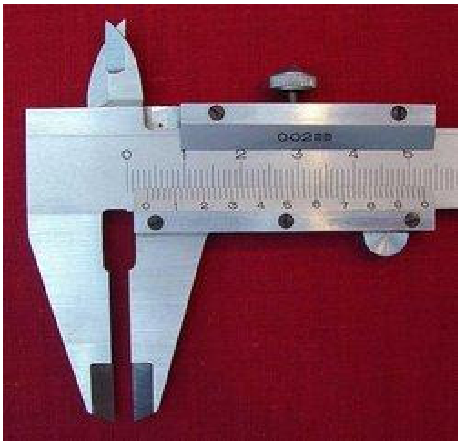
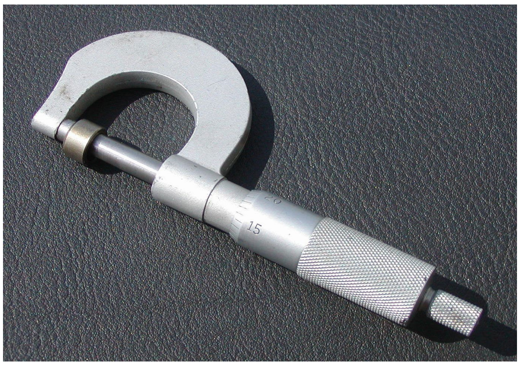
UNIT I

MEASUREMENT

Structure





I.1 INTRODUCTION

In the Foundation Course in Science and Technology, you have studied about the nature of scientific investigations. You know that scientists use devices to measure physical quantities and thereby quantify them. In a physics laboratory, we perform experiments, which involve measurement of various physical quantities. It is very important to understand the correct way to make measurements and obtain correct readings. It is also true **that even the best of the measuring instruments do not yield exact values because of their limited accuracy and precision**. We therefore express all measurements as approximate numbers such as 3.2 cm or 3.20 cm. Do you know as to why do we use different number of digits and what distinguishes them? At this stage, we can only say that these are results of measurements with different devices. While doing computations with these numbers, special care is required. For example, the ratio of two measurements such as 32.1/12 is expressed as 2.7 rather than 2.68 or 2.675. Do you know the reason? The number of digits used in a measurement express some significance regarding the quality of measuring instrument.

In this unit, you will learn the meaning and usage of measured numbers, with particular reference to precision and accuracy. You will also learn the techniques of computation using these numbers and expressing the results of experiments. Normally, **we express a result in what is known as 'scientific notation' with appropriate units**.

Objectives

After studying this unit, you should be able to:

- explain why measurements result in approximate numbers;
- distinguish between precision and accuracy;
- express a measurement in scientific notation; and
- use significant figures to add, subtract, multiply and divide approximate numbers.

I.2 ERRORS IN MEASUREMENTS

You are familiar with at least two reasons why all measurements are inexact. Firstly, an error is caused by the measuring instrument itself, such as the zero error. Secondly, an error can be due to limitations of human judgement and perception, such as in aligning the end of a rod whose length is to be measured with the zero of the scale, or parallax in reading a value. To enable you to better appreciate the inexact nature of measurement, let us reflect on the process of measurement, say of length. Let us assume that we have a 'perfect' centimetre scale which has clear and equal markings of millimetres. We wish to measure the length of three arrows *A*, *B* and *C* shown in Fig. I.1. Let us suppose that we are able to perfectly align the tails of the arrows with zero marking on the scale. (As such, this is impossible to achieve in practice, but we always begin by considering an ideal situation to gain an insight into the process of measurement.)

