UNIT 10 LEARNING ABOUT CHANCE

10.1 INTRODUCTION

Probable impossibilities are to be preferred
to improbable possibilities.
Aristotle (384 – 322 BC)

In the previous units on data handling we have talked about the need to introduce data handling early on in the primary school. We found that children are aware of many forms of data. They may also be using the data to make generalisations, choices and decisions. Can we help children develop their abilities to analyse data and make certain predictions? In fact, is there even a need for children to think about questions requiring inferring from data? Would they be involved in predicting outcomes of events taking place in their environment? Would there be situations in their lives where they would need to decide about what is more probable and what is equally likely? In this unit we shall be answering such questions.

In Sec. 10.2, you will see how all of us, including very young children, have a very varied understanding of chance. This understanding is, of course, based on each person's experience.

In Sec. 10.3 we aim to bring out the need for introducing children to elements of probable outcome, chance and related ideas. We shall discuss how this can help all of us analyse our biases, social or otherwise. We shall also see how a proper understanding of chance can help all of us take better decisions.

We assume that, by the end of Sec. 10.3, you will realise that it is important to include a study of chance and elementary notions of probability in the primary school syllabus. Assuming this, in section 10.4, we suggest some activities that may help children develop an understanding of chance, 'likely events', 'impossible events' and 'possible events'.

Through this unit, we hope to encourage you to bring in elements of data and chance in your teaching. Some of you may not recall what probability is about. Therefore, we end the unit with an **appendix** on what probability is, very simplistically put.

Objectives

After studying this unit, you should be able to
- explain the need for primary school children to learn about chance and related concepts;
- suggest effective methods of communicating these concepts to the children.

10.2 CHILDREN'S NOTIONS OF CHANCE

In conventional primary schools, the children are not given any opportunity to explore probability and understand implications of chance. And yet, the world around us requires us to keep making decisions on the basis of the likelihood of something happening. A knowledge of probability helps us make informed decisions in such situations. So, do we not expose children to probability, even informally, because we think they don't deal with decision making or 'chance'? Let us see whether this is so.

In a survey we had undertaken, we asked 50 children (between the ages of 9 and 15) questions about

- picking balls of a particular colour from a bag having balls of different colours;
- events that can happen by chance; and
- what 'luck' is.

Read on to find some of the data we gathered.

One of the questions was: I have a bag in which there are 7 black balls and 3 white balls. If I take out one ball, what would its colour be? Give reasons for the colour you have said.

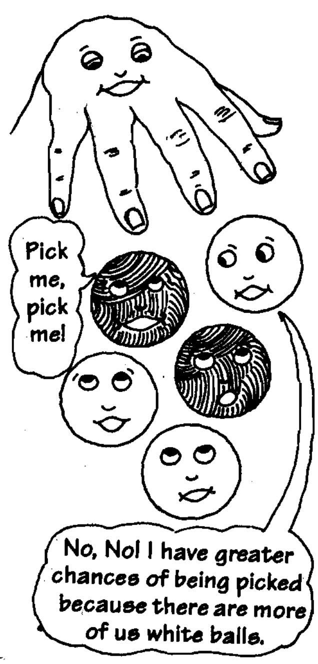


Fig.1: Which coloured ball has a greater probability of being chosen?

Roughly 30% of the children said that a black ball would be picked out because the number of black balls is more. Many other children also said a black ball would be picked, but gave different reasons like 'because I like the colour', or 'because it is the colour of my dog', etc. Some children said the ball would be white 'because white is a symbol of peace', or 'because white is nice to touch'! A few children were bewildered and said that they couldn't answer the question until they actually did the experiment.

Now we asked all the kids: In a bag I had 7 white and 5 red balls. I drew one ball and found it to be white. I put it back into the bag and shook the bag well. Then I picked another ball from the bag. What was its colour?

