EXPERIMENT

5

DETERMINATION OF YOUNG'S MODULUS BY BENDING OF BEAMS

Structure

5.1 INTRODUCTION

A **beam** is a bar of uniform cross-section (circular or rectangular) of a homogeneous, isotropic (same properties at all points and in all directions) elastic material.

If you press a rubber ball or a piece of sponge, you will observe that their shape undergoes a change. What happens when you stop pressing them? You will observe that they regain their original shape. In fact, all bodies can, more or less, be deformed by a suitably applied force and when the deforming force is removed, they tend to recover their original state. The simplest case of deformation is observed when we stretch a wire fixed at one end. Addition of further weight at its lower end increases its length. When the suspended weight is removed from the wire, it tends to come back to its original length. You may similarly have observed that a train running over the rails produces a depression in them. However, they attain their normal state once the train has passed. It means that a body opposes any change in its shape and/or size by an external force. And once the external force is removed, the body tends to regain its original normal state. This property is called **elasticity**. Greater the force necessary to produce deformation in the body, more elastic it is said to be.

When a body is subjected to a deforming force, an opposing force comes into play and tends to resist the effect of applied force. In equilibrium state, the restoring force is equal to the applied external force. The restoring force per unit area set up inside the body is called **stress**. The fractional change in the length, volume or shape of the body is termed as **strain**. For example, when a wire is stretched by applying a force along its length, i.e. normal to its cross-sectional area, the change occurs in its length. The change in length per unit original length of the wire is called **longitudinal strain**. The ratio of stress to longitudinal strain, within the elastic limit, is called **Young's modulus**. The value of Young's modulus depends on the nature of the material rather than the physical dimensions of the sample.

The maximum stress that a material can sustain without undergoing permanent deformation is termed as its **elastic limit**.

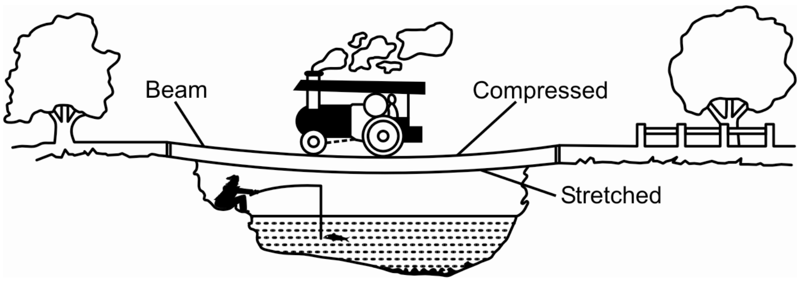

Fig.5.1: A railway engine moving over a railway bridge produces depression in the beam

Knowledge of Young's modulus is vital for bridge design as we need to know the precise deformation (depression) in a loaded structure and its parts. Refer to Fig. 5.1. When a train passes over the bridge, the beam bends. Its upper surface is compressed whereas the lower surface is stretched. These deformations are transmitted to other parts of the bridge also. Young's modulus also enables us to know the stress which a body, say the connecting rod or piston of a steam engine or a girder, can bear. (You must have observed the girders and beams used in bridges and high rise buildings. The girders are manufactured with their cross-section in the form of the letter I. In a beam of rectangular cross-section, the longer side is used as the depth.) In this experiment you will learn to determine Young's modulus of a material by the method of bending of beams.

Objectives

After performing this experiment, you should be able to:

- focus a microscope and a telescope on a given object;
- remove parallax error;
- measure small depressions;
- compare accuracies of the methods used for measurement of depression of the beam using (i) a microscope and (ii) a telescope and optical lever arrangement; and
- calculate the value of Young's modulus of elasticity.

5.2 DEPRESSION OF A BEAM

When a beam is supported near its ends and loaded at the centre, it shows maximum depression at the loaded point. However, the depression produced in the beam depends on its material; in a steel beam, it is so small that you cannot observe it with unaided eye. Refer to Fig. 5.2, which shows a beam supported on two knife-edges indicated by *A* and *B*. Suppose that it is loaded in the middle at *C* with a weight *W*. The reaction at each knife edge can be taken to be ($W/2$) in the upward direction. In this position, the beam may be considered as equivalent to two inverted cantilevers, fixed at *C*. The bending in these two cantilevers will be produced by the reaction load – acting upwards at *A* and *B*. Therefore, it is important for us to know how bending is produced in a cantilever and on what factors it depends.

