UNIT 3 HELPING CHILDREN LEARN MATHEMATICS

Structure **Page No.**

3.1 INTRODUCTION

Do you remember your school-going days, particularly your mathematics classes? What was it about those classes that made you like, or dislike, mathematics? In this unit we will be raising certain issues related to these questions. It is intimately related to the previous unit, where we discussed some aspects of children of preschool and primary school age. There we observed that:

i) A young child's way of thinking is qualitatively different from that of an adult's.

ii) Children follow a certain pattern in their overall development, and this pattern is more or less universal in nature. Individual children, however, differ in the pace at which they develop.

iii) Each child evolves her own way of `making sense' of things around her.

iv) By the time a child enters formal school, she already knows some mathematics.

v) Young children use play and other activities to evolve strategies to understand the physical world around them.

vi) Older children also learn with concrete materials and games, and can make sense' of the formal knowledge given to them in school through such learning experiences.

vii) Unfortunately, most mathematics teachers emphasise algorithms and memorisation, rather than understanding. The "rules" of mathematics may be comprehensible to the adult mind, but need to be communicated to children in ways that the children can comprehend.

viii) We are intuitively aware of the long and arduous process that children go through while learning a single mathematical concept or skill. But the time-frame that the formal school system allows for "covering" the syllabus doesn't take this into account.

This list is not exhaustive. Why don't you quickly run through Unit 2 and complete the list? You may find this useful, because the present unit focusses on the implications of those points for teaching.

In this unit, we have made an attempt to highlight some of the principles that need to be kept in mind while teaching mathematics to children of preschool and primary school. Doing this would help in creating a learning environment for a preschool or primary school child that is appropriate for her stage of development, her needs, her ways of thinking and learning, and her pace of learning.

We have also given some examples of the kind of activities or opportunities that can be given to children to help them develop mathematical thinking. Unfortunately, the examples of activities that we suggest are mostly from an urban situation. In fact, it is also difficult to think of examples common to all urban areas. We hope that you will adapt the activities to suit the needs of your learners.

Please go through the previous unit again, so that you find it easy to make the connections with the teaching points in this unit.

Objectives

After studying this unit; you should be able to

- explain why a teacher needs to know the level of development of hi; her learners;
- identify the ways in which you can help children develop their ability to think mathematically;
- explain the sequence of learning that would enable a child to acquire abstract mathematical concepts;
- list the principles you need to keep in mind while devising activities to teach specific mathematical concepts and skills effectively.

3.2 BUILD UPON THE CHILD'S BACKGROUND

As you read in Unit 2, each child is unique. Individual children vary in age, level of cognition, background, etc. What implications does this have for a teacher? Doesn't e 'or she need to take these differences into account, as well as differences in taming styles? In the following exercise we ask you to consider this aspect of your learners.

Fig.1 : A teacher should be aware of the differences in the learners.

E1) What are the other differences between learners that a teacher needs to keep in mind, while teaching?

Let us see an example in which a teacher took the pupil's background into account to help him learn.

Sumit, a fifth standard child in a rural school, was being introduced to the formal procedure of addition and subtraction. The teacher tried to gauge how much he knew, before-she taught him the formal method.

Teacher *How much is 8 plus 11?*

Sumit *19*

Teacher *How did you do it?*

Sumit *I rowed. I took 11 and added on 8.*

Sumit had used the strategy of "counting on " from the larger number, and could even describe his method in words.

Teacher *What about 22 plus 19?*

Sumit *(writing 22+19): 41?*

Teacher: *Did you count from 22 by ones?*

Sumit : *I took the 10 from 19 first, and that's 32, and then I took the 9, and that's 41.*

This time Sumit had used "regrouping" to facilitate his addition.

Thereafter, he was given the written problem:

$$\begin{array}{r} 18 \\ \underline{+5} \\ \underline{41} \end{array}$$

His answer was 41

How did he get this answer? He added the 8 and 5 in the units column correctly to get 13, put 1 below them and "carried over" the 3. Then he added 3 to 1 in the tens column to get 4. Hence, his answer!

He was quite convinced that his answer was right. The teacher decided to pose the question differently. She said, "If you had eighteen marbles and you got five more, how many would you have altogether?" Sumit counted on his fingers and said 23. Raven the teacher pointed out his written answer to him, he slowly agreed that it was wrong. Isn't it interesting that he was willing to accept his own intuitive method (the informal procedure) as right, rather than the formal written method?

