EXPERIMENT 4

RELATION BETWEEN WAVELENGTH AND FREQUENCY OF STATIONARY WAVES

Structure

4.1 INTRODUCTION

You all must have enjoyed the pleasing music produced by string instruments like sitar, violin, guitar, ektara, etc. at a concert or on a radio or a television. Do you know how stringed instruments produce music? When the string of such an instrument is plucked, bowed or struck, it begins to vibrate and produces sound. The quality of sound depends on the frequency of vibration of the stretched string. Now the question arises: What factors determine the frequency of vibration of a string? How are these factors related to frequency? In this experiment, you will discover answers to such questions.

Tuning a given musical instrument with another means adjusting the frequency of the given instrument so that it is the same as that of the other one.

You may have observed that in an orchestra, a violinist tightens or loosens the pegs of the instrument while tuning with other musicians. (As the peg is tightened or loosened, a portion of the string is either wound or unwound around the peg.). As a result, tension in the string changes. This suggests that the frequency produced by the string of the violin depends on the tension in it. Can you think of other parameters that may influence the frequency of vibration of a string? What happens if you take strings of same material having different thicknesses or strings of different materials but same thickness? Well, we expect that the frequency of vibration of the string in each case should differ. This means that the mass per unit length of the string also influences its frequency of vibration. Is it why the strings of guitars are wrapped with a metal winding?

You may have seen a veena. In this musical instrument, strings of unequal lengths are tied between two fixed ends. You may have also seen that once a musician has tuned the instrument, she moves her fingers along its string to produce music. In this way, she varies the vibrating length in order to produce different notes. This suggests that the frequency of vibration of the string depends on its vibrating length as well. We know that the length of the vibrating segment of the string is related to the wavelength of the stationary waves set up in it. Hence, we expect that there exists a definite relationship between the wavelength and frequency.

The aim of this experiment is to know how frequency of vibrations of a stretched string depends on tension, mass per unit length and its vibrating length. You may recall from your investigations with the simple pendulum (Experiment 1) that when a physical quantity depends on more than one parameter, it makes sense to vary only one parameter at a time. In this case, any change in the frequency can be attributed to the change in that particular parameter.

It is possible to set up waves of known wavelength in a wire. But it is more convenient to make a wire vibrate with a known frequency. So you will discover the effect of tension and mass per unit length of the wire on the wavelength, keeping the frequency constant. Therefore, you should do this experiment in three parts. In the first part, you should investigate how the wavelength changes with tension in the wire while the frequency of vibration of the wire and its mass per unit length are kept fixed. In the second part, you will investigate how the wavelength varies when wires of different thicknesses (but same material) or different materials (but same thickness) are used. That is, you will learn how wavelength varies with mass per unit length of the wire when tension in the wire and frequency are not changed. In the third part, you will establish the relation between frequency and wavelength, keeping the tension and mass per unit length of the wire fixed.

Objectives

After performing this experiment, you should be able to:

- set up stationary waves in a stretched string;
- investigate the dependence of wavelength of stationary waves on tension in a string and its mass per unit length;
- establish the relation between wavelength and frequency; and
- obtain the expression for velocity of transverse waves on a string.

4.2 STATIONARY WAVES IN A STRETCHED WIRE

The measurement of tension (T) and mass per unit length (μ) of a stretched wire are rather routine exercises. But to make a precise determination of wavelength, we set up *stationary waves*. Stationary waves are formed when two identical progressive waves moving in opposite directions are made to superpose. The stationary waves do not move with time in either direction. (For this reason, they are also sometimes referred to as *standing waves.*) From your school physics, you will recall that stationary waves can be produced in air columns as well as stretched strings. Here we intend to set up stationary waves in a sonometer wire.

A wave which transports energy as it propagates in space is said to be progressive. In a stationary wave, no energy is transported.

