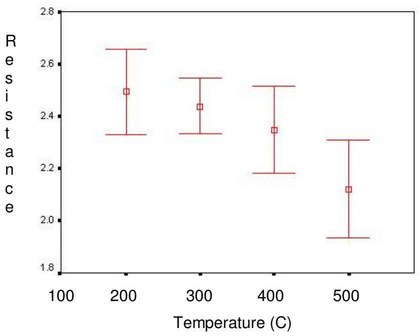
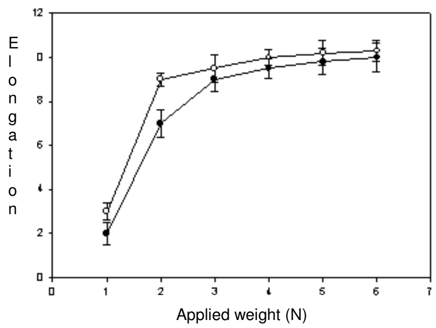
UNIT

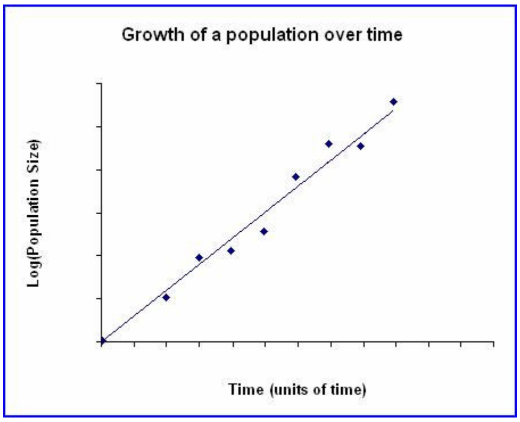
II

ERROR ANALYSIS

Structure







II.1 INTRODUCTION

In Unit I you have learnt that the result of any measurement is expressed in the form of numbers. You also know that every measuring device has a least count, which is an indicator of its ability to measure a physical quantity upto a particular accuracy limit. It means that the number obtained as a result of (a series of) measurement(s) cannot be said to be 'exact'. Further, there can be defects in measuring instruments and even a very careful experimenter is susceptible to certain personal errors.

Since limitations of instruments arise in the form of their least counts, it is important that an instrument with appropriate accuracy is selected for measurement. Note that even then it may not be possible to measure a quantity exactly. That is, the measured value of a physical quantity is normally different from its true value. The uncertainty in any number obtained from a measurement constitutes what is referred to as **error**. It is important to note that within an experiment, the error accumulates in different measurements.

In this unit, you will learn about the types and sources of errors in detail. You will also learn to estimate and possibly eliminate or minimise and account for such errors. In most of the physics experiments, our objective is to determine relationship among physical quantities. Therefore, we estimate errors in measurement of various physical quantities and try to establish valid relationships. In the experimental write-ups of this course, you will learn to apply the knowledge of errors and their propagation in actual measurements. You will first perform measurements of fundamental quantities such as mass, length and time, and then do experiments involving two or more physical quantities.

Objectives

After studying this unit, you should be able to:

- identify the types and sources of errors;
- distinguish between random errors and systematic errors;
- compute errors in the measurement of various physical quantities;
- establish functional relationship between various physical quantities;
- use the criterion of best fit in a straight line plot; and
- interpret a graph and determine the values of physical quantities of interest.

II.2 TYPES OF ERRORS

In Unit I you have learnt that errors can arise due to limitations of measuring instruments as they cannot measure smaller than their least count. For instance, a metre scale cannot measure less than 0.1 cm, a vernier callipers measures a minimum of 0.01 cm and a screw gauge cannot measure distances less than 0.001 cm. Similarly, a thermometer can measure temperature to an accuracy of half a degree Celsius. When measuring angles, a simple protractor measures to an accuracy of one degree. But when a vernier is attached to the protractor, as in a spherometer or spectrometer, we can measure angle more precisely, up to 30″.

In addition to the limitations listed above, which are inherent in the measuring devices, other sources of error could be (i) changes in environment, (ii) faulty observation techniques, (iii) malfunctioning of measuring devices, etc.

The errors in any measurement can be classified in two broad categories: Systematic Errors and Random Errors. Let us now learn about these in detail.

II.2.1 Systematic Errors

The systematic errors, also called 'determinable' errors, arise due to identifiable causes. For this reason, these can, in principle, be eliminated or corrected. These errors result in measured values being either consistently high or low, i.e., different from the true value. The systematic errors include:

Zero Error arises due to wear and tear caused by extensive use. The zero of the vernier scale may not coincide with the zero of the main scale when the jaws are put in contact. The magnitude and nature (positive or negative) of the zero error can be easily determined and corrected. In case of positive zero error, the zero of the vernier scale is on the right of the zero of main scale and vice versa. So, for positive zero error, we subtract (and add in case of negative zero error) the value of error from the measured value.

Backlash Error in a screw gauge, a travelling microscope or a spherometer, can arise due to wear and tear or defective fitting in the instrument. In this case, a forward or backward rotation may not produce the same result. This is minimised by rotating the screw head of the measuring device in only one direction from the initial to the final point of measurement. Note that forward and backward rotations, if resorted to, in the experiment even for finer adjustments can induce errors.

End Correction arises when the edge (zero marking) of a scale is not distinctly visible due to worn out. This leads to an error if one tries to keep the zero of the scale at the starting point. This can be eliminated easily by shifting the reference point of the scale to a definite and distinct point (say, 1 cm mark).

Errors due to changes in a physical quantity can take place during the course of the experiment. For example, the resistance in electrical circuits can change due to the heat generated on passing current through it. This leads to errors that are generally difficult to calculate or compensate for. However, this can be avoided to some extent by allowing the current to flow in the circuit only when observations are being taken.