In this example, Sumit demonstrates a well developed skill of using appropriate and efficient strategies to add numbers. However, he finds the formal manipulation of symbols difficult, perhaps due to various reasons. It could be that Sumit has yet to develop an understanding of `place value'. It could also be that Sumit does not find the given task of addition of numbers with the algorithm. meaningful. The moment the teacher posed the problem in a context and ' with reference to concrete objects (counting marbles), Sumit was able to understand it, and hence solve it

Place the problems in a relevant context.

The example above clearly demonstrates that Sumit had evolved his own strategies of doing addition intuitively: `counting on' and `regrouping'. He was aware of patterns 41 in numbers, and hence was able to regroup to add some - large numbers with ease. The example also shows how the teacher tried to assess Sumit's background, and use this knowledge to make the problem comprehensible to him in two ways:

i) by giving it a relevant context, and

ii) by concretising it for him.

Now, here's an exercise for you to see how important it is that a teacher should know the child's level and background.

E2) Ravi is a teacher of Class 4 in a municipal school in Delhi. When the new school year started, he opened the textbook and started teaching the children how to write 4-digit numbers. After copying part of the text on the board, he gave them several simple exercises, and left the room. Later, he was surprised to find that most of the children couldn't do the exercises. Why do you think such a situation arose?

What we have tried to say in this section is that when teaching a child a concept, it is important to identify the pre-requisites, and see if the child has attained them.

Let us now talk about some principles of teaching, specifically in the context of learning mathematics.

3.3 MAKING CONNECTIONS

In Unit I you have read about what the ability to think mathematically involves. In this section we shall discuss ways of developing this ability in children.

Fig.2 : "Why does it have horns? Well! When you need to .... That's why."

As you know, the emphasis of mathematical learning needs to be on the **process of finding the answer**, and not merely on getting the answer. Therefore, you as a teacher need to encourage children to observe, question, explore and move logically towards an answer. You need to encourage them to systematise their reasoning. How would you do this?

To start with, you can **encourage children to ask questions**. When children ask you "Why are some leaves green and some brown?", or "How come the moon moves along with us?", or "Where do people go when they die?", you need to answer to their satisfaction. However difficult or silly you may feel the questions are, consider them seriously and help the children to move logically towards an answer.

Another thing that helps is to pose open-ended questions to children, like "In how many different ways can you fold this paper into the shape of a square?". They can be given the opportunity to frame their own questions, as in guessing games that we have mentioned in Sec.3.5. Such opportunities help in making learning less rigid, and allow children's minds to unfold their potential. They would also help the children realise that **there can be several solutions to a problem**.

You could think of several types of activities for guiding and encouraging children to systematise their reasoning. For example, they could be asked to select criteria for sorting a set of objects, and helped to apply the criteria consistently. Or, they could make hypotheses on the different ways their schoolmates travel to school. Then they could collect record and analyse data to prove or disprove their hypotheses.

You can think of some more activities to help children explore and learn, while doing the following exercises.

E3) a) Choose a topic in measurement, and design two activities in your context to help your pupils explore and learn the concept.

b) Try these activities out on a few children, and write down the ways in which they promote children's mathematical thinking.

A very important aspect of mathematical thinking is **the ability to recognise patterns and links.** How can you as a teacher foster this ability in a child? Let's see how Aditi's teacher did this.

10-year-old Aditi was busy with problems of mental arithmetic. While calculating 2 x 74 she arrived at 432. She was asked to do it again, and again she said 432. The teacher realised that Aditi had calculated 72 x 6 rather than 76 x 2. She describes how she helped Aditi to arrive at the correct answer. She asked her the following questions in the given order:

What is 2 x 100? Aditi replied 200.

2 x 90? 180

2 x 80? 160

2 x 76? (*After a slight pause*) 432

2 x 70? 140

2 x 80? 160

2 x 76? 432

2 x 100? 200

2 x 200? 400

2 x 76? 432

Suddenly Aditi stopped, thought a bit and said, "Wait. I think something's wrong. Let me work it out. "She got her paper and pencil, and worked out that 2 x 76 equals 152, and felt very pleased with herself

What is striking about the situation above is what the teacher did not do.

- She did not give Aditi the correct answer immediately.
- She did not tell her that she had made a mistake.
- She did not call her a fool and try to hurt her ego.
- She did not threaten her.
- She did not get the class to snigger at her.

