EXPERIMENT

8

FREQUENCY RESPONSE OF AC CIRCUITS

Structure

8.1 Introduction
  Objectives

8.2 Characteristics of a Series-*LCR* Circuit

8.3 Frequency Responses of a Resistor, an Inductor and a Capacitor

8.4 Frequency Response of a Series-*LCR* Circuit

8.5 Dependence of Quality Factor on Resistance

8.1 INTRODUCTION

In your 10+2 physics curriculum, you have learnt about a resistor, an inductor and a capacitor. You now know how these are used as passive elements in an electrical circuit. For instance, a resistor is used to control current in an electric iron; a capacitor filters ac component and an inductor and a capacitor are combined to tune to a particular frequency in a radio circuit. These elements are said to be *passive* since they cannot provide any power amplification to a signal. Basically all these components offer opposition to flow of current through them.

Component	Impedance	Reactance
Resistor	$Z_R = R$	$X_R = R$
Capacitor	$Z_C = -\frac{j}{\omega C}$	$X_C = \frac{1}{\omega C}$
Inductor	$Z_L = j\omega L$	$X_L = \omega L$

The measure of opposition to current in a dc circuit is specified in terms of **resistance** and for an ac circuit, we use the term **impedance**. The impedance of a resistor is independent of frequency. The impedances offered by a capacitor and an inductor are frequency dependent and are respectively expressed in terms of their **reactances** (capacitive and inductive). Due to the frequency dependence reactances, L and C play an important role in ac circuits when placed individually, together or in combination with R. In this experiment, you will get an opportunity to study the behaviour of these components with variable frequency signals. You will also study the frequency responses of these components individually as well as when all of them are connected in series.

You may recall that in case of R, the current and voltage are in phase with each other. But while in case of L, the voltage leads the current in phase by $\pi/2$, in case of C, it lags by $\pi/2$, as shown in Fig. 8.1.

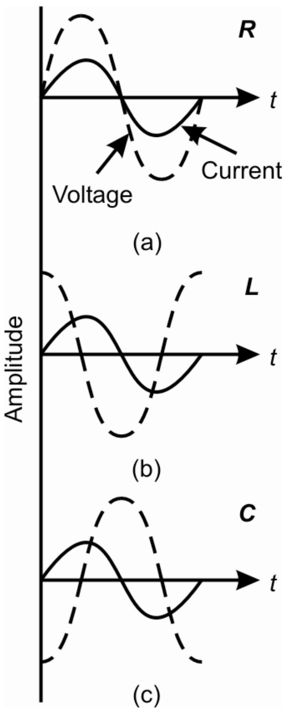

Fig. 8.1: Phase responses of a) *R*; b) *L*; and c) *C*, respectively

In case of a series-*LCR* circuit, the frequency response curve exhibits a resonance frequency with a spread around it. And the spread, determined by the total circuit impedance, is a measure of the quality factor, Q of the circuit. It is defined as the ratio of the resonance frequency and the bandwidth of the resonance curve (at half-power points). In this experiment, you will also study the dependence of Q on R in a series-*LCR* circuit.

Objectives

After performing this experiment, you should be able to:

- study frequency responses of a resistor, an inductor and a capacitor;
- draw the frequency response of a series-*LCR* circuit;
- calculate the quality factor of a series-*LCR* circuit; and
- study the dependence of Q on R in a series-*LCR* circuit.

8.2 CHARACTERISTICS OF A SERIES-*LCR* CIRCUIT

From your earlier classes, you may recall that combination of *RC*, *RL* and *LC* are used to filter out unwanted frequencies from a desired signal. A frequency in a very narrow band can be selected by an *LCR* series or parallel combination. To understand this, refer to Fig. 8.2, where we have depicted frequency dependence of reactances of R, L and C. Note that at lower frequencies, capacitive reactance X_C is large and inductive reactance X_L is small. Most of the voltage-drop in a circuit containing L, C and R in series combination is then across the capacitor. At high frequencies, the inductive reactance is large but the capacitive reactance is low and most of the voltage drop is then across the inductor. In-between these two extremes, there is a frequency at which the capacitive and inductive reactances are exactly equal but act in opposition and cancel each other. This frequency is called **resonance frequency**. We have denoted it by f_r.

