EXPERIMENT

2

OSCILLATIONS OF A SPRING-MASS SYSTEM AND A TORSIONAL PENDULUM

Structure

2.1 INTRODUCTION

We find many uses of spiral springs in daily life. In a transistor set and a pocket calculator, springs hold dry cells in proper position. Springs are used as shock absorbers in automobiles and railway wagons. You may have also used yourself a bull-worker or seen body-builders using it. Do you know that it essentially consists of springs? In wrist watches, springs control oscillations of the system. In all these cases, the basic difference in the springs being used is in their spring constants. So to decide on the type of a spring for a particular purpose, you must know its spring constant. In a physics laboratory you can determine the value of spring constant, k, by:

i) knowing extension in the spring for a given load (static method), and

ii) determining the period of harmonic oscillations of a spring-mass system (dynamic method).

In a physics laboratory, you will come across many instruments which involve torsional oscillations. The most familiar of these are the torsional pendulum (used to calculate modulus of rigidity), inertia table (used to determine moment of inertia) and the moving coil galvanometer (used to measure charge and current). When wire in a torsional system is twisted, due to elasticity a restoring couple is set up in the wire. It tends to oppose twisting of the wire. The restoring couple per unit twist, measured in radian, is known as **torsional rigidity** or **torsional constant**. While choosing the suspension wire (fibre) for a specific purpose, we should have prior knowledge of torsional rigidity. In this experiment, you will learn to measure torsional rigidity by a simple arrangement.

Objectives

After performing this experiment, you should be able to:

- acquire skills of measuring small thickness with precision using a micrometer screw gauge;
- measure extension of a spring for a given load and calculate its spring constant (static method);
- measure the period of oscillation of a spring-mass system for different loads and calculate k (dynamic method);
- compare the accuracies of static and dynamic methods;
- calculate torsional rigidity (k_t) and modulus of rigidity of the given wire; and
- predict the material of the wire.

2.2 DETERMINATION OF SPRING CONSTANT

In Experiment 1, you investigated the question: What determines the values of T for a simple and a bar pendulum? You may now ask: Can we make similar investigations for a spring-mass system? It makes sense and you can do so along the lines outlined in Experiment 1. But now we intend to calculate the spring constant of a spring in two different ways: (i) by knowing extension for a given load, and (ii) by measuring the period of harmonic oscillations of a spring-mass system.

The apparatus required for this purpose is listed below:

Apparatus

A spiral spring, slotted weights in multiples of 100g, stop watch, a laboratory stand and a 50 cm scale

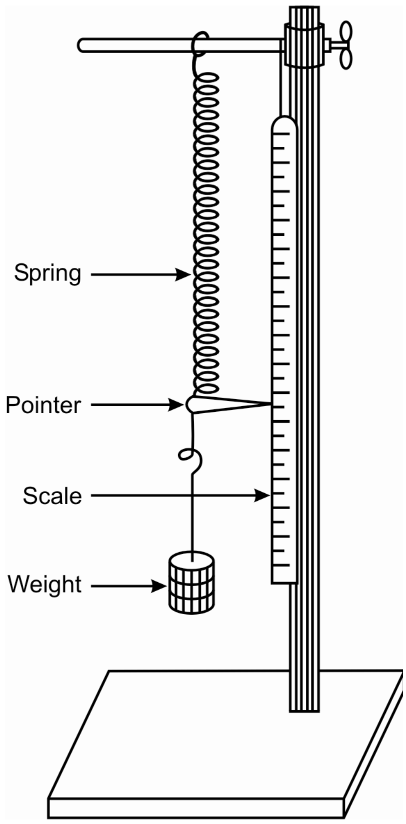

Fig.2.1: A spring-mass system

Refer to Fig. 2.1, which shows a spring and a metre scale suspended on the stand side by side. Fix a sharp-tipped pointer (needle) at the lower end of the spring. In case you do not get a needle, you can make a pointer of cardboard by cutting it in the shape of a triangle. Then you have to attach its base to the straight end of the spring so that its vertex moves in contact with the scale. This helps in minimising parallax error also. Suspend a hanger (which itself is a known weight, equal to any other slotted weight) in the hook of the spring. (Alternatively, you can tie a pan to the lower end of the spring and put weights.) Normally, it is advisable to put an initial load on the hook as it will take care of the kinks and other such inhomogeneities in the spring. This implies that the choice of the initial position does not matter.

