UNIT 2 KNOWING YOUR LEARNER

Structure **Page No.**

2.1 INTRODUCTION

The other day I overheard 6-year-old Ahmed explaining to his older sister about why swallowing the seeds of an orange is harmful. He said, “The seed will become a plant in your tummy and then a big tree, and then you'll burst. So be careful!”

Well! Well! Doesn't this raise questions like how often do we adults make assumptions about the ways in which children think?

And then, how many of us think about questions like **how** a primary school child learns? Do most children follow a similar learning path from preschool through the primary years of education? Do they go through the same stages of development? In this unit, we shall examine these questions.

We will explore the qualitatively different ways in which children think and learn, as well as the general nature of young children. We will also examine how the adult-child gap shapes our attitudes towards children. We begin with the generally accepted fact that a **child starts learning from the time she is born.** Therefore, she already knows quite a bit when she joins school.

In Sec. 2.4 we outline the major developmental stages that children go through from the preschool through the primary years of education. Although these stages are characteristic of children's general cognitive development, we have discussed them with particular reference to Mathematics learning. We make a case for **viewing the teaching of preschool and primary school mathematics from the perspective of the child,** and not from the viewpoint of pure subject content and pedagogy. We bring you instances to show you that as children explore the world around them, mathematical experiences present themselves alongside others.

Through this unit we also hope to sensitise you to issues raised by the following questions: what factors influence a child's attitude towards mathematics? Why does a child start being afraid of, and feel disinterested in, mathematics? How does classroom teaching influence or cause these attitudes?

Thus, the thrust of the unit is that a teacher of primary school children must be sensitive to issues that determine a child's ability to learn mathematics, as well as issues that influence a child's attitudes towards mathematics. We will reinforce what we say in this unit through the examples that we'll discuss in the rest of the course.

One point that we'd like to mention about the unit is that we have tried to present arguments to support our understanding. Please feel free to disagree with us. But make sure that you too have sound arguments to back your opinions.

Objectives

After studying this unit, you should be able to

- briefly describe the developmental stages of children's thinking and learning processes;
- assess the levels at which various concepts of primary school mathematics should be taught;
- argue why it is necessary to know your learner in order to teach mathematics effectively.

2.2 HOW DOES A CHILD THINK?

Children learn from any activity

You must have interacted with children of various ages. From your experience, do you feel that children start learning from a very early age, and continue learning? Or, do you believe that children are a `blank slate' when they enter formal school, and everything has to be taught to them in the school? In fact, **children learn from anything and everything they see and act upon.** They have learnt a lot before they join school, and they continue to learn outside the school hours. If we believe that children learn only in school, it is because of what we wrongly regard as learning. When a child spends hours on trying to solve a jigsaw puzzle (say), she is often reprimanded by adults for wasting study time. Little do the grown-ups realise that it is through such interesting games that this child may be increasing her understanding of shapes and size. And, this learning is taking place outside the. school hours, without formal instruction. A curriculum built upon assumptions about children's learning, that ignore this aspect, is also responsible for children losing interest in mathematics or in any formal learning.

From the time a child is born, her interaction with the world around her starts. She perceives things around her, and gradually makes sense of them. She slowly begins to recognise people and objects, relate more and more to the environment, and observe things through the senses of touch, sight, taste, smell and sound.

Fig.1: "I wonder if this tastes like the thing Mummy gave me in the morning."

When an infant wanders around on the floor, picking up a tiny bit of dirt from the floor and sticking her finger into the tiniest hole, what is going on in her mind? Isn't she asking herself questions, thinking up possible answers, making theories, hypothesising, and then testing her hypothesis by further questions or observations? This unending curiosity, continuous urge to make sense of the things around, and the power to explore makes the child an investigator, an explorer, a decision maker, a little scientist! And this is true of older children too! Doing the following exercise will give you and opportunity to think about this aspect of children.

E1) List some illustrations of exploration by four or five-year-olds that you know.

Let us now look at another aspect of the way a child's mind works. The other day 4-year-old Akash had gone for a walk with his father. After some time Akash said, "Father I am tired. Let us go back." The father responded, "Already tired! So soon? Let's walk some more!" The father didn't realise that, while he had walked only 75 steps, the child had walked 225 steps!

Now let's look at eight-year-old Rahul, whose teacher used to beat him regularly.

One day he decided that he had had enough. He told his mother, "I am not going to school." She said, "OK", not caring to ask why. She merely assumed that he wanted a change from the routine.

Children's thinking is different from that of adults.

Would you agree that in both these examples the adults did not try to understand what the children were trying to communicate? They simply made assumptions, based on their own perception, about what the children felt and what they were trying to say.

While teaching mathematics, you would often come across similar situations. For example, on being asked to give half of her slice of bread, a child may give only a small piece. For her, anything less than one is half, and half may just mean a piece. But how many adults bother to try and understand, the child's viewpoint?

