u}$ because the total final momentum of the system in Fig. 2.17b is a vector sum and not the difference of the momenta of M and $(M - \Delta M)$. Let us now apply Eq. 2.29. If we take the vertically upward direction as positive, the impulse is $-M\mathbf{g}\,\Delta t$ and is equal to the change in linear momentum.

So, $\quad -M\mathbf{g}\,\Delta t = (M - \Delta M)(\mathbf{v} + \Delta\mathbf{v}) + (\Delta M)\mathbf{u} - M\mathbf{v}$

$$= M(\Delta\mathbf{v}) + \Delta M(\mathbf{u} - \mathbf{v} - \Delta\mathbf{v})$$

We may use Eq. 1.35 to simplify the above relation:

$$-\mathbf{g} = \frac{\Delta\mathbf{v}}{\Delta t} + \frac{1}{M}\frac{\Delta M}{\Delta t}\mathbf{u}_{rel},$$ where $\mathbf{u}_{rel} = \mathbf{u} - (\mathbf{v} + \Delta\mathbf{v})$ is the relative velocity of the exhaust with respect to the rocket.

Now, in the limit $\Delta t \rightarrow 0$, we have

$$-\mathbf{g} = \frac{d\mathbf{v}}{dt} - \frac{1}{M}\frac{dM}{dt}\mathbf{u}_{rel} \qquad (2.32)$$

The negative sign on the right-hand side of Eq. 2.32 appears as

$$\lim_{\Delta t \to 0}\frac{\Delta M}{\Delta t} = -\frac{dM}{dt},$$ because M decreases with t.

So, when we apply Eq. 2.32 in numerical problems we just replace $\frac{dM}{dt}$ *by its magnitude.*

On integrating Eq. 2.32 with respect to t, we get

$$\int_0^t \frac{d\mathbf{v}}{dt}\,dt = -\mathbf{g}t + \mathbf{u}_{rel}\int_{M_0}^{M}\frac{dM}{M},$$

where M_0 is the initial mass of the rocket and M is its mass at time t. Now, if $\mathbf{v}_0$ is the initial velocity, then we get

$$\mathbf{v} - \mathbf{v}_0 = \mathbf{u}_{rel}\ln\frac{M}{M_0} - \mathbf{g}t. \qquad (2.33)$$

We shall illustrate Eq. 2.33 with the help of an example.

Example 7

The stages of a two-stage rocket separately have masses 100 kg and 10 kg and contain 800 kg and 90 kg of fuel, respectively. What is the final velocity that can be achieved with an exhaust velocity of 1.5 km s^{-1} relative to the rocket? (Neglect any effect of gravity).

Since we are neglecting gravity Eq. 2.33 reduces to

$$\mathbf{v} - \mathbf{v}_0 = \mathbf{u}_{rel}\ln\frac{M}{M_0} \qquad (2.34)$$

Now, let the unit vector along the vertically upward direction be $\hat{\mathbf{n}}$. So, Eq. 2.34 can be written as

$$v\hat{\mathbf{n}} - v_0\hat{\mathbf{n}} = -(u_{rel}\,\hat{\mathbf{n}})\ln\frac{M}{M_0},$$ where $\mathbf{u}_{rel} = -u_{rel}\,\hat{\mathbf{n}}$, as the relative velocity of the exhaust points vertically downward.

$$\text{or}\quad v - v_0 = -u_{rel}\ln\frac{M}{M_0} \qquad (2.34a)$$

For our problem,

$u_{rel} = 1.5$ km s^{-1}.

For the first stage $v_0 = 0$

$$M_0 = (800 + 90 + 100 + 10)\ \text{kg} = 1000\ \text{kg}$$

$M = (90 + 10 + 100)$ kg = 200 kg, as the 800 kg fuel gets burnt in the first stage.

Hence, from Eq. 2.34a, we get

$$v = -(1.5\,\text{km s}^{-1})\left(\ln\frac{200}{1000}\right)$$

$$= (-1.5\ \text{km s}^{-1})\ (\ln 2 - \ln 10)$$

$$= 1.5 \times 1.6\ \text{km s}^{-1}$$

$$= 2.4\ \text{km s}^{-1}.$$

Note that the above will be the initial velocity for the second stage. Also note that at the beginning of the second stage there occurs another drop in mass to the extent of the mass of the first stage (i.e. 100 kg).

