EXPERIMENT 1

DEPENDENCE OF THE PERIOD OF A PENDULUM ON LENGTH, AMPLITUDE AND MASS

Structure

1.1 INTRODUCTION

As a 15 year old boy, Galileo by chance, observed the motion of chandeliers in a banquet. He monitored the time periods of the chandeliers with the help of his pulse rate, as clocks were unknown at that time. He concluded that chandeliers of same length had more or less the same period. Moreover, the period was larger for chandeliers of larger length. This led him to the idea of simple pendulum. What do you learn from this incident? It tells us that keen observations help to innovate and discover. We would like you to inculcate such a spirit.

In your school, you must have worked with a simple pendulum. A simple pendulum is a heavy (point) mass suspended from a rigid support by a weightless, inextensible string. In practice, a simple pendulum is realised by suspending a heavy metallic bob from a rigid support by means of an ordinary string. (So you must appreciate that what we have in practice is not an ideal simple pendulum!) It can freely oscillate to and fro about the point of suspension in a plane. The maximum displacement of the bob on either side of its equilibrium position is called the **amplitude** of oscillation. The time taken by the pendulum to complete one oscillation is called **time period**. As we examine the motion of a simple pendulum, some questions that immediately come to our mind are:

- Do the length and/or thickness of the string influence the time period of a pendulum?
- How does the time period change with the amplitude of the swing?
- Do the material, shape and size of the bob affect the time period of the pendulum? If so, how?
- How does the air dragged by the bob of a pendulum influence its period?

You will discover answers to some of these questions here.

> You may recall that in your school physics course, you studied only the dependence of period on the length of simple pendulum.

You may think that this experiment is far too simple to perform at your level. But our purpose of starting with a simple and familiar arrangement is to help you in understanding simple harmonic motion and also to give you experience of planning an experiment. That is, we want you to learn the art of setting up the apparatus, taking measurement, making simple calculations and analysing the result. In this way, we intend to give you training in scientific method of learning, which would lead to development of investigative skills.

A pendulum, as you know, happens to be the main equipment inside a wall clock. Even the most precise time measurements are done by clocks using quartz crystals as oscillators like pendulums in wall clocks.

You may now think that a simple pendulum is an ideal arrangement for time measurement. But it is not so; a practical simple pendulum has some inherent drawbacks. For example, the bob drags air, the string is not strictly inextensible, motion about the point of suspension may have rotational component, etc. Use of a compound pendulum eliminates some of these drawbacks. A compound pendulum is a rigid body capable of oscillating freely about a horizontal axis passing through it. In your laboratory, you will find it in the form of a metallic bar having a series of holes. These holes allow us to make the pendulum oscillate freely when suspended from a knife-edge. In some cases, a cylindrical rod is also used as compound pendulum. In this experiment, we shall restrict ourselves to the oscillatory motion of a simple and a bar pendulum. In the next experiment, you will make similar investigations with a spring-mass system.

Objectives

After performing this experiment, you should be able to:

- establish the relation between the time period and the length of a simple pendulum;
- discover the dependence of the period on the amplitude of oscillation and the mass of the bob;
- compare the values of acceleration due to gravity obtained by using a simple pendulum and a bar pendulum; and
- compute the radius of gyration of a bar pendulum.

1.2 INVESTIGATIONS WITH A SIMPLE PENDULUM

In the first part of the investigations with a simple pendulum, you are required to study the dependence of time period on its length, amplitude of oscillation and mass of the bob. Since we are interested to know the way in which *three* different parameters affect the period, it makes sense to vary only one parameter at a time, keeping the other two constant. Then any change in period can be attributed to the change in the parameter that has been altered. (If two or all three parameters are changed simultaneously, we will have no way of knowing how much of the change in period is due to one particular parameter.) Therefore, we advise you to make investigations in three steps. The apparatus with which you will work is listed below.