Fig. 2: Bridging the adult-child gap.

E2) List some more examples to illustrate the difference between an adult's and a child's way of thinking.

E3) Give examples of situations where an adult imposes his logic on the child.

E4) a) In what ways does the adult-child gap show up in the primary classroom

b) List five things that you, as a teacher, can do to minimise such a gap.

Now let us consider another aspect of the adult-child gap. Most preschool children of educated parents learn to recite the alphabet and numbers at home. Some parents even boast of their children having learnt numbers up to 100 by the age of 3 or 4! But, what does this "learning" really mean? Does this mean that the child can count 100 objects? Does the child appreciate the difference between 50 objects and 100 objects?

What small children learn by constant repetition is only a string of number names, without necessarily understanding what they mean. Although knowing the number names is part of the process of learning numbers; it does not reflect the understanding of numbers. (We have discussed this in detail in Unit 5.)

When an adult assumes that a child has understood the meaning of number, just because she can recite number names, then he pushes the child to "learn" more things like addition, etc. The child hasn't been given enough time to understand the concept of number. But she wants to get appreciation from the adult. So she manages, by memorising without understanding. This is how the **expectations of adults can hamper children's real learning.**

Unfortunately, at present most teaching is limited to making children arrive at answers mechanically and fulfil expectations of adults by rote learning. In this way we also reduce all school knowledge to mere information rather than understanding. This is true not only for preschoolers, but also for children in higher classes. For example, we adults often reduce the concept of "area" to "length x breadth = area", or more complicated versions of the same formula. Very few of us actually bother to help the child understand what "area" really means. I'm sure you can think of many such examples.

E5) Give one example each of a concept that a Class 1, Class 2,........Class 5 child is made to learn by heart without really understanding the full meaning of the concept. Also list what aspects are normally missed.

So far, we have been questioning the assumptions that adults make about the way children learn. if children do not learn in this way, then how do they learn?

2.3 HOW DO CHILDREN LEARN?

Have you ever tried teaching a young child what "ball" means? Did you do it by a lot of verbal description" Or did you let *the* child actually handle a ball, throw it, watch it bounce and play with it? If you used the flatter way, you would definitely have succeeded in teaching the child what a ball is. The child learns about the shape, size, texture, bounce and other qualities of-the ball by playing with it and feeling it.

Fig. 3: "I was only trying to learn about glass."

Does, Al-S instance give you some idea about how children acquire knowledge? In this section we, shall consider this question, especially in the context of learning mathematics.

2.3.1 Children Learn By Experiencing Things

One view about learning says that children construct knowledge by acting upon things. They pick up things, throw them, break them, join them, and learn about their properties. A child's natural urge to explore and touch things helps her to develop an understanding of various aspects of these things, like their shapes, sizes and other material properties. This helps her to slowly understand and use the spatial properties of things, that is, she can decide what thing can go under what, or how toys can be fitted into her toy box, etc.

Of course, a child may not always be able to explain what-,he has understood, for example, the difference between a ball and a stone. She may know the difference and may even demonstrate it, but she may not be able to articulate it. Just as you may know how to ride a bicycle, but can you explain in 10 sentences how you do it?

A child constructs knowledge.

In dealing with and thinking about concrete materials, children do many things. The games they play and the way they interact with adults gives them opportunities to deal with concepts and skills' that they are trying to master in different ways. For example, while learning the meaning of 'half, when a child is made to find halves of a variety of objects, she will slowly **construct the understanding** of 'half '.

To **synthesise** different ideas or experiences means to combine them for developing a single idea or impression.

Thus, by doing varied activities, and by analysing and synthesising what happens in the course of these activities, the child constructs a framework for understanding the phenomena around her.

Now, for an exercise for you to assess how much you've understood about how children learn.

E6) Which of the following statements do you think are true about young children? Indicate with a "T" for True and "F" for False.

a) Children know more than they can articulate.

b) Children know no mathematics when they enter formal school.

c) The ability to count means the ability to recite number names in a sequence.

d) When children use the correct word to express a concept, they know the concept.

Let us now focus on a child's ability to learn mathematics.

2.3.2 Coming To Grips With Mathematics

How does a child acquire mathematical concepts? Can any concept be presented to a child at any stage in such a manner that the child gets some idea about it? If this is so, then we adults need to be very sensitive about the level of development a learner has already reached, and present the concept accordingly. We need to gauge the "readiness" of the child for learning that particular concept.

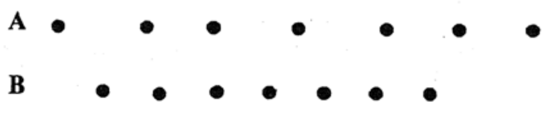
There is an optimum time for learning specific concepts/skills.