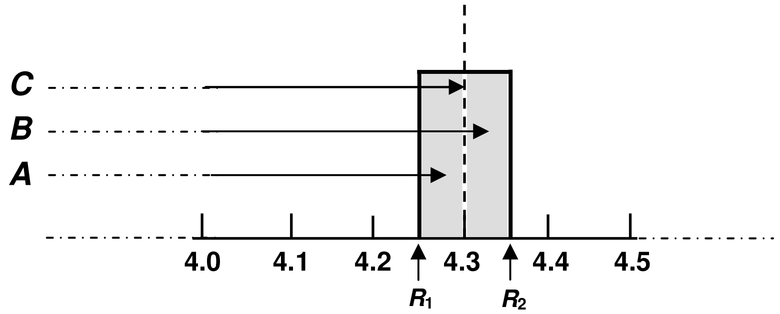


Fig.I.1: The length of all the three (unequal) arrows *A*, *B* and *C* is reported as 4.3 cm. The shaded portion on the scale represents the range of error in this measurement. (The scale is highly magnified.)

To measure the length of these three arrows, we look at the positions of arrow heads. The head of arrow *A* is closer to the 4.3 cm mark than to the 4.2 cm mark. We will report the length of arrow *A* as 4.3 cm to the nearest millimetre. Let us now measure the length of arrow *B*. The head of arrow *B* is closer to 4.3 cm mark than to 4.4 cm mark. Therefore, we will report its length also as 4.3 cm to the nearest millimetre or simply 4.3 cm. Similarly, the length of arrow *C* would be reported as 4.3 cm. Thus the lengths of all arrows, though different, will be reported as 4.3 cm.

We can conclude that a measurement which is reported as 4.3 cm (which is in the middle of R_1 and R_2) might possibly be in error by 0.05 cm (or one-half of the unit of measure, which is 0.1 cm in this case) or less. It means that in the measurement 4.3 cm, the last digit 3 is in error. We can generalise this result: **no measurement can ever be exact**; there will always be deviation from the true value due to the limited accuracy of the measuring device/instrument. The inaccuracy is expressed in the last digit.

I.2.1 Probable Error and Precision

We have seen that the maximum error, barring an error in a measurement, is half of the unit of measurement. The possible error is thus due to inherent imprecision in measuring devices. The measurements having less probable error are more precise. **Since probable error is proportional to the unit of measure, the instrument having smaller unit of measure gives more precise measurement.** A measurement reported to one-hundredth of a centimeter, such as 5.32 cm is more precise than a measurement reported to one-tenth of a centimeter, such as 5.3 cm.

The *probable error* is half of the unit of measurement.

SAQ 1 : Precision in measurement

Spend 3 min.

Consider the following pairs of measurement. Indicate which measurement in each pair is more precise:

a) 17.9 cm or 19.87 cm
b) 16.5 s or 3.21 s
c) 20.56 °C or 32.22 °C

I.2.2 Relative Error and Accuracy

$$\text{Relative Error} = \frac{\text{Probable Error}}{\text{Total Measurement}}$$

So far we have considered measurement of nearly equal lengths with emphasis on precision. Let us now consider measurement of much different lengths. Suppose that two measurements yield 3.2 cm and 98.6 cm using the same metre stick. The probable error in both these measurements is equal to 0.05 cm. But the measurement 98.6 cm is bigger than measurement 3.2 cm. Will you say that the measurement 98.6 cm is more accurate? Again, let us consider measurement of time in seconds. How do the accuracies of 7.4 s and 98 s compare? To compare such measurements, we introduce the term **relative error**, which is defined as the ratio of probable error to the total measurement. In Table I.1, we have calculated relative error in a few typical measurements. The exact method of expressing the relative error will be discussed in
section I.5.

Table I.1: Calculation of relative error

Measurement	Unit of measure	Probable error	Relative error
3.2 cm 98.6 cm	0.1 cm 0.1 cm	0.05 cm 0.05 cm	0.02 0.0005
7.4 s 98 s	0.1s 1s	0.05 s 0.5 s	0.007 0.005

Note that in the measurement of 3.2 cm and 98.6 cm, the unit of measure is same and we say that both measurements are equally precise. But the relative error is less in the larger measurement (0.0005 compared to 0.02) and it is said to be more accurate.

Comparison of measurement 7.4 s and 98 s is more revealing. The measurement 7.4 s is more precise than the measurement 98 s (possible errors 0.05 s and 0.5 s respectively) but less accurate (relative error 0.007 as compared to 0.005).