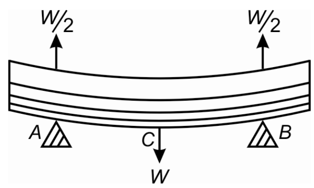

Fig.5.2: A beam supported near the two ends and loaded at the centre

5.2.1 Working Principle of a Cantilever

A Cantilever is a beam fixed horizontally at one end.

Consider the cantilever shown in Fig. 5.3a. Suppose that weight W_1 is acting at the free end. As soon as the beam is loaded, it bends. Do you know why? To discover answer to this question, consider the section P_1QRP_2 of the beam. Since the load W_1 has been applied at the free end of the beam, the restoring force acts vertically upward along P_2 P_1. These two forces, being equal and opposite, form a couple. You will recall from your elementary physics classes that couple has a tendency to rotate a body. However, a cantilever cannot rotate because it is fixed at one end. Therefore, the beam bends in the clockwise direction. This is indicated by the arrow. (For this reason, this couple is called **bending couple** and the moment of this couple is called **bending moment**.)

You may now ask: How can a beam be in equilibrium when a couple acts? This can happen when a balancing couple is acting on the beam. To know how balancing couple is formed, let us understand what changes take place in the interior of the beam when its free end is loaded. For this purpose, imagine the beam to be made up of a large number of small elements placed one above the other. These small elements are called **filaments**. When a cantilever is loaded, the filaments in the upper-half of the beam are stretched and the filaments in the lower-half are compressed. However, a surface (or filament) exists in the middle, which is neither stretched nor compressed. This surface, known as **neutral surface,** is denoted by *EF* in Fig. 5.3b.

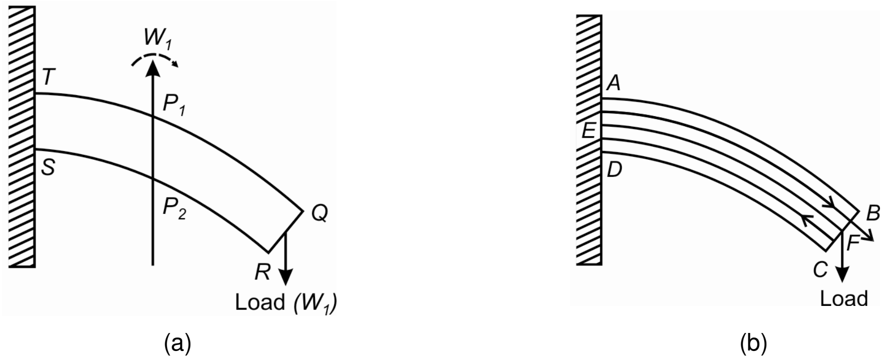

Fig.5.3: a) When a cantilever is loaded, it bends; and b) filaments in the interior of a cantilever under the action of a bending couple

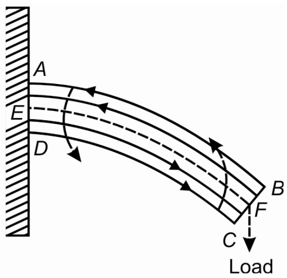

Fig.5.4: The moments of forces about the neutral axis indicated by dotted arrows oppose bending

Due to changes induced by the couple, restoring forces develop in the filaments, as shown in Fig. 5.4. Above the neutral surface, these forces act towards the fixed end of the beam and tend to oppose extension. On the other hand, below the neutral surface, restoring forces act towards the loaded end and oppose further contraction. These two sets of forces act in opposite directions and their moments about the neutral surface are directed in the anticlockwise direction (indicated by dotted arrows). This direction is opposite to that in which the beam has been bent due to the bending couple. This set of forces constitutes balancing couple and tends to restore the beam to its original condition. When the beam is in equilibrium, the moment of couple is equal to the bending moment. You may now like to know the factors on which the bending moment depends.

5.2.2 Bending Moment

Consider a small portion of the beam shown in Fig. 5.5a. It is bent in the form of an arc. Suppose that an element *ab* on the neutral surface subtends an angle θ at the centre of curvature. Also let *R* be the radius of curvature of the part *ab* of the neutral surface. Then the length of portion $a'b'$ of a filament, which is at a distance *z* from the neutral surface (filament), can be expressed as $a'b' = (R + z)\,\theta$.