Stretch the spring by pulling the hanger downwards through a small distance and then let it go. The spring-mass system will execute vertical oscillations. Ensure that the pointer does not stick anywhere and the oscillations are free. Now your apparatus is ready and you can start your experiment. But before you do this, do spend a few minutes making qualitative observations as to how extension/period changes when the mass is changed within **elastic limits**. This limit will be different for different springs. So you should consult your counsellor before putting a load on the spring.

2.2.1 Static Method

Load the spring by putting a weight and record the corresponding equilibrium position of the pointer. Due to elasticity, a restoring force is set up in the spring. It tends to oppose the applied force and bring the system back to its original state. If extension is small compared to the original length of the spring, the magnitude of restoring force exerted by the stretched spring on the mass is given by

$$F = -kx, \qquad (2.1)$$

where x is extension in the spring and k is spring constant.

From Eq. (2.1) it is clear that once you know extension as a function of load, k can be calculated easily. It is with this purpose that we attach a pointer to the lower end of the spring. This method of determining k is known as *static method*.

When an external force is applied on a body, it tries to retain its shape and size. And as soon as the applied force is removed, the body regains its original state. This property is called **elasticity**. Its maximum limit is called **elastic limit**. If applied force exceeds elastic limit, it produces permanent deformation and the body fails to regain its original shape and size even when the applied force has been removed.

Treat the equilibrium position of the pointer on the scale as your initial observation. Record your reading in Observation Table 2.1. Now increase the load in steps by adding equal weights each time. For each load record the position of the pointer. Before taking a reading, you should wait for some time so that the pointer comes to rest. Take at least six observations.

Observation Table 2.1: Extension as a function of load

S. No.	Load on the spring (g)	Reading of the pointer (cm)		
		Increasing load	Decreasing load	Mean reading
1.				
2.				
3.				
4.				
5.				
6.				

To ensure that you are working within the permissible elastic limit, you should record the position of the pointer by unloading the spring in the same steps. Again tabulate your readings in Observation Table 2.1. Do these readings differ from those recorded while loading the spring? If observations for a given weight are nearly the same, both while loading and unloading, you can be sure that you are certainly working within the elastic limit. Note that you have to observe the mean reading of the pointer for a given load.

If the extension corresponding to mass M is x_0, we can write

$Mg = kx_0$. (i)

Note that we are considering magnitudes only.

Let δ be the elongation corresponding to an additional load m. Then we have

$(M+m)\, g = k\,(x_0 + \delta)$. (ii)

From (i) and (ii), we get

$mg = k\delta$

or $k = \dfrac{mg}{\delta} = \dfrac{g}{\text{slope}}$. (iii)

Now you should plot a graph between the load and the corresponding elongation. Conventionally, we plot the independent variable along the x-axis and the dependent variable along the y-axis. Which physical quantity will you plot for this experiment along the x-axis? Obviously, load should be plotted along x-axis. Draw the best fit line through observed points as shown in Fig. 2.2. (For a good steel spring, we expect the graph to be linear.)

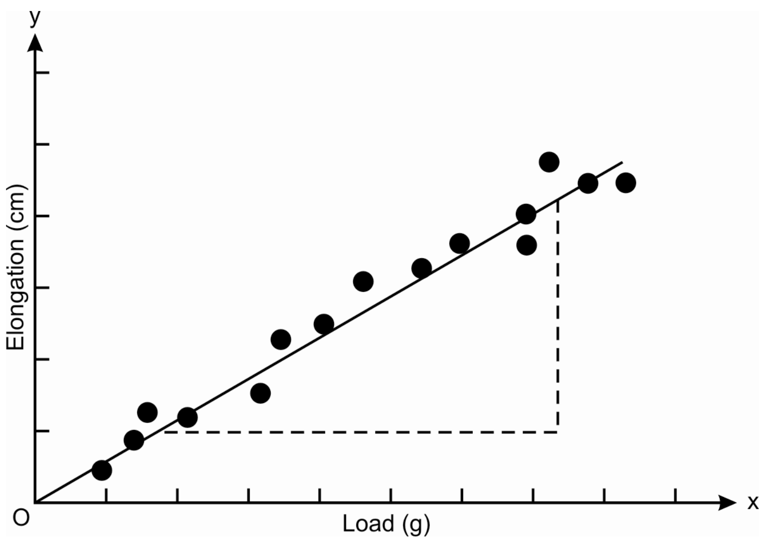

Fig.2.2: Best fit line through observed points

Does your straight line pass through the origin? The inverse of the slope of the straight line is a measure of the spring constant. To calculate the slope, you should use two widely separated points on the straight line. These should be other than observation points. Use g = 9.8 ms^{-2} to calculate k and express your result in SI units.