In response to this, only about 20% of the children said that it was white because there were more white balls than red. Some others said that it would be white because white is the colour of peace, or because it is a good colour, or because white balls do not explode. Many other children said that they would have got a red ball because:
- I like red
- red coloured balls have a different kind of a feel

- white balls are more, and whatever is less probable usually happens (from a 15-year-old)
- after joy comes sorrow!
- when we shake the bag whatever is at the bottom will come on top (a 15-year-old in Class 9)

The answers most children gave showed some form of association. It does appear that children form notions about these issues as well as others very early on.

Why don't you see if the children around you have a different response?

E1) Try a similar exercise with 10 or 15 children in Classes 3 to 5. Note down your observations. How different are they from the data we got?

Continuing with our survey, we asked the children to give us an example of an event which they associate with 'chance'. The answers to this were very interesting. They consisted of descriptions of various types of accidents on the road or at home, or of someone winning a lottery or meeting a long lost friend, etc. Almost all the responses were of events that do occur, but only rarely. This shows us that **the notion of 'chance' is associated in these children's minds with the occurrence of events with a very low probability**. (If you are unsure of what the mathematical definition of probability is, please read the **appendix** to this unit).

Fig.2

Do you agree with this understanding of 'chance'? Or, would you use 'chance' as in 'There is a better chance of getting rain in monsoon time, than at other times of the year'? In fact, this statement was made by a Class 4 child while talking to us about what 'chance' means. Other children have also used it intuitively to mean the possibility (not just a remote one) of an event happening. We need to build on and strengthen such an intuitive understanding.

Now, why don't you find out what the people around you associate with 'chance'?

E2) Ask a few 8- or 9-year-old children, as well as some adults, which events they associate with 'chance'. What understanding do their answers show? What differences do you find in the type of answers the children gave you and those that the adults gave?

Let us now come to 'luck'. We asked the 50 children whether they considered themselves lucky or not, and why. We also asked them if they knew anybody whom they considered to be unlucky.

Most children said that they are lucky. The more common reasons given were:

- I live in a good house,
- I study in a good school,
- I get good marks in the examination,

- I am a human being.

The few children who said that they are unlucky gave reasons like

- I have done badly in the examination,
- My father does not work,
- I write so well in the examination, yet I do not get good marks.

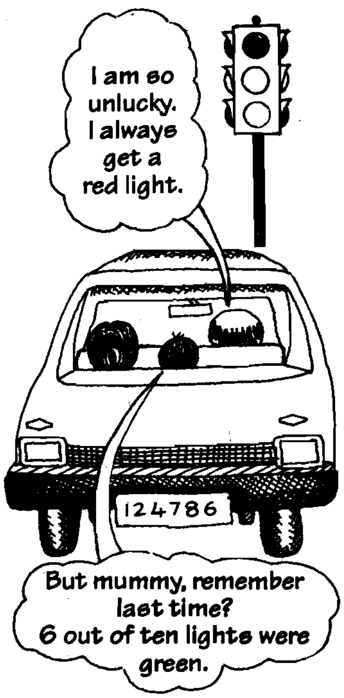


Fig. 3

We also asked them whether they know anyone who is unlucky. About 70% children said that those who do not do well in life, who fail in exams and those that are poor and have nothing to eat and wear are unlucky. From the children's answers, it was clear that good things were **certain** to happen to lucky people. In other words, **the chance of something good happening to a lucky person is very high.**

Now here's an activity for you.

E3) What is the relationship between 'chance' and 'luck', according to you? Also ask your friends this question and note down their responses.

What we have just seen is that children do make associations. This is based on their capability for making generalisations by observing many particular cases. But, for them, the generalisations are certain, and not just probable. As they grow older, they do develop some notion of measuring the likelihood of something happening, i.e., its probability. This understanding needs to be modified and/or reinforced. Why this is important is what we shall now discuss.

10.3 WHY TEACH CHILDREN ABOUT CHANCE?

In the previous section you read that children have some understanding of chance. We shall discuss two major reasons here for trying to develop their understanding that the chance of something happening varies and can be measured.