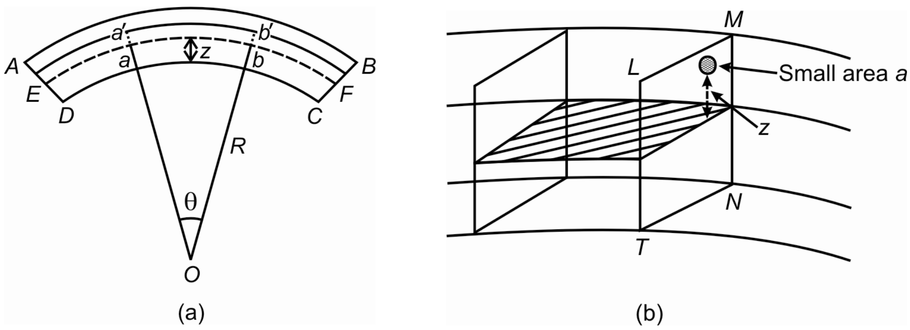

Fig.5.5: a) A small portion of the beam in strained condition; and b) *LMNT* is cross-section of the beam perpendicular to its length and the plane of bending.

When the beam is not bent, the length of this filament is equal to the length $R\theta$ of the neutral filament. Therefore, increase in length can be written as

$$a'b' - ab = (R + z)\,\theta - R\,\theta = z\,\theta. \qquad (5.1)$$

Hence

$$\text{Longitudinal strain} = \frac{\text{Increase in length}}{\text{Original length}} = \frac{z\theta}{R\theta} = \frac{z}{R}. \qquad (5.2)$$

Now consider a section *LMNT*, which is perpendicular to the length of the beam and its plane of bending, as shown in Fig. 5.5b. In this section, consider a small element of area *a* at a distance *z* from the neutral surface. The strain produced in the filament passing through this area will be *z/R*.

From the preceding sub-section, you may recall that whenever the length of a filament increases, a force acts on the filament towards the fixed end of the beam. You can calculate the magnitude of this force by noting that

$$Y = \frac{\text{Stress}}{\text{Longitudinal strain}},$$

so that stress is a product of Young's modulus of the material of the beam and longitudinal strain. This shows that stress on area *a* is

$$S = Y\frac{z}{R}. \qquad (5.3)$$

And the magnitude of force acting on area *a* is given by

$$F = \text{Area} \times \text{Stress} = aY\frac{z}{R}. \qquad (5.4)$$

Moment of this force about the neutral surface is equal to the product of force and its distance from the neutral surface:

$$M = Ya\frac{z}{R}z = Ya\frac{z^2}{R}. \qquad (5.5)$$

The total moment of the forces acting on all the filaments in the section *LMNT* (or in the beam) is given by:

$$\sum\frac{Yaz^2}{R} = \frac{Y}{R}\sum az^2 = \frac{Y}{R}I_g \qquad (5.6)$$

where $I_g = \sum az^2$ is the moment of inertia of the beam. Thus, the bending moment of the beam is given by $\frac{Y}{R}I_g$.

You may now like to know the relation between moment of the restoring couple and the depression at the free end of the cantilever.

5.2.3 Depression at the Free End of a Cantilever

Refer to Fig. 5.6. It shows a cantilever of length ℓ loaded at the free end. *AB* represents its neutral axis. Let us choose x-axis along its length and *y*-axis

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$\sum az^2$ is moment of inertia, I_g of the beam about the neutral surface. Therefore, it is equal to AK^2, where *A* is area of cross section of the beam and *K* is its radius of gyration about the neutral surface. For a rectangular cross-section, $A = b \times d$ and

$K^2 = \frac{d^2}{12}$, where *b* is length and *d* is width of the rectangular portion.

$$\therefore\ I_g = AK^2 = \frac{bd^3}{12} \qquad \text{(i)}$$

For a circular cross-section,

$A = \pi r^2$ and $K^2 = \frac{r^2}{4}$

where *r* is its radius.

$$\therefore\ I_g = AK^2 = \frac{\pi r^4}{4} \qquad \text{(ii)}$$

Refer to any elementary book on differential calculus. The complete expression for radius of curvature is given by

$$\frac{1}{R} = \frac{(d^2y/dx^2)}{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}$$