Suppose that the graph paper is somehow not available in your laboratory. You may then ask: How to calculate *k*? You will have to use (iii). Suppose you have taken seven readings. Then calculate extension δ for load difference between readings 4 and 1;5 and 2; 6 and 3; and 7 and 4. Calculate mean value of δ and hence *k*.

Error Analysis

Calculate the change in slope of the straight line caused by drawing lines of maximum and minimum slopes. This gives maximum error in the slope. Using g = 9.8 ms^{-2} calculate the error in the value of k in SI units.

Conclusion: The spring constant of the given spring = ………. ± …..… Nm^{-1}

SAQ 1 : Static method

Spend 2 min.

From your graph, calculate the extension for a load of 2 N.

2.2.2 Dynamic Method

In the preceding section, you learnt to determine spring constant by measuring extension of spring as a function of load. You may now ask: Is there some other method also for determining *k*? We can use the so-called *dynamic method*. It is based on observing the period of harmonic oscillations of the spring-mass system.

In Unit 1 of Oscillations and Waves course, you have learnt that a spring-mass system executes SHM like a simple pendulum, provided the extension is not large. Another question that comes to our mind immediately is: Does gravity affect the frequency of oscillations? Gravity has no effect on the frequency of oscillations. The period of oscillation is given by

$$T = 2\pi\sqrt{m/k} \ . \qquad (2.2)$$

This relation shows that we can determine *k* by knowing the period of oscillations for a given mass. The value of *m* will depend on the nature of spring. For a thin spring, *m* could be a few gram. In this experimental method, you will be required to measure the period of simple harmonic oscillations. You must ensure that oscillations of the system hanging vertically are longitudinal. That is, there should be no lateral oscillations. Otherwise, the motion will not be simple harmonic.

Put a load on the hanger and note the position of the pointer on the scale. Take it as the equilibrium position. Now stretch the spring by pulling the hanger slightly downward and then release it. The system will begin to oscillate. In case there is no lateral oscillation, your apparatus is set. Bring it to rest. Also ensure that the spring executes 20-30 oscillations before their amplitude shows visible decrease.

Note the least count of the stop watch and record it in Observation Table 2.2. Now set the spring-mass system into oscillations. Allow the first few oscillations to pass so that there is no anharmonic component. Begin your counting through the equilibrium position and simultaneously start the stop watch. Note the time for *N*, say 30, complete oscillations. To minimise the error in *T*, it is desirable to take time for 50 or more oscillations. However, you must ensure that the amplitude of oscillations does not decay significantly. Enter your reading in the Observation Table 2.2. Add more weights in the hanger and repeat the procedure at least five times. Tabulate your observations. How does the time period change? As before, the procedure may be repeated by decreasing the load in same steps. Calculate the mean time for each load.

Observation Table 2.2: Measurement of time as a function of load

Least count of stop watch = …………….. s
Number of complete oscillations counted each time (*N*) =……….

S. No.	Load on the spring *m* (g)	Time for *N* complete oscillations (s)			Time period $T = \frac{t}{N}$ (s)
		with load increasing	with load decreasing	mean (*t*)	
1.					
2.					
3.					
4.					
5.					
6.					

Plot T^2 versus *m*. Draw the best possible straight line as shown in Fig. 2.3. Does it pass through the origin? From the slope of the straight line, you can easily calculate *k*. Check if this value agrees with that obtained by the static method. The two values should be same or nearly equal. (In case you get to know the standard value of *k* from your counsellor or a book for the material of spring, you can judge whether the dynamic method is more accurate than the static method or not.)