10.3.1 Weighing the Options

As we have seen, children and adults are constantly having to decide between alternatives that seem to have the same amount of plus and minus points. Many of us decide by tossing a coin. For instance, while playing football or hockey, a coin is used to decide who will have the first strike. Again, haven't you come across children saying, "Let's toss a coin. Heads — we go to the movies, tails — we go to the park"?

And then, if a choice between more than two options is required, a dice or fingers on a hand are used. Children play several board games with dice, moving their pieces on the board according to the number they get in the throw.

Have you thought about what children think about the different possibilities involved in tossing a dice or a coin? Do they think of them as events that can be controlled by the person throwing the dice or coin? While interviewing some Class 3 children about this, we asked them, "If you throw a dice, which is the most difficult score to get?" Most of them said 6 is the most difficult. Some said that the possibility of 6 coming is the same as that of any other number. One child said, "We never get the number we need." Several other children, both younger and older, agree with 9-year-old Vibha. She says, "The number we want is the most difficult to throw, and getting it depends on luck. If we are lucky, we always get the number we want. Also, there is a trick to throwing a six whenever you want it."

In response to the question: "Can we get 5 heads in 5 tosses, one after the other?", Vibha said, "Yes. It may even happen that we get 10 heads at a stretch, we can't say." When asked what we can say about the number of heads in 1000 tosses, she first said that there would be 500 heads. When asked why, she thought about it some more and said, "In 1000 tosses also all of them may be heads, or 600 heads may come, or any number near 500 may come."

From a survey we did with children of Classes 3, 4 and 5, we found that most of them believed that if a coin is tossed 5 times, getting 5 heads in a row is less possible than getting a head and a tail alternately. Most of them also believed that, unless a person is unlucky, for every bad thing that happens to her, a good thing will happen to make up for it. We shall not stop here to discuss whether they are right or wrong, but would like you to think about **how these beliefs and associations may have developed**. Could it be their experiences, could it be based on what they hear from the people around them? We nead to find out the reasons if we want to help them change these fixed ideas from early on?

It is necessary for us to help children understand, for instance, that the number of heads tends to be about half the number of tosses as the number of tosses increases. They need to realise that in a fair game each player wins and loses to the same extent, more or less. Though the word 'probability' need not be used, children need to understand that **it is a way of measuring uncertainty**. They need to understand that if there are events that involve a risk, how much of a chance is really being taken with the occurrence of the event. For example, any vaccine is not a 100% protection, and sometimes the inoculated person can have side effects — so should the vaccine be given or not? This depends on how effective it is versus what its side effects are. Suppose the vaccination against a disease is effective in 800 out of a 1000 cases and if there is a strong probability of getting the disease if one is not vaccinated. Then, weighing the risk in both the options — should we or should we not get vaccinated, , it would be better to be inoculated. This kind of rough understanding and use of probability is what we need to develop in children and adults. Think about other reasons for teaching about chance while doing the following exercise.

E4) Many situations in the real world involve chance. List at least 5 that children would come across.

We need to ensure that children meaningfully relate to the idea of 'chance' in the various situations they encounter. They should not equate it with 'accident'. This is why these notions need to be introduced early to children.

We also need to help children look at their biases more objectively, which the study of chance can help in doing. This is what we shall discuss now.

10.3.2 Social Biases

All of us make associations and generalise about the world around us. These generalisations serve as a basis for superstitions, beliefs and biases. These messages are all around the children, becoming a part of their persona and their beliefs. Many of these social beliefs are passed on from generation to generation. For example, in places where the birth of a son is hailed, the mother is blamed if a daughter is born to her. Then, again, you may find people believing that girls are not capable of learning science or mathematics, or that poor people are irresponsible and lazy. There are many such biases you would also have about other social groups.