Apparatus

Three identical bobs of different materials, protractor, strings of varying lengths, stop watch, metre rod, clamp stand, cork pads, vernier callipers

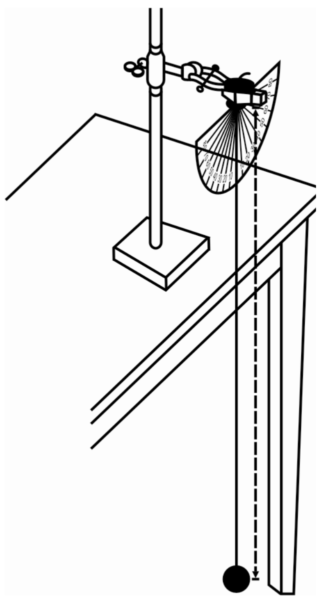


Fig.1.1a: A schematics of a simple pendulum

Take a long piece of string, nearly 2 m long, and tie it to the pendulum bob. Fix the top of the string between cork pads placed in the jaws of the clamp, as shown in Fig.1.1a. Displace the bob to one side and then release it. The pendulum will begin to oscillate. You should ensure that the bob neither spins nor experiences jerks. That is, the pendulum should execute free oscillations.

Then you would be sure that your set-up is ready and you can begin your investigations.

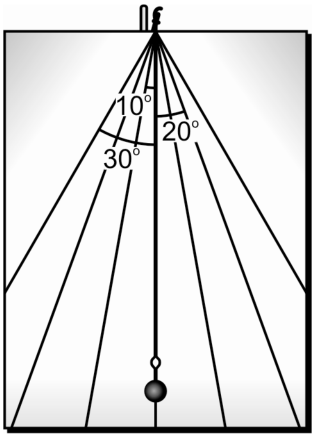


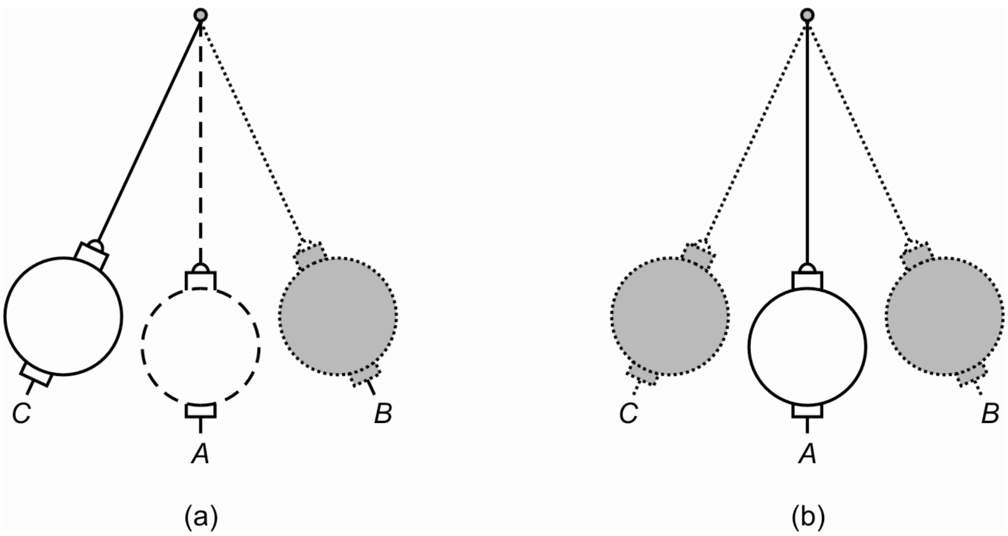
Fig. 1.1b: Simple pendulum with angles graduated sheet

1.2.1 Dependence of the Period on Length

Make a reference mark using a pointer at the equilibrium position of the bob as well as at the maximum displacement of oscillation. You should keep the amplitude constant in each observation and it should be such that at no time, the small angle approximation is violated ($\theta \leq 10°$). That is, the motion is simple harmonic. This may be ensured by using a protractor. (If a protractor is not available in the laboratory, you can make angle markings on a separate sheet of paper. Place the graduated scale behind the pendulum in such a way that the zero angle line coincides with the equilibrium position of the pendulum. Moreover, the origin of angular scale should be aligned with the point of suspension, as shown in Fig. 1.1b.)