You will therefore appreciate that a **smaller measurement can be more precise yet less accurate**. And for a given accuracy, small measurement should be more precise. This is why when measuring the dimensions of a room, metre is used as unit of measure while in measuring inter-city distances, the unit kilometer is used for the same accuracy.

Spend 3 min.

SAQ 2 : Accuracy in measurement

Consider the following pairs of measurements. Indicate which measurement in each pair is more accurate:

a) 40.0 cm or 8.0 cm
b) 0.85 m or 0.05 m

You now know that in a measurement, errors can be introduced by

- the measuring instrument due to its inherent imprecision;
- limitations of an experimentalist; and
- external condition.

Depending on the situation, the errors are classified in two broad categories: **systematic errors**, which arise mostly due to the instruments used in the measurement, and **random errors**, which arise from accidental reasons in the measurement process. The systematic errors are identifiable and can, in principle, be minimised or corrected. To minimise random errors, we repeat measurements many times and take their arithmetic mean as the true value. You will learn about these in detail in the next unit.

I.3 SCIENTIFIC NOTATION

In the scientific notation, a measurement is expressed in decimal numerals. You may recall that in inter-atomic distances, very small numbers are obtained, whereas in measuring interstellar distances, we have to deal with very large numbers. In scientific notation, these numbers are written as a number between one and ten multiplied by an integral power of ten. For example, the diameter of the sun is 1,390,000,000 metre and the diameter of hydrogen atom is only 0.000000000106 metre. In scientific notation, we write the diameter of the sun as 1.39×10^9m and the diameter of the hydrogen atom as 1.06×10^{-10}m.

SAQ 3 : Expressing results in scientific notation

Spend 2 min.

Express the mass of a water molecule, measured as 0.000 000 000 000 000 000 000 03g, in scientific notation.

You will appreciate that writing numbers in scientific notation makes representation more convenient. Moreover, computations become easier because we can apply the laws of exponents readily.

I.4 SIGNIFICANT DIGITS

In Sec. I.2.1, you have learnt that a measurement reported as 5.32 cm is more precise than that reported as 5.3 cm. The number of digits in these measurements is three and two, respectively. This suggests that the number of digits used in reporting a measurement have some significance. Whenever you report any measurement, it is important to express it in “correct” number of significant digits. For example, suppose that you are measuring time with a stop watch with least count of 0.1 s and reporting the reading by taking average of (say) 5 events. Then the average value of 7.635 s, say, should be expressed as 7.6 s since each measurement has accuracy of only 0.1 s.

There are certain rules for counting the significant digits. We now state these with some examples.

- All zeros appearing between two non-zero digits are significant. The measurement 107.005 m has six significant digits, whether it is written as 0.107005 km or 10700.5 cm.

- All zeros appearing on the immediate right of a decimal point, i.e. in front of non-zero digits are not significant, when there is no non-zero digit on the left of the decimal. Thus, 0.003 kg has one significant digit, as does 0.7s. However, 0.103 m has three significant digit and so does 0.00783 m.
- All zeros following a non-zero number and to the right of the decimal point are significant. The measurement 47.000 m has five significant digits while 700.000 kg has six significant digits. It means that to have exact idea about the number of significant digits, particularly when a number ends with zeros, we must use decimal point as an indicator. Otherwise, there would always be an ambiguity. For example, 700 m may be taken as a number with 1 or 3 significant digits. However, expressing such numbers in scientific notation resolves such ambiguities.

You may now like to answer an SAQ.

Spend 6 min.

SAQ 4 : Significant figures

Complete the Table I.2 and answer the following questions:

Table I.2

S.No.	**Measurement (m)**	**No. of significant digits**	**Unit of measurement (m)**	**Probable error (m)**	**Relative error**
1.	0.2	1	0.1	0.05 m	$\frac{0.05\,\text{m}}{0.2\,\text{m}} = 0.25$
2.	0.20	2	0.01	–	–
3.	0.2000	–	–	–	–
4.	25	–	–	–	–
5.	250	–	–	–	–
6.	25000	–	–	–	–
7.	102	–	–	–	–
8.	1002	–	–	–	–
9.	0.02	–	0.01	–	–
10.	0.00	–	0.01	–	–

a) What is the significance of ‘trailing’ zeros in the first three measurements?

b) What is the significance of the zeros in the fifth and sixth measurement?

c) What is the significance of zeros between non-zero digits in the seventh and eighth measurements?