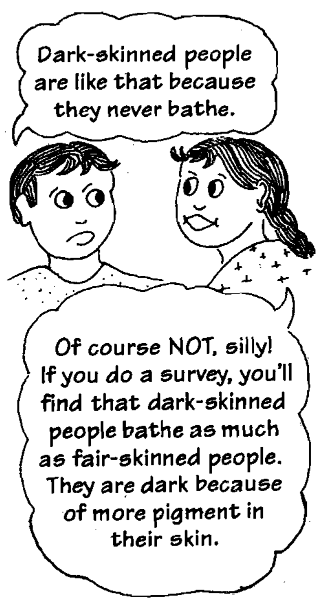


Fig.4

And what about the many superstitions that children and adults believe in? Many of us feel upset if a cat crosses our path or if somebody sneezes just before an important event. Children, while preparing for exams, use a lot of omens that they think will help them to do well in the exams. Are these beliefs reasonable? Similarly, when we believe that a certain class of people are dirty or illiterate, is it reasonable?

These social biases and superstitions may change because of an exposure to probability. For this to happen, the children need to think about whether their beliefs are reasonable. And, one way of doing this is to ask the children to compare these statements with all their experiences and those of the people around them. They could, then, analyse this data to see how false or true their beliefs are. In fact, you could pick some of these social biases, convert them into hypotheses and ask the children in your class to collect data to find out how probable these statements are. The following exercise is one such activity for you.

E5) Pick 2 social biases for conversion into hypotheses. Ask the children in your class to gather related data about them. How did you help them verify the truth or falsity of the hypotheses?

If we want our children to think logically, if we want them to form better and more valid notions, then we need to induce them to think about 'chance', 'luck', 'possibilities' and 'impossibilities'.

10.4 ACTIVITIES FOR CHILDREN

In this section we shall give some suggestions for activities that can help children grasp concepts related to probability. Please try them out with the children you are teaching, and see how effective they are. Also, please keep in mind the points made in Unit 7 while going through this section.

Activity 1

Divide the children into groups of 3 or 4, and give each group a coin. Ask the children to toss the coin several times, and record whether they got a head or tail each time. The record can be kept by each group in a table with two columns as given in Table 1.

Table 1

Toss No.	What we get
1	x
2	x
⋮	⋮

They can put a tick when they get a head and put a cross when they get a tail. After 15 tosses you could ask them to look for the longest repetition. For example, did they get a head continuously six times? Or, for how many tosses did they have tails and heads alternately? You could ask each group to note down its longest chain.

You could also ask the students to look for patterns in the outcomes of the throws. For example, do we see a head following two tails? Or, vice-versa? Or, do heads and tails appear alternately all the time? Or, is there no pattern at all? Ask them to increase the number of throws and record the result after every 10 tosses. What is the longest chain now? Is there a pattern now in the tosses?

E6) What do you think the children are learning in this activity? In what way would it contribute to their understanding of probability?

The activity above could be modified to be made more challenging to the learners in several ways. We have suggested one way below.

Activity 2

Once the children have made a list of what they got in each throw, you could ask them to try and predict what they would get in the next throw. Before any throw, each group member would be asked to predict how the coin will fall. Their predictions, including the 'don't knows', can be noted down.

Let each group repeat this 20 times, and then ask the children to see whether anyone could predict all the 20 throws correctly. What was the maximum number of correct throws predicted by any person? Ask them to speak about what would happen if the number of trials was increased to 50. Would it make the predictions better? If necessary, the children could try this out in groups to see any changes in outcomes.

Along with the activities, you could give children story problems like the one below.

> Naresh and Seema were playing ludo. Seema said, "I will throw a six", and she produced a six. This happened three or four times in the same game, making Naresh very angry. He started crying and protested that Seema was cheating and using trick throws to win. Seema kept saying, "I am shaking and rolling the dice without looking. I am not holding it in my fingers either. How can I be producing 6 deliberately?"

Naresh did not agree, and kept accusing her of using trick throws.