To begin with, note the least count of the stop watch and record it in Observation Table 1.1. Now set the bob in motion by displacing it on one side. To count the number of oscillations, you can choose your reference point in two ways, as shown in Fig. 1.2. (To help you to visualise these, we have exaggerated the size of the bob and the angular displacement). We prefer the second option because the reference point remains unaltered in this case.

If you are working with another student, one of you can count while the other keeps time. The 'counter' should begin countdown two, one, "go", one, two… and so on. This gives the timekeeper a warning about the 'Go" signal. The end of counting may be indicated by saying 'stop'. Make sure that each one of you takes at least one complete observation individually.



One complete oscillation
C→A→ B → A→C

One complete oscillation
A→ B→ A→ C→ A

Fig.1.2: Two different ways of counting the number of oscillations

The *reaction time* is the time interval between the input stimulus and its response.

Begin your counting through the *equilibrium position* of the bob. It is important to simultaneously start the stopwatch. (There can be time lag between the starting/stopping the watch and the oscillation count due to *reaction time*, which is, on an average, 0.3s. This can introduce some error in the value of time period *T*.) An important point to consider here is to know the degree of accuracy that is necessary. Another point is to measure a time interval in which the amplitude of swing does not diminish significantly. To see this, you can note time for 1, 10, 20, 30, 50, 70, 100 oscillations. You should take at least three observations in each case. Record your readings in Observation Table 1.1. Calculate the period of oscillation, *T*.

To decide on the optimum number of oscillations, observe the variation in the value of *T*. When the difference between two successive values of *T* is less than 0.1 percent, it is acceptable. We expect the optimum number of oscillations to be 30. However, do not consider the number '30' to be sacrosanct. Make your own decision.

The least count of an ordinary stop-watch is 0.1 s. So whenever you have to measure time of the order of one second or so, you should use a more accurate automatic switching device, such as digital timer.

Observation Table 1.1: Determination of optimum number of oscillations

Least count of stop watch = ……………………s

S. No.	No. of oscillations (*N*)	Time (s)				$T = \frac{\text{Mean time}}{N}$ (s)
		(i)	(ii)	(iii)	(Mean)	
1.	1					
2.	10					
3.	20					
4.	30					
5.	50					
6.	70					
7.	100					

Conclusion: The optimum number of oscillations is…………….

You have now decided on the number of oscillations (*N*) to be counted. After this, measure the diameter of the bob using vernier callipers shown in Fig. 1.3a. Record the least count in Observation Table 1.2(a). Take observations in different directions. Calculate the radius. (The length of the string plus the radius of the bob defines the length of the pendulum.)