Defective or *improper calibration* in instruments such as ammeter or voltmeter leads to errors in the measurement. In this case, there will be a constant difference between measured and true values. This arises due to manufacturing defect. The best option in such a situation is to calibrate the instrument against standard equipment.

When an observer experiences relative movement of an object and its image, there exists a parallax between them.

Faulty observation can also arise due to parallax. To minimise error due to parallax, you should note the reading along the line, which is normal to say, both the scale and the edge of the table on which scale is placed. In fact, the error due to parallax can be avoided, if the metre scale is placed on the edge of the table while making measurement.

II.2.2 Random Errors

You must have noticed that if the same measurement is repeated for the same quantity, you may get different readings with a scatter of values distributed about some mean value. These are called random errors and can arise due to accidental errors in the measurement process. These errors are unavoidable, though sources of random errors can not always be identified.

The *Observational* random errors arise due to error of judgment of the observer while reading the smallest division in the scale (like the coincident vernier division with the main scale division). To minimise observable random errors, you should always take more readings and calculate their mean or draw the best fit graph.

If the values obtained in several measurements are x_1, x_2, x_3, …, x_N, the average value is determined by adding all the values and dividing their sum by the total number of observations:

$$\bar{x} = \frac{x_1 + x_2 + x_3 + \ldots + x_N}{N}. \qquad \text{(II.1)}$$

The *Environmental* random errors can arise due to unpredictable fluctuations in line voltage, sudden changes in temperature or mechanical vibrations, etc. There could also be a random spread of readings due to friction or wear and tear of mechanical part(s) of the system.

Random error can also be induced by a careless experimenter, who does not concentrate in his/her work in the laboratory. The errors arising out of this situation can not be determined in any way. That is why, we have, in the Course Introduction, highlighted the importance of observing decency in the laboratory and careful handling of various instruments while performing an experiment.

When inexact values are used in a calculation, some error or uncertainty in the result is inevitable. The quality of a measurement and reliance on the result so obtained are determined by the magnitude of the estimated error. In scientific work, it is customary to quote a result along with associated error in measurement (with proper units) and upto the same order of magnitude.

The random errors can be quantified by statistical analysis and expressed as **absolute error** or **relative error**.

SAQ 1 : Classification of errors

Spend 3 min.

Classify the following according to the type of error involved by putting a tick in the appropriate column:

Sl. No.	Measurement	Type of Error	
		Systematic	Random
i)	A time interval measured using a stop watch that is running slow		
ii)	The length of a piece of steel rod is measured by several students in a laboratory		
iii)	A steel scale expands on a hot day to give a short reading of length		
iv)	The needle of a voltmeter is bent such that it does not rest on zero		
v)	The number of nuclear particles emitted per second by a lump of radioactive element		

Let us now learn to estimate the magnitude of errors.

II.3 ESTIMATING THE MAGNITUDE OF ERROR

Refer to Table II.1, where typical values of measurement of time period are given. As such, by looking at the data, we cannot identify the "true" value.

Table II.1: A set of typical values of measurement

S. No.	Data (x_i) (s)	Deviation $\Delta x_i = \lvert x_i - \bar{x} \rvert$ (s)
1.	2.69	0.01
2.	2.67	0.01
3.	2.68	0.00
4.	2.69	0.01
5.	2.68	0.00
6.	2.69	0.01
7.	2.66	0.02
8.	2.67	0.01
	$\bar{x}$ = **2.68**	$\overline{\Delta x}$ = **0.009**

The magnitude of the difference between the actual (or the mean) value of a physical quantity and its individual measured value is known as **absolute error** in the measurement. Let us denote it by Δx_i. If N independent measurements of a quantity are labelled as $x_1, x_2, \ldots, x_N$, the average value is given by Eq. (II.1). In summation notation, we can write

$$\bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i .$$

Read the symbol Σ as sigma and it represents the sum of all the measurements. As you can see from Table II.1, the average value of time period is 2.68 s. To calculate absolute error, we calculate the modulus of individual deviations $\Delta x_i = |x_i - \bar{x}|$ from the average value. Then, these deviations are added and their sum is divided by the total number of observations. Mathematically we can write,

$$\overline{\Delta x} = \frac{\sum_{i=1}^{N} |\Delta x_i|}{N}. \qquad \text{(II.2)}$$

For the data given in Table II.2, the average value of absolute error is 0.009s. So we express the result of measurement as (2.68 ± 0.01) s. Note that absolute error has the same units as the quantity measured.

In error analysis, a useful measure of deviation is the ***variance***. That is, variance is a measure of the spread of a distribution. For *N* observations, the variance in summation notation is given by

$$\sigma^2 = \frac{\sum_{i=1}^{N} (\Delta x_i)^2}{N}. \qquad \text{(II.3)}$$

Once variance is known, its square root gives **standard deviation**.

The standard deviation represents the range over which measurements vary. In other words, the standard deviation equals the magnitude of the uncertainty in the measurements.

You must be wondering as to why we consider standard deviation and not the average of mere deviations. This is because the individual deviations (which is also an indication of error involved in measurement) may be positive or negative. Since errors are additive in nature, it is more appropriate to take average of squares of the deviations and calculate standard deviation.

To give you a feel of the numbers, we would like you to answer the following SAQ.

SAQ 2 : Standard deviation

The measurement of the length of a table yields the following data:

ℓ_1 = 135.0 cm, ℓ_2 = 136.5 cm, ℓ_3 = 134.0 cm, and ℓ_4 = 134.5 cm

Calculate the standard deviation and express the final result with possible error involved.

