UNIT 13 TYPES OF FRACTIONS

Structure **Page No.**

13.1 INTRODUCTION

It is usually seen that children's perception of a fraction is very restricted. They find it virtually impossible to think of a fraction as a number and consequently are unable to relate it with any number. If you ask a child what 'half' means, more often than not she will say that it is a part of a cake, or a bun, or some such object. This kind of answer is typical of what she would say for one-third, one-fourth, or any other fraction. Children fail to recognise any link between fractions and numbers. This could be due to the restricted and imprecise use of language in describing fractions. They might have never heard their teacher saying that fractions are numbers. They see fractions only in terms of their concrete representation and hence are unable to name it or symbolise it.

In this unit we have suggested some activities that you can use to help children in understanding fractions as abstract quantities and how to symbolise them. We have also talked about equivalence and ordering of fractions.

Objectives

After studying this unit you should be able to

- use teaching methods that help children to recognise a link between fractions and numbers and use notations for naming a fraction ;
- use various child-centred methods to teach the concept of equivalent fractions ;
- use teaching methods that help children identify fractions greater than one, less than or equal to one, and list them in required order.

13.2 NAMING FRACTIONS

The other day I was talking to my niece Ankita and her friend Rama both students of Class IV. Just then I remembered my friend a primary teacher who once told me that children find it difficult to name fractions. I thought of testing it on these two girls. Look at this situation

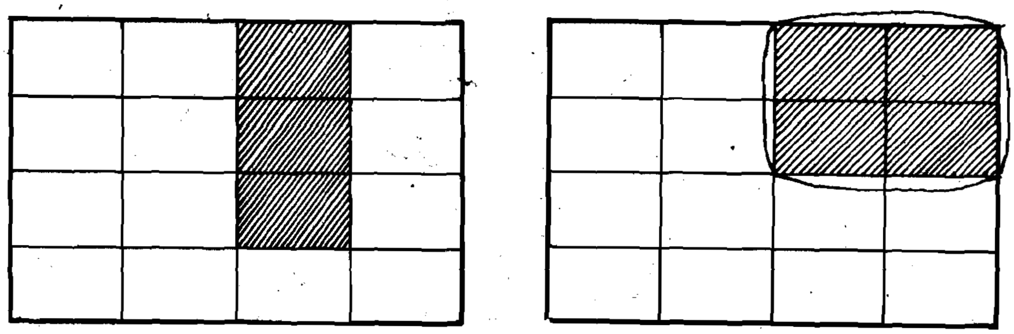
Example 1

I : What fraction is the shaded region in each case?

Ankita : 1(a) is three-sixteenth and 1(b) is four-sixteenth.

Rama : 1(a) is three-sixteenth and 1(b) is one-fourth.

It was really interesting to get these answers. I thought of assessing Rama's depth of understanding of naming fractions.



1(a) 1(b)

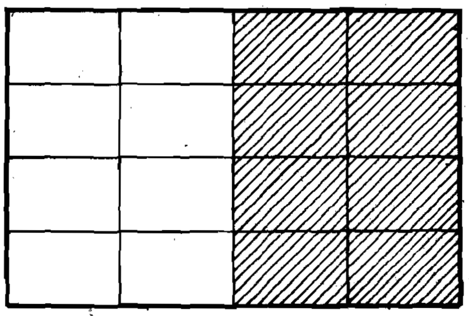
Fig.1

I : Rama why do you say that 1(b) is one-fourth?

Rama : (A bit hesitantly). Because there's ..... there's sixteen on a sheet, and there's four each.... each side of the ..... If you split it up then that's a fourth and that's a fourth and that's a fourth (pointing to sets of four squares at a time).

But as I watched her a doubt crept into my mind. Because her action as she said 'and there's four each.... each side of the ...' indicated that she was actually counting the four rectangles in the ringed region. Was she saying the ringed region was a fourth because there were four rectangles in it, or because there were four areas of the same size on the whole sheet? This doubt was confirmed when Ankita and Rama tried to explain why a certain area was one-eighth.