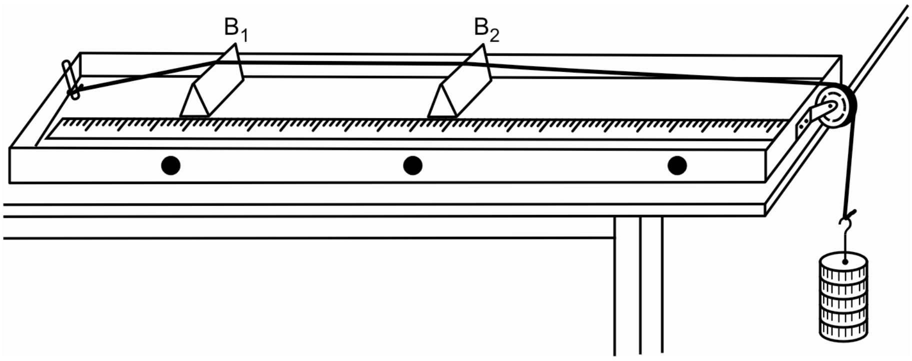

Fig.4.1: Stationary waves in a stretched string of a sonometer

Refer to Fig. 4.1. It shows a sonometer, which consists of a hollow wooden box with two circular holes, a peg at one end and a pulley on the other. One end of a wire is fixed to the peg and the other end, passing over a smooth pulley, carries a hanger. (In place of hanger, you can also use a pan.) By placing weights on the hanger, the string can be stretched. The wire passes over two bridges B_1 and B_2. While performing experiments with a sonometer,

The sonometer wire is said to vibrate in *unison* with the source of sound when the natural frequency of the wire equals the frequency of the source.

the string is made to vibrate in **unison** with the source of sound, which may be a tuning fork or an electromagnet. To achieve this, the vibrating length of the wire between the bridges is adjusted by sliding them between the peg and the pulley. This condition (of unison) is said to be ensured when the wire vibrates with maximum amplitude and a V-shaped paper rider placed in the middle of the wire between the bridges falls down.

The vibrations are said to be forced vibrations when a body vibrates with the frequency of the applied periodic force. In this condition, the energy fed from outside equals the energy lost by the body.

In your school physics, you have learnt that when a vibrating tuning fork is placed on the sounding board of the sonometer, the air inside the sonometer begins to vibrate. It makes the wire to execute forced vibrations leading to formation of transverse waves. In the region B_1 B_2, these transverse waves are reflected at the fixed points B_1 and B_2. As a result, we obtain a set of incident and reflected waves travelling in opposite directions. Their superposition gives rise to stationary waves. The wire between the bridges can vibrate in one or more well-defined segments, as shown in Fig. 4.2. Note that there are some points at which the wire remains motionless at all times. On the other hand, at some other points, the waves reinforce strongly and the wire vibrates vigorously. The points corresponding to zero amplitude of vibration are called nodes (*N*), whereas points with maximum amplitude are called antinodes (*A*). The simplest mode of vibration occurs when the string vibrates in a single loop (Fig. 4.2a). The frequency of vibration corresponding to this mode is known as the **fundamental frequency** of vibration.

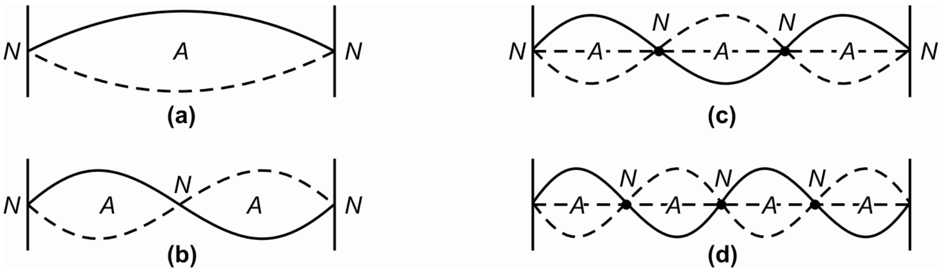

Fig. 4.2: Stationary waves set up in a wire fixed at both ends

The apparatus required for the experiment is listed below:

Apparatus

Four iron wires of different thicknesses (Alternatively 4 wires of different magnetic materials), sonometer, hanger, slotted weights, an electromagnet with a 6 volt a.c. transformer, six tuning forks of known frequencies, rubber pad, metre scale, screw gauge, a chemical balance and a weight box.