For small bending, $\frac{dy}{dx} << 1$ and we can ignore it in comparison to one in the denominator of above expression. This leads us to simple expression

$$\frac{1}{R} = \frac{d^2y}{dx^2}$$

vertically downwards. When the free end of the cantilever is loaded with a load W_1, the maximum depression occurs there. The neutral axis takes new position AB' and end B is depressed by δ. Consider a section P of the beam at a distance x from end A. Due to the load W_1, the bending moment acting on this section is given by

$$W_1 \times PB = W_1 (\ell - x).$$

Since the beam is in equilibrium, this must be equal to $\frac{YI_g}{R}$, the moment of resistance to bending. Thus, we can write

$$W_1(\ell - x) = \frac{YI_g}{R}. \qquad (5.7)$$

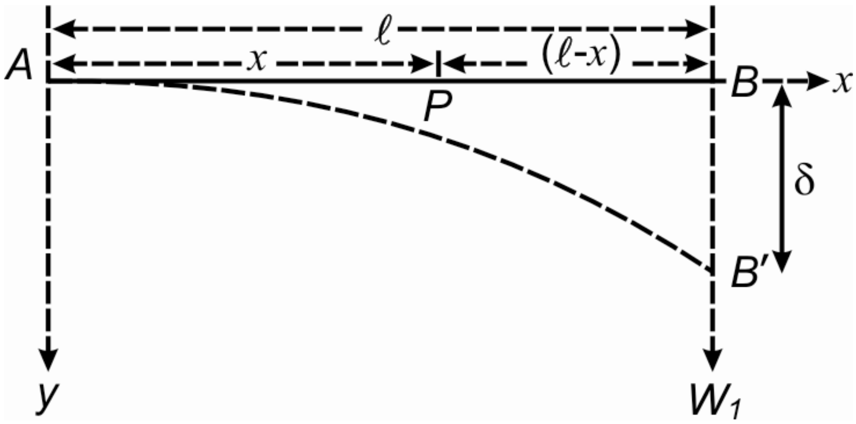

Fig.5.6: A cantilever of length ℓ loaded at the free end

Since the neutral surface remains unstretched, its radius of curvature (R) at any given point is given by the relation $\frac{1}{R} = \frac{d^2y}{dx^2}$.

Substituting this value of R in Eq. (5.7), we get

$$W_1(\ell - x) = YI_g \frac{d^2y}{dx^2}$$

or

$$\frac{d^2y}{dx^2} = \frac{W_1}{YI_g}(\ell - x). \qquad (5.8)$$

Integrating Eq. (5.8) twice with respect to x, we get the value of depression (δ) at the free end:

$$\delta = \frac{W_1 \ell^3}{3YI_g} \qquad (5.9)$$

That is, the depression at the free end of the cantilever is $\frac{W_1\ell^3}{3YI_g}$.

Integrating Eq. (5.8) with respect to x we get

$$\frac{dy}{dx} = \frac{W_1}{YI_g}\left(\ell x - \frac{x^2}{2}\right) + C_1$$

where C_1 is constant of integration.

When $x = 0$, $\frac{dy}{dx} = 0$.

Hence $C_1 = 0$.

$$\therefore \frac{dy}{dx} = \frac{W_1}{YI_g}\left(\ell x - \frac{x^2}{2}\right).$$

At the free end of the beam ($x = \ell$), $y = \delta$. Hence again integrating, between the applicable limits we have

$$\int_0^{\delta} dy = \frac{W_1}{YI_g}\int_0^{\ell}\left(\ell x - \frac{x^2}{2}\right)dx$$

$$\therefore \delta = \frac{W_1}{YI_g}\left(\frac{\ell^3}{2} - \frac{\ell^3}{6}\right)$$

$$= \frac{W_1\ell^3}{3YI_g}.$$

SAQ 1 : Depression in a cantilever

Spend 3 min.

By looking at Eq. (5.9), list the factors on which the depression at the free end of a cantilever depends.

Again refer to Fig. 5.2. If the length of the beam *AB* is *L*, the length of both cantilevers (*AC* or *BC*) will be *L* / 2. Since the reaction at each knife-edge is *W* / 2, we can assume that each cantilever (*AC* or *BC*) is loaded at the free end by a load *W*/2. Then Eq. (5.9) can be used to calculate elevation Δ of *A* or *B* above *C* by substituting $W_1 = W/2$ and $\ell = L/2$:

$$\Delta = \frac{\frac{W}{2}\left(\frac{L}{2}\right)^3}{3YI_g}$$

$$= \frac{WL^3}{48YI_g}.$$

The elevation of *A* or *B* above *C* is the same as the depression of *C* below *A* and *B*. Therefore, on rearranging the above result, you can write