A better index of the accuracy of a measurement or equipment is **relative error**, also called percentage error. In fact, we quite often express our result by quoting the relative error rather than the absolute error. The relative error is the ratio of absolute error to the mean measured value of the quantity expressed in percent:

$$\delta x = \frac{\overline{\Delta x}}{\bar{x}} \times 100. \qquad \text{(II.4)}$$

Note that we have written the relative error as δx to differentiate it from the absolute error. You will note that the relative error covers all or most of the readings.

II.4 PROPAGATION OF ERRORS

You now know how to calculate error in the measurement of a directly observable physical quantity. But in most experiments, you will be required to measure two or more independent quantities to determine a physical quantity of interest. Therefore, the error in the final result depends on the errors in the measurement of individual quantities. In other words, the error in each measurement will "propagate" and get reflected in the final result. The actual analysis of propagation of errors is beyond the scope of this laboratory work. We shall, therefore, quote some useful rules.

II.4.1 Error Propagation in Basic Operations

To understand how error propagates through basic mathematical calculations, we first consider addition and subtraction of two or more numbers.

Addition and Subtraction

Suppose that two physical quantities A and B have measured values $(A \pm \overline{\Delta A})$ and $(B \pm \overline{\Delta B})$, respectively, where $\overline{\Delta A}$ and $\overline{\Delta B}$ are their absolute errors. Let us calculate the error $\overline{\Delta Z}$ in their sum $Z = A + B$.

We have by addition

$$Z \pm \overline{\Delta Z} = (A \pm \overline{\Delta A}) + (B \pm \overline{\Delta B}).$$

The maximum possible error in Z is therefore

$$\overline{\Delta Z} = \overline{\Delta A} + \overline{\Delta B}.$$

For the difference $Z = A - B$,

$$Z \pm \overline{\Delta Z} = (A \pm \overline{\Delta A}) - (B \pm \overline{\Delta B})$$
$$= A - B \pm \overline{\Delta A} \pm \overline{\Delta B}$$

or

$$\pm \overline{\Delta Z} = \pm \overline{\Delta A} \pm \overline{\Delta B}. \qquad \text{(II.5)}$$

The maximum value of the error $\overline{\Delta Z}$ is sum of individual errors $(\overline{\Delta A} + \overline{\Delta B})$. Hence the rule for propagation of errors for a sum or a difference is: **The absolute error in the final result is the sum of the absolute errors in individual quantities.**

As such, Eq. (II.5) over-estimates the error. A more useful expression for $\overline{\Delta Z}$ based on statistical analysis is

$$\overline{\Delta Z} = \sqrt{\left(\overline{\Delta A}\right)^2 + \left(\overline{\Delta B}\right)^2}\ . \tag{II.6}$$

Let us now calculate a propagating error in the following example.

Example II.1
Propagation of errors in addition

The measured values of two lengths L_1 and L_2 are (1.746 ± 0.001) m and (1.507 ± 0.001) m, respectively. Calculate the total error in measurement.

Solution:

The error in measurement would be equal to the sum of errors in L_1 and L_2. Thus

$$\overline{\Delta L} = (0.001 + 0.001)\ \text{m} = 0.002\ \text{m}$$

and you can express the result as

$$L = (3.253 \pm 0.002)\ \text{m}$$

If you use statistical analysis, you will obtain

$$\overline{\Delta L} = \sqrt{\left(\overline{\Delta L_1}\right)^2 + \left(\overline{\Delta L_2}\right)^2}$$

$$= \sqrt{(0.001\text{m})^2 + (0.001\text{m})^2}$$

$$= 0.0014$$

$$= 0.001\ \text{m}$$

Note that we have kept only positive root because errors are cumulative.

Multiplication and Division

If a quantity $E = A \times B$ and the results of measurement of A and B are $(A \pm \overline{\Delta A})$ and $(B \pm \overline{\Delta B})$, respectively, we can write

$$E \pm \overline{\Delta E} = (A \pm \overline{\Delta A}) \times (B \pm \overline{\Delta B})$$

$$= AB \pm B\ \overline{\Delta A} \pm A\ \overline{\Delta B} \pm \overline{\Delta A}\ \ \overline{\Delta B}\,.$$

Dividing by $E = AB$ throughout, we obtain

$$1 \pm \frac{\overline{\Delta E}}{E} = 1 \pm \frac{\overline{\Delta A}}{A} + \frac{\overline{\Delta B}}{B} + \frac{\overline{\Delta A}\,\overline{\Delta B}}{AB}. \tag{II.7}$$

Since $\overline{\Delta A}$ and $\overline{\Delta B}$ are small, the term $\frac{\overline{\Delta A}\,\overline{\Delta B}}{AB}$. can be neglected. Hence Eq. (II.7) can be rewritten as

$$\overline{\Delta E} = \pm A\,\overline{\Delta B} \pm B\,\overline{\Delta A}.$$

The maximum error in E is given by

$$\frac{\overline{\Delta E}}{E} = \frac{\overline{\Delta A}}{A} + \frac{\overline{\Delta B}}{B}. \qquad \text{(II.8)}$$

Let us now consider the propagation of errors when the operation 'division' is carried out. If we write $E = \frac{A}{B}$, the error $\overline{\Delta E}$ will be given by Eq. (II.8). We leave this as an exercise for you.

If you take logarithm of $E = AB$ and differentiate it, you will get $\frac{\overline{\Delta E}}{E} = \frac{\overline{\Delta A}}{A} + \frac{\overline{\Delta B}}{B}$. This is generally known as the **logarithmic error**.