4.3 VARIATION OF WAVELENGTH WITH TENSION

In this part of the experiment, you have to keep mass per unit length (μ) of the wire and its fundamental frequency of vibration constant. Working with a wire of uniform cross section ensures constancy of μ. To achieve the latter, you can use either a tuning fork or an electromagnet. We advise you to use an electromagnet, if available, because it can make the wire execute sustained vibrations.

In case you are not provided an electromagnet, choose a tuning fork of known frequency. (You may also discuss with your counsellor.) As you know, we have “musical ears”. You can get close to the condition of unison using your ear. To do this, strike the tuning fork on the rubber pad and hold it near your left ear. Strike the sonometer wire between the bridges with your finger and hold your right ear near the sonometer wire. As long as the frequencies produced by the tuning fork and sonometer wire are not in unison, you will hear two distinct sounds with different frequencies. But by adjusting the position of bridges, gradually you can attain near unison condition. Next strike one of the prongs of the tuning fork with a rubber pad and press the stem of the fork on the sounding board of the sonometer. Do not touch its U-part. (If you do so, the vibrations of tuning fork will die rapidly.) You will observe that the wire begins to vibrate resulting in stationary waves. The paper rider placed in the middle of the wire will fall when it resonates with the turning fork.

While working with a tuning fork, you may observe that vibrations may not be sustained for long. Then you should strike the prong of tuning fork again with the rubber pad and place it on the sounding board to determine the resonating length for each load. Moreover, since the energy supplied by the tuning fork to the vibrating wire is many-fold less than that given by the electromagnet, the wire will not vibrate vigorously. Therefore, in this case, you have to rely more on the paper rider, which falls off or vibrates vigorously when unison occurs.

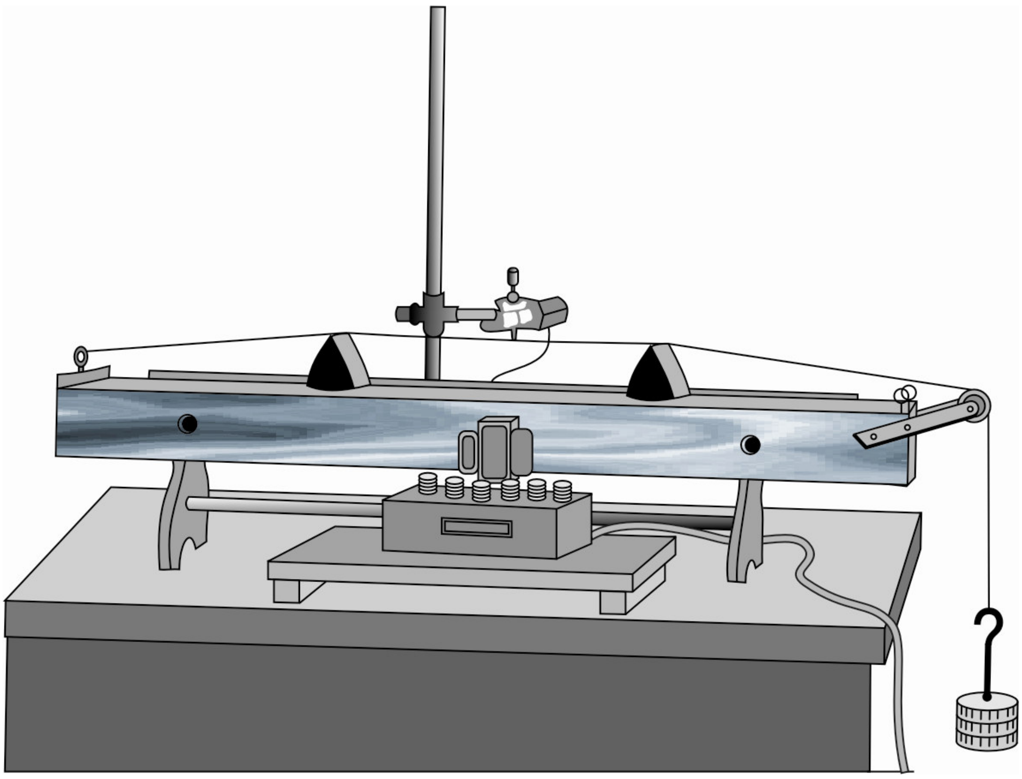

Fig.4.3: Experimental arrangement for generating transverse stationary waves in a sonometer wire using an electromagnet