Q1) Do you think Seema was cheating? Why?
Q2) Can you throw a dice or toss a coin so as to produce a result that you want every time? How?
Q3) Is it possible that in a game involving only roll of dice or tossing a coin, one person would keep winning all the time? Why?

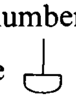
Here is an exercise about the activity now.

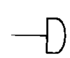
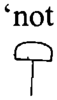
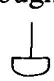
E7) a) What kind of abilities does Activity 2 help children develop?

b) Give another activity that would help children develop the same kind of abilities.

c) Create a story problem to help children realise that chance is not only associated with events with a low probability of happening.

Now, here's an example of how teachers can help children clarify their understanding of less likely and more likely events. It also brings out **the links between gathering data and inferring from them** about the possibility of a related event happening.

Example 1: One day Surja Devi divided up the 20 children in Class 3 into groups of three or four. To one group she gave a die, to another 2 dice, to a third an ordinary drawing pin, and so on. Then she asked each group to throw the die (or the 2 dice, or the drawing pin) and see what number comes (or combination of numbers or whether the pin falls like  or ). She asked them to repeat the experiment several times, and record the outcomes.

After a bit, Surja asked them if they saw a pattern in the data gathered? On this basis could they suggest whether it is likely, for instance, that they can get 7 with throwing one die? In each group, she would talk to the children, asking them to suggest what was an impossible event or a very likely event. For instance, she was talking to Durga in a 'pin group'. She asked Durga if the pin could fall like  . Durga laughed and said, "Of course not, Teacher." Then she asked her to suggest another 'not possible position', and a 'very possible position'. Durga suggested  as 'not possible' after some thought. About 'possible positions', she said, "Yah! It can be  or  ."

Similarly, Surja went around the other groups and got examples from them of impossible, very likely and equally likely events. For instance, a child said that throwing ten with one die was impossible, but with 2 dice it was possible. Another child said, "Out of 10 throws I did with a die, I got 4 three times, 6 twice, 5 twice, 2 three times. So, getting 1 is an impossible event." Others started arguing with him, throwing other dice for him to see that 1 can come.

The class ended on a note of excitement and chatter. Surja promised them that they would do something as exciting the next day.

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Here are some related exercises for you now.

E8) What do you think Surja Devi's aim was in giving the children those activities?

E9) Give children of Class 3 and Class 6 the following activity to do: Toss 5 coins at a time and observe the number of heads and tails after each toss. Repeat this 10 times and observe the pattern of heads and tails. Repeat it 50 times. Ask them to see if there is any different pattern now. Also ask them if they can, after 50 times, predict the number of heads in the next throw, or in the next 3 throws.

Observe the children while they are doing the activity. Is there any difference in the way the Class 3 children deal with it and the way the Class 6 children deal with it? If so, what are the differences?

Let us now continue with some more suggestions for activities.

Activity 3 (Race of heads and tails)

This game can be played by one or more children. Each child tosses a coin. She moves up one step if she gets a head, and moves down one step if she gets a tail in a pictorial ladder. This ladder can be drawn on a length of paper where the zero line is drawn in the middle (as shown in Fig.5). So, for instance, if the child gets a head, she moves up to point 'A'. If she again gets a head, she moves up to 'B'. In the third throw she gets a tail, so she comes down to A.

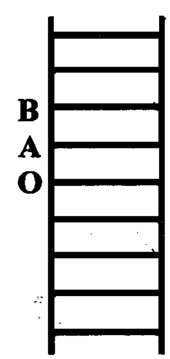

Fig.5

Whoever goes up 7 steps or comes down 7 steps from the zero line first wins the race. You could have all the children keeping their records. Then you could place them all on the wall and ask the children to observe the patterns, if any, in them. For instance, they may notice that a larger number of children are only one or two steps away from the zero line. (Why is this so)?

If each child is required to make 100 tosses, would there be a greater number of children further away from the zero line? You could ask the children such questions and you could make extensions of this activity.