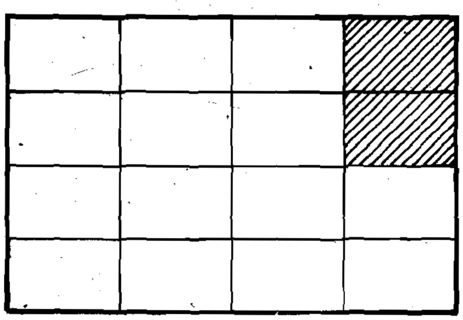
Rama :



It's one eighth because it's got eight squares.

Fig.2

Ankita :



No, this is one-eighth because the whole can be split into eight pieces like this.

Fig.3

So, naming a fraction and understanding the notation by which fractions are symbolised is another big problem with children.

E1) Which of the illustrations and explanations above is correct? What would you do to focus on the correct solution?

Any confusion in a child's mind about the procedure by which a fraction is named should be cleared before proceeding further with the topic. When children are thoroughly familiar with fraction names such as a 'half', 'third', 'quarter', and with the meaning of those names in relation to various wholes, for instance, a third of a cake, they are ready to be introduced to symbols for fractions. It is seen that we usually tell them that in order to write 'two-third' they can draw a line under '2' and write '3' beneath the line as $\frac{2}{3}$ without explaining why it is written this way. Children often get confused and write $\frac{3}{2}$ for two-thirds. We need to tell them that the numeral at the bottom of a fraction (the denominator) shows the total number of parts into which the whole is divided, and the numeral at the top (the numerator) is the number of parts that we are considering. For instance, the fraction $\frac{2}{3}$ tells us that the 'whole has been divided, into three equal parts, and we are looking at two of those parts. The denominator of the fraction $\frac{2}{3}$ gives the fraction its name 'third' and the numerator tells us the number of thirds to be considered.

We need to stress the writing of fractions as numbers. You could help children associate a number with the given fraction. For instance, a paper is divided into two equal parts and one part out of these two parts is represented by a number $\frac{1}{2}$. Children can visually see and link numbers with the given fraction.

They also need a lot of practice in translating fraction names to fraction symbols. My experience is that, to start with it is best to consider only fractions whose numerators are '1'. This helps to focus their attention on the significance of the denominator. Children might be asked to complete a table like this one:

Number of equal parts of the whole	2	3	4	5	6
Name of each part	1 half	1 third	1 quarter	–	–
Symbol for one part	$\frac{1}{2}$	$\frac{1}{3}$	–	–	–

Gradually, some more exercises/activities involving different numerators can be done by the children.

E2) Can you suggest some group activities for children in a class of 40, to see if the concept of naming and symbolising a fraction has got across to them?

Once children realise that fractions are numbers they may wonder about operations on them. But before you teach them how to add and subtract fractions, it is necessary that they have a clear understanding of other concepts like equivalence and ordering. These concepts play a central role in understanding addition and subtraction. Many teachers have found that children do not understand these concepts easily. In the next section we shall give some suggestions for communicating the idea of equivalent fractions to a child.

13.3 EQUIVALENCE OF FRACTIONS

Children do come across many instances where the same fraction can be named in

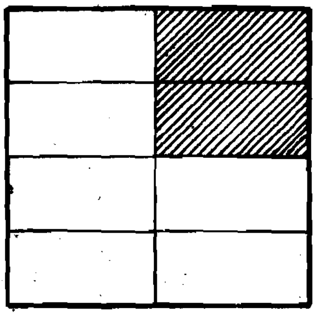


Fig. 4

more than one way. For example, many children say that $\frac{2}{8}$ of the square in Fig.4 is shaded and many others say that $\frac{1}{4}$ of the square is shaded.

Both these answers are correct. But children sometimes are not ready to accept the fact that different fractions may represent the same measure or equal amount of whatever object they are talking about. For example, with units attached, the fractions $\frac{2}{3}, \frac{4}{6}, \frac{6}{9}, \frac{8}{12}, \ldots$ represent equal measures. In this respect they are therefore equivalent. This concept requires a lot of practice and children take a long to grasp this idea.