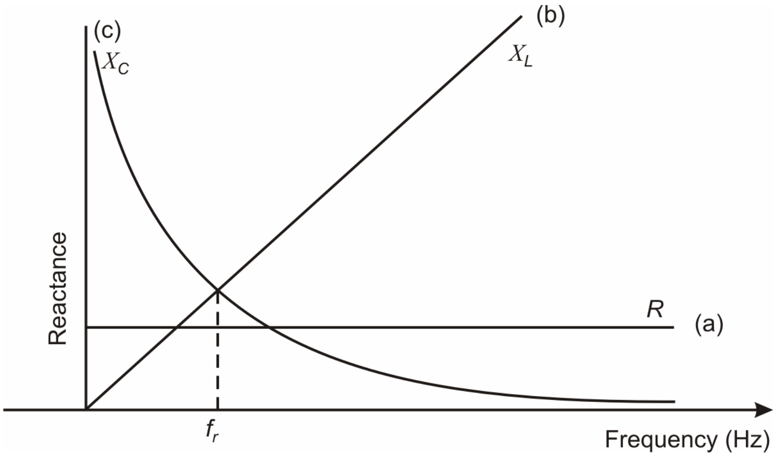

Fig. 8.2: Frequency dependence of reactance for a) *R*; b) *L*; and c) *C*

The resonance frequency is defined by

$$f_r = \frac{1}{2\pi\sqrt{LC}} . \qquad (8.1)$$

In resonance condition, the impedance is minimum as only the resistance *R* in the circuit opposes the flow of current. The current at resonance frequency is equal to the applied voltage divided by the circuit resistance, and thus can be very large if the resistance is low.

For a fixed applied voltage, the expected qualitative variation in circuit current with frequency is shown in Fig. 8.3. Note that current is maximum at the resonance frequency and decreases on both sides around it, giving a bell-shaped curve.

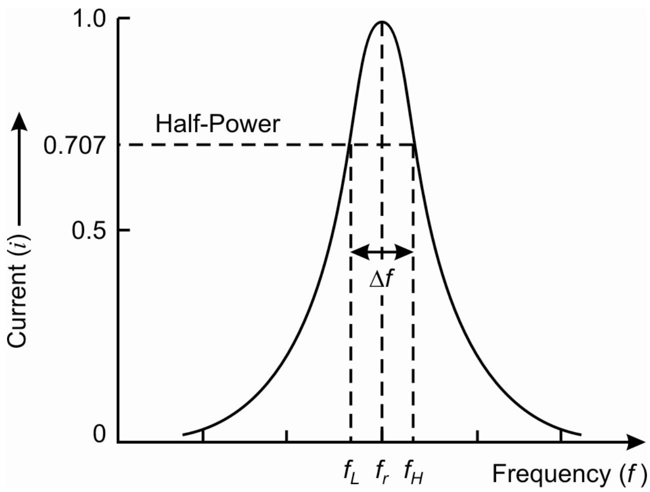

Fig. 8.3: Resonance curve for a series-*LCR* circuit. f_r and Δf (= $f_H - f_L$) respectively denote resonance frequency and bandwidth. f_H and f_L are the higher and lower frequencies respectively of the bandwidth.

At resonance frequency, the maximum power in a series-*LCR* circuit is given by

$$P_{max} = i_{max}^2 / R \qquad (i)$$

and output power is

$$P_o = i_o^2 / R . \qquad (ii)$$

At half power points, we can write

$$P_o = \frac{1}{2} P_{max} .$$

On combining (i) and (ii), we get

$$i_o^2 = \frac{i_{max}^2}{2}$$

or

$$i_o = \frac{i_{max}}{\sqrt{2}}$$

$$= 0.707 \; i_{max}$$

That is, the current in an *LCR*-series circuit at half power points is 0.707 times the maximum current at resonance frequency.