For example, for a small child to appreciate the relation between addition and subtraction, she should be able to realise that actions are **reversible.** And, she can do this if she understands how **conservation** operates. How would you gauge this understanding?

Reversibility: the principle that action taken on objects, if reversed in sequence, will return the object to its original state.

Conservation: the principle that quantity (number, mass, liquid) remains the same regardless of the spatial shape it may assume.

In a typical task to check if a child can conserve number, the child is shown two rows of buttons of the same length, with the same number in each row. While the child looks on, the buttons in one row (Row A below) are spread out by the adult. The other row (Row B) is left untouched. Then the child is asked if the two rows have the same number of buttons.



The Swiss psychologist **Jean Piaget** (1896 - 1980) observed the following:

Children who can conserve argue that the two rows have the same number of buttons because you have neither taken away from the untouched row nor added to the first row. They also argue that one can bring the buttons in the first row closer so that the positions of the buttons in both the rows are the same. This argument indicates the ability of the child to reverse her thought process.

In this activity the child and the adult (Piaget) were engaged in a conversation in which the adult was willing to listen to the child, tried to understand her and made her think. The adult was treating the child as an independent thinking person.

The adult must treat the child as a thinking person

By such interactions children learn to articulate reasons and construct arguments. When a child is exposed to several interactions of this kind, she gradually develops the ability to think mathematically.

How often do you have such interactions with children? This is what the following exercise is about.

E7) Give an example of a good adult-child conversation of the kind mentioned above, aimed at developing the understanding of a concept.

As you know, formal mathematics depends very heavily on the use of symbols and algorithms. How comfortable are primary school children with them? Consider an example.

A Class 4 teacher was going to teach her class fractions. At the beginning of the term she asked the children, "If you had three chocolates, and wanted to divide them equally among five people, how would you do it?" Most of the children could think of one or more ways of doing this. By the end of the term, when the children had been taught how to deal with fractions, the teacher again asked them the same question. And this time, most of the children couldn't do it! Instead of reality and their own common sense, they now had "rules", which they could never understand or remember how to apply.

This example reminds us that only supplying readymade rules to children, without explaining why the rules work, usually blocks their thinking. Often, if children are encouraged to see patterns themselves, they find it easy to accept the formal rules of arithmetic that you may be trying to teach them.

Coming to symbols, various experimental studies show that even children as old as 9 have difficulty in representing the operations of addition and subtraction (+ and - signs). Most primary schoolchildren are uncomfortable with the conventional operator signs of arithmetic. This is because symbols (and algorithms, etc.) are taught in a way that makes no sense to the children, as they are not related to the children's reality. Therefore, the mechanics of dealing with the symbols, etc., doesn't interest them.

What can we do to help our learners acquire abstract concepts? To begin with, we mast remember that no amount of explanation will enable any of us to relate an unfamiliar symbol system with reality. We must go the other way, that is, from **concrete examples** to the **symbol** system. Relating abstract concepts and symbols to the everyday experiences of our learners seems to be the easiest way to learn/teach them. Indeed, we all learn this way. Or don't we? Try the following activity and judge for yourself.

E8) Add 4 and 5 in base 5. What processes did you follow in making sense of this task? What difficulties did you face? Do you think the task of a learner beginning to learn mathematics is more or less difficult than this?

While doing this exercise, how much of the difficulty that you faced was because you felt that you didn't have enough previous knowledge to do the task? The point brought out by this question is important, namely, **the readiness of the** learner to comprehend a particular concept or to do a particular task. For example, Class 2 children cannot completely grasp the idea that the digit 2 in 26 means 20, even though they can write and recognise 26 and can also identify it as a number smaller than 62. But the teachers often assume that the children have understood the concept of place value, and force them to start solving problems with "large" numbers by using standard algorithms. This is of no pedagogic value. In fact, teaching children strategies and methods of solving problems that they are not ready for stops them from thinking, simply because they get preoccupied with the mechanical task of arriving at an answer.

What we have discussed so far also adds weight to the following observation of child psychologists.

2.3.2 Children Have Their Own Strategies For Learning

Vibhor, aged 7, was once asked if he knew what `seven lots of eight' are. He said he didn't. He was then asked, "Can you work it out?" There was a long pause. Then Vibhor said, "56". "How did you get that?" "Well", answered Vibhor, "ten eights is 80. Then take away 8, that is 72, again take away 8, 64, take away 8, 56."

Fig. 4: A child developing her mathematical thinking

Shanta, a Class 3 child, was asked to solve 189 - 67. She said it was 3 + 30 + 89, that is, 122. This strategy of hers, was considered `wrong' by her teacher, because his method to get the answer was `189 take away 7, take away 60'.