For the second stage

$$v_0 = 2.4\ \text{km s}^{-1}$$

$$M_0 = (90 + 10)\ \text{kg} = 100\ \text{kg},\ M = 10\ \text{kg}.$$

$$v = \left(2.4 - 1.5\ \ln\frac{10}{100}\right)\text{km s}^{-1}$$

$$= (2.4 + 1.5 \times 2.3)\ \text{km s}^{-1} = 5.85\ \text{km s}^{-1} = 5.8\ \text{km s}^{-1}.$$

The final result of Example 7 has to be rounded off to two significant digits. Here we have a special case as the digit to be discarded is 5. By convention, we have rounded off to the nearest even number.

Let us now follow up this example with an SAQ.

SAQ 7

Find the final velocity of the rocket in Example 7 taking it to be single-stage, i.e. its mass is 100kg and it carries 890 kg of fuel. Hence comment whether the two-stage rocket has an advantage over single stage or not.

Let us now sum up what we have learnt in this unit.

2.4 SUMMARY

- Newton's first law states that "*Every body continues in its state of rest or of uniform motion in a straight line unless it is compelled to change that state by forces impressed upon it.*"
- Newton's second law gives a relationship between force and linear momentum and can be expressed as

  $$\mathbf{F} = \frac{d\mathbf{p}}{dt}.$$

  For a system of constant mass it becomes

  $$\mathbf{F} = m\mathbf{a}.$$

- Newton's third law states that "*To every action there is an equal and opposite reaction.*" Forces of action and reaction act on different bodies.
- A particle is said to be in equilibrium if the net force acting on it is zero. For coplanar forces, this implies that

  $$\Sigma F_x = 0,\ \Sigma F_y = 0.$$

- The total linear momentum of a system is conserved if no net external force acts on it.
- The impulse of a force on an object equals the change in its linear momentum and is given by

  $$\mathbf{J} = \int_{t_1}^{t_2} \mathbf{F}(t)dt,$$ where the force acts during the time interval $\Delta t = (t_2 - t_1)$.

- For a variable mass system, Newton's second law of motion is expressed as

  $$\mathbf{F} = m\frac{d\mathbf{v}}{dt} + \mathbf{v}\frac{dm}{dt}.$$

- The increase in the velocity of a rocket within a time t of its take off is given by

  $$\mathbf{v} - \mathbf{v}_0 = \mathbf{u}_{rel}\ \ln\frac{M}{M_0} - \mathbf{g}t.$$

2.5 TERMINAL QUESTIONS

1. A block of mass 100 kg is placed on an inclined plane of height 6 m and base 8 m (Fig.2.18). The coefficients of static and kinetic frictions are 0.3 and 0.25, respectively. (a) Would the block slide down the plane? (b) What force parallel to the plane must be applied to just support the block on the plane? (c) What force parallel to the inclined plane is required to keep the block moving up the plane at constant speed? (d) If an upward force of 882 N parallel to the plane is applied to the block what will be its acceleration? (e) What will happen if an upward force of 490 N parallel to the plane is applied? (f) What will happen if an upward force of 254.8 N parallel to the plane is applied?

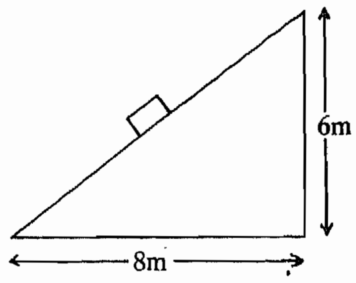


Fig. 2.18

2. A boy of mass 20 kg is standing on a flat boat of mass 30 kg so that he is 3 m from the shore (Fig. 2.19). He walks 1 m on the boat toward the shore and then halts. How far is he from the shore at the end of this time?
3. Explain why it is less dangerous to fall on a mattress than on a hard floor.
4. A fire-fighter directs a stream of water against the door of a building in flames. The water is delivered by the hose at the rate of 45 kg s^{-1}. Water moving horizontally at 32 ms^{-1} hits the door. After hitting the door, the water drops vertically downward. What is the horizontal force exerted on the door?