If you make statistical analysis, you will get the following result:

$$\frac{\overline{\Delta E}}{E} = \sqrt{\left(\frac{\overline{\Delta A}}{A}\right)^2 + \left(\frac{\overline{\Delta B}}{B}\right)^2}. \qquad \text{(II.9)}$$

You can now conclude that when independent measurements are multiplied or divided, the fractional error in the result is the square root of the sum of the squares of fractional errors in individual quantities. These results hold for absolute errors as well as relative errors.

Now study the following example.

Example II.2
Propagation of errors in division

In an experiment, we calculate velocity from measurements of displacement and time. If the displacement is $\mathbf{S} \pm \overline{\mathbf{\Delta S}}$ = (0.63 ± 0.02) m and time is $t \pm \overline{\Delta t}$ = (1.71 ± 0.10) s, calculate the error in velocity measurement.

Solution:

We know that velocity is rate of change of displacement:

$$\mathbf{v} = \frac{\mathbf{S}}{t} = \frac{0.63}{1.71} = 0.37\,\text{ms}^{-1}.$$

The fractional errors in displacement and time are given by

$$\frac{\overline{\Delta \mathbf{S}}}{\mathbf{S}} = \frac{0.02\,\text{m}}{0.63\,\text{m}} = 0.03,$$

and

$$\frac{\overline{\Delta t}}{t} = \frac{0.1\,\text{s}}{1.71\,\text{s}} = 0.06.$$

Hence the fractional error in **v** is

$$\frac{\overline{\Delta \mathbf{v}}}{\mathbf{v}} = \sqrt{\left(\frac{\overline{\Delta \mathbf{S}}}{\mathbf{S}}\right)^2 + \left(\frac{\overline{\Delta t}}{t}\right)^2}$$

$$= 0.07$$

so that

$$\overline{\Delta \mathbf{v}} = 0.37 \text{ ms}^{-1} \times 0.07 = 0.03 \text{ ms}^{-1}.$$

Hence

$$\mathbf{v} \pm \overline{\Delta \mathbf{v}} = (0.37 \pm 0.03) \text{ ms}^{-1}.$$

Note that we have given the velocity to the second decimal. This is because the error is up to the second decimal

Let us now see how error propagates in calculations involving operations of both multiplication and division.

Example II.3
Error propagation in multiplication and division

A particular physical quantity, in an experiment, is computed from the relation $B = \frac{XY}{Z}$. Take the values of *X*, *Y* and *Z* measured in the laboratory as:

$X = 17 \pm 10\%$

$Y = 100 \pm 6$

and

$Z = 15 \pm 3$

Let us see how error propagates in such a situation.

Solution:

By converting the uncertainties to percentage, you will get

$$B = \frac{(17 \pm 10\%) \times (100 \pm 6\%)}{(15 \pm 20\%)}$$

$$= 113.3333 \pm 36\%,$$

where we have added the uncertainties.

Proceeding further, you will note that 36% of 113.3333 = 40.799988 so that

$$B = 113.3333 \pm 40.8000$$

This means that the uncertainty in the value of *B* is about 41. So, it makes no sense to retain the digits after the decimal in the value of *B* as well as the uncertainty. It is therefore more sensible to write

$$B = 113 \pm 41.$$

II.4.2 Error Propagation in Angular Measurements

In your B.Sc. physics laboratory, you will get an opportunity to make very accurate and high precision measurements of angles. This is particularly true of experiments involving small physical quantities such as wavelength using a grating and a spectrometer. A spectrometer has a fixed circular protractor with a vernier moving over it. Usually the least count of a spectrometer is 1 min of an arc or $\frac{1}{16}^{\text{th}}$ of a degree. The calculations of error propagation are the same as in other measurements. This is illustrated below.

In a diffraction grating experiment, the wavelength $\lambda = N \sin\theta$, where N is the number of rulings in the grating and θ is the angle of diffraction for that λ. Then

$$\frac{\overline{\Delta\lambda}}{\lambda} = \frac{N\cos\theta}{N\sin\theta} = \cot\theta\,.$$

II.4.3 Error Propagation due to Exponent of a Measured Quantity

Suppose we have to calculate the area of a square piece of land of side A. It is given by $s = A \times A = A^2$. From Eq. (II.8), it readily follows that

$$\frac{\overline{\Delta s}}{s} = \frac{\overline{\Delta A}}{A} + \frac{\overline{\Delta A}}{A}$$

$$= 2\frac{\overline{\Delta A}}{A}. \qquad \text{(II.10)}$$

That is, the error in A^2 is twice the error in A. You will obtain the same result if you take the logarithm of both the sides:

$$\log s = 2 \log A.$$

On differentiating and changing the differentials to 'deltas', we get Eq. (II.10).

For a wire, the diameter d is measured as (1.02 ± 0.01) mm. Therefore, the error in the area of cross section $\left(= \frac{\pi d^2}{4}\right)$ will be twice the error in d, i.e. nearly 2%.

In general, if a quantity appears in an expression with a power n (> 1), its error contribution increases as many fold. This means that you should measure quantities appearing with power 2 or more with a higher degree of accuracy. Moreover, take a large number of readings. In case its magnitude is small, you should take readings at different points/ perpendicular directions.

Now study the following example.

Example II.4

Error propagation in angular measurement

The period of oscillation of a simple pendulum is $T = 2\pi\sqrt{\frac{L}{g}}$. L is about 100 cm and is known to 1 mm accuracy. The period of oscillation is about 2 s. The time of 100 oscillations is measured with a wrist watch having least count of 1 s. Let us calculate the percentage error in the value of g.