Discouraging children from evolving their own strategies results in blocking their ability to think, to build connections and look for patterns in mathematics. Instead, they learn mathematics as a series of disconnected and meaningless facts and rules to be blindly memorised and applied (for example, the multiplication tables).

If you allow children to solve mathematical problems by their own methods, you would find an amazing variety of thought processes. Consider what this friend of mine has to say.

"While teaching children mathematics, I have often been surprised by the manner in which children arrive at answers to questions. Between the problem and the answer there is a string of arguments and logic which is often a creation of the child. I once taught children multiplication of a two-digit number by a two-digit number. I worked out a few examples and explained the algorithm to them a number of times. Then I gave them the problem,

$$\begin{array}{r} 12 \\ \times 13 \\ \hline\hline \end{array}$$

The first child to report having done the problem gave the answer 156. On looking at her notebook I found the answer written just below the problem. I asked the child for the rough work She produced the following figures 100 20 30 6. On repeated questioning the girl said, "I multiplied 10 by 10 first, and then 2 by 10, and so on."

On another occasion I was doing problems in simple interest that involved finding out the rate of interest. Giving them a problem, I was just beginning to relax when a hand shot up. I was amazed at the speed, and asked for the answer. He gave the correct reply, 5%. I mentally patted myself for a successful presentation of the complex algorithms. Then I suddenly thought, "Let me look at the notebook and find out how he has solved the question." There was a lingering doubt in my mind that he was perhaps coached at home. The child came with a blank notebook I asked him where he had solved the problem. "Oh! I solved it using what you had said yesterday", he said. "You had said that banks give 5% interest on money deposited." "

What do you deduce from these two examples? Would you agree that they add to the evidence that **children develop their own strategies to solve problems?** They may either be correct or wrong, from the adult's point of view. But, for the child they are always correct. A child continues modifying old strategies and developing new ones to match and understand her mathematical experiences. Given the right environment, this process goes on, and enables the child to become capable of doing and thinking mathematics naturally. But forcing children to follow one single strategy, without deviation or creativity, gradually dampens their urge and ability to create their own strategies and conceptual thinking.

Here are some exercises based on what we have just discussed.

E9) Give some children around you a task in mathematics. The task should be in an area in which they' have not been given a large dose of algorithms and strategies. Do all of them follow similar strategies? What are the various strategies adopted by them?

E10) Here are four problems. Four children solved one problem each, as given below. Identify the strategies the children have used while solving them.

a) 8 + 6 = 8 + 2 + 4 = 14

b) 4 + 9 = 5, 6, 7, 8, 9, 10, 11, 12, 13

c) 3 + 12 = 12, 13, 14, 15.

d) 7 + 5 = 1 + 1 + ... + 1 (seven times) + 1 + ...+1 (5 times) = 12

Thus, we can say that standard algorithms discourage children from thinking and constructing their own understanding of mathematics. They also reduce mathematics to a mere mechanical activity of observation and manipulation of symbols. This is reinforced by most teachers who give children marks only for the mechanical steps they follow in applying an algorithm.

Another very important aspect of children learning mathematics is language. Let's see how.

2.3.4 Speaking Mathematically

A Class 2 teacher was explaining the concept of place value to his students, using the number eleven. He started by saying "One and one make eleven." Some of the children, who had till now learnt that one and one actually make two, were thoroughly confused. Why did this confusion arise? Could it be because of the language used?

The language of mathematics and mathematics as a language, both place heavy demands on children.

Clearly, language is needed for conveying mathematical notions to children. Also, language itself is something that children are trying to master. Hence, in learning mathematics, children have to cope with trying to understand language as well as mathematics. And therefore, when you find that a child is not able to understand a particular mathematical concept, it may just be due to confusion created by the language used for explaining the concept.

E11) Give some examples, from your experiences, of confusion arising in a child's understanding of mathematical concepts because of language interference.

Sometimes children coming from certain backgrounds may not be familiar with some words that are used in the textbooks and by the teachers. For example, not knowing the meanings of terms such as `shorter', `wide', `same', `different', `few', `as many as', `equal to', `each', etc., can obstruct their understanding of mathematics. Another source of confusion is when many different words express the same mathematical concept. For example, `equals', `makes' and `is the same as' are all represented by the sign '='.

Even older children often have to face this kind of problem. This is because the language used in conveying mathematical ideas at any level places heavy demands on the children's ability to comprehend language. Getting children to talk about the mathematics that they are doing helps them to tackle this problem, and to learn the language of mathematics.

At another level, children can be confused by the grammatical complexity and sentence length of a word problem. For example, the question "What number between 25 and 30 cannot be divided exactly by 2 or 3?" is indeed complex. Wouldn't a child find it easier to understand if it were reworded as "Look for a number between 25 and 30. You cannot divide this number exactly by 2 or by 3. What is the number?"?