At resonance frequency, the maximum power in the circuit is given by

$$P_{max} = \frac{i_{max}^2}{Z} = \frac{i_{max}^2}{R} \qquad (8.2)$$

From Fig. 8.3, you may conclude that a series-*LCR* circuit will sustain only those frequencies which fall within the width of the bell-shape. This phenomenon facilitates frequency selectivity, which is quantified in terms of the **bandwidth** of the circuit. It is defined as the range of frequencies corresponding to half-power points. Physically, it means that the *LCR*-circuit operates and delivers more than half the maximum power in this frequency range. In Fig. 8.3, the half-power points correspond to 0.707 times the value of maximum current.

The frequency difference $\Delta f = f_H - f_L$ is a measure of the bandwidth of the resonance curve. In terms of resonance frequency f_r and bandwidth Δf, we characterise the quality of a circuit by defining the **Quality Factor** as $Q = f_r/\Delta f$. As such, Q determines the sharpness of resonance. Q is usually used in designing electronic circuits in communication engineering and the typical values are of the order of 10^2 to 10^5.

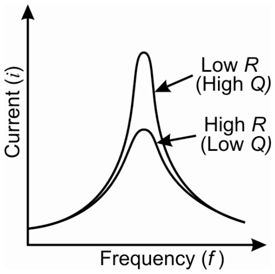

Fig.8.4: Dependence of *Q* on resistance

A series-*LCR* circuit is also called a *series resonant circuit*. It enables us to select the signals of only one frequency and reject all the others. The sharper is the *i*-*f* curve of an *LCR* circuit, the more selective it is for a particular frequency. The selectivity of an *LCR* series resonant circuit depends on the resistance in the circuit. In Fig. 8.4, we have compared *i*-*f* curves for two values of *R*. You will note that the curve corresponding to smaller *R* is sharper. That is, the bandwidth is smaller and frequency selectivity would be better in a low resistance ac circuit.

We use a series-*LCR* circuit in the antenna circuit of radio and TV receivers. By suitably adjusting the values of *L* and *C*, we tune to the desired radio or TV station.

Spend 3 min.

SAQ 1 : Tuning using LCR circuit

You want to tune your radio to listen to IGNOU programmes. The frequency of Gyan Vani radio station is located in a densely populated frequency range that is being used by many broadcasters. What type of series-*LCR* circuit should it have: one with a low or a high value of *R*? Justify your answer, giving reason.

8.3 FREQUENCY RESPONSES OF A RESISTOR, AN INDUCTOR AND A CAPACITOR

You may have realised that while using the alternating current, three quantities may change in an *ac* circuit: voltage, current and frequency, depending on the nature of passive element. To study frequency responses of different circuit components, we must have a suitable variable frequency source. For this, in a physics laboratory, we normally use an audio oscillator, which generates signals of frequencies in the range 20 Hz to 20,000 Hz. For studying the frequency responses of *R*, *L* and *C*, we keep the current constant and measure voltages V_R, V_L and V_C across *R, L* and *C*, respectively for different values of *f*. For studying the frequency response of a series-*LCR* circuit, we keep the voltage fixed and measure current for different values of *f*.

An audio oscillator usually has a number of knobs. The three knobs with which you will work in this experiment are the voltage selector, the range selector and the frequency selector. The voltage selector determines the voltage of the oscillator, while the other two knobs deal with frequency. When the frequency of the output current is varied, the voltage also changes. For keeping the output voltage fixed, it is safer to vary only frequency and that too in the range 100 Hz to 10,000 Hz. Before starting the experiment, it is important that you familiarise yourself with its various knobs and input and output leads. To fully convince yourself, you could also consult your counsellor.

Now, let us list the apparatus required to perform this experiment.

Apparatus

Audio frequency (AF) oscillator, resistors, inductors (in mH range), capacitors (in µF range), and ac milliammeter, an ac voltmeter, connecting wires and sand paper.