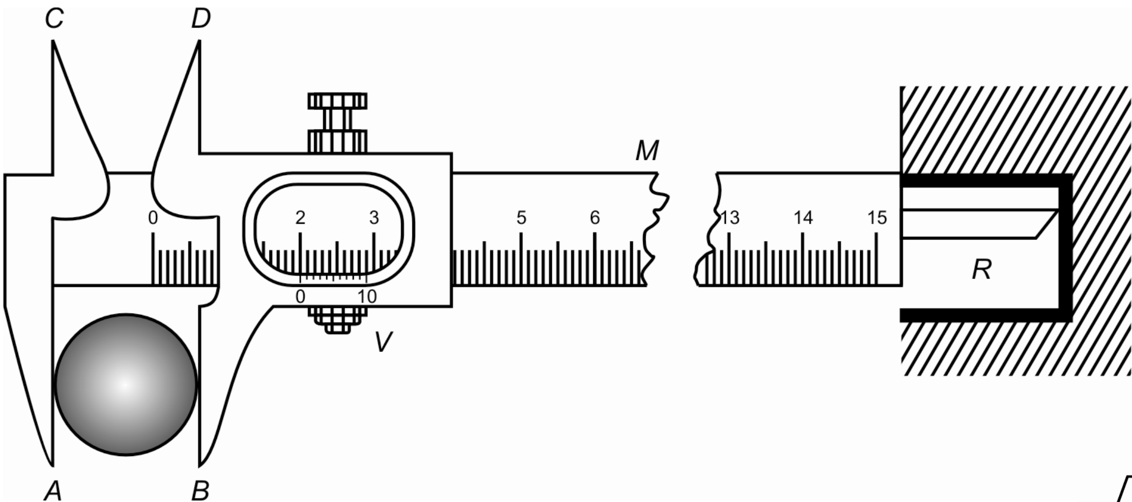


Fig.1.3a: Vernier callipers with bob in the jaws

In case you do not get vernier callipers, you can determine the diameter of the bob using a graph paper. Place the bob on a mm-graph paper, like the one given in your laboratory manual. Make some arrangement for keeping the bob between two tangential planes as shown in Fig. 1.3b. Make sure that

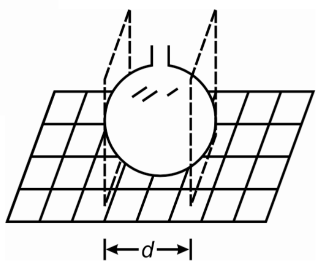


Fig.1.3b: Determination of diameter of bob using graph paper

these tangential planes are parallel to each other. Take the readings of the two tangential planes from the mm-graph paper. The difference of the readings gives the diameter. This observation may be repeated by reorienting the position of the bob and taking fresh readings by readjusting the positions of the tangential planes.

Normally, we measure the length of the string using a metre scale whose accuracy is up to one significant place after the decimal point. So, even though the accuracy in measuring the diameter is up to two significant digits, you should round it off to the first place after the decimal. Do you know the reason? To understand it, note that the length (ℓ) of the pendulum is given by

$$\ell = \ell_0 + r, \quad (1.1)$$

where, ℓ_0 is length of the string and r is radius of the bob.

From Unit II, you may recall that the quantities being added (ℓ_0 and r in this case) must be rounded off upto the same number of significant digits after the decimal point.

Note the time for N (say, equal to 30) complete oscillations. Repeat this observation at least three times. Measure the length of the string from the point of suspension to the point of attachment to the bob using a metre scale. Enter this data in Observation Table 1.2 (b).

Decrease the length of the pendulum by about 25 cm and repeat the experiment, keeping the amplitude of swing as before. That is, you should not change the position of the reference mark at the maximum displacement. Record the length of the pendulum and the time for the same number of complete oscillations. Does the time period change with change in length?

Repeat the procedure at least five times by varying the length of the string. What do you conclude?

Least Count = Value of 1 MSD – Value of 1 VSD

$$= 1\text{ MSD} - \left(\frac{9}{10}\right)\text{ MSD}$$

$$= \left(\frac{1}{10}\right) \times \text{value of 1 MSD}$$

For the vernier callipers, the value of 1 MSD is 1 mm. Hence

$$\text{LC} = \left(\frac{1}{10}\right)\text{ mm}$$

$$= 0.01\text{ cm}$$

Observation Table 1.2: Effect of length on the period of the simple pendulum

a) Diameter of bob

Least count of vernier callipers = …………………………………….. cm

S. No.	Diameter, d (cm)	Radius, $r = d/2$ (cm)
1.		
2.		
3.		

Mean radius =..............cm.

b) Measurement of time for different lengths of the pendulum

Least count of the stop watch = ...……………………………… s
Least count of metre scale = ……………………………….. cm
No. of complete oscillations (*N*) = ………………………………..

S. No.	Length of string (ℓ) (m)	Time (*t*) for *N* complete oscillations (s)				Time period, $T = \frac{\text{Mean time}}{N}$ (s)
		(i)	(ii)	(iii)	Mean *t*	
1.						
2.						
3.						
4.						
5.						
6.						

Conclusion: The period of the pendulum……..…..as its length increases.

To investigate the exact relation between the time period and the length of the pendulum, note whether *T* increases or decreases as length increases. (An increase in time period suggests that *T* is directly proportional to the length.) A variation in *T* suggests its connection with the length of the pendulum; i.e. $T \propto \ell$. From your observations, you can not quantify this proportionality. To know the exact dependence of *T* on ℓ, we write

$$T = A\,\ell^{\,n}, \qquad (1.2)$$

where *A* is constant of proportionality and *n* is some constant. Theory predicts that *n* should be ½. To verify this, you should plot *T* versus $\ell^{1/2}$ or T^2 versus ℓ and the graph is expected to be a straight line passing through the origin.