$$Y = \frac{WL^3}{48\Delta I_g}.$$

For a beam with a rectangular cross section of width *b* and depth (or thickness) *d*, $I_g = bd^3/12$. Hence, in terms of the dimensions of the beam, the expression for Young's modulus simplifies to

$$Y = \frac{WL^3}{4\Delta bd^3}. \qquad (5.10)$$

From this result, you will note that to determine Young's modulus you have to measures Δ, the depression at the center of the beam when it is loaded with a known weight *W*. For steel bars, the magnitude of depression is very small, and has to be measured very accurately. For this purpose, you will learn to use a microscope as well as a telescope and optical lever arrangement. Now, we discuss the methods used to obtain Δ.

The apparatus required for this purpose is given below:

Apparatus

A rectangular steel beam, two knife-edges, a microscope, a pin, an optical lever, a telescope, lamp and scale arrangement, metre scale, a hanger, a set of half-kilogram weights, vernier callipers and a screw gauge.

5.3 MEASUREMENT OF DEPRESSION IN A BEAM USING A MICROSCOPE

Suppose n divisions of the vernier scale coincide with α divisions of the main scale, each of which is x mm. Then, value of n vernier divisions = αx
or

1 vernier division = $\frac{\alpha x}{n}$

$\therefore$ least count = 1 MSD – 1 VSD

$$= \left(1 - \frac{\alpha}{n}\right) x = \left(\frac{n-\alpha}{n}\right) x$$

Normally $n - \alpha = 1$. Hence,

least count = $\frac{x}{n}$

$$= \frac{\textbf{value of one MSD}}{\textbf{number of VD}}.$$

If $x = 0.05$ cm and $n = 50$,

Least count = $\frac{0.05\text{cm}}{50} = 0.001\,\text{cm}$

To set up the experiment, place the given beam horizontally on the knife-edges, as shown in Fig. 5.7. See that equal (but small) portions of the beam project beyond the knife-edges and the smaller side of its cross-section is vertical. Suspend a hanger for loading the beam, exactly at the centre, between the two knife-edges. Attach a small pin (vertically) at the centre of the beam with wax for reading the position of the beam. Focus the microscope on the pin and coincide its horizontal cross-wire with the tip of the pin. If you are not able to focus the microscope on the pin, you should seek the help of your counsellor. Before you start taking observations, you should calculate the least count of the microscope. For this purpose, note the value of the smallest division of the main scale of the microscope and the number of divisions on the vernier scale. The difference between the value of the smallest division of the main scale and the value of one division of vernier scale gives its **least count**.

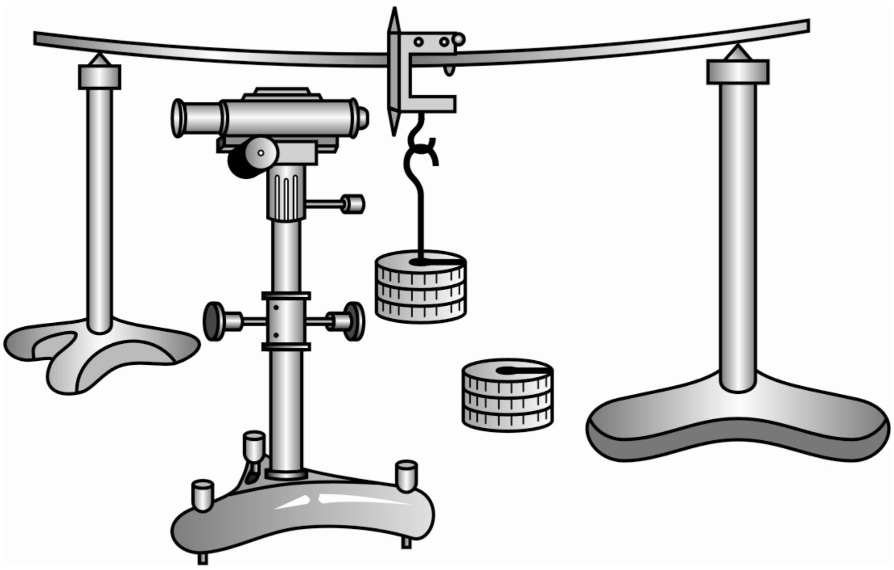

Fig.5.7: Experimental arrangement for measuring depression of the beam using a microscope