From this SAQ, you must have noted that a digit is significant if and only if it affects the relative error. In other words, a measurement possessing greater number of significant digits has greater relative accuracy.

Sometimes we take a sequence of whole number measurements such as 32, 30, 28, 26. All these measurements have two non-zero significant digits, except the measurement 30. In such special cases, zero is also significant without any ambiguity.

We express the result of any measurement in a **standard form** along with error using the following rules:

(1) The error is stated up to *one significant digit* only.

(2) The measurement is rounded off to the *same* order of accuracy as the error.

(3) The result of measurement is written with the decimal point after *the first significant figure*.

(4) The error is multiplied by *the same power as the measurement*.

If only step (1) and (2) are followed, the form of result is correct but not standard. Steps (3) and (4) convert the result to standard form. For example, the standard form of result for measurement of length where metre scale is used should be written as (4.6 ± 0.1) cm.

I.5 COMPUTATIONS WITH APPROXIMATE NUMBERS

In section I.2, you have learnt that the reported measurements have error in the last digit. For example, a measurement reported as $3.\bar{2}$, has error in the digit 2 which has been indicated by placing a bar (–) over this digit. In calculating values of physical quantities from experimental data, we use some rules for expressing the results of basic operations with approximate numbers. Let us learn these now.

I.5.1 Addition and Subtraction

Study the addition given below:

$$\begin{array}{r} 2.135\bar{6} \\ 2.5\bar{3} \\ 1.0\bar{2} \\ \hline 5.6\bar{8}5\bar{6} \end{array}$$

Note that the sum has two error-containing digits. Therefore, we round off the sum to 5.69 so that it contains only one digit containing error. Rounding off is necessary because the sum cannot be more precise than individual measurements. We note that the sum 5.69 has the same unit of measure as the least precise addend. Thus we can say that

While adding (or subtracting) approximate numbers, round off the sum (or difference) to the same unit of measure as the least precise measurement.

It is also important to note that we have used the rule of more than half and less than half in the digit next to the least precise digit.

Spend 3 min.

SAQ 5 : Addition and subtraction of approximate numbers

Calculate i) the sum of $2.154\bar{6}$ m, $2.1\bar{1}$ m and $2.12\bar{5}$ m and

ii) the difference of $2.154\bar{3}$ m and $2.1\bar{1}$ m.

Having answered this SAQ, you will agree that before adding (or subtracting), we could round off the individual numbers so that they contain one more digit of precision than the number of precision digits in the least precise number. Thus the addends in SAQ 5 would become 2.155m, 2.11m and 2.125m.

I.5.2 Multiplication and Division

Let us now consider multiplication of approximate numbers. We want to multiply $1.2\bar{3}$ by $2.\bar{3}$. At each step of the computational process, we put a bar (–) over a significant digit which arises from computation with a digit containing error as shown below:

$$\begin{array}{r} 1.2\bar{3} \\ \times\ \ 2.\bar{3} \\ \hline .\bar{3}\bar{6}\bar{9} \\ 2.4\bar{6}\times \\ \hline 2.\bar{8}\bar{2}\bar{9} \end{array}$$

We see that the product contains three digits which contain errors. Since we report the result in a number having only one digit containing error, we should round off the product to $2.\bar{8}$. Thus the product has two significant digits. This is also equal to the number of significant digits contained in a factor having the least number of significant digits, namely $2.\bar{3}$. Therefore,

the product (or quotient) of two measurements should be rounded off to contain as many significant digits as the measurement having a fewer number of significant digits.