Activity 4

You could form the children into groups. Keep a bag of 10 balls of two colours, say, black and white. To start with you can tell the children how many balls of each colour there are in it. You could ask them to predict what colour a ball that you pick out would be. Each group could make guesses, and points could be given for guesses that worked. You could ask the children: Is there a 50-50 chance of getting a black ball? Is the chance of getting a white ball, more than that of getting a head when a coin is tossed? Getting a good stimulating discussion going would help them to articulate and clarify their ideas. It would also bring forth the views they have formed so far.

As a next step, you could do the reverse. You could tell the children the total number of balls in the bag. Then ask the groups to guess how many white and how many black balls there are, based on what you pull out 10 times, and then 40 times.

How could you extend this activity? Think about it while trying the following exercise.

E10) What concepts do Activities 3 and 4 help the children learn?

Other activities could be asking the children to watch the number plates of the vehicles going by their houses. Every time it is odd, they could mark √; and if even, they could mark × in their record. Based on about 20 vehicles, they could be asked to predict what to expect on the 21st vehicle.

Other variations of such activities can be thought of too. What is important is not just doing the activity, **but having a very active discussion between the children and the teacher about how likely or unlikely certain events are. They should be encouraged to generate reasonable conjectures, and check them through experimentation to accept, discard or alter them.**

Similar activities and discussions are also important to generate a better understanding of social biases. Think of this while doing the following exercise.

E11) Talk to a group of children to find out what prejudice they may have about a group of people living in their town. Design an activity that would allow them to gather data related to this bias. How would you use this to help children realise whether the bias is valid or not?

Let us now summarise what we have done in the unit.

10.5 SUMMARY

In this unit we have made the following points:

1. Children make certain associations with the occurrence of events, based on their experiences. Accordingly, they develop a certain understanding of 'chance' and 'luck'.

2. The notions children have about 'chance' need to be developed along lines that will allow them to see this concept more realistically. This will allow them to make more informed decisions about apparently equivalent choices that they are often required to make. It will also allow them to analyse their prejudices, social or otherwise, as well as superstitions, to see how valid they are.

3. Several interesting activities, appropriate for young children, have been suggested for developing their understanding of probability.

4. If you do not remember what 'probability' means in mathematics, we have briefly explained it in an appendix to the unit.

10.6 COMMENTS ON EXERCISES

E1) If you don't have red and white balls, you can use marked and unmarked 25p coins, or something else.

E2) The important thing in this is to ask the question to people who will answer freely and naturally. Record their answers in your mind and write them down whenever you can. What understanding is reflected in the answers? Do they reflect the belief that chance is the happening of something that is rare? Is there any difference between the answers and examples given by children and those given by adults?

E3) While noting your observations, also find out if people have an informal way of measuring the 'quantum' of chance or luck.

E4) Recall your childhood, or that of your siblings or of your own children. Or observe children around you. Think of instances where they would use chance. For instance, in games involving dice. Some children even say that getting 'out' while batting is a matter of chance!

E5) For example, they could have a belief that girls are better than boys at mathematics. A related hypothesis could be
"The average female student is better than the average male student of the class in mathematics."
How would you help children to test this? Would you get them to ask various people their opinions? Or would you get them to see the marks of the students in different mathematics classes?

E6) For one, they are learning to record the data in a symbolic form. What else are they learning? One of the things they may find is that a tail need not follow a head. They may also realise that as the number of throws increases, the number of times a head shows up, or a tail shows up, tends to ½.

E7) a) For one, the ability to observe patterns, and generalise on that basis. Would they be able to realise that the chance of getting a head is ½? Write down some other abilities. Do you think the children would enjoy this activity or get bored?

b) Once you have your list of objectives that the activity can meet, think of other activities that can achieve them too.

E8) For example, distinguishing certain (possible, likely) outcomes from uncertain (impossible, unlikely) outcomes. The children could also see familiar events having equal chances of occurring. This also helped children to collect, sort and tabulate data in order to see the chances of an event happening.