Theoretically, the slope of the straight line obtained on plotting T^2 versus ℓ should be $4\pi^2/g$. Therefore, by calculating the slope from your $T^2 - \ell$ graph, you can easily calculate acceleration due to gravity. Compare your value of *g* with the standard value at your place and compute the percentage error in your result.

You may wonder why we started with a relation like the one given in Eq. (1.2). Since the dimensions of *T* and ℓ are not the same, we cannot think of exponential, logarithmic or trigonometric relation.

The equation of a straight line is $y = mx + c$

A mathematically more elegant formulation is obtained by taking natural logarithm (to the base *e*) of the expression given in Eq.(1.2) . This gives

$$\ln T = n \ln \ell + \ln A. \qquad (1.3)$$

This is the equation of a straight line.

Now you may plot ln *T* versus ln ℓ. The slope of the curve will give you *n*.

Note that this method of calculation involves use of log-log graph paper, which may not be available in your laboratory or convenient to work with. However, we leave it to your choice.

You can now conclude that the time period of a simple pendulum is related to its length through the relation

$$T = A\,\ell^{1/2}. \quad (1.4)$$

Spend 10 min.

SAQ 1 : Length and time measurement

i) In your observations, you are required to record time with respect to the reference mark (by maintaining proper record about the direction of motion) at the equilibrium position of the bob. Why is it necessary?

ii) Why is it necessary to add the radius of the bob to the length of the string to obtain the length of the pendulum?

iii) Can we use metre scale or a micrometre screw to measure the radius of the bob? Justify your answer.

iv) Read the time period from your graph for a length of 100 cm. What is its significance?

1.2.2 Dependence of the Period on the Amplitude of Oscillation

To study the effect of amplitude of oscillation on the period of the pendulum, we have to keep the length of the string and the mass of the bob constant in this part of the experiment. First, fix a protractor as shown in Fig.1.4. You may work with a length of about 1.5 m and in the beginning take the angular amplitude in the range 4°–10°. This ensures simple harmonic motion (SHM). Note the time for 30 oscillations and record it in Observation Table 1.3.

Repeat it at least three time and compute the period of oscillation. Compare your observations. Are they different? Next, take larger angular amplitudes of say, 20°, 30°, 40°, 50° and 60° and determine the time period in each case. Is it different from that in the small angle approximation? If so, quantify the difference by calculating the relative change. What do you infer about the motion of the pendulum?

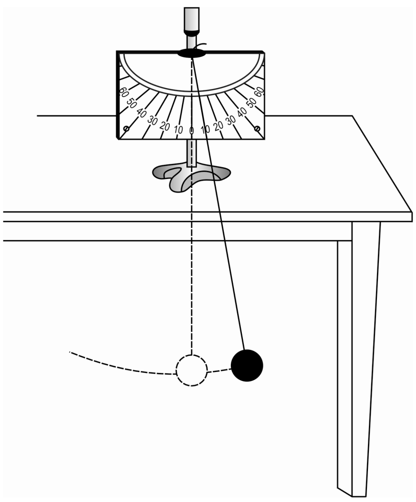


Fig.1.4: Dependence of the period on amplitude of oscillation

Observation Table 1.3: Variation of time period with angular amplitude

Number of complete oscillations counted each time (*N*) = 30

Length of the pendulum = ……….……………………m

S.No.	Angular amplitude (degree)	Time for *N* (=30) oscillations (s)				Time period (s)
		(i)	(ii)	(iii)	(Mean)	
1.						
2.						
3.						
4.						
5.						
6.						

Conclusion: 1. For small angular amplitudes, the period of the simple pendulum is…………s

2. For large angular amplitudes, the period of the simple pendulum is………….s

Comments: ………………………………………………………………………

...……………….……………………………………………………...