Now consider the multiplication of three numbers:

$$5.286\bar{5} \times 3.\bar{8} \times 19.6\bar{2} = 2\bar{0}.\bar{0}\bar{8}\bar{8}\bar{7} \times 19.6\bar{2}$$

$$= 3\bar{9}\bar{4}.\bar{1}\bar{4}\bar{0}\bar{2}\bar{9}$$

which must be round off to $3.\bar{9} \times 10^2$.

You could have obtained the same result by rounding off these numbers first as shown below:

$$5.2\bar{9} \times 3.\bar{8} \times 19.\bar{6} = 2\bar{0}.\bar{1} \times 19.\bar{6} = 3\bar{9}\bar{3}.\bar{9}$$

which can also be rounded off to $3.\bar{9} \times 10^2$.

Here we have rounded off $2\bar{0}.\bar{1}\bar{0}\bar{2}$ (the product of $5.\bar{2}\bar{9}$ and $3.\bar{8}$) to $2\bar{0}.\bar{1}$ before multiplying it with $19.\bar{6}$. We can generalise this as follows:

Before multiplying (or dividing), round off the numbers to one more significant digit than (the number of significant digits) in the least precise factor.

You may now apply this rule to answer the following SAQ.

SAQ 6 : Division of approximate numbers
Divide $9.536\bar{2}$ by $3.\bar{2}$.

Spend 3 min.

In the next unit we will discuss the different types of errors which arise due to the defects in measuring instruments, fluctuations in the quantity to be measured and such other reasons. You will also learn how these errors get propagated and influence the final result.

Let us now sum up what you have learnt in this unit.

I.6 SUMMARY

- Precision of any measurement depends on the least count of the measuring instrument.
- The result of every measurement is expressed in numbers such that only the last digit contains error.
- In scientific notation, a measurement is expressed as a decimal number between one and ten multiplied by power of ten.
- Relative error is the ratio of probable error to total measurement. Accuracy is related to relative error.
- A digit is significant if and only if it affects the relative error.
- While adding (or subtracting) approximate numbers, round off the sum (or difference) to the same unit of measure as the least precise measurement.
- The product (or quotient) of two measurements should be rounded off to contain as many significant digits as the measurement having the least number of significant digits.

I.7 SOLUTIONS AND ANSWERS

Self-Assessment Questions

1. a) 19.87 cm

   b) 3.21 s

   c) Equally precise

2. a) The relative errors are:

$$\frac{0.05}{40} = \frac{5}{4000} = \frac{1}{800}$$

and

$$\frac{0.05}{8} = \frac{5}{800} = \frac{1}{160}$$

Therefore, the measurement 40.0 cm is more accurate. However, both measurements are equally precise.

b) The measurement 0.85 m is more accurate but as precise as 0.05 m.

3. 3×10^{-23}g

4.

S.No.	Measurement (m)	No. of significant digits	Unit of measurement (m)	Possible Error (m)	Relative error
1.	0.2	1	0.1	0.05	$\frac{0.05}{0.2} = 0.25$
2.	0.20	2	0.01	0.005	$\frac{0.005}{0.20} = 0.025$
3.	0.2000	4	0.0001	0.00005	$\frac{0.00005}{0.2000} = 0.00025$
4.	25	2	1	0.5	$\frac{0.5}{25} = 0.02$
5.	250	3	1	0.5	$\frac{0.5}{250} = 0.002$
6.	25000	5	1	0.5	$\frac{0.5}{25000} = 0.00002$
7.	102	3	1	0.5	$\frac{0.5}{102} = 0.0049$
8.	1002	4	1	0.5	$\frac{0.5}{1002} = 0.000499$
9.	0.02	2	0.01	0.005	$\frac{0.005}{0.02} = 0.25$
10.	0.00	2	0.01	0.005	–

a) They are significant.

b) They are also significant. As a rule, only those zeros are significant which come from a measurement. Since the unit of measurement is 1 m in both these cases, the zeros trailing the numbers are arising out of the measurement and hence are significant.

c) Significant.

5. i) $6.3\overline{9}$ m

ii) $0.0\overline{4}$ m

6. $3.\overline{0}$