Doing the following exercise may give you some more insight into the importance of using language that a child is familiar with.

E12) Identify the different ways in which you can explain the following mathematical problem to a Class 2 child and to a Class 4 child.

`Why is one-fourth less than one-half?

Observe the language you use.

And finally, a point to keep in mind about the learning environment, that holds for any of us, child or adult.

2.3.5 High Self-esteem Helps Learning

Consider Ajay, a student of Class 2. He is constantly told by his irritated father, "How stupid you are! You don't even understand this! Even your sister understands it." How do you think Ajay feels? And, if his sister hears this, how would **she** feel?

Children, like all of us, learn best when they think well of themselves. A child who has poor self-esteem is likely to be constantly worried about her inability to please the teacher, and consequent rejection. The importance a teacher gives to children's feelings and her attitude towards children greatly influences the way children learn. Beginning with what children can do, providing feedback to them in terms of how and why they are `wrong' or `right', and allowing them to seek clarifications and ask questions, enables them to develop high self-esteem. The most important thing is that the teacher needs to have faith in her learners. And she should show them that she has faith in their ability to learn.

Children who are constantly anxious about being reprimanded and being compared unfavourably with other `brighter' ones are likely to develop withdrawal tendencies. They avoid answering questions and actively participating in classroom discussions for fear of making mistakes. This decreases their self confidence and their capacity to think and learn independently. Most children within the present competitive system of education have this problem. Such children continue to learn mechanically and depend on algorithms rather than develop mathematical thinking. They even become obsessed with the `one correct answer' approach, and the desire to learn and create gets permanently stifled.

Children learn best in an emotionally secure environment.

Do you agree with what we've said in this sub-section? The following exercise may help you to decide.

E13) Give instances from your childhood, of you or your friends disliking a subject because of the reasons mentioned in this sub-section. How would you correct the situation, if you had been in the teacher's place?

There is an important social attitude that we have hinted at when talking of Ajay's sister in the example above. It shows up in remarks like "How come, being a girl, you know maths so well?" Why do people believe that girls can't learn mathematics? In fact, gender differences in students' attitudes towards mathematics arise from what society expects boys and girls to do. For example, it has often actively encouraged boys to pursue studies in mathematics and discouraged girls from doing so. Attributing failure in mathematics to gender has led to a decrease in the self-esteem of the girl child, and a consequent mental block against mathematics at a very early age.

Girls and boys are equally capable of learning mathematics.

So far we have taken a brief look at the factors that need to be considered while examining the way a child learns. Let us now look at the intellectual growth of a child.

2.4 DEVELOPMENT IS CONTINUOUSLY GOING ON

Think of any two children around you. Would you say that they are alike? Do they learn the same things the same way? It is very unlikely because children are extremely varied. In fact, no two children are alike. They learn at different paces, respond to situations differently and go through different experiences in life. But, in all this variation you must have noticed a pattern, a pattern in their development and learning abilities.

Piaget describes children's' development in four stages, beginning with the sensorimotor stage (from birth to 2 years).

Children follow certain broad stages of development. Each stage builds on the one before it. The stages of cognitive development have been categorised by

Piaget as the sensorimotor, pre-operational, concrete operational and formal operation stages.

For the purposes of this course and this section, we shall concentrate on the second and third stages.

2.4.1 Pre-operational Stage

'Cognitive' means relating to the process of learning, understanding, and representing knowledge.

This period of a child's cognitive development usually begins at the age of 2, and lasts until about the age of 6. Thus, it usually coincides with the preschool age. In the **pre operational stage** the child's judgements are based on how things appear, rather than on adult logic. For example, a child may not realise that certain actions can be reversed. And therefore, she thinks that the volume of milk decreases as it is poured out of a thin long glass into a shorter wider one. Or that the same objects placed further apart in a lines to form a longer line, are more in quantity. Haven't you, at one time or another, exploited this inability of the preschooler to conserve? I have done so, for instance, when my four-year-old nephew insists on having a full cold drink. I pour the drink out from the half-full tall glass into a small glass. This way he gets a full glass of the cold drink, which makes him happy.

E14) Give two more examples from your own experience, where a child's judgement is based on visual perception, rather than on logic.

Fig.5: Very young children can make out which set of objects is bigger, if the difference in size is obvious.

It is at this stage that an average child learns how to handle some numbers consistently and how to apply them to a variety of everyday situations. A two-year-old learns to distinguish between 2 toy cars and 3 toy cars merely by looking at the group of cars, in just the same way as she learns to use her visual perception to distinguish between a car and a bus. However, she cannot make out the difference between a collection of 8 cars and one of 9 by just looking at them. She will have to learn to count to compare these sets or larger ones.