Procedure

1. Place the oscillator on the table and connect with it the resistor and milliammeter, as shown in Fig. 8.5a. V is an ac voltmeter, which is connected across the resistor to measure output voltage.

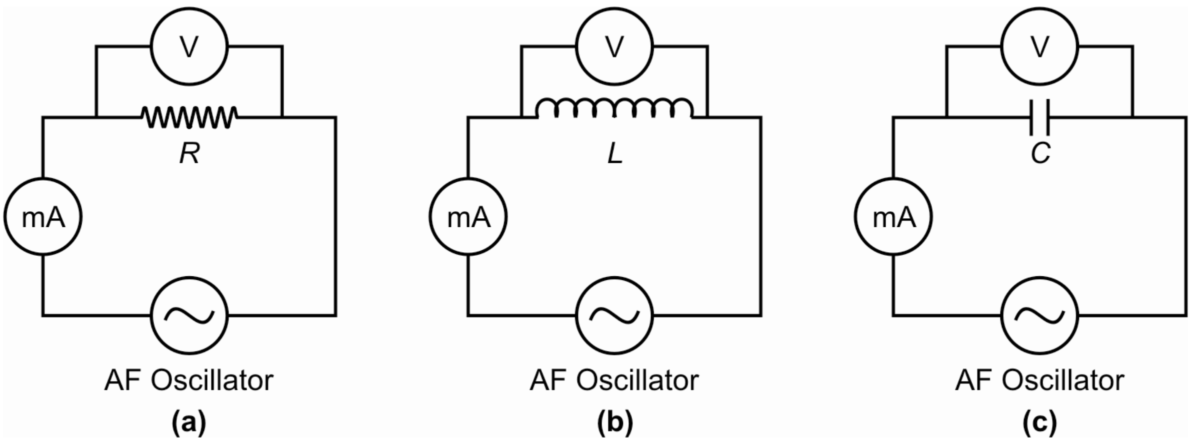

Fig. 8.5: Circuit diagram for studying frequency responses of a) *R*; b) *L*; and c) *C*

2. Connect the power supply cord of AF oscillator to the ac mains supply.

3. Switch on the main supply. Fix the output voltage at a (low) value so that current i in the circuit is within the mA range. This may be determined by using Ohm's law.

4. Note the value of current i in the circuit from the milliammeter and record it in Observation Table 8.1. You should keep this value of current unchanged in this part of the experiment.

5. Now for a frequency *f,* measure the voltage V_R across the resistor *R*. Record both *f* and V_R in Table 8.1. Repeat the process for at least ten different frequencies.

Observation Table 8.1: Frequency response of a resistor

Value of resistance =......................Ω
Current through the resistor, i =......................mA

S. No.	Frequency, *f* (Hz)	Voltage across the resistor, V_R (V)
1. 2. 3. . . 10.		

6. Replace *R* by an inductor *L* (Fig. 8.5b) and repeat steps 3-5. Note that the output voltage has to be low so as to limit the flow of current through the inductor circuit. Record your readings in Observation Table 8.2.

Observation Table 8.2: Variation of voltage across an inductor with frequency

Value of self inductance =.......................mH
Current through the inductor, i =.......................mA

S. No.	Frequency, *f* (Hz)	Voltage across the inductor, V_L (V)
1. 2. 3. . . 10.		

7. Replace the inductor *L* by a capacitor *C* (Fig. 8.5c) and repeat steps 3-5 taking the same precautions. Record your readings in Observation Table 8.3.

Observation Table 8.3: Variation of voltage across a capacitor with frequency

Value of capacitance =…………………..μF
Current through capacitor, i =………………….…..mA

S. No.	Frequency, *f* (Hz)	Voltage across the capacitor, V_c (V)
1. 2. 3. . . 10.		

8. Plot V_R versus *f*, V_L versus *f* and V_C versus *f* .

How does the nature of frequency response curve change for different passive elements in an ac circuit? Do your graphs match with the plots in Fig. 8.2? If not, discuss your results with your counsellor.