Once you have focused the tip of the pin and coincided it with the horizontal cross-wire, you are ready to perform your experiment. Read the main scale and the vernier scale readings. This is the reading when no load is placed in the hanger. Record it in Observation Table 5.1. Next, without disturbing anything at all, place a weight of half-a-kilogram in the hanger. Is the tip of the pin visible in the field of view of the microscope? If so, does the tip of the pin coincide with the horizontal cross-wire? We expect that the tip will not coincide with the horizontal cross-wire because the beam has been depressed at the centre. You should observe a gap between the tip of the pin and the horizontal cross-wire. Move the microscope vertically downward and make the tip of the pin to again coincide with the horizontal cross-wire of the microscope. Note the main scale and the vernier scale readings. Record these in Observation Table 5.1.

Increase the load in equal steps of half-a-kilogram. Note the position of the pin by coinciding it with the horizontal cross-wire every time. Now remove the weights gently in the same steps and note the microscope readings again. This is to be repeated till there is no weight on the hanger. Note that **the weight should be placed or removed from the hanger very gently**.

Observation Table 5.1: Measurement of depression using a microscope

Value of 1 MSD of the microscope (x) = …. cm
No. of vernier scale divisions (n) = ...cm
Least count of the microscope (x/n) = …..cm

S. No.	Load placed on the hanger W (g)	Microscope reading when the tip of the pin coincides with the horizontal cross-wire			Depression Δ (cm)
		with load increasing (cm)	with load decreasing (cm)	Mean (cm)	
1.	0				
2.	500				
3.	1,000				
4.	1,500				
5.	2,000				
6.	2,500				
7.	3,000				
8.	3,500				

This will give you two readings for each load: one with load increasing and the other with load decreasing. Calculate the mean of these two readings for a given load. Calculate the depression produced in the beam for each load by subtracting the initial mean reading from the mean reading for that particular load.

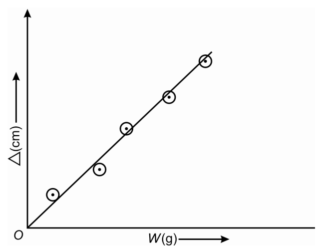

Fig. 5.8: Graph between load *(W)* and depression (Δ)

Plot a graph between the load (along x-axis) and depression (along y-axis). We expect the plot to be a straight line. Draw the best straight line passing as closely as possible through the observed points, as shown in Fig. 5.8. Calculate the slope of the straight line by choosing two widely separated points. The slope will give you the value of Δ/W.

SAQ 2 : Elastic limit

Spend 2 min.

Why is it necessary to take readings with decreasing load as well?

5.4 MEASUREMENT OF DEPRESSION IN A BEAM USING A TELESCOPE AND AN OPTICAL LEVER

To measure depression in a beam using a telescope, you will require an optical lever and a lamp and scale arrangement. (An optical lever consists of a plane mirror mounted on a tripod stand.) To set up the apparatus, place the beam as in the previous part of this experiment. Remove the vertical pin and replace it by an optical lever such that the two legs supporting the mirror *M* rest on the fixed horizontal base *F* behind the beam and the third leg *L* rests on the beam at its center *C*, as shown in Fig. 5.9. What will happen if you place the two legs supporting the mirror on the beam and the third leg on the base? If you do so, the depression will not correspond to the one at the centre. It is important to adjust the mirror so that it is vertical and parallel to the length of the beam.

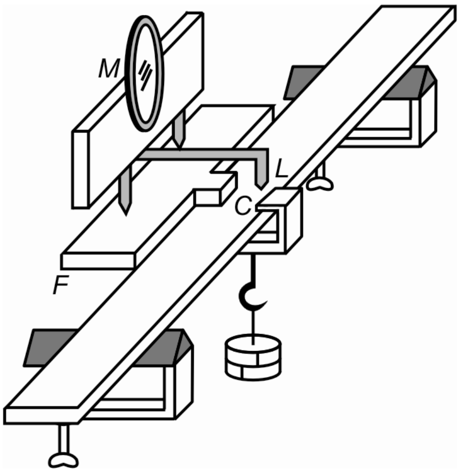

Fig.5.9: Experimental arrangement for measuring depression of a beam using a telescope and an optical lever

When a load is placed on the hanger, depression is produced in the beam. As a result, the leg of the optical lever touching the centre of the beam would go down. This would tilt the mirror forward. So, to know the depression in this part of the experiment, you have to measure the angle through which the mirror tilts. This requires the use of a telescope and a lamp and scale arrangement.