Children appreciate the concepts of `more' and `less' much before they can even count or conserve quantity. This is because **preschoolers think in patterns,** and therefore, rely a lot on their own perception. Children can recognise shapes much before they can reproduce them. They can `read' pictures much earlier than they can read words. You must have observed young children recognising human and animal drawings with ease, and yet not being able to reproduce even a square or a triangle with the same ease. Similarly, a young child who can remember how to go from one place to another often finds it very difficult to describe the route verbally or pictorially. It is a child's ability to recognise patterns that enables her to `read' words that she has often seen, without recognising even a single alphabet.

Preschoolers can add and subtract small numbers.

If you give preschoolers some objects to count, you will be surprised to learn that they can recognise, count and even add and subtract small numbers **before** they can conserve number. They work with a small number of objects perceptually, and deal with them without counting. This is why numbers up to 4 or 5 are also called **perceptual numbers.**

See Unit 5 for more details on this.

But the usual preschooler does not understand the idea of relative size. She can only compare two objects at a time. For her, quantity can be either "More" or "less", not "more than this, but less than that", and also not "more, less and lesser". For her, size is also either big or small, there's no in-between. Because of this, she may also not be able to order objects according to, size and length, or sequence events.

Now for an exercise!

E15) I have a three-year-old friend. He has a lot of toy cars to play with. Playing with him once, I divided the cars into two sets. One set was more spread out and had 14 cars in it. The other set had 15 cars, but they were placed more closely. He had the choice of taking the set with more cars. He made the

correct choice. From this, which of the following statements would you deduce? What are the reasons for your choice?

a) He can count upto 20.

b) He can perceptually distinguish between large sets.

c) He just made the choice by chance, and may not be able to repeat it.

d) He may be able to do a lot of things with cars, but not with other objects.

As a child gets older, she moves from an intuitive understanding of number to a level higher than that of recognising numbers by mere perception. A good way of enabling the older among the pre-operational children to make connections, see relationships, and thereby, increase their mathematical understanding, is to involve them in games played with a small number of objects in which they have to `add' and `take away' objects.

Let us now shift our attention to the older, more developed child.

2.4.2 Concrete Operational Stage

Piaget describes a five-year-old boy playing with a collection of pebbles. First, he laid them in a line and counted along the line from left to right. There were ten. Then he counted them from right to left. To his great astonishment, the total was again ten! He put them in a circle, counting them clockwise, and then anti-clockwise. Full of enthusiasm he found that there were always ten. Whichever way he counted, the number of objects was always the same. He was discovering that the number of objects remains the same even if the way they are placed changes. Slowly the child was ridding himself of his own earlier idea that the number of objects in a set depends on the way they are laid out. He was now ready to conserve. He had also achieved the understanding of the mathematical idea that one conserves quantity even when one partitions a set of things into subsets. Conservation is achieved by a child at the **concrete operational stage,** which a child passes through approximately between the ages of 6 and 10. It is this understanding that creates a qualitative difference in the concrete operational child's thinking.

Around the age of six or seven, the child can count and compare two sets of objects, and perform the more complex operations of adding and subtracting objects. The numerical operations gradually become internalised, but not at the level of abstraction. 7 to 10-year-olds remain essentially related to physical objects. They can conserve and intuitively grasp many basic ideas of mathematics. **But this grasp is only in terms of concrete** operations. This is why Piaget refers to this stage as the stage of `concrete operations'.

Thus, if you give a concrete operational child the choice of solving a problem of addition by adding objects rather than numerals, she would prefer to count objects because she trusts her intuition and concrete experience rather than symbolic operations. For example, take six-year-old Kavita. She worked out the following problem of subtraction:

$$\begin{array}{r} 16 \\ -\ \underline{31} \\ \underline{25} \end{array}$$

When asked how she did it, she replied, "1 take away 6 is 5, 3 take away one is 2." When asked if it was right, she said, she did not really know. However, given the problem of `31 take away 16' to do the way she wanted to, she soon came up with the answer, 15. She even exclaimed that this was done by a different method.

Similarly, Amit (8 years) could not do the division problem '$45 \div 3$', but found it easy to share 45 sweets between three people.

Children have problems with conventional methods because **the formal code is much too abstract to master at this stage.** Relating the problem to a concrete real-life experience helps children to rely on their own intuitive understanding, and thus invent a strategy to arrive at a solution.

There is also another reason why children in early primary school have a problem with handling formal arithmetic. In the formal code of arithmetic the operations proceed from right to left, whereas reading in English proceeds from left to right. Many primary school children continue to make the error in going from left to right while doing arithmetical operations. This indicates the need to develop their spatial thinking ability before and along with other arithmetical abilities.

Here's an exercise which will help sensitise you to the abilities of your learners.