The experimental arrangement for generating stationary waves in a sonometer wire using an electromagnet is shown in Fig. 4.3. Connect the electromagnet to a 6 V transformer and place it near the middle of the wire. When the electromagnet is connected to a source of ac power supply, the core of the electromagnet will be magnetized twice with opposite polarities in each cycle. As a result, the sonometer wire gets attracted towards the electromagnet twice in each cycle and begins to vibrate. Since the frequency of ac is 50 Hz, the wire will vibrate with a fixed frequency of 100 Hz.

Spend 3 min.

SAQ 1 : Inducing forced vibrations

Suppose that the electromagnet is connected to a source of direct current. Will the wire vibrate? If so, what will be the frequency of its vibrations?

Stretch the wire by putting an appropriate weight on the hanger. (You should consult your counsellor in determining this.) If the mass of the hanger is m kg and a weight of M kg is used in stretching the wire, the tension in the wire will be $T = (M + m)g$ newton; g is acceleration due to gravity.

Keep the bridges B_1 and B_2 on the sonometer at the largest possible separation and switch on the current through the electromagnet. The wire will begin to vibrate. This means that the apparatus is now in working condition and you can begin your investigations.

Your objective is to determine the length of the wire for which the sonometer vibrates in the fundamental mode. This happens when the wire vibrates in a single loop with maximum amplitude. This length represents the separation between two consecutive nodes and is equal to half of the wavelength of the stationary wave in the wire.

When you switch the current on, the wire is supposed to vibrate with a frequency of 100Hz. But you may not see the wire vibrate at all. Do you know why? This is likely to happen if the length of the wire between B_1 and B_2 is much different from that corresponding to the fundamental frequency and the amplitude of forced vibration set up in the wire is extremely small. So you have to adjust the length of the vibrating wire. To do this, keep one of the bridges (say, B_1) fixed and move the other bridge (B_2) towards it slowly. What do you observe? Does the amplitude of vibration increase? If so, continue to decrease the vibrating length of the wire by moving the bridge B_2 closer to B_1 until the amplitude of vibration becomes maximum. You will then clearly see that the wire is vibrating in a single loop of significant amplitude. If you place a paper rider gently in the middle of the wire now, it will be thrown off. Note the weight and the corresponding length between B_1 and B_2 by noting their positions on the metre scale attached to sonometer board. Record the readings in Observation Table 4.1. Next, move the bridge B_2 closer to B_1 by a small distance (2-3 cm). What do you observe? Does the amplitude of vibration change? If so, the frequency of the vibrating wire is not 100 Hz. Then, slowly move the bridge B_2 away from B_1 and locate the position where wire vibrates in unison again. You should repeat this act 3-4 times for a given tension to minimize the error in your observation. You will also hear maximum sound when the vibrating wire is in unison with the forced frequency.

Now, you change the tension in the wire by adding weights of 0.2 kg or 0.5 kg in equal steps and measure the resonating length of the wire in each case following the procedure outlined above. You will observe that the resonating length increases with increasing load. Enter your data for each step in Observation Table 4.1. You should not load the wire beyond its elastic limit. (Consult your counsellor to know this value.)

To check that you are working within the permissible range of tension, you should repeat the above procedure by unloading the wire in the same equal steps. Measure the corresponding resonating length of the wire. Tabulate each reading. Do these lengths differ from those obtained for corresponding tension while loading the wire? We expect these to be almost the same.

Observation Table 4.1: Dependence of wavelength on tension

Frequency of vibration of the wire =……………………….Hz

Least count of metre scale =…………………...….cm

Weight of the hanger (m) =……………………….kg

S. No.	Weight placed on hanger, M (kg)	Tension $T = (M+m)\,g$ (N)	Resonating length of the wire between the bridges B_1 and B_2 (m)				Mean resonating length for a given load, $\ell = \frac{\bar{L}_1 + \bar{L}_2}{2}$ (m)	Wave-length $\lambda = 2\ell$ (m)
			Load increasing		Load decreasing			
			L_1(m)	Mean value $\bar{L}_1$	L_2(m)	Mean value $\bar{L}_2$		
1.								
2.								
3.								
4.								