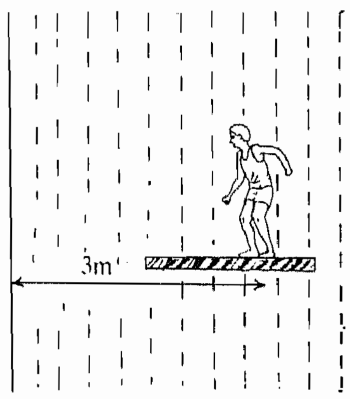


Fig. 2.19

2.6 ANSWERS

SAQs

1. Let the mass of the astronaut in kg be m. Then from the given conditions and Eq. 2.4 b, we get

$$\{(15 + m)\text{ kg}\}\ (2.04 \times 10^{-2}\text{ms}^{-2}) = 2.07\text{ N}$$

$$\text{or } m = \left[\frac{2.07\text{N}}{(2.04 \times 10^{-2})\text{ms}^{-2}} - 15\right]\text{kg} = 86.5\text{ kg}$$

2. a) The reaction force acts on the man. Due to the large mass (inertia) of the man the force is not able to make him move.

   b) Apple also attracts the earth with a force of magnitude F. The acceleration of the apple and the earth are, respectively, F/m_a and F/m_E, where m_a and m_E are the masses of the apple and the earth, respectively. Since $m_E >> m_a$, $F/m_E << F/m_a$. Hence the earth does not move appreciably.

3. Refer to Fig. 2.20. Let the tension be **T**. The forces are resolved along the directions of x and y-axes. The former is along the floor and the latter is perpendicular to it. **N** is the normal reaction and correspondingly the magnitude of the force of kinetic friction F_k is equal to $\mu_k N$. It is in a direction opposite to the tendency of motion. Since there is no motion in the vertical direction, the resultant of the forces along the y-axis must be zero. Moreover, as the body moves with a uniform velocity, the resultant force along the x-axis is also zero. So we have

$$T \sin\theta + N - mg = 0 \quad (2.35)$$

$$\mu_k N - T\cos\theta = 0. \quad (2.36)$$

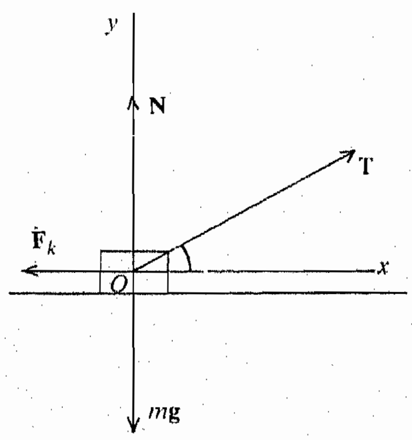


Fig 2.20

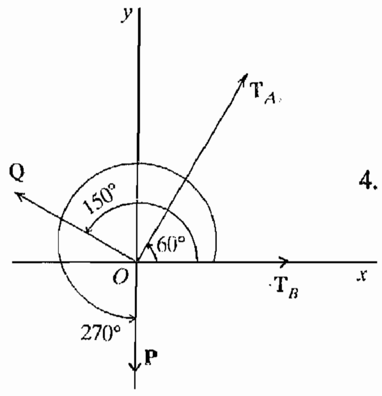


Fig 2.21

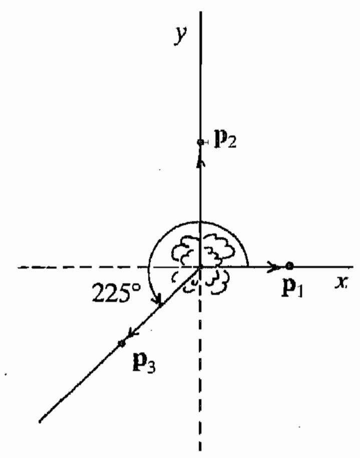


Fig. 2.22

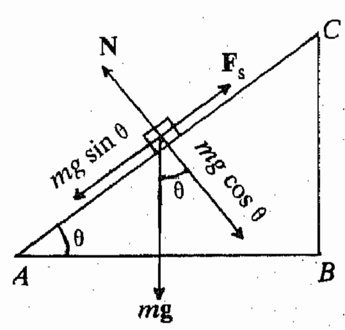


Fig. 2.23

To find T we have to eliminate N between Eqs. 2.35 and 2.36. So, we have

$$T\left(\sin\theta + \frac{\cos\theta}{\mu_k}\right) = mg,$$

$$\text{or} \quad T = \frac{\mu_k mg}{\cos\theta + \mu_k \sin\theta}$$

4. Refer to Fig. 2.21. We define a two-dimensional rectangular coordinate system with the common point of action of **P**, **Q**, $\mathbf{T}_A$, $\mathbf{T}_B$ as origin. We shall now apply Eq. 2.16c to obtain the condition of equilibrium.