E16) From your experience, and what you have studied so far, by which age would-you expect an average child to be ready to acquire the following concepts?

i) simple classification,

ii) conservation of length,

iii) commutativity of multiplication, i.e., ab=ba for any two numbers a and b,

iv) time,

v) chance and probability

Formal operational stage (above 11 years).

The final stage of logical development occurs at the post-primary age, that is, 11 years and above. In this stage a child becomes capable of using **formal operations.** She is now capable of using words or symbols to denote quantity or objects. The child can now work with hypothetical statements, and explore logical relationships between statements. The ability to cope with abstractions such as concepts of proportionality, variables and algebraic equations depends on how well developed the child's formal operational thinking is.

The problem with learning mathematical concepts is that concrete references are generally not made available to help primary school children learn them. The methodology of primary school mathematics teaching, as practised today, is more appropriate for a formal operational child. But, what the child requires is concrete and meaningful learning experiences for grasping mathematical concepts and skills.

And now for an exercise.

E17) Which of the following statements do you think are true about children? Indicate with `T' for true and for false. Give reasons for your choice.

a) Most primary school children are in the concrete operational stage.

b) To understand number, children must be able to classify and order.

c) Pre-operational thinking is characteristic of the primary school child.

d) The ability to conserve is fundamental to the development of mathematical thinking.

In this section we seem to have divided children very neatly into different stages of development, based on their ages. Does this mean that no 9-year-old has reached the formal operational stage, or that a 6-year-old can't be at the pre-operational stage? Let's find out.

2.4.3 Each Child Is Unique

Although every child goes through similar stages of development, the process may vary from one set of children to another, and also from one child to another. This difference may be related to the socio-economic and cultural background that children come from, apart from individual differences like those in intelligence, attitudes, etc. For example, it is common to find a street vendor's six or seven-year-old calculating the cost of three or four items in a very short time.

For another example, girls in rural households are expected to cook from a very early age on. Consequently they have an intuitive grasp of proportionality, conservation of mass, and many other mathematical concepts. This wouldn't be true of a child of the same age from an urban middle-class background, or even a boy of the same age from a rural background.

Apart from social, cultural and economic factors, there are psychological aspects that vary from child to child. This is often reflected in the different strategies that a child evolves for grasping new concepts. For example, a child may either use a `count all' or `count on' strategy to begin with, and proceed to more efficient ones as she grows. Some children regroup numbers in fives, while others may do so in threes, or even tens.

Why don't you try this exercise now?

E18) "Children of the same age can be at different operational stages, and children of different ages, can be at the same developmental stage." Do you agree with this statement? If so, give examples to justify it. If not, give reasons for your disagreement.

Most child psychologists believe the statement in E18 to be true. Therefore, we need to observe our learner very carefully, assess her capabilities, and not just assume that she has or doesn't have certain abilities because of her age. The activities you want your learner to do and the learning situations you want to present her with must match her abilities at that point of time. There is no point in expecting a pre-operational child to compare the number of objects of two large sets. Her answer to "Which is more?" would depend on her visual perception. It may turn out to be right, but there is no way of knowing how the answer was arrived at.

Over here I would like to make an important remark about the stages we have discussed. They appear to be like steps while climbing a ladder. In other words, it appears that we finish with one stage, and only then go to the next. They seem to be quite separate. But, this is not so. **There** is **a continuity in the cognitive development of a child.** Each stage builds upon the previous one. Also, there **are phases of development within each stage.** For example, while a child is struggling to conserve number, she is also struggling to conserve area, or volume, or deal with 'sedation. While she is struggling to compare two sets by one-to-one correspondence, she may also be learning to count. The different stages are important for the adult's understanding of children, and for planning curriculum content and instruction.

As discussed earlier, each child develops at her own individual pace. Some may reach the concrete operational stage faster than others for various reasons, including different backgrounds. As a teacher, you need to think of tasks and learning situations that are appropriate for your learner's developmental stage and background. Of course, you need to be aware of the general pattern of development while formulating general teaching strategies.

As a teacher, you would also need to keep in mind what we will talk about now.

2.4.4 The Invisible Effort

Although the development of children is a process, what is noticed and given recognition to is the end-product. We usually speak of children having achieved the ability to subtract, multiply or divide, that is, of **having reached a certain stage of development.** The long and arduous process that the child goes through in arriving there is not usually noticed. For example, have you ever thought about the processes that a child goes through while learning how to count? The following activity may give you some idea.

E19) Imagine a time in history when the number system had not yet evolved. A farmer needed to keep track of his cattle. What would he do to figure out whether all his rattle returned home safe? List the processes that you can think of.

If you've done E 19, you must have realised that subsumed in a specific skill (ability to count), there are a number of sub-skills. This fact should be kept in mind when you're going through the next unit.

Let us now round up what we have said about the learner.