Let me share my experience with you in this regard. Once I got a chance to interact with four children aged between 8 and 10 years in my neighbourhood. I took them for a treat to a nearby cafe. They all wanted big pastries. Consider this situation.

Example 2

I : If I buy 3 pastries how would you share them equally among yourselves?

One child : I will cut each of the three pastries into four equal parts and then give one part from each pastry to each of us. So, each of us will get an equal amount.

Another child : No, first I would give one pastry each to three of us and then ask each of these three to give one-fourth of their pastry to the fourth person. Then each of us will get an equal share.

I : O.K. Both of you are dividing 3 pastries into four equal shares like this (Fig.5).

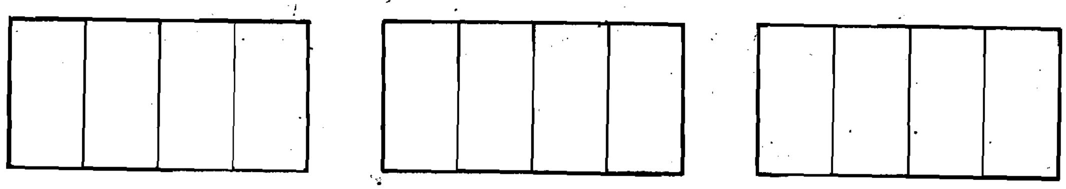


Fig.5(a)

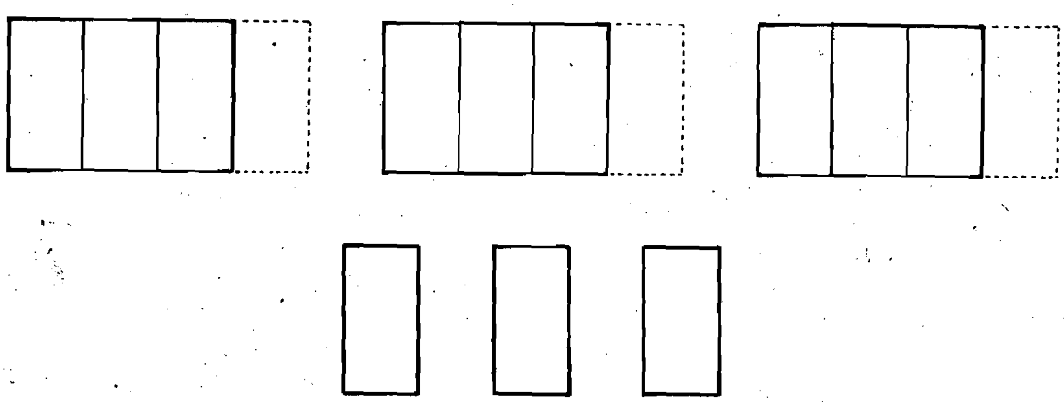


Fig.5 (b)

Why don't you all write down the fraction of pastry that each of you will get?

1st child : Each will get $\frac{1}{4}$

2nd child : Each will get $\frac{3}{4}$

3rd child : Each will get $\frac{3}{12}$

? What would you say about the responses of these children? Who was wrong and who was correct, and why?

In the above situation the first child looked at the three pastries as a single whole and so he got $\frac{1}{4}$ of it. For the second child the whole consisted of 3 discrete objects —the pastries and his share was $\frac{1}{4}$ of these three pastries. The third child again viewed at three pastries as a single whole divided into 12 equal parts out of which he got 3 parts.

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So from the above observations you would agree that it is necessary that children realise the need to clearly state what 'whole' they are referring to while naming a fraction. Once they are clear about it they will not have problems in representing the same measure by different fractional numbers.

To start with we can ask children without introducing any formal name, to do 'detective work' on a particular fraction to see how many other names they can find for it. But this detective work needs to be structured. You could start with the following activity.

Example 3 : Take a strip of paper fold it into eighths. Then invite children to find another way of naming one-half of the strip. Let them find that

$\frac{1}{2}$ of the strip is $\frac{2}{4}$ of the strip, and also $\frac{4}{8}$ of the strip.