From the Observation Table you will observe that λ changes with T. Mathematically, we can write

$$\lambda = f(T).$$

Can you quantify this relation exactly by looking at your observations? Probably you can not. To discover the exact relationship between λ and T, we write

$$\lambda \propto T^a$$

or

$$\lambda = k_1 T^a, \quad (4.1)$$

where k_1 is constant of proportionality and a is another constant.

Taking logarithms on both the sides, we get

$$\log \lambda = \log k_1 + a \log T. \quad (4.2)$$

If log-log graph papers are not available in your laboratory, you should calculate $T^{1/2}$ and T^2 and plot λ versus $T^{1/2}$, T, T^2, etc. The graph which gives a straight line will correspond to Eq. (4.1).

Now, take a log-log graph and plot λ along y-axis and T along x-axis. You will obtain a straight line. Its intercept on the y-axis is a measure of constant of proportionality and the slope of the straight line gives the value of a. Calculate the slope by using two well separated points. We expect the value of a to be one-half.

So we can write Eq. (4.1) as

$$\lambda = k_1 \sqrt{T}. \quad (4.3)$$

Spend 5 min.

SAQ 2 : Variation in wavelength with tension

Plot a graph between λ and $T^{1/2}$. Choose the points corresponding to T_1 = 9.8 N and T_2 = 34.3 N to calculate the value of wavelength from your graph.

4.4 VARIATION OF WAVELENGTH WITH MASS PER UNIT LENGTH

Table 4.1: Densities of some typical metals

Material	Density ($\times 10^3$ kg m^{-3})
Iron	7.86
Steel	8.03
Nickel	8.912
Copper	8.96
Aluminium	2.698

To investigate the dependence of wavelength on mass per unit length of the wire, take four wires of different thicknesses but of the same magnetic material. For each wire, you first determine the mass per unit length (μ). To do so you have to weigh each wire in a physical balance and measure the corresponding lengths. The ratio (m/ℓ) will give you μ. For more precise work, you should measure their diameters (d) using a micrometer screw gauge. Note its least count and observe whether or not there is any zero error. Measure the diameter at several places. In this way you can account for the inhomogeneities, if any, in the wire. Record your readings in Observation Table 4.2(a). Calculate the mass per unit length by the relation $\mu = \frac{\pi d^2}{4} \rho$, where d is the mean diameter of the wire and ρ is the density of the material.

Table 4.2 (a): Determination of mass per unit length of a wire

Least count of the micrometre screw gauge = ……cm

Sample wire	Diameter				Mean diameter *d* (m)	Density ρ (kg m^{-3})	Mass per unit length of the wire (kg m^{-1})
	Obs. No.	Main scale reading	Circular scale reading	Total reading			
A	(i)						
	(ii)						
	(iii)						
B	(i)						
	(ii)						
	(iii)						
C	(i)						
	(ii)						
	(iii)						
D	(i)						
	(ii)						
	(iii)						

In this part of the experiment, you have to keep the tension in the wire constant. To do this, place weights of 2 kg, say, on the hanger. Do not change this weight during this part of the experiment. Now, following the procedure given in Sec. 4.3, adjust the distance between the bridges B_1 and B_2 so that the wire vibrates in one loop with the maximum amplitude. Measure the distance and record it in Observation Table 4.2(b).

Repeat this procedure for other wires, keeping the tension in the wire constant. Record your readings in Observation Table 4.2(b).

Observation Table 4.2(b): Dependence of wavelength on mass per unit length

Frequency of tuning fork/electromagnet = …………Hz
Tension in the wire = …………N

S. No.	Mass per unit length μ (kg m^{-1})	Length corresponding to unison ℓ (cm)		Mean value of ℓ (cm)	Wavelength $\lambda = 2\ell$ (m)
1.		(i)			
		(ii)			
		(iii)			
2.		(i)			
		(ii)			
		(iii)			
3.		(i)			
		(ii)			
		(iii)			

Does λ change with μ? To quantify this dependence, we write

$$\lambda = k_2\, \mu^b, \qquad (4.4)$$

where k_2 is constant of proportionality and b is another constant.