What the teacher **did do** was:

- She talked to Aditi.
- She thought about and realised why the error was being made.
- She thought of a method which, according to her, would help Aditi realise her error.
- She was patient and gave more than one opportunity to Aditi to help her realise that something was wrong with her way of solving the problem.
- She encouraged Aditi to observe, think, recognise patterns and make connections.

In short, the teacher was trying to create a meaningful learning environment.

Unfortunately, a lot of teachers do not do this. As-was discussed ,in Unit 2, most children are compelled to mechanically use the algorithms taught, rather than helped to understand the process.

Here's an opportunity for you to see how you would help a child to observe patterns.

E4) After a lot of effort, 8-year-old Hari worked out 2 x 88 = 176. When asked to say what 2 x 89 was, after a lot of hard work, he produced the answer 178. How would you help him to realise that there is a simpler way of getting to the answer?

Most children are like Hari. They have not been trained to observe, explore, recognise connections and generalise patterns that they find. For them 2 x 88 is as much a piece of information or `fact' to be retrieved, as 2 x 89 is. This is in spite of the fact that children perceive patterns in their environment and see links between things that they relate to. How can we correct this situation?

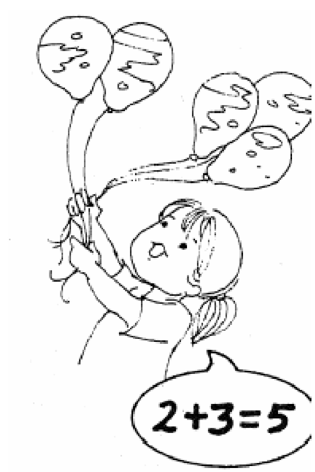


Fig.3 : Using concrete materia helps learning.

Much of mathematics teaching is actually about encouraging children to become more aware about the patterns they find, to articulate the rules and to use them in their thinking. So, to start with, let us consider some ways in which we can help children find patterns in mathematics. A good way is to provide them with exercises involving play with sequences of objects. When children place object after object in a sequence, they begin to learn to express and develop a sense of pattern, a sense of generality. Slowly, they can be shifted from games dealing with objects to games that involve sequences of pictures of objects that they are familiar with. Gradually, children will be able to find patterns in number as well. Once children begin to. identify, understand and create patterns, they can use them to do operations in arithmetic also.

A word of caution! It is not enough to just show children a pattern, as in multiplication, say, and then move on to another topic. Children need time to explore for themselves, and get a feel of what they are just finding out. Thus, while doing the following exercise, you need to also think about how long the tasks will take.

E5) List some activities/tasks/exercises that you would give a class of 50 children to do to make them aware about patterns, and to articulate what the patterns are.

You must be wondering about why we haven't spoken about one of the essentials of mathematical thinking, that is, the ability to go from particular to general and vice versa. For a child to develop this ability, after a certain stage she needs to learn how to deal with formal mathematics. What are the processes she needs to go through to become comfortable with symbols and abstract concepts? We discuss this in the following section.

3.4 E - L - P - S

Has the title of this section stumped you? Children, similarly, don't understand new symbols that are thrust upon them without giving them an adequate grounding. You need to-offer a child carefully sequenced learning experiences in mathematics to build up understanding. Mathematics, like any other learning, is a continuous process. Children need to move from concrete experiences to visual, symbolic and abstract activities. This sequence is characterised by

(**E**) **experience** with physical objects (e.g., stones, sticks, or any easily available objects)

(**L**) using spoken **language** to describe the experience (e.g., by using word/story problems, games);

(**P**) representing this experience through **pictures** (e.g., represent quantity through pictures);

(**S**) generalising the experience through written **symbols** (e.g., numerals).

Let us trace this sequence in the context of a child learning the concept of 'half', assuming that she is familiar with whole numbers.

(**E**) She divides her sandwich/chapati, or pieces of coloured paper, or other such objects into two halves. Later, she divides, say 6 objects, into two sets.

(**L**) She starts associating the word `half' with the quantity. You can devise games to get her acquainted with the names of various fractions.

(**P**) You can show her various pictures, as in Fig.4,

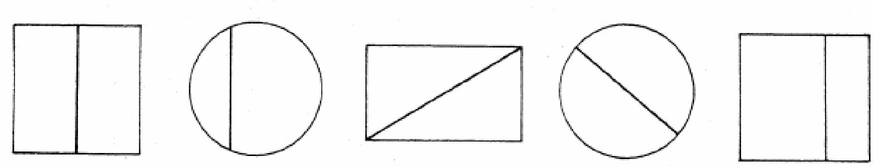


Fig.4

and ask her to say in which figures the line divides the figure in half, There can be several variations (see Block 4).