In the same way they could divide a square into eighths and find that

$\frac{1}{2}$ of the square is $\frac{2}{4}$ of the square, and also $\frac{4}{8}$ of the square.

At this point you could explain the equivalence of one-half, two-fourths, and four-eighths, etc. and write $\frac{1}{2} = \frac{2}{4}$, which means that one-half is the same as two-fourths. You can ask the children to record the general findings they have made above by writing

$$\frac{1}{2} = \frac{2}{4} = \frac{4}{8}.$$

Children can study the above list, and may notice a pattern in the denominators and numerators of the fractions in particular, that the denominator of each fraction is twice the numerator. You can ask children to suggest other fractions that follow the same pattern. You yourself can initiate and suggest $\frac{6}{12}$.

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Once, the children have got enough experience of finding the fractions equivalent to a given fraction and have observed the pattern followed in each case, let them try and discover the rule for writing equivalent fractions, namely, **multiply the numerator and denominator by the same number or divide the numerator and denominator by their common factor.**

E3) Suggest one activity each to help children find fractions equivalent to one-third, one-fourth and two-thirds.

Here is another activity which a teacher took up in her class to teach the concept of equivalent fraction using different concrete objects.

Example 4

Teacher : Here I have 32 marbles. I am giving 16 marbles each to Mohan and Sohan (two students in the class)

Now Mohan, you divide these 16 marbles into four equal parts.

Mohan : (Dividing like ○○ ○○ ○○ ○○ / ○○ ○○ ○○ ○○

and showing it to the teacher) Here are four equal parts.

Teacher : How many marbles are there in one part?

Mohan : 4 marbles.

Teacher : What fraction of marbles is this one part?

Mohan : It is $\frac{1}{4}$ of 16 marbles.

Teacher then calls another student Rani.

Teacher : Mohan, give two parts out of four to Rani.

Mohan : O.K. madam (giving ○○ ○○ / ○○ ○○ i.e. two parts to Rani).

Teacher : Mohan, tell me, what fraction of marbles have you given to Rani?

Mohan : I have given her $\frac{2}{4}$ of 16 marbles.

Teacher : Fine! Now Sohan, you divide your 16 marbles into 8 equal parts.

Sohan : (Dividing like ○○ ○○ ○○ ○○ ○○ ○○ ○○ ○○

and showing to teacher) Here I have divided them.

Then the teacher made him answer questions like how many marbles are there in one part? What fraction of marbles is this one part? Then she calls another girl Rita and tells Sohan to give her four parts out of 8. Sohan gave ○○ ○○ ○○ ○○ 4 parts to Rita. Fig. 4.

Teacher : Sohan, tell me, what fraction of marbles have you given to Rita?

Sohan : I have given her $\frac{4}{8}$ of 16 marbles.

Teacher : Alright! So Rani has got $\frac{2}{4}$ of 16 marbles and Rita has got $\frac{4}{8}$ of 16 marbles. How many marbles each one of you have? (addressing to Rani and Rita).

Rani : 8 marbles madam.

Rita : I also have 8 marbles.

Teacher : Does it mean that $\frac{2}{4}$ of 16 marbles is the same as $\frac{4}{8}$ of 16 marbles?

Entire class roared: Yes Madam.

Teacher : Yes, you are right. The only thing is they are grouped differently.

In the same way she explains that if she has 2 chapatis split into 4 (see Fig.6)