8.4 FREQUENCY RESPONSE OF A SERIES-*LCR* CIRCUIT

You have studied the frequency responses of individual circuit components in Sec. 8.3. Let us now investigate the behaviour of a circuit obtained by series combination of *L*, *C* and *R*, as shown in Fig. 8.6. Choose the values of *L* and *C* such that the resonance frequency f_r lies between 100 Hz and 10,000Hz. You can calculate f_r using Eq. (8.1).

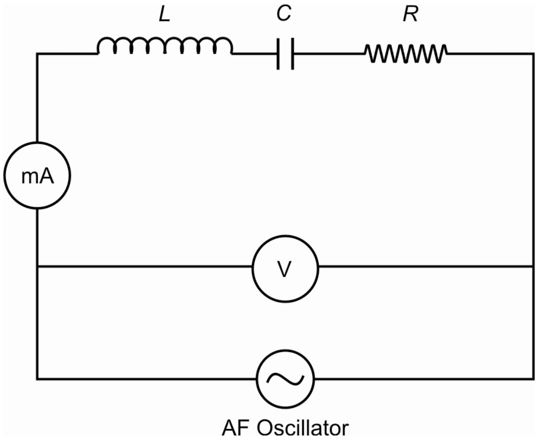

Fig. 8.6: Circuit diagram for studying the frequency response of a series-*LCR* circuit

Connect the circuit as shown in Fig. 8.6 and note down the values of *L*, *C* and *R*. Now you can start the experiment. Start with the frequency of 100 Hz and record the corresponding circuit current in Observation Table 8.4.

Observation Table 8.4: Variation of current with frequency in a series *LCR* circuit

Value of resistance *R* in the circuit =......................Ω
Value of self-inductance *L* of the coil =......................mH
Value of capacitance *C* of the capacitor =......................μF

S. No.	Frequency, *f* (Hz)	Current, *i* (mA)
1.		
2.		
3.		
.		
.		
10.		

Resonance frequency f_r =.....................Hz

Resonance current i_{max} =......................mA

Frequencies corresponding to half power points ($i = 0.707\ i_{max}$):

f_H =......................Hz, and f_L =......................Hz

Bandwidth, $\Delta f = f_H - f_L$ =......................Hz

Quality factor $Q = \dfrac{f_r}{\Delta f}$ =.....................

You must keep voltage constant during the whole experiment. Vary *f* and measure current in the circuit for each value of *f*. You will note that current in the circuit increases initially, attains a maximum value and begins to decrease thereafter. Plot *f* along the x-axis and i along y-axis. Is the curve bell shaped?

Determine the maximum current i_{max} corresponding to resonance frequency f_r from the graph. Record the values of f_H and f_L corresponding to $i = 0.707\ i_{max}$, on both sides of the resonance peak. Calculate the bandwidth $\Delta f = f_H - f_L$ from the graph. You can now calculate the quality factor *Q* of the resonance circuit from the ratio of f_r and Δf.

8.5 DEPENDENCE OF QUALITY FACTOR ON RESISTANCE

Now let us study the effect of *R* on the bandwidth and effectively the quality factor of the series-*LCR* circuit. For this purpose, you should use at least three different values of resistance and take the observations of circuit current with frequency variation as in Sec. 8.4. Record your readings in Observation Table 8.5.

Observation Table 8.5: Frequency response of a series-*LCR* circuit for different resistances

Value of self-inductance *L* =....................mH

Value of capacitance *C* =....................μF

S.No.	Frequency, *f* (Hz)	Current (i) in the circuit (mA)		
		R_1 = ... Ω	R_2 = ... Ω	R_3 = ... Ω
1. 2. 3. . . 10.				

Plot frequency versus current graph for all three cases. Next, note down the values of f_r , f_H and f_L for each curve and calculate quality factor in each case. What is your conclusion about the dependence of *Q* on *R*?