(**S**) Later, she learns the written symbol for representing `half

E6) Do you agree with the necessity of the sequencing E - L - P - S for learning? If not, then what do you suggest as an alternative path for understanding and internalising mathematical concepts?

In Unit 2 you read that primary school children are at the concrete operational stage of development. To help your learner proceed to the next stage, you would need to emphasise the links between the concrete and the formal.

The concrete mode to be linked to the formal mode

You may feel that once a child has understood a particular abstract concept or process, henceforth she does not need concrete learning experiences to grasp other concepts or processes. But this is not true. Even after becoming capable of doing mental and formal arithmetic successfully, children may need to check their understanding of concepts, operations, problems, etc., by using actual objects. This spiral nature of their development is characteristic of mathematics learning.

For example, children need to understand `place value', first when double digit numbers are introduced. For this, they would need a lot of concrete experiences of grouping, and so on (see Unit 6). This would help them to slowly progress towards an understanding of `tens' and `ones'. After this they would be ready to learn how to formally multiply and divide small numbers. And then, they would again need to be exposed to a variety of concrete learning situations for developing an understanding of `place value' in the context of larger numbers.

Mathematics learning **is not linear**

This way of dealing with the concept in the context of smaller numbers first, and then with larger numbers, also gives children a chance of developing a better understanding of the concept. For example, consider a child who is grappling with a new concept like commutativity of addition. To start with she only needs to see how the property works in the context of small numbers, which she is already familiar with. At this stage she can do without the extra burden of handling large numbers, which she is not comfortable with.

You can think of many more examples of what we have just discussed while doing the following exercises. .

E7) Can you give some more examples of the spiral development of the mathematics curriculum?

E8) A Class 3 child was asked to add 1/4 + 1/5. She wrote 2/9. Why do you feel this happened? How would you help her to sort out the error?

One way of helping the learner in E8 would be through word problems that are set in a context that the child relates to. For example, if you are trying to teach a preschooler the meaning of "two", a good way would be to give her problems such as "give me two pencils". By dealing with such problems the child practises and gradually internalises the meaning of "two". Similarly, while doing word problems such as "You had five pencils, I gave you twelve more, how many pencils will you have altogether?", the child constructs the notion of addition.

'Unfortunately, word problems or story problems are usually taught at the end of Class I. This is probably because many of us wrongly believe that word problems are a means of practising the algorithm. The adult logic decides that formal symbols should be dealt with first. Do you agree?

Text books and syllabi are designed according to adult logic

E9) When do you think you should introduce word problems-before children master the formal algorithm, or after? What are your reasons for your choice?

In any case, no textbook can start from where the individual child is. If teachers wish to lay a solid foundation for mathematical thinking and abilities, it is important that they use teaching material other than the textbook for preschool and primary.school children. In fact, it may be better to use a workbook instead of a textbook to supplement an activity based curriculum, especially for the younger children.

In this section, and earlier, we have been repeatedly suggesting that a child needs to be presented with a sequence of learning experiences to help her move from concrete experiences to understanding a concept at the abstract level. The sequence has to be understood as a broad and flexible sequence. And, at each stage of the sequence, you need to know at what stage of understanding the child has reached. Only then must you go further, building on that understanding.

Throughout, we have also been stressing the need for making mathematics learning enjoyable. Let us now consider one way of doing this.

3.5 PLAY AND LEARN!

Mathematical concepts can be communicated through games

Children can learn many basic mathematical concepts through games. They enjoy Mathematical concepts can be playing within familiar contexts. Their games also generate a good deal of communicated through games. mathematical activity in a spontaneous and enjoyable way. New ideas and concepts can be introduced through games and situations which young children find familiar, enjoyable and non-threatening. This is also true for older (primary stage) children.

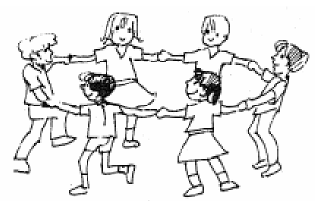


Fig.5: Ring'a ring'a roses.

When young children divide things amongst themselves, they are matching one-to-one. When they play with blocks, they are experimenting with different shapes. When they sing about "five little monkeys", they are learning number names.

Children also enjoy word games. They are usually good at detecting verbal patterns. Since pattern recognition lies at the heart of mathematical thinking, children are really doing mathematics at the same time as developing their language skills.