We know that when a beam of light is incident on a plane mirror, which is turned through an angle θ about a vertical axis in its plane, the reflected ray turns through twice the angle.

Fix a vertical scale in front of the mirror at a distance of about one metre on a rigid stand so that its image is visible in the mirror. Place the telescope close to the scale and at the same height as the mirror. Focus the eye piece so that the horizontal cross-wire of the telescope is distinctly visible. Now focus the telescope on the image of the scale in the mirror. For this focusing, you may have to turn the mirror slightly about its horizontal axis. If you are not able to focus the image of the scale clearly, you should not waste time. Seek guidance of your counsellor and you should practice it a few times on your own thereafter. When you can clearly see the image of the scale in the mirror through the telescope, note the position of the horizontal cross-wire on the image of the division of scale and record it in Observation Table 5.2.

What does the position of the horizontal cross-wire signify? Refer to Fig. 5.10. Here M_1 is the initial position of the plane mirror. This means that what you have recorded is in fact division *A* of the scale. Now gently place a load of 500 g on the hanger. This would depress the beam slightly. As a result of this, the mirror will tilt forward through an angle, say θ. Now, Instead of division *A* of the scale, you will see another division on the scale, say *B* (see Fig. 5.10) in the telescope after reflection from the plane mirror. Record its position in Observation Table 5.2.

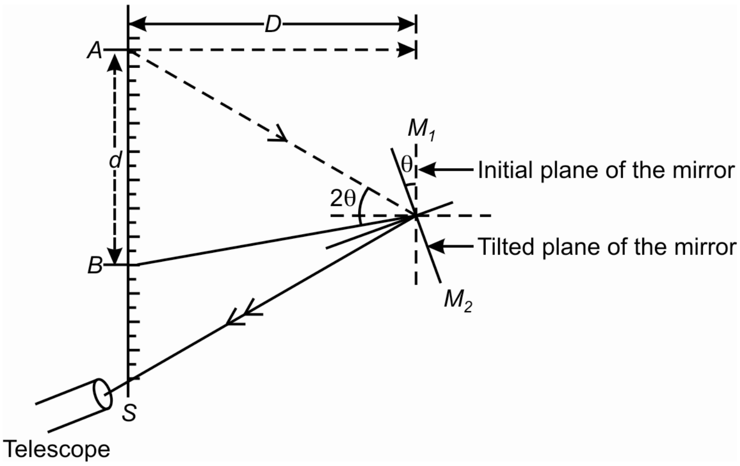

Fig.5.10: Illustrating the principle of optical lever

If distance between the two divisions *A* and *B* on the scale is *d* and *D* is the distance between the mirror and scale, then

$$2\theta = \frac{d}{D}.$$

If the third leg is at a perpendicular distance x from the hind legs *P* and *Q*, the depression, Δ, of the beam for the given load is given by

$$\Delta = x\theta \; = \frac{xd}{2D}. \qquad (5.11)$$

Note that once x, *d* and *D* are known, Δ can be readily computed.

Measure the distance *D* between the mirror and the scale. To measure x, you should place the optical lever on a sheet of paper and press it lightly so that impressions of its feet are obtained on it. From these impressions, determine the perpendicular distance of the front foot of the optical lever from the line joining the hind legs. Using Eq. (5.11) you can readily know the depression of the beam for a given load.

Increase the load on the hanger in equal steps of half-a-kilogram. Note down the position of the horizontal cross-wire of the telescope on the image of the scale after each addition of load.

Next, decrease the load on the hanger in the same steps and note the position of the cross-wire on the image of the scale in the mirror every time. Record it in Observation Table 5.2. For each load, calculate the mean value of the two readings – one taken while increasing the load and the other while decreasing the load – of the cross-wire thus obtained. Calculate d for each load by subtracting the initial mean reading (d_0) from the mean reading for that particular load. Using Eq. (5.11), calculate the depression of the beam (Δ) for each load and record it in Observation Table 5.2.