Now, $\Sigma F_x = T_B \cos 0° + T_A \cos 60° + Q \cos 150° + P \cos 270°$

$$= T_B + \frac{T_A}{2} - \frac{\sqrt{3}Q}{2} \text{ and}$$

$\Sigma F_y = T_B \sin 0° + T_A \sin 60° + Q \sin 150° + P \sin 270°$

$$= \frac{\sqrt{3}}{2}T_A + \frac{Q}{2} - P.$$

Hence, from Eq. 2.16c, we get

$$T_B + \frac{T_A}{2} - \frac{\sqrt{3}}{2}Q = 0, \qquad (2.37a)$$

$$\frac{\sqrt{3}}{2}T_A + \frac{Q}{2} - P = 0. \qquad (2.37b)$$

From Eq. 2.37b, $T_A = \frac{2}{\sqrt{3}}\left(-\frac{Q}{2} + P\right) = \frac{2,000}{\sqrt{3}}\text{N} = 1155\text{N}$

And from Eq. 2.37a, $T_B = \frac{\sqrt{3}}{2}Q - \frac{T_A}{2} = \frac{1}{2}(6928 - 1155)\text{N} = 2886\text{N}$

5. As $p_1 = p_2 = mu$ and $p_3 = 3mv$, we get from Eq. 2.25b

$$\cos\theta = \sin\theta = -\frac{u}{3v}. \qquad (2.38)$$

∴ $\tan\theta = 1$. Again from Eq. 2.38 we understand that $\cos\theta$ and $\sin\theta$ are both negative as $\frac{u}{v}$ is positive. So θ must lie in the third quadrant. Hence, θ = 225°. (see Fig.2.22).

6. a) $J = \Delta p = (0.25\text{ kg}) \times \{40 - (-20)\}\text{ m s}^{-1} = 15\text{ kg m s}^{-1}$

$$\Delta t = 10^{-2}\text{ s}, \bar{F} = \frac{J}{\Delta t} = 1500\text{N}$$

b) The gravitational force of attraction between sun and earth is very weak but it has been acting since their formation and so it can generate a substantial impulse.

7. Had it been a single stage rocket, then

$v_0 = 0$

$M_0 = (890 + 100)\text{ kg} = 990\text{ kg}, M = 100\text{kg}.$

$$v = (-1.5\text{ km s}^{-1})\left[\ln\frac{100}{990}\right]$$

$= (-1.5\text{ km s}^{-1})(\ln 10 - \ln 99)$

$= 3.4\text{ km s}^{-1}$ which is 41% less than the value of velocity (5.8 km s^{-1}) attained in a double-stage rocket. Hence double-stage has an advantage over the single-stage.

Terminal Questions

1. Refer to Fig. 2.23. $BC = 6\text{m}$, $AB = 8\text{m}$. $AC = \sqrt{6^2 + 8^2}\text{ m} = 10\text{m}$, $\sin\theta = 0.6$, $\cos\theta = 0.8$.

a) We have resolved the forces along and perpendicular to the plane. Since there cannot be any motion perpendicular to the plane, we have $N = mg\cos\theta$, where m = mass of the block. The magnitude of the force of static friction

$F_s = \mu_s N = \mu_s mg\cos\theta.$

$F_s = (0.3)(100\text{ kg})(9.8\text{ m s}^{-2})(0.8) = 235.2\text{ N}$, and

$mg\sin\theta = (100\text{ kg})(9.8\text{ m s}^{-2})(0.6) = 588\text{ N}$. So $mg\sin\theta > F_s$ and hence the block will slide down the plane.

b) The required magnitude of force parallel to the plane = $mg \sin\theta - F_s = 352.8$ N.

c) When the block is urged to move up the plane, the force of kinetic friction $\mathbf{F}_k$ of magnitude $\mu_k N$ comes into play and it acts down the plane. So the magnitude of total force down the plane will be ($\mu_k N + mg \sin\theta$). Now, in order that the block moves up at a constant speed, the magnitude of the applied force parallel to the plane must be equal to ($\mu_k N + mg \sin\theta$) = mg ($\mu_k \cos\theta + \sin\theta$)