2.5 SUMMARY

In this unit, we have tried to present some of the different thinking and learning processes of preschool and primary school children, in the context of mathematics learning. We have specifically focused on:

1) how children attempt to build their understanding of early mathematical concepts. In particular,

   i) the interactions of a child with materials and with other people forms an important basis of learning. And children are doing it all the time. Therefore, they're learning all the time, unless we stop them by interfering!

   ii) acquisition of knowledge and parroting expected information is not the same thing.

2) the ways in which children construct knowledge.

3) individual differences in children.

4) the concept of mathematics readiness.

5) the importance of self-esteem for the cognitive development of a child.

6) the different stages that preschool and primary school children undergo in developing their understanding of mathematical thinking.

7) the fact that development can be facilitated by exposing younger children to various real-life situations that involve the use of mathematical concepts, but it certainly cannot be pushed. Any attempt to hurry children can lead to a serious loss of self-confidence.

And now, you may like to go back to the **objectives** listed in Sec.2.1, to see if you feel that you've achieved them. You may also like to go through the following section in which we have given some hints and suggestions about most of the exercises in the unit.

2.6 COMMENTS ON EXERCISES

El) For example, when a child goes for a walk with you, and asks, "Do flowers have a mummy and a daddy?"

E2) For example, for a 6-year-old child, the oldest that a person can be is 20 years old. The understanding of the concept of time is a source of a lot of examples.

E3) For example, when a child paints a drawing of a at blue, and the adult insists that a cat can't be blue.

E4) a) For example, when a child can recite numbers upto ten (or twenty), the teacher assumes that she knows what each number means, while, in fact, the child may not be able to count even 6 objects.

You can list many other examples.

b) For one, help the child to talk about what she thinks is the concept or the process that you are teaching. This can help you to see her point of view.

You can list several other points that you would keep in mind while trying to communicate with children.

E5) For example, consider, the algorithm for multiplying two 2-digit numbers, that is taught to a Class 3 child. Suppose the child uses this to multiply 35 by 42. She places a cross in the right-hand corner of the second row, without knowing why it should be done. She doesn't realise that what she is actually doing is writing 42 as 2 + 40, first multiplying by 2, and then by 40. Multiplication by 40 just means multiplying by 4 and shifting the resulting numeral's place by one leftwards. And why this shift? Answering this requires a proper understanding of place and place value.

E6) a) T

b) F

c) F

d) F

E7) As mentioned in the exercise, an example is given in the preceding paragraphs.

E8) When I did it, I first wrote the numerals in a different number system. For example, 4 is still written as 4, but 5 is represented as 10. The place values have changed to `units', `fives', and so on. Note down the problems you face at each stage. How did you try and overcome them? (See Unit 6 for details on place value.)

E9) For example, when I asked some 8-year-olds to multiply 12 by 8, I found one of them adding 12 eight times, another doing 12 x 10 - 12 - 12, and so on.

E10) a) Re-grouping and adding.

b) Counting-on.

c) Using commutativity and counting on

d) Counting all.

E11) When a Class 1 child was told that `3 - 1' denotes `3 take away 1', and was asked to do this sum, he merely rubbed off 1, and left 3. When asked why, he said, "you wanted me to take away 1, so I rubbed it out."

You could think of many other examples.

E12) I would try and explain to a Class 2 child with several concrete examples, and to a Class 4 child using pictures and symbols, of course depending on her level of understanding. I would talk to the children and get them to talk about what they are experiencing. This way their clarity of 1/2 and 1/4 could be gauged.

E14) Take two pieces of string of the same size. Keep one straight and curl the other in front of a child of 5. She is very likely to think that the curled one is shorter.

You can think of several other examples.

E15) (c), because a 3-year-old is not likely to have understood the number concept.

E16) In an urban middle-class set up, it would probably be

i) 4 years

ii) 7 years

iii) 9 years

iv) 9 to 11 years

v) At a simple level, 9-year-olds.

E17) a) T. They still require concrete experiences to understand concepts and processes.

b) T. This is discussed in detail in Unit 5.

c) F. When a child can conserve, she is ready to enter the concrete operational stage. Thus, most primary school children, especially 6-year-olds and above, are in the concrete operational stage.

d) T. The ability to conserve subsumes the ability to reverse thought processes, which is essential to mathematical thinking.

E18) This is a true statement, because it is not age alone that determines a child's level of cognitive development. This development also depends on the socio-economic background and kind of exposure a child has had, among other factors.

E19) When I tried this activity I got the following list.

- recognise the animals by their characteristics,
- keep a stone/twig for each animal as they return,
- if there are too many, group these into fives, and give the group a symbol,
- restrict each one to its respective space and identify the missing ones.

These processes involve evolving a means to identify with a name (number name), partitioning (`counted' and `to be counted'), grouping (base 10 in the present system of number), symbolising (numeral), and the use of space (spatial layout of objects).