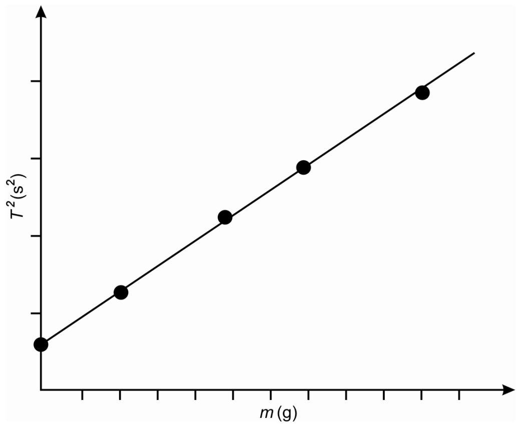

Fig.2.3: Expected plot of T^2 versus *m*

As before, you can compute error in *k* by drawing lines of maximum and minimum slopes. What is the relative change in the value of *k*?

Result: Spring constant of the given spring = ………… ±……………Nm^{-1}

Spend 4 min.

SAQ 2 : Dynamic method

i) Extrapolate the graph between T^2 and m backward and interpret the intercept.

ii) Use your graph to determine T for a load of 2N.

2.3 DETERMINATION OF TORSIONAL RIGIDITY OF A WIRE

As mentioned before, you are required to use a torsional pendulum shown in Fig. 2.4 to measure torsional rigidity. The necessary apparatus required for this purpose is listed below:

Apparatus

Torsional pendulum (inertia table), stop watch, rigid circular cylinder, vernier callipers, micrometre screw, spirit level, physical balance and a weight box

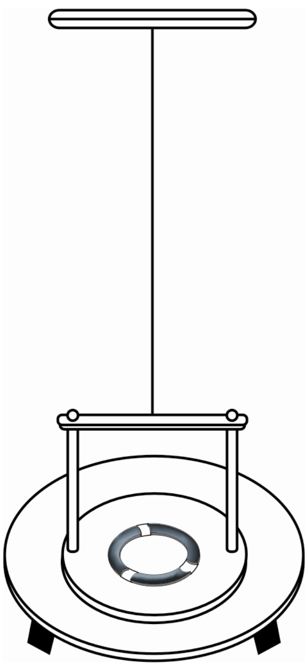

Fig.2.4: A torsional pendulum

In a torsional pendulum, one end of a long and thin metallic wire is clamped to a rigid support. The other end of the wire is fixed to the centre of a projection coming out of the central portion of the circular disc. Normally, this disc is made of aluminium or brass. You can observe concentric circles on the upper face of the disc and a groove near the circumference. The concentric circles help us in putting loads symmetrically. The concentric groove helps in setting the disc horizontal by placing balancing weights. The iron table below the disc is provided with three levelling screws.

Think of what happens when a cylindrical wire is clamped at one end and the other end is twisted in a plane perpendicular to its length. Due to elasticity, an equal and opposite torque is developed in the wire. The restoring torque per unit radian, k_t is given by

$$k_t = \frac{\pi n r^4}{2\ell}, \qquad (2.3)$$

where n is modulus of rigidity, r denotes the radius of given wire and ℓ is its length. In the apparatus given to you, if you rotate the disc in a horizontal plane (keeping the wire vertical) and then release it, the system will execute torsional oscillations in the horizontal plane. These torsional oscillations are simple harmonic. The period of oscillations is given by

$$T_0 = 2\pi\sqrt{\frac{I_0}{k_t}}, \qquad (2.4)$$

where I_0 is moment of inertia about the axis of rotation.

If an auxiliary body of known moment of inertia I is placed on the disc such that its centre coincides with the centre of the disc, the period of oscillations will change. Do you know why? It is because of the redistribution of mass about the axis of rotation. If you denote the period of the system now by T, you can write

$$T = 2\pi\sqrt{\frac{I_0 + I}{k_t}}. \tag{2.5}$$

On combining Eqs. (2.4) and (2.5), we obtain an elegant expression for torsional rigidity:

$$k_t = \frac{4\pi^2 I}{T^2 - T_0^2}. \tag{2.6}$$

On comparing this result with Eq. (2.3), we get an expression for the modulus of rigidity:

$$n = \frac{8\pi I \ell}{(T^2 - T_0^2)\, r^4}. \tag{2.7}$$

Eqs. (2.6) and (2.7) show that you can readily calculate k_t and n, once T, T_0, I, ℓ and r are known.