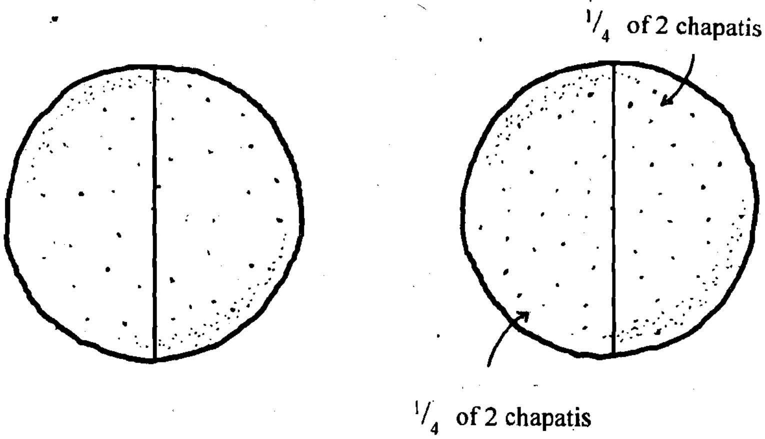

Fig.6

then $\frac{2}{4}$ of 2 chapatis is 1 whole chapati and if she splits 2 chapatis into 8 (see Fig.7)

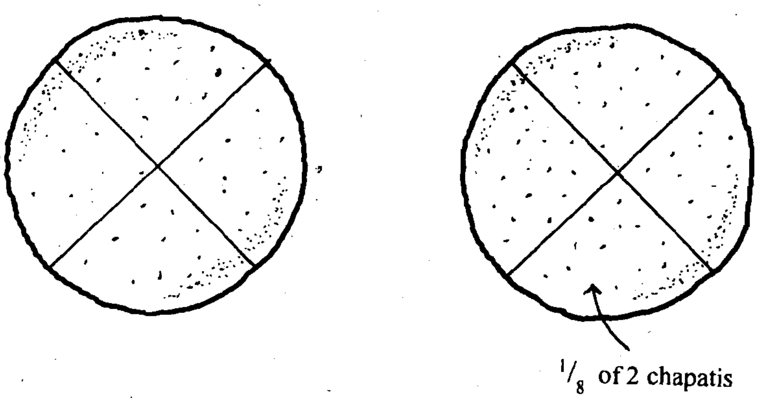

Fig.7

then $\frac{4}{8}$ of 2 chapatis is 1 whole chapati. That means $\frac{2}{4}$ of 2 chapatis is the same as $\frac{4}{8}$ of 2 chapatis.

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Such activities may help the children know two-fourths to be indisputably equivalent to four-eighths or six-twelfths etc. whatever the whole may be.

E4) Do you agree that such activities help children to have a better understanding of the concept of equivalence? If so, why? If not, what would you suggest?

Have you ever asked children to write fractions equivalent to a given fraction? I have, and find that they usually adopt the multiplication method i.e. multiplying numerator and denominator by the same number. They find it difficult to obtain an equivalent fraction by dividing by a common factor. Many people I have asked agree with this observation. They also say that it is probably because of this that children have problems in reducing a fraction to its lowest terms. In this regard many of you would have had the kind of experience I had with my niece Ankita, a fourth standard student when we were discussing fractions, the other day.

Example 5

I : Can you give me some fraction equivalent to $\frac{8}{12}$?

Ankita : Yes.

Then she tried to remember as I noticed her talking to herself.

$$\frac{8 \times 2}{12 \times 2} = \frac{16}{24}$$

$$\frac{8 \times 3}{12 \times 3} = \frac{24}{36}$$

$$\frac{8 \times 4}{12 \times 4} = \frac{32}{48}$$

Ankita : What did teacher tell me to do here? Yes, now I remember. Multiplying by 2, I get $\frac{16}{24}$. Multiplying by 3, I get $\frac{24}{36}$. Multiplying by 4, I get $\frac{32}{48}$ and like this I can write many more fractions equivalent to $\frac{8}{12}$.

I : Fine! You are right. Now, tell me, is $\frac{2}{3}$ also equivalent to $\frac{8}{12}$?

Ankita : Let me check Auntie.

As I watched her I could notice that she was again trying to check it by the multiplication method. She multiplied $\frac{2}{3}$ (both numerator and denominator) by 2, it did not work. She multiplied it by 3, it did not work. She multiplied it by 4 and shouted.

$$\frac{2 \times 4}{3 \times 4} = \frac{8}{12}$$

Ankita : Yes Auntie, even $\frac{2}{3}$ is equivalent to $\frac{8}{12}$. We multiply 2 by 4, and 3 by 4, and get $\frac{8}{12}$.