You could devise several games to teach any mathematical concept. These games can be played with the whole class, or in smaller groups. The games could also be so designed that-the children learn the related mathematical language as well.

Here are a few examples of some team games. The teams can be small (1-3 children) or big (15-20 children). We start with some games for small children.

a) One team places a number of stones in front of itself. The other team has to:
1st game - place as many stones, or

2nd game count and say how many they are, or

3rd game - add as many stones as necessary to make 14 stones, say, or
4th game - take away some stones to leave 3, etc.

As the game progresses you could get them to learn the number names, too.

b) One team throws two dice (with dots or with numerals on them) and picks up, from the collection of stones in the centre, as many stones as the sum (or difference, or product) of the numbers shown on the dice. The other team does the same. After two turns whoever has more stones wins.

Again, they could get used to the language like `six plus two equals eight'.

c) With stones, twigs, dice, cards or beads you can design games to teach 'place value'. With 10 stones (for base 10) being equivalent to one card or one bead, exchanges can be made and records can be kept. Once they are able to grasp the notion of tens and ones in concrete forms, they can be exposed to games using numerals.

For example, you could take two sets of ten cards each, numbered from zero to nine, to be used by two groups. The children shuffle the cards and place them face down on the table. Then they take turns to select one card at a time and place it on the board, in the column marked `units' or `tens'. The aim is to make the largest possible number, and once a card is placed, it cannot be removed. As they play, they loudly say the number they are making. For example, if the first group turns over 3, and places it under `tens', they should say `thirty', and so on.

This game can also be played with two dice, instead of cards.

You can think of several more games, while doing the following exercise.

E10) Write down a game each to teach children

i) multiplication,

ii) what a circle is,

iii) estimation skills.

Also say what you expect the child to know before you try to teach them these concepts.

You can also create a variety of guessing games like the following. They give children ample opportunity to develop their mathematical thinking and language.

a) One group can think of a number and whisper it to the teacher. The other groups have to guess the number. A clue is provided to enable them to guess, such as: This number is a prime number between five and nine.', or `This is the number of sides of a cube.' or `This is 2/5th of 90% of the girls in this class.' The questions would depend, of course, on the level of the children, because children are setting them.

b) In another game, an assortment of familiar objects are placed before the class. One group or child (as you may wish to conduct the game) is asked to choose an item and whisper its name to the teacher. The other children/groups take turns to guess which object was chosen, based on clues which refer to the size, shape or position of the object in relation to the others. For example, it is taller, it is heavier, it is in front, it is not round in shape, and so on.

Games can be used for developing many related skills of mathematical thinking.

c) You can think of guessing games in which the guessers are allowed to ask only a limited number of questions, the answers to which must only be `yes' or `no'. Such games give children the opportunity to shift from asking specific questions (Is it a door? a book?) to more general questions (Can you sit on it? Is it as big as me? Are there more than one in the room?). This helps cut down the number of questions needed to reach the answer. For example, suppose the object chosen is a number from 1 to 100. At first, children may ask if it is a particular number, like 4 or 26. After a while, they learn to ask questions like : Is it bigger than 4?, Is it even?, etc. Others will soon pick up this strategy. Or, if the object chosen is, say, a cylindrical container. Then questions will lead to the use of mathematical terms such as - is it **curved**? Is it **symmetrical** about a line? Does it have 4 angles?

Formulating better question is part of developing mathematical thinking

Such games enable children to develop mathematical thinking, by developing their skills of **generalising**, **particularising**, **estimating and recognising patterns.** All these, in effect, add to their power of mathematical thinking and reasoning.

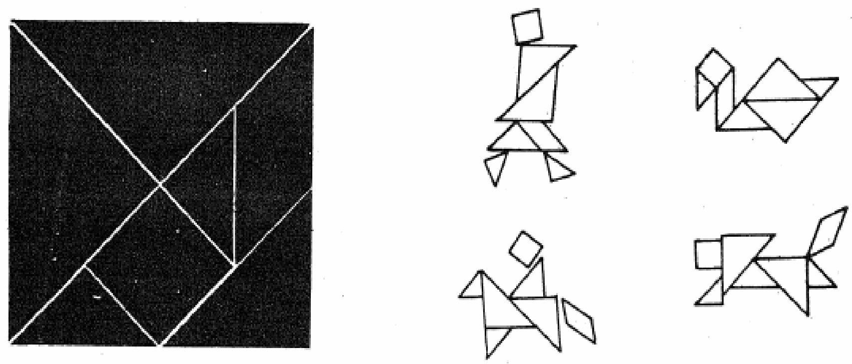
And what about the use of sports activities, group dancing, etc., for fostering the mathematical growth of a child? Why don't you think of such examples now?