Taking logarithms on both the sides, we get

$$\log \lambda = \log k_2 + b \log \mu$$

If you plot λ versus μ on a log-log graph, you will obtain a straight line. Is the slope of the straight line positive or negative? A negative value signifies that λ decreases as μ increases. The slope of the straight line gives us the value of the exponent b. We expect $b = -\,0.5$. (Discuss your result, if there is significant deviation from the quoted value, with your counsellor.) Thus we can write

$$\lambda = k_2 \mu^{-1/2}. \qquad (4.5)$$

On combining the results contained in Eqs. (4.3) and (4.5), we obtain

$$\lambda = k\left(\frac{T}{\mu}\right)^{1/2}, \qquad (4.6)$$

Spend 10 min.

SAQ 3 : Dependence of wavelength on mass per unit length

i) How would the result of Eq. (4.6) be influenced if the wire stretched on the sonometer were hollow?

ii) Suppose you have adjusted the length of the string (of steel) to be in unison with a tuning fork. Now you replace the string with a similar one of nickel. Will the same length of the string be in unison with the fork? Why?

iii) From the graph obtained by plotting log λ versus log T from the data recorded in Observation Table 4.1, calculate the intercept on y-axis. How is it related to mass per unit length of the wire? Compare this value with the value estimated from its radius and density.

4.5 RELATION BETWEEN WAVELENGTH AND FREQUENCY

To establish the relation between wavelength and frequency for a given wire, the tension in the wire is kept fixed. To vary the frequency, you will require a set of tuning forks of different frequencies. Obviously, an electromagnet will not be appropriate for this part of your investigations because it makes the wire to vibrate with only one frequency.

To begin with, stretch the wire with an appropriate load, 2 kg weight, say. Now, out of the set of tuning forks, select the tuning fork with the lowest frequency. Keep the bridges B_1 and B_2 maximum distance apart on the sonometer. Now, as discussed in Sec. 4.3, keep B_1 fixed and shift B_2 to adjust the distance between the bridges so that the wire vibrates in one single loop of maximum amplitude. This means that the wire and the tuning fork are in unison. Measure the length and record it in Observation Table 4.3

Keeping the tension fixed, repeat the procedure for other tuning forks. Measure the length each time and record it in Observation Table 4.3.

Observation Table 4.3: Dependence of wavelength on frequency

Tension in the string =…………N

S. No.	Frequency of the tuning fork f (Hz)	Length corresponding to unison (m)			Mean length ℓ (m)	Wavelength $\lambda = 2\ell$ (m)
		(i)	(ii)	(iii)		
1.						
2.						
3.						
4.						
5.						

How does wavelength of stationary waves depend on the frequency of tuning fork (and hence fundamental frequency of the string for a fixed tension)? We expect the wavelength to decrease as frequency increases. To quantify this dependence, we express it mathematically as

$$f = k_3 \lambda^c, \qquad (4.7)$$

where k_3 is a constant of proportionality and c is some other constant.

Now, if you plot f versus λ on a log-log graph paper, you should obtain a straight line. From the slope, you can calculate the value of c. We expect the value of c to be –1. What is your result?

Also from the intercept on the y-axis, you can calculate ln k_3 and hence k_3. Compare this value of k_3 with the ratio $\sqrt{T/\mu}$ for this wire. Are the two values same? Theoretically, they should be. What does it suggest? It implies that frequency and wavelength of stationary waves on a string are connected by the relation

$$f = \frac{1}{\lambda}\sqrt{\frac{T}{\mu}}. \qquad (4.8)$$

The dimensions of the product $f\lambda$ are those of velocity (ms^{-1}). From this you can conclude that the velocity of stationary waves in a stretched string is given by $v = \sqrt{\frac{T}{\mu}}$.

Now you may like to attempt on SAQ.

Spend 2 min.

SAQ 4 : Relation between wavelength and frequency

What will be the change in frequency if the unison length of the string between the bridges is doubled?