1.2.3 Effect of Mass of the Bob on the Period

To determine whether or not the period of a simple pendulum depends on the mass of the bob, take three bobs of different materials. These should be identical (in shape and size) so that (i) the air-drag experienced by every bob is the same and (ii) the length of the pendulum is same in all cases.

Note the time for 30 complete oscillations as you have done for recording your observations in Table 1.2(b). Repeat the procedure for at least two other bobs of same size but different materials. Record your readings in Observation Table 1.4. Compute the time period. Is it influenced by the mass of the bob? If yes, how much? To quantify this change, calculate the difference between the values of time periods for bobs of minimum and maximum masses. *Theoretically, we do not expect any change in the time period as the mass of the bob changes*. Discuss it with your counsellor and point out the possible reasons.

Observation Table 1.4: Variation of time period with mass of the bob

Length of the pendulum =……………..m

S.No.	Mass of bob (g)	Time for *N* (=30) oscillations (s)				Time period (s)
		(i)	(ii)	(iii)	(Mean)	
1.						
2.						
3.						

Conclusion: The period of the pendulum varies/does not vary with mass of the bob.

1.2.4 Damping and Relaxation Time

You must have observed that the amplitude of oscillations of the pendulum **does not** remain constant with time. It gradually decreases and becomes smaller with time. This is because the pendulum loses energy due to air resistance. Such a motion is said to be **damped**. In practice, every oscillating system experiences damping to a varying extent. In the B.Sc. Physics elective course on Oscillations and Waves, you have learnt that damping in an oscillating system can be characterized by **relaxation time** of the system. So in the second part of the investigations with a simple pendulum, you are required to calculate relaxation time.

A systematic way of introducing damping in case of a simple pendulum is to put a fan on and let the pendulum oscillate. We assume that frictional force F_d is small and can be expressed by linear proportionality to velocity. That is, we write $F_d = \gamma v$, where γ is constant of proportionality.

If x (t) denotes the displacement at any time t, the motion of a damped oscillator is described by the equation

$$\frac{d^2x}{dt^2} + 2b\frac{dx}{dt} + \omega_0^2 x = 0, \qquad (1.5)$$

where $\omega_0 = \sqrt{g/\ell}$ is angular frequency of undamped oscillations and $b = \gamma/2m$ is a measure of damping experienced by the bob of mass m. It has dimensions of frequency. The inverse of this quantity, $b^{-1} = \frac{2m}{\gamma}$ is called *relaxation time* and is denoted by the symbol τ. The values of τ are smaller in case of a heavily damped system.

When damping is small, the solution of Eq. (1.5) is

$$x\,(t) = a_0 \exp(-\,t/\tau)\cos(\omega_d\, t + \phi), \qquad (1.6)$$

where $\omega_d = \sqrt{\omega_0^2 - b^2}$ is the angular frequency of damped oscillations and ϕ is the initial phase. $a_0 \exp(-\,t/\tau)$ is the amplitude of oscillation in the presence of damping. a_0 is amplitude of oscillation in the absence of damping. (You can find this solution in Unit 3 of Oscillations and Waves course.) Note that Eq.(1.6) represents an oscillatory motion but it is not simple harmonic. After n oscillations, the amplitude will be

$$a_n = a_0 \exp(-nT_d/\tau),$$

where T_d is the period of damped oscillations. Taking logarithms, we get

$$\ln a_n = \ln a_0 - \left(\frac{T_d}{\tau}\right) n. \qquad (1.7)$$

This equation shows that if you measure a_n and plot a graph between ln a_n versus n, the curve will be a straight line. Its intercept on the y-axis gives ln a_0.

The slope of the straight line gives T_d/τ. Using this relation, relaxation time can be readily calculated, once T_d is known for a given length of a pendulum.

To measure a_n, you should fix a scale on the table. Displace the bob to one side and release it. Note the amplitude after 10, 20, 30,….oscillations and record it in Observation Table 1.5. In case it is not convenient to do so in one go, you can do it in steps. But in each case, the initial amplitude of swing should be kept the same.