$= (100 \text{ kg}) (9.8 \text{ m s}^{-2}) (0.25 \times 0.8 + 0.6) = 784$ N.

d) The acceleration $= \dfrac{(882 - 784)\text{N}}{100 \text{ kg}} = 0.98 \text{ m s}^{-2}$

e) Since 490 < 784, we understand from (c) that the block will not move up. In this case the magnitude of the resultant force down the plane = (588 – 490)N = 98 N, which is less than the magnitude of the maximum force of static friction F_{sm} (=235.2 N). So the force of static friction will adjust itself equal to 98 N and the block remains at rest.

f) Now, the resultant force down the plane has a magnitude (588 – 254.8)N = 333.2 N. This exceeds F_{sm}. So the block will move down the plane. Its motion will be opposed by the force of kinetic friction $\mathbf{F}_k$. Now $F_k = \mu_k mg \cos\theta$

$= (0.25) \times (100 \text{ kg}) \times (9.8 \text{ m s}^{-2}) \times (0.8) = 196$ N. So its acceleration down the plane

$= \dfrac{(333.2 - 196)\text{N}}{100 \text{ kg}} = 1.4 \text{ m s}^{-2}$

2. Let the masses of the boy and boat be m and M, respectively (Fig.2.24). Let the velocity of the boy relative to the boat be $\mathbf{v}$ and that of the boat with respect to the shore S be $\mathbf{u}$. So the velocity of the boy with respect to the shore is $\mathbf{v} + \mathbf{u} = \mathbf{v}_1$, say. Now, before the boy started walking on the boat, the total linear momentum of the system (boy and boat) with respect to the shore was zero. The motion of the boat due to his walking arises out of the mutual forces of action and reaction. We shall neglect the forces of friction between the boy and boat as well as between the boat and the surface of water. So, no external force acts on the system. Hence, from the principle of conservation of linear momentum, we have

$$m(\mathbf{v} + \mathbf{u}) + M\mathbf{u} = \mathbf{0} \qquad (2.39)$$

It is evident that as the boy walks on the boat towards the shore the boat moves in the opposite direction. Let the unit vector along the direction of motion of the boy be $\hat{\mathbf{n}}$.

So $\mathbf{v} = v\hat{\mathbf{n}}$, $\mathbf{u} = -u\hat{\mathbf{n}}$. Hence, from Eq. 2.39, we get

$$m(v - u)\hat{\mathbf{n}} - Mu\hat{\mathbf{n}} = \mathbf{0}.$$

$$\therefore m(v - u) - Mu = 0, \text{ or } u = \frac{mv}{m + M}$$

or $\dfrac{m}{m+M} = \dfrac{u}{v} = \dfrac{s}{L}$, where s is the distance travelled by the boat and L is the length covered by the boy on the boat. In our problem, $m = 20$ kg, $M = 30$ kg, $L = 1$ m.

$\therefore$ $s = 0.4$ m. So after he halts, the boy is at a distance (3 – 1 + 0.4) m, i.e. 2.4 m from the shore.

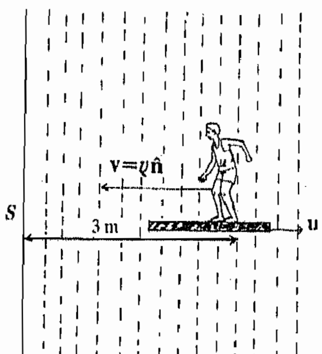


Fig. 2.24

3. In either case the person comes to rest finally and has the same velocity at the point of hitting the mattress or the floor. So the impulse, i.e. change of linear momentum is same.But the mattress being soft, the duration of impact is greater than that in the case of the hard floor. So, from Eq. 2.29 the average force is smaller in the former case. Hence, it is less dangerous to fall on a mattress than on a hard floor.

4. Water drops vertically downward after hitting the door. So the horizontal motion of water stops abruptly at the door. Since water is moving at 32 m s^{-1}, each kg of water loses 32 kg m s^{-1} of linear momentum. But water strikes the door at the rate of 45 kg s^{-1}. So the rate of loss of linear momentum is equal to (45 kg s^{-1}) × (32 kg m s^{-1} kg^{-1}) = 1440 kg m s^{-2} = 1440 N, which is the horizontal force on the door.