Solution:

You can rewrite the expression for T as

$$g = \frac{4\pi^2 L}{T^2}.$$

Therefore, the percentage error in g can be calculated using the relation

$$100\frac{\overline{\Delta g}}{g} = 100\frac{\overline{\Delta L}}{L} + 2\times 100\frac{\overline{\Delta T}}{T}.$$

The percentage error in $L = 100\frac{\overline{\Delta L}}{L} = 100\times\frac{0.1\text{ cm}}{100\text{ cm}} = 0.1$

and

the percentage error in $T = 100\frac{\overline{\Delta T}}{T\times n} = 100\times\frac{1\text{s}}{2\times 100\text{ s}} = 0.5$.

Hence

$$100\frac{\overline{\Delta g}}{g} = 0.1 + 2\times 0.5 = 1.1\%.$$

Note that we have multiplied T by n while calculating the percentage error in T. Do you know why? This is because the actually measured quantity is time for 100 oscillations ($n\times T$) rather than T. If you take time for one oscillation, the percentage error in T will be 50%. It means that **taking more observations helps us to reduce error in a measurement.**

II.5 ERROR PROPAGATION IN GRAPHING

II.5.1 Plotting a Graph

Many a time, it is not possible for us to visualise the functional relationship between two physical quantities by looking at the experimental data. But if we plot a graph, it becomes very easy, quick and convenient to predict the nature of relationship. In fact, graphs can also be used to minimise errors or locate inaccuracy in results. It means that *a graph* is **not** *a game of joining experimental points*.

A straight line graph represents a linear function and is the easiest graph to use. The equation of a straight line is $y = mx + c$, where m signifies the gradient and c is the intercept on the y-axis. In case of non-linear functional relationship, it is more convenient to plot log y versus x (semi-log) or log x versus log y on (log-log) graph. In such a situation, the gradient gives the power in exponent. In case of functions involving exponentials, a semi-logarithmic graph may be used to get a straight line graph.

When plotting a graph, we must observe the following guidelines:

- Draw axes clearly and label with:
  - **name** of the physical quantity being plotted
  - its **symbol**
  - the **unit**
  - the **scale**.

  It is customary to plot the independent variable along the x-axis and the dependent variable along the y-axis.

- You should choose the scales so that the points are suitably spread out on the entire graph paper rather than being cramped into one corner. To ensure this, you should check the minimum and maximum values of the data to be plotted. You may then round off these two numbers to slightly less than the minimum and slightly more than the maximum. The resulting difference should be divided by the number of divisions on the graph paper. For example, if the measured values are between 5.2 cm and 17.7 cm, you should choose the scale to run from 5 cm to 20 cm rather than from 0 to 18 cm.

- Use a plotting symbol such as a dot, small circle, etc. to show the measured values. In more accurate experiments, we use **error bars** to show the uncertainty associated with each point (Fig. II.1).

- Give the graph a **title** (including identification of components used, and other relevant information).

- If there is more than one curve on the graph, label the different curves. Use different notations (•, Δ, , o) to denote different sets of data points. In case you are not using error bars, the symbol used to depict measured data should in no case be bigger than the size of the smallest square on the graph paper.

- The curve drawn should be the simplest mean curve that fits the data. Note that the curve may not necessarily pass through each observed point. However, it should pass through the region of uncertainty for each point. And the graph should be smooth, even if all observation points do not fall on the curve.

In Fig. II.1, we have plotted speed v along y-axis and time period T along x-axis on a linear graph paper, since these quantities are linearly related. In some experiments, we may get data where the relationship between the measured variables is not linear and we have to plot a graph where the variables of interest are related through a power-law as in a simple pendulum $T = k\ell^{1/2}$. In such cases, to draw a graph between the time period and the length of the pendulum, you will have to calculate square root for each value. This introduces another step and is obviously cumbersome.

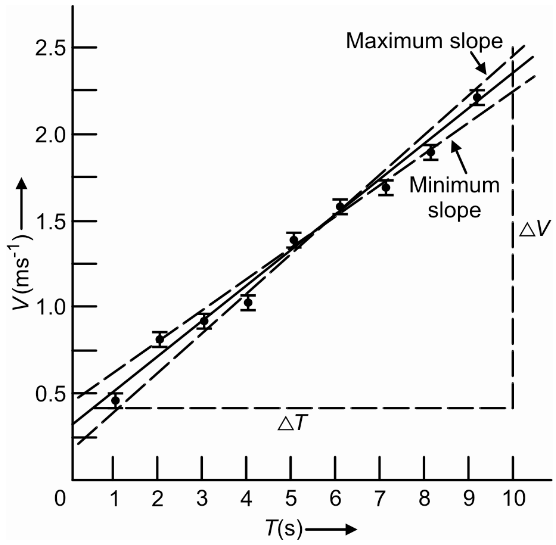


Fig.II.1: Graph between speed and time

Many a times, the variables may range over several powers of ten. For example, according to Keplar's law, the semi-major axis of the orbit of a planet (R) is related to its period (time for one revolution around the sun) T by the relation

$$R^2 = kT^3, \tag{II.11}$$

where k is constant.

If you consider the experimental data which shows how T depends on R, you will observe that the latter varies by two orders of magnitude and T varies by three orders of magnitude. In other words, the experimental data follows Eq. (II.11). For a moment suppose you do not know the exact relationship between the variables T and R. Then you can write

$$R = kT^n, \tag{II.12}$$

where n is constant. In such cases, you can obtain the value of n by taking logarithm of (Eq. II.12):

$$\log R = \log k + n \log T \tag{II.13}$$

Now you can plot log R versus log T on a linear graph paper. The slope of straight line obtained will give the value of exponent n. But again, as mentioned above, taking logarithm of each experimental data is rather tedious. For convenience, we use semi-log (Fig. II.2) and log-log (Fig. II.3) graph papers in such cases.