Observation Table 5.2: Measurement of depression using a telescope and an optical lever

Distance D of the scale from the mirror of optical lever =……………………… cm

Perpendicular distance x of the front foot of the optical lever from the line joining the other two legs =……………………… cm

S. No.	Load (W) placed on the hanger (g)	Position of the horizontal cross-wire of the telescope on the image of the scale (cm)			$d = s - d_0$ (cm)	$\Delta = \frac{dx}{\quad}$ (cm)
		with increasing load	with decreasing load	Mean (s)		
1.	0			$d_0 = \ldots$		
2.	500					
3.	1,000					
4.	1,500					
5.	2,000					
6.	2,500					
7.	3,000					
8.	3,500					

Plot a graph between load (W) along the x-axis and depression (Δ) along the y-axis. You should preferably use the same scale as was done in case of W–Δ graph for a microscope. Calculate the slope of the straight line thus obtained. We expect it to be same as that obtained in Sec. 5.3.

5.5 COMPARISON OF ACCURACIES

To compare the accuracies in measurement of depression using a microscope and a telescope, you have to calculate Young's modulus in both cases. To do so, you should measure the thickness and width of the beam and its length between the knife edges. To measure the length of the beam between the knife edges, you can use a metre scale. Using different parts of the scale, repeat the measurement several times and get the mean value. Record your readings in Observation Table 5.3(a).

Observation Table 5.3(a): Length (*L*) of the beam between knife-edges *A* and *B*

S. No.	Scale reading for the knife-edge *A* x_1 (cm)	Scale reading for the knife-edge *B* x_2 (cm)	Length $(x_2 - x_1)$ cm
1. 2. 3. 4. . . .			

Mean length *L* (cm) = ………………….

Use a screw gauge to measure the thickness of the beam at several places along its length. Make your own Observation Table 5.3(b) and calculate mean thickness. Similarly take a number of readings to measure the width of the beam with vernier callipers at several places. Record the readings in Observation Table 5.3(c). Calculate the mean value.

Observation Table 5.3(b): Measurement of thickness (*d*) of the beam using a screw gauge

Least count of the screw gauge =……...cm

Zero error (if any) =……...cm

Zero correction (if there is zero error)=……...cm

Mean thickness = ………..cm

Corrected value (if zero correction is made) = ………..cm

Observation Table 5.3(c): Measurement of width (*b*) of the beam using vernier callipers

Least count of the vernier callipers = ……..cm

Zero error (if any) =……...cm

Zero correction (if there is zero error) =……...cm

Mean width = ………cm

Corrected value (if zero correction is made) = ………cm

Spend 5 min.

SAQ 3 : Measurement of small lengths

Suppose that a good screw gauge or vernier callipers is not available in your lab to measure *d* and *b*. Which device – a metre scale, a graph paper, a microscope or a telescope – will you use or recommend. Justify your answer.

Knowing *L*, *b*, *d* and the slopes of the straight lines obtained using a microscope and a telescope, you can easily calculate Young's modulus of the material of the beam using Eq. (5.10):

$$Y = \frac{L^3}{4bd^3} \times \frac{1}{\text{slope}} = \ldots\ldots\ldots\ldots \text{ dynes cm}^{-2}$$

$$= \ldots\ldots\ldots\ldots \text{ N m}^{-2}$$

Result: Young's modulus of the material of the given beam using microscope

=……..N m^{-2}

Result: Young's modulus of the material of the given beam using telescope and optical lever =……. N m^{-2}

Which of these results is closer to the standard value? Theoretically, the accuracy to which the depression is measured using a microscope is equal to the least count (L.C.) of the microscope. Suppose that L.C. of microscope is 0.001 cm.

In the case of optical lever arrangement, the least count of vertical scale is 0.1 cm. This is multiplied by the factor x/D (see Observation Table 5.2).

If D = 1m = 100 cm and x = 3 cm, then $\frac{x}{2D} = \frac{3}{200} = 0.015$. So the least count of measurement of depression by the optical lever arrangement = 0.1 × 0.015 = 0.0015 cm.

This shows that measurement of depression with microscope (and hence value of *Y*) is more accurate than with an optical lever arrangement. But the optical lever method can be made to give better results than microscope method. For this you have to think of a way to improve the least count for the measurement of depression by optical lever arrangement. You may, for instance, use a half-millimetre scale instead of mm scale. The least count of the measurement of depression with the optical lever arrangement depends on (i) the length of tilting arm of the optical lever, x, and (ii) the distance between the mirror and the scale *D*. It may not be possible to adjust x unless you can use another optical lever. However, if we use a high power telescope so that *D* can be as large as possible, say 3 m, the optical-lever method can yield more accurate results.