Fig.6: Children can learn mathematical skills from Origami.

E11) Create a. guessing game for children of Class 2, to familiarise them with the concept of a time interval

E12) How could you use group dancing to teach concepts of geometry?

There are many other enjoyable activities that can be utilised for familiarising children with various geometrical ideas. For example, children can learn about symmetry by creating symmetrical "rangoli" patterns on paper. They can be introduced through origami, the art of paper folding, to various two and three-dimensional shapes. While demonstrating, the teacher can emphasise the terms used at each step, such as `now fold the paper in half ,' Next, make it into a square by folding', `When you fold this end like this (demonstrate), it becomes a triangle'. Tangrams can also be used for the same purpose.



(a) (b)

Fig.7:(a) A tangram, (b) some shapes made from a tangram.

So far we have stressed the importance of going from concrete to abstract, spending a lot of time on the concrete mode; and using enjoyable activities for teaching mathematics. This is not all that goes into building a learning environment. In the next section we will discuss some more aspects.

3.6 OTHER WAYS TO AID LEARNING

In this section we shall pay particular attention to the need for repetition, learning from other children, and utilising errors for learning.

3.6.1 Repetition Need Not Be Boring

Repetition, and not rote learning, helps children learn.

From an early age on, children engage in and learn from repetitive behaviour, such as dropping and picking up things, opening and closing boxes and tins, repeating the same words, playing `peek-a-boo' repeatedly, urging adults to repeat the same stories, and so on. Would you call any of these activities rote learning? Thus,

repetition need not be rote learning

Repetition can be imaginative. It can involve the children in enjoyable activities, which could even be initiated by the children themselves. In these repetitions the participating children observe and experience something new and different each time.

Rote learning, on the other hand, does not allow for variety because it is not the process which is being repeated, but the `information' which is being repeated mechanically (for example, memorising multiplication tables mechanically).

If you look around you, you will notice that repetition happens with natural variety in a child's living environment. But it has to be consciously created in a formal learning environment, with enough variety to sustain the interest of children. How would you meet this challenge? Maybe, the following example can give us some ideas.

Children often consider multiplication tables to be the bane of their existence. Is it really necessary to go on and on mechanically repeating them? And does this memorising by rote help a child understand what the tables mean? Is it not true that learning by rote usually stays at the superficial level of repeating tables in a given order? The fluency of using them is absent, which you can observe if you ask them to find the multiples in a different order.

Instead of memorisation, isn't it better to help the child to see the underlying pattern? You could think of several activities to enable children to establish the notion of multiplication and recognise patterns in the multiplication tables. For example, children can he asked to identify groups of 2, 4 or 5 apples each, and then answer simple questions like `How many groups of '4 apples each are there?' and `How many apples in all is that?'. And this can be done with a variety of objects.

You can think of some more activities of this kind, while doing the following exercise.

E13) What kinds of classroom activities can you think of for helping children to make groups of 5 and 10?

We have discussed activities to help children understand multiplication, in Block 3.

Once they have enough practice with such activities, children can be helped to record what they are doing in mathematical terms. For example, they could write four groups of two apples each as 4 x 2 = 8. This could be represented pictorially as well.

Now the children will be in a position to enjoy discovering patterns in multiplication tables. You could ask them to fill up a 10 x 10 grid, like the one below. To fill in each square the child will have to put the product of the number of the row it is in and the number of the column it is in.

X	1	2	3	4	5	6	7	8	9	10
1										
2										
3										
4						24				
5										
6										
7										
8										
9										
10										

This kind of an activity could be done over a period of time. Children could be given as much time as they need or as much time as they retain interest in the activity. Let them talk to each other and discover patterns for themselves.

Here are two related exercises for you.

E14) Try and see the order in which different children fill numbers in the grid above. My claim is that all of them would fill in the ones, the fives and the tens first. Test my hypothesis. Did you find any evidence to the contrary?

E15) Can repetition help children understand the concept of time? Give an activity in detail which you would give a group of five children to do for this purpose.

Repetition must be interesting and varied so that children continue to want to learn.