Observation Table 1.5: Variation of amplitude with number of oscillations

Length of the pendulum =……………..……………..m
Period of the pendulum =……………………………....s
Mass of the bob =………………………………g

S.No.	n	a_n (m)	ln a_n
1.	10		
2.	20		
3.	30		
4.	40		

Result: The relaxation time of the given pendulum oscillating in a viscous medium like air is……s

SAQ 2 : Damped oscillating systems

Spend 3 min.

Name one physical system where linear damping model holds.

1.3 INVESTIGATIONS WITH A BAR PENDULUM

We know that a simple pendulum suffers from the drawback that some air is always dragged by the bob. Similarly, the string may not be perfectly inextensible leading to non-planar oscillations. These sources of error sometimes lead to variation in the value of T. Can you suggest a way to overcome these problems? The remedy lies in the use of a compound pendulum.

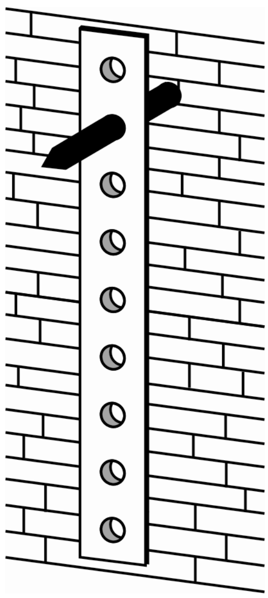


Fig.1.5: A bar pendulum

A compound pendulum is a rigid body capable of oscillating freely about a horizontal axis. In the physics laboratory, it is normally available in the form of a bar of length nearly one metre and width about 2.5 cm. A series of circular holes, 5-6 mm in radius, are drilled symmetrically about its centre of gravity (C.G.), i.e. along the length of the bar. (You can make a bar pendulum by taking a metre scale and drilling equidistant holes in it, as shown in Fig. 1.5.) The centres of any two consecutive holes are at equal distances of about 5 cm. These holes allow the bar to be suspended from a knife-edge. Usually, two movable knife-edges are provided with the bar pendulum. These can be fitted successively in various holes, one on each side of C.G. and at equal distances from it. You may now realise how deficiencies in a simple pendulum are taken care of in a compound pendulum.

As the bar pendulum is made to oscillate about a horizontal axis, its motion is simple harmonic and the time period is given by

$$T = 2\pi\sqrt{\frac{k_r^2 + \ell^2}{\ell g}}, \tag{1.8}$$

where ℓ is the distance between the point of suspension and C.G. and k_r is the radius of gyration of the body.

The radius of gyration of a body about an axis is the distance between the said axis and the point at which the whole mass of the body could be considered to be placed without bringing about any change in its moment of inertia about that axis.

Eq. (1.8) is the same as Eq. (1.34) of Unit 1, Block 1 of the course on Oscillations and Waves. We define

$$L = \frac{k_r^2}{\ell} + \ell$$

and call it the length of an equivalent simple pendulum. Combining this result with Eq. (1.8), we get

$$T = 2\pi\sqrt{\frac{L}{g}}. \tag{1.9}$$

In this part of the experiment, you are required to investigate how the period of oscillation varies with distance between the point of suspension and C.G. of a bar pendulum.

The apparatus with which you will work is listed below:

Apparatus

Bar pendulum, stop watch, metre scale, telescope.

1.3.1 Variation of the Period with Length

Fix one knife-edge in the hole nearest to one end of the bar pendulum. The other knife-edge is fixed in the hole nearest to the other end so that the two knife-edges are equidistant from and symmetrically placed with respect to the C.G. of the bar. Now suspend the pendulum vertically by resting it on one of the knife-edges on a horizontal rigid support. As before, put a reference mark to denote the mean position of the pendulum. Place the telescope at a distance of about one metre and focus its vertical cross-wire on the reference mark. Displace the bar slightly aside and let it oscillate. You should ensure free oscillations in the vertical plane. Now you are ready to perform the experiment.