E9) You could also generate a discussion in the two groups, and see what different points emerge about the levels of understanding.

E10) Play the game in Activity 3 yourself, and then get a few children to play it. See what they do in the process. Observe them, talk to them and get them to talk to find out what they may be learning by participating in this game.

Think about the task children have been given in Activity 4. In making the guess regarding the colour and then looking at the result, what is the pattern they would find and what can they conclude from it? Answering this will help you to extract the concept that this activity would help children learn or practise. For example, does it indicate that guesses in this case can go either way? Would the children realise that any ball can be drawn? Does it make them see that if the number of balls is not the same, then the chances of getting a black or white ball are not 50-50?

E11) This is a difficult and sensitive study. Therefore, you need to be careful to not bring in your own biases. The biases could be that men are not as intelligent as women, or that all rich people are nasty, or that people of a particular caste are more intelligent, or something else.

To help the children verify their hypotheses, you could design a questionnaire or an observation task for them, and have them collect information and data from a reasonable number of respondents. Of course, the questionnaire and observation done should be such that the observer records what she actually sees, and not her interpretation of it.

Let the children analyse the data to see whether they confirm the bias. They could do this in small groups or as a whole class activity.

APPENDIX: WHAT IS PROBABILITY?

Suppose you ask your friends what the probability is of their getting a head when they toss a coin. You may find a whole range of answers — $\frac{1}{2}, \frac{1}{6}, \frac{1}{10}, \frac{1}{7}$, etc. If you probe your friends further, each of them will have different reasons for their answers. When I did this exercise with some adults in my neighbourhood, I got answers like

- It is $\frac{1}{6}$ by my experience.
- It is $\frac{1}{10}$. It could be $\frac{1}{9}$ or $\frac{1}{11}$, but 10 is a round figure.

- It is $\frac{1}{2}$ because the total number of possibilities are 2 — H or T. Of these, only 1 possibility is the one we want — H. So, it is $\frac{1}{2}$.
- It is $\frac{1}{2}$ because there are only 2 possibilities — either you get a head or you don't.

Do you agree with any of these reasons? Are there other answers that your friends came up with that you accept? I agree with the third response given above. Let us see why.

Suppose I toss a fair coin. There are two possible outcomes, head or tail. As the coin is fair it is reasonable to assume that these are **equally likely.** So, if we denote the probability of tossing a head by P(H), then

$$P(H) = \frac{\text{Number of outcomes which are Head}}{\text{Total number of outcomes}} = \frac{1}{2} = 0.5$$

In the same way, when I throw a fair dice, there are 6 possible outcomes — 1, 2, 3, 4, 5 and 6. It is reasonable to assume that these are all equally likely. So, suppose we want the probability of throwing an even number,

$$P(\text{Even}) = \frac{\text{Number of outcomes which are Even}}{\text{Total number of outcomes}} = \frac{3}{6} = 0.5$$

i.e., 2, 4 or 6. Then

In general, when all the outcomes are equally likely the probability that a particular event E occurs, that is, **the likelihood of E happening** is

$$P(E) = \frac{\text{The number of outcomes in which E occurs}}{\text{Total number of possible outcomes}}$$

So, as you can see from the definition, the probability is measured by a fraction lying between 0 and 1. If some event has probability 0 of happening, then it can never happen (for instance, the probability of a person never dying is zero). If an event has probability 1, it means that it is sure to happen (for instance, the probability of the sun rising every morning is 1). All other probabilities lie in between. For instance, if a person knows that she is going to have twins, the probability that both will be girls is 1 in 4, that is, $\frac{1}{4}$ or 0.25. (Why?)

Notice that all the examples given above rely entirely on the assumption that **each outcome is equally likely.** There are many situations (for example, gambling with loaded dice), for which this is not the case. In such situations the measurement of uncertainty gets a little complicated. Any simple text on probability will have material related to this, which you can look up if you are interested.