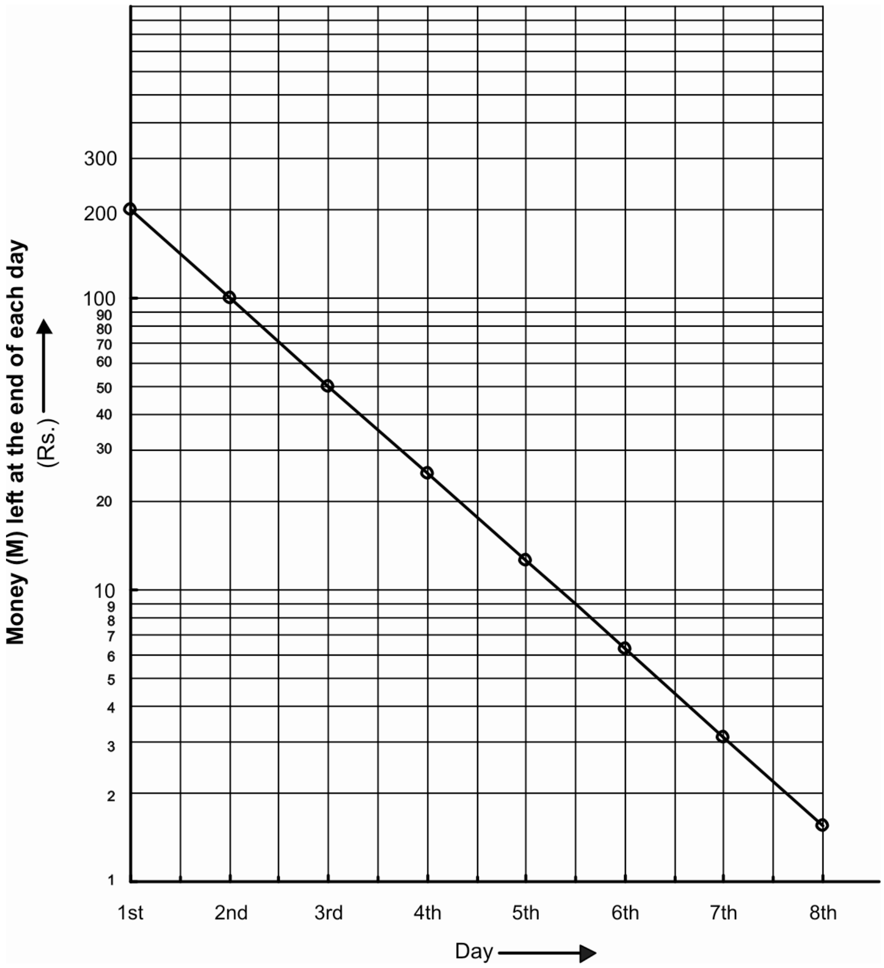


Fig.II.2: Semi-log graph

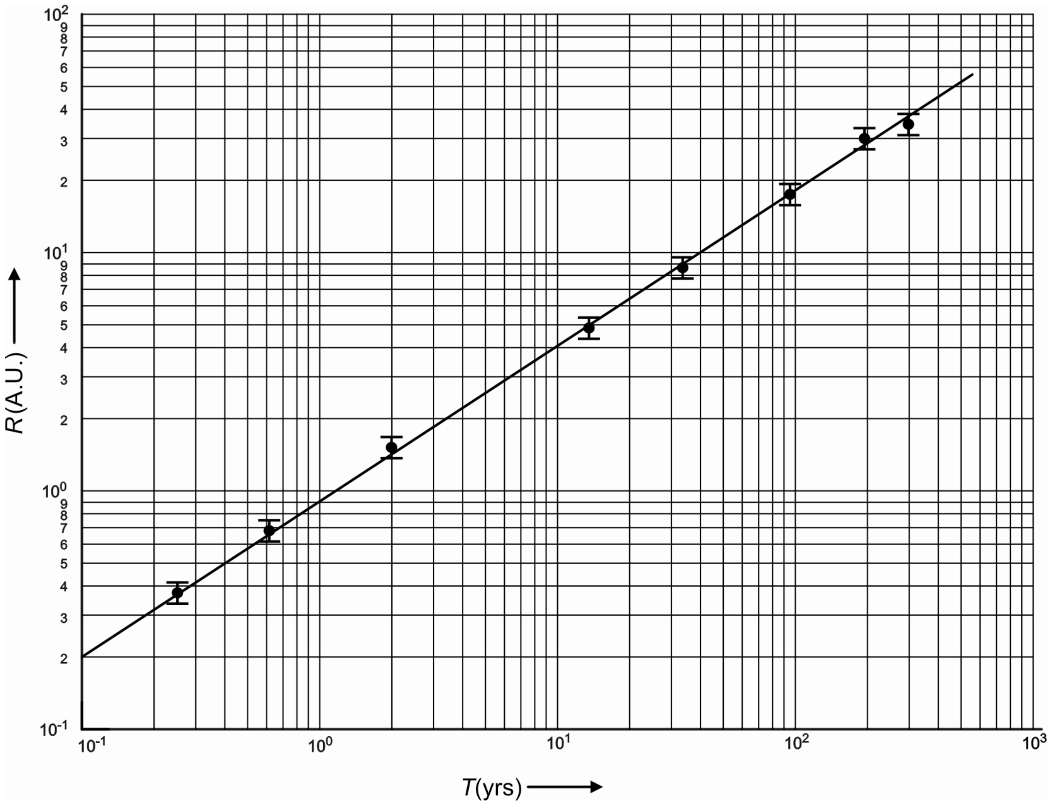


Fig.II.3: Log-log graph

II.5.2 Error Estimation on Graphical Plots

To determine the error in the value of the slope of the straight line drawn on any graph paper (linear, semi-log or log-log), you should draw two lines representing the greatest and the least possible slopes which reasonably fit the data. For the graph in Fig. II.1, the error in the slope is defined as

$$\text{Error in slope} = \frac{\text{Maximum slope} - \text{Minimum slope}}{2}$$

Similarly,

$$\text{Error in intercept} = \frac{\begin{array}{c}(\text{Intercept of minimum slope line} \\ - \text{Intercept of maximum slope line})\end{array}}{2}$$

Let us now sum up what you have learnt in this unit.