A point that we made in Unit 2 was that children learn best in a non-authoritarian environment. We now talk about one way in which such an environment can be built.

3.6.2 Children Learn From Each Other

The other day I had gone to a, nearby school to observe the teacher-children interaction. The children were working on a problem that the teacher had asked them to solve in their copybooks. The teacher walked around for a bit, stopping to ask individual children how they got a particular answer. Most children reacted by reaching for a rubber, to erase whatever they had done even if they were proceeding correctly. Haven't you come across such behaviour? What does it show? Doesn't it show that children lack confidence in their own ability to solve problems? This lack of confidence is there in adults also when faced with a figure of authority.

Children who lack confidence in their own ability to think will find it difficult to develop this ability later. So, it is necessary to tailor the learning environment towards building confidence in children.

But how can we do this? If you look closely at children playing in groups, you may get_ a clue. While playing together, children check each other's thinking. Games and activities give children the opportunity to interact with each other in a non-threatening, autonomous and easy atmosphere. They give feedback to each other, in the form of an answer or a suggestion. A child takes such an input by another child as just another opinion to be viewed, examined, accepted or ignored.

On the other hand, the adult normally gives his opinion as. the truth, to be accepted without doubt or question. If the child appears to doubt it or looks uncertain, the adult repeats what he has said patiently, and then edgily and, finally, angrily. The eventual judgement is "You fool, you cannot understand it!" The child, who is already dwarfed by the adult, accepts this opinion and starts losing confidence.

Thus, the non-threatening peer interaction is very important for children's learning. Such interaction is important for ether reasons too. We ask you to think about them in the following exercise.

E16) Can you think of some more advantages of peer interaction and child-to-child learning?

If you agree that children learn a lot from each other, then how can we maximise such opportunities? The important thing is that these interactions should be informal, joyful and non-threatening. Just telling a child to teach another, by such statements as "Why don't you teach your neighbour/friend/brother/sister this?", doesn't usually work. This is because the child-tutor then tries to ape the adult, and the learner becomes as defensive as with an adult.

To set up a child-to-child learning situation that natural, and therefor6v, productive, is not very easy. Maybe, one should watch children, without their knowing, and see how they naturally interact. This may give us an idea of how to simulate peer-learning in the formal classroom.

Here's a related exercise for you.

E17) After seeing some children interacting naturally, write down those features of such interactions that make peer learning potentially a better way of learning.

Another point that we adults need to keep in mind while building an environment favourable for learning is the following. We have also hinted about this in Sec.3.3.

3.6.3 Errors Are Useful

While teaching children, you must have found theft making mistakes off and on. How do you respond to the errors'? What do they tell you about the child-failure to learn, or an attempt to understand and internalise? Or is it both? If so, how do you distinguish between these two and decide what it is in a particular situation?

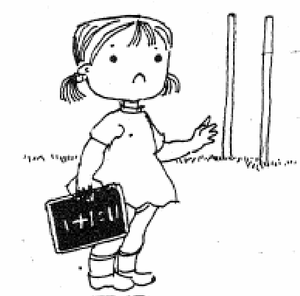


Fig.8: Children's errors are a window into their world.

Children's errors are a natural and inevitable part of their process of learning. In the process of grasping new concepts, children apply their existing understanding, which may not match with the method and content of formal instruction. Sunlit (in Sec.3.3) is an example of this.

Children's errors. are also reflections of how children think and learn. They are often a window into the child's world. For example, writing 51 for 15 tells us that the child has not grasped the concept of place value as yet, and needs a lot more practice with grouping.

This kind of analysis of a child's error can play a highly constructive role in helping the teacher to guide the learner to develop mathematical thinking. Making errors, and learning from them, is part of the process of developing a sound understanding. In fact, this is more important than producing the right answer. Unfortunately, the traditional teacher still tends to view learning as having occurred only when correct responses are given.

Analysing error helps the teacher

1n the foll6wing exercise we ask you to look closely at a child's error.

E18) A Class 5 child fills in the box in $3 \div \square = -3/2$ with 9/2. What is your diagnosis of the error? How woula you remeay the situation?

With this we come to the end of our discussion on ways of building a good learning environment for children. Let us now briefly touch upon what we have discussed in this unit.

3.7 SUMMARY

In this unit we have focussed on how mathematics learning can be made meaningful for primary school children. We have done this through examples of how children learn and how we can create appropriate games and activities for them. We have given examples to show that mathematics learning relates closely to how children think. We have particularly stressed the following points.