Begin the experiment by setting up the apparatus for determination of T and T_0. You should first level the iron table using the levelling screws. You should test this using a spirit level. Next you should adjust the balancing weights in the groove of the disc so that the disc is horizontal. To ensure this, you should again use spirit level. You should also make sure that the suspension wire is free from kinks. Now place a vertical pointer in front of the disc and just put a mark on the disc when it is at rest. This denotes the equilibrium position and reference for counting the number of oscillations. Next, rotate the disc slightly in horizontal plane so that the wire gets twisted and then release it. The system will begin to oscillate. How are these oscillations different from those of the simple pendulum? After a few (five or so) oscillations, you may begin oscillation count through the equilibrium position and simultaneously start the stop watch. Note the time of N (20 or 30) oscillations. Record your readings in Observation Table 2.3. Repeat the observations at least five times. Calculate the mean value of time period. This gives T_0.

Observation Table 2.3: Determination of T_0 and T

Least count of stop watch = ……………… s

No. of oscillations counted each time (N) =……………….

S. No.	Time for N oscillations (s)		Time period (s)	
	Without cylinder	With cylinder	Without cylinder T_0	With cylinder T
1.				
2.				
3.				
4.				
5.				

Mean T_0 =....................s

Mean T =.......................s

Now place a right circular cylinder at the centre of the disc such that its axis coincides with the axis of suspension of the wire. Again twist the inertia table and record the time for the same N number of oscillations. Repeat the process at least five times. Calculate the period of oscillations. This gives you T.

From Eq. (2.6) you will recall that to calculate k_t, you must know I also. From the B.Sc. Physics elective course on Elementary Mechanics, you may recall that the moment of inertia of a right circular cylinder of mass M and radius R about an axis passing through its centre is given by

$$I = \frac{MR^2}{2}. \qquad (2.8)$$

This shows that I can be calculated if we know M and R. You can measure the mass by weighing the cylinder in a physical balance. And to measure its diameter, use vernier callipers. Record your readings in Observation Table 2.4. Take at least five readings. Calculate the mean value.

Observation Table 2.4: Radius of cylinder

Least count (LC) of vernier callipers = …….. cm

S. No.	Diameter of the cylinder (cm)					Radius of cylinder $R = d/2$ (cm)
	Main scale reading (s)	Vernier scale reading (v)	$h = v \times LC$	$d = s + h$	Mean d	
1.						
2.						
3.						
4.						
5.						
6.						

Mean radius of cylinder = …….cm =........m

Mass of right circular cylinder = ……..kg

Moment of inertia of right circular cylinder =…….. kg m^2

Result: The torsional rigidity k_t of the given wire is ……....Nm

Once k_t is known, n may be computed if you measure the length of the wire and its radius. From the value of n so obtained you should be able to predict the material of the wire by consulting some practical physics textbook.

To find r, use a micrometre screw. Take readings at several points along the length of the wire and record these in Observation Table 2.5. By doing so, you account for any non-uniformity in the diameter of the wire. For greater accuracy, measure the diameter of wire in two mutually perpendicular directions (Refer to Fig. 2.5).

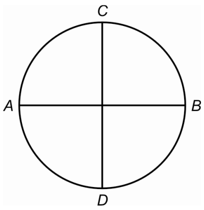

Fig.2.5: Measurement of diameter in two mutually perpendicular directions

Observation Table 2.5: Radius of wire

Least count of micrometre screw = ………………cm

S.No.	Diameter of the wire (cm)			Radius of wire (cm)
	Along *AB*	**Along *CD***	**Mean**	
1.				
2.				
3.				
4.				
5.				
6.				

Mean radius of the wire = ………..m.

Length of the wire = …….......m.

Compute the error following the procedure outlined in Unit 2 on error analysis. Do your results differ from the standard values of k_t and n within these error limits only? If not, you should discuss the reasons of deviation with your counsellor.

Result: i) The modulus of rigidity of the wire = ………… ±………………Nm^{-2}

ii) The material of the given wire is……….

If time permits, you can investigate the relation between k_t and the radius of the wire by working with another wire of the same material but different radius. Similarly, you may study dependence of k_t on the material of wire. For this you will have to take another wire of different material but having the same radius.