II.6 SUMMARY

- **Systematic errors** can arise due to zero error, backlash error, end correction, defective calibration or faulty observation procedures. Such errors are identifiable. So these can be eliminated or accounted for.
- **Random error** can arise due to error of judgement and environmental factors during the performance of results. Such error results in a scatter of values and to minimise these, we take a large number of observations.
- The magnitude of errors can be computed statistically. It is usually expressed as a mean of deviations of observed values from the final value or through standard deviation.
- Errors are cumulative and propagate in an experiment depending on the number of measurements and measuring devices.
- A properly drawn graph helps to minimise errors.

II.7 TERMINAL QUESTIONS

Spend 10 *min*

1. A physical quantity x is related to three other physical quantities *a*, *b* and *c* through the relation

   $$x = ab^2c^{-3}$$

   If the errors in *a*, *b* and *c* respectively are 1%, 3% and 2%, calculate the percentage error in x.

2. In the measurement of viscosity of a liquid, we determine the rate of flow of the liquid (volume (V) flowing per second) through a capillary tube of radius *a* and length ℓ under constant pressure difference *P*. The expression for viscosity is given by

   $$\eta = \frac{\pi P a^4}{8\ell V}.$$

If the percentage errors in P, a, V and ℓ respectively are 1%, 1%, 2% and 1%, calculate the percentage error in η.

II.8 SOLUTIONS AND ANSWERS

Self-Assessment Questions

1. i) Systematic, ii) random, iii) systematic
   iv) systematic v) random

2.

Sl. No.	Length (x_i) (cm)	$\Delta x_i = x_i - \bar{x}$ (cm)	$(\Delta x_i)^2$ (cm^2)
1.	135.0	0	0
2.	136.5	+1.5	2.25
3.	134.0	−1.0	1.0
4.	134.5	−0.5	0.25
	$\bar{x} = \frac{540}{4} = 135.0$		$\Sigma(\Delta x_i)^2 = 3.5$

$$\sigma = \sqrt{\frac{\Sigma(\Delta x_i)^2}{N}} = \sqrt{\frac{3.5}{4}} = \sqrt{0.9} \approx 1.0\,\text{cm}.$$

The final result can be expressed as length = (135.0 ± 1.0 cm)