1. A child comes to you with a certain level of mental development and knowledge. You need to build on that.

2. Children should be helped to develop skills involved in mathematical thinking, like making connections, going from particular to general and vice versa, making hypotheses and proving or disproving them.

3. Formal mathematics should be related to children's intuitive understanding, using concrete objects.

4. While teaching a child mathematics, we must sequence learning experiences appropriate to the child's level of development, and not only according to the logic of the subject. The sequencing is broadly:

   experience---------language-------pictorial representation-------symbols.

5. Repetition is necessary for the child to become comfortable with a new concept. You need to make repetition interesting. Repetition is not the same as rote learning.

6. Children learn from their peers very easily, because their interaction is usually non-authoritarian and non-threatening.

7. Children's errors must be analysed and utilised for teaching them.

3.8 COMMENTS ON EXERCISES

E1) Their first languages, language competence and educational level of parents may differ, among many other factors that you should list.

E2) No attempt was made to assess how much the children know. Did they understand `place' and `place value'? Could they write 3-digit numbers? Could they even represent numbers less than 100? Further, one can't teach large numbers (i.e., 3 or more digits) with concrete objects. Were the children ready for the formal approach? All these questions, and many others, need to be considered while discussing the situation in this exercise.

E3) a) We have discussed this topic in Block 5.

b) You need to note how they improved the ability of the children to hypothesise, to verify their hypothesis, to alter the hypothesis if necessary, to try and generalise from their observations.

E4) He should be encouraged to observe that multiplication is repeated addition -with concrete objects and small numbers, to start with. Once he understands this, he would realise that 2x89 is simply 2x88 + 2.

E5) You could divide the class into groups of 5, say. Then, each group could be asked to find and continue patterns in a set of objects (like twigs, or stones, or buttons) that you start sequencing in some way.

If the children are ready to deal with pictures, you could distribute picture cards to the various groups. They could be asked to develop a pattern, or one group could begin a pattern for another group to continue, and so on.

If you're teaching the children multiplication (or division), you could develop guessing games on the board, by which all of them could simultaneously work towards recognising the connection between multiplication with addition (or of division with subtraction).

E7) Take, for example, the learning of measurement of length and area. When you go through Block 5 you will realise that a child would first need concrete experiences related to measuring length. She would gradually realise the need for standard units and the abstract concept of length. Then, going further, while learning about measuring area, you would again need to give her concrete experiences, especially to show why one needs another kind of unit for measuring area. Similarly, while learning about the volume of objects, she would need to go back to her earlier learning of length and area.

E8) Clearly, the child had tried to mechanically follow an algorithm that was fed to her by the teacher. Since she didn't know why the algorithm worked, she found it difficult to remember. Therefore, she resorted to what seemed logical to her - the numerator is the sum of the numerators, and the denominator is the sum of the denominators.

For more details you can refer to. Block 4.

E9) According to us, for reasons we have explained in the paragraphs before E9, properly constructed word problems should be introduced as part of the concrete learning experiences.

El0) You can find details of (i) in Block 3, (ii) in Block 5, and (iii) in Block 3 and 5.

Ell) You could refer to Block 5 for details.

E12) Children could sing songs that involve different two-dimensional shapes. As they mention a shape, for example, a circle, they could hold hands to form a circle. Next, if they sing about a triangle, they would form a triangle, and so on. All this could be done to rhythm.

E13) The Class 1 children could be asked to pick up several pebbles (or leaves, or sticks) on their way to school. Then they could be asked to sort them out into groups in such a way that each group has as many pebbles/leaves/sticks as the number of fingers on one hand. Very young children appear to know that `five' represents the number of fingers on one hand. By this matching activity, they could make groups of five. This activity could be done with a variety of objects.

Older children could make groups of ten by counting them out. They could be given coloured paper to tie a flag on each group, and group games, could be organised to use the bundles for teaching the concept of place value, or multiplication by ten.

E15) For related discussions, you could look up Block 5.

E16) It can help in developing their ability to think on their own, without looking towards an adult for support. It's also important for developing their ability to think critically.

Peer interaction also allows a child to see where she is placed vis-a-vis her friends.

You can add more points, especially after you do E17.

E18) The child doesn't seem to have understood the formal symbols that are involved. Probably, the child also doesn't understand how to divide 3 by 9/2. To remedy this, I would go back to the concrete mode to try and explain to the child the process of dividing one fraction by another. For a detailed discussion on this, refer to Block 4.