Measure the distance between the point of suspension (centre of the hole) and the C.G. of the bar. This gives us ℓ. Now measure the time for 30 complete oscillations. Record your readings in Observation Table 1.6. Invert the pendulum and note the time for the same number of oscillations. Now insert the knife-edges in the adjacent holes so that they are symmetrical about C.G., as before. You will note that now the length of the pendulum has been changed and the time of N oscillations is expected to be different from the preceding value. Repeat observations by inserting the knife-edges in different holes. At all times, the knife-edges should be symmetrical about C.G. What happens as you approach the centre of the bar? You will observe that the time for *N* oscillations first decreases, takes a minimum value and then begins to increase. As you near the C.G. of the bar, it becomes very large. See what happens when the knife-edge in put at the central hole. You will note that the bar will not oscillate; it just gets struck up on one side.

Observation Table 1.6: Variation of time period with distance of a hole from C.G.

Least count of the stop watch =……………..……….s

S. No.	Distance of the point of suspension from *C.G.* ℓ (cm)	Time for oscillations *N* = 30				Time period *T* (s)	ℓT^2 (cm s^2)
		(i))	(ii)	(iii)	Mean		
1.							
2.							
3.							
4.							
5.							
6.							

Plot a graph between *T* and ℓ. You will get two curves which are symmetrical about the C.G. of the bar. Now you draw a line parallel to the x-axis. At how many points it cuts these curves? The number of points should be four, say at *J*, *K*, *M* and *N*, as shown in Fig. 1.6. At all these points, the period of the pendulum is the same.

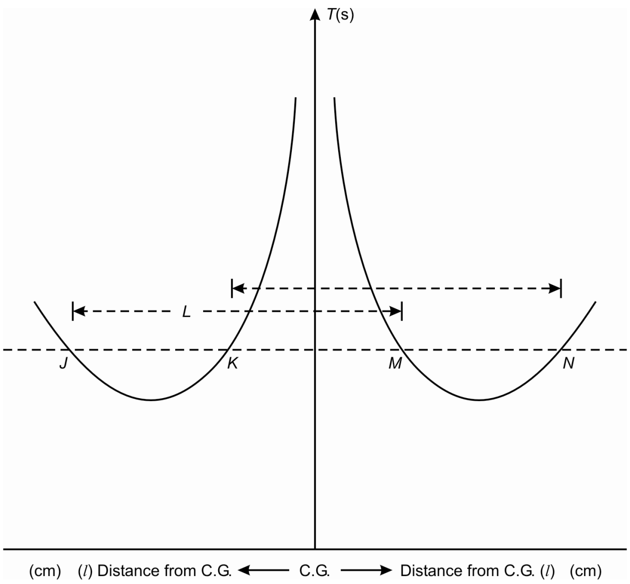


Fig.1.6: Plot of time period with distance of point of suspension from *C. G*.

Measure distances *JM* and *KN*. How do you interpret these? Each of these distances represents the length of an equivalent simple pendulum, *L*. Using Eq. (1.9), you can compute the acceleration due to gravity.

Result: Acceleration due to gravity = ……………………………………......

1.3.2 The Radius of Gyration

To calculate the radius of gyration, we rewrite Eq. (1.8) as

$$\ell T^2 = \left(\frac{4\pi^2}{g}\right)\ell^2 + \frac{4\pi^2}{g}k_r^2 . \qquad (1.10)$$

This equation suggests that if you plot ℓT^2 versus ℓ^2, you will obtain a straight line, whose slope is $\frac{4\pi^2}{g}$ and intercept is $\frac{4\pi^2}{g}k_r^2 = c$, say.

Hence

$$g = \frac{4\pi^2}{\text{slope}}, \qquad (1.11)$$

and the radius of gyration is given by

$$k_r^2 = \frac{cg}{4\pi^2}$$

or

$$k_r = \frac{\sqrt{cg}}{2\pi}. \qquad (1.12)$$

Result: i) The radius of gyration of the bar pendulum is ……..... m.

ii) The acceleration due to gravity is …………………..…...ms^{-2}.

You may now compare the values of acceleration due to gravity obtained from a bar pendulum (based on Eqs. (1.11)) and that by using a simple pendulum. Which one is closer to the standard value?

Spend 5 min.

SAQ 3 : Bar pendulum

i) Why is it necessary to put the knife-edges symmetrically about *C.G.*?

ii) Name two sources of error in your experiment.