Terminal Questions

1. To calculate the percent error, we note that

$$a = (a_0 \pm 1\%)$$

$$b = (b_0 \pm 2 \times 3\%)$$

and

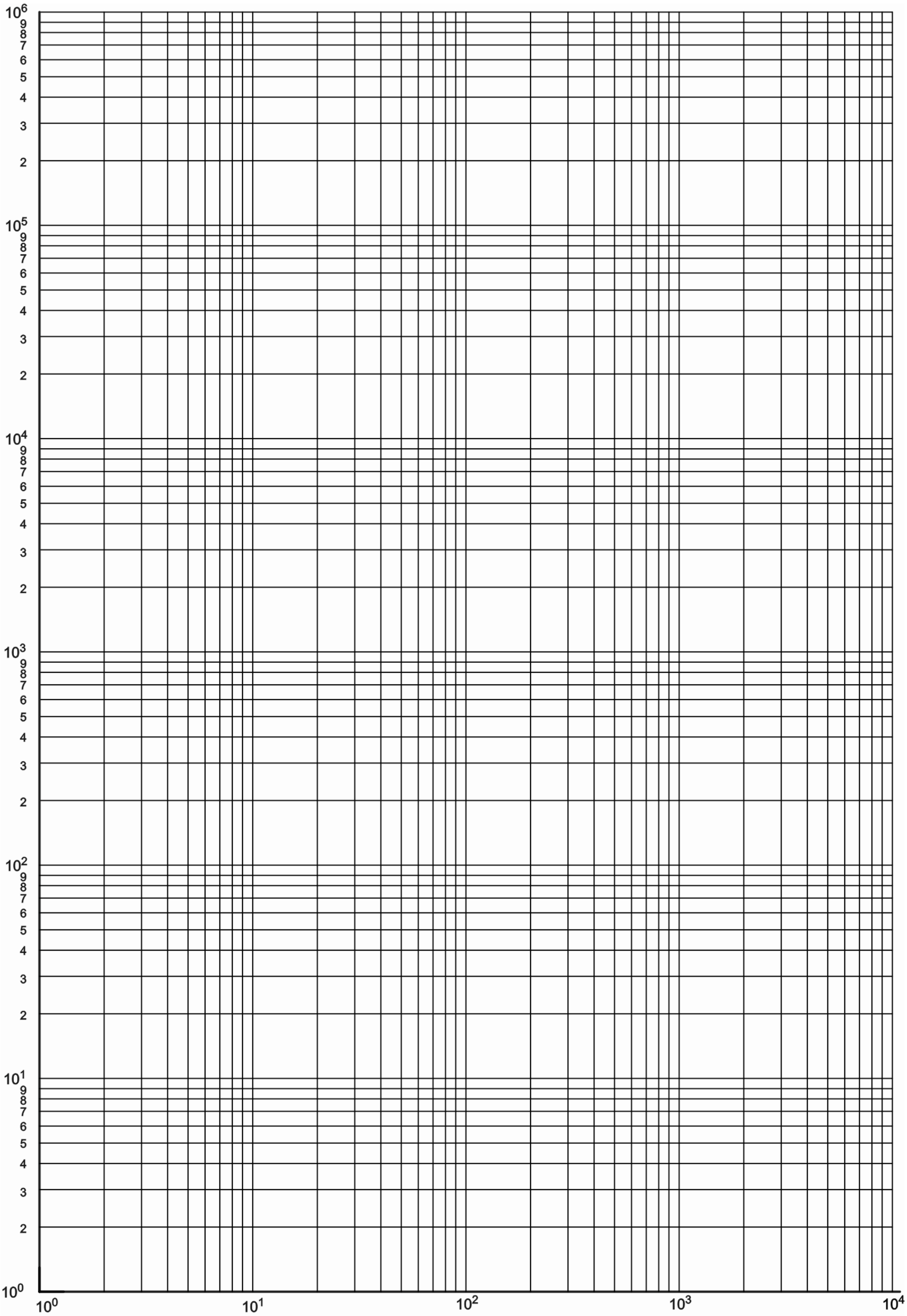
$$c = (c_0 \pm 3 \times 2\%)$$

So the total percentage error in x is1+6+6 = 13%.

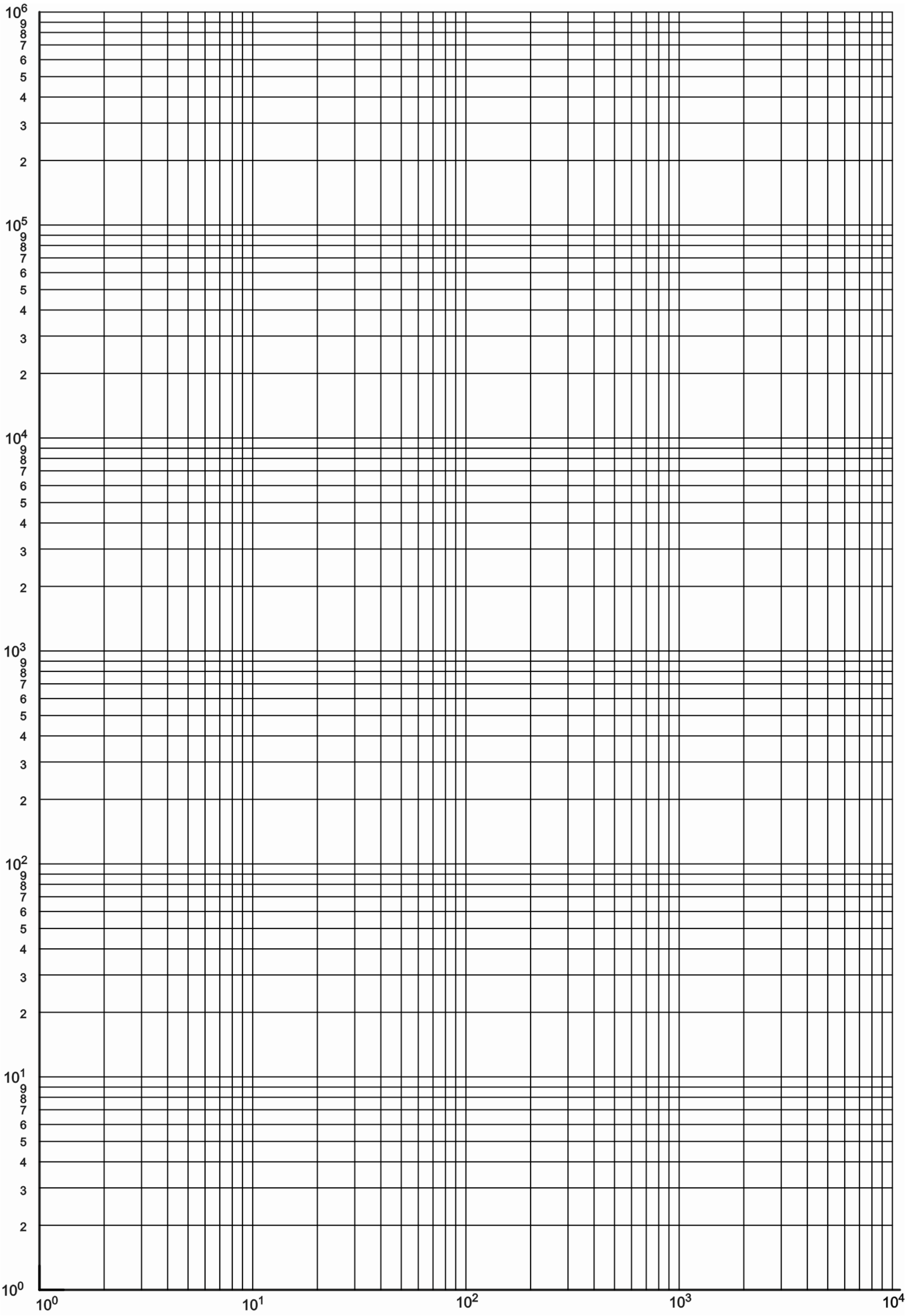
2. $$\frac{\Delta\eta}{\eta} = \frac{\Delta P}{P} + \frac{\Delta a}{a} + \frac{\Delta V}{V} + \frac{\Delta l}{l}$$

$\therefore$ percentage error in η is = 1+4+2+1 = 8%.

Log-log graph sheet



Log-log graph sheet



Log-log graph sheet