I : Why don't you do it the easier way? You know 4 is a common factor of 8 and 12. So divide 8 by 4 and 12 by 4 and we get $\frac{2}{3}$.

Ankita : Oh..... it looks easy, but can we do it this way?

I : Why not? Let me show you something. Look at this paper. I fold it so that I get these two equal parts (as in Fig.8). Tell me, what fraction is each part?

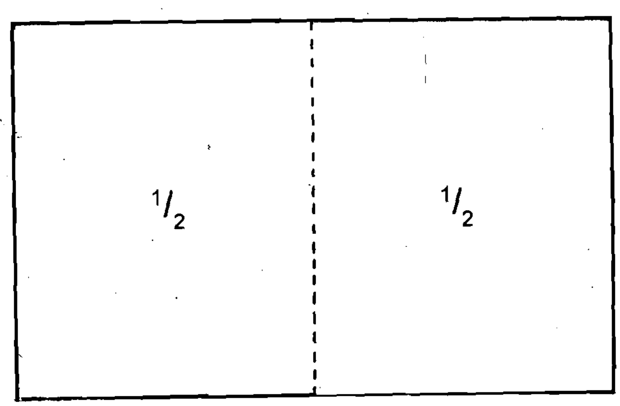

Fig.8

Ankita : $\frac{1}{2}$.

I : Again fold it so that there are 4 equal parts (see Fig.9). Now how many parts is the paper divided into?

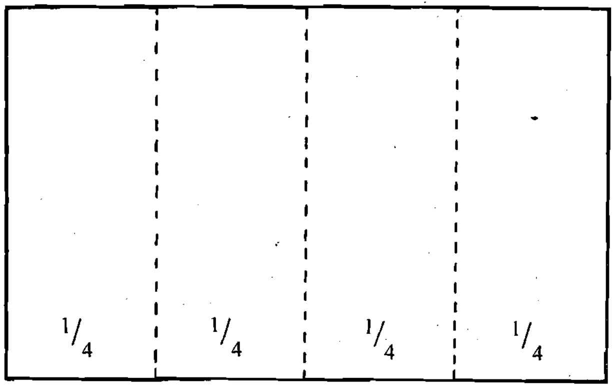

Fig.9

Ankita : Four parts. Each part is $\frac{1}{4}$.

I : Let me open the folds. Do you agree that $\frac{1}{2}$ the paper is the same as 2 one-fourth parts?

Ankita : Yes.

I : So, I can write $\frac{1}{2} = \frac{2}{4}$. Now if I fold this paper once again and then unfold it we get something like this (as in Fig.10).

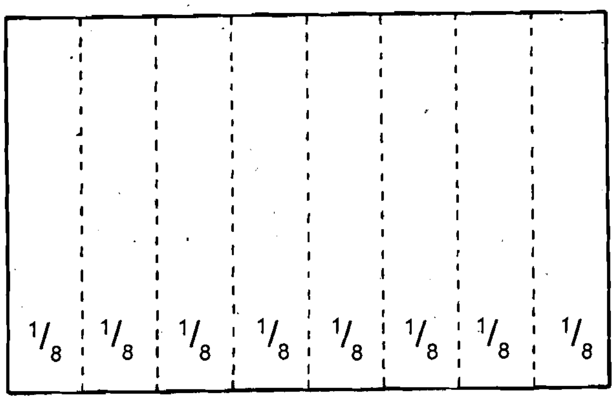

Fig.10

Tell me how many one-eighth parts are there in $\frac{1}{2}$ part of the paper?

Ankita : One two ..... four. Four.

I : So $\frac{4}{8} = \frac{1}{2}$

Ankita : Yes

I : Now can I say that $\frac{4}{8}$ is equivalent to $\frac{2}{4}$ and $\frac{1}{2}$ and write $\frac{4}{8} = \frac{2}{4} = \frac{1}{2}$?

Ankita : Yes, we can do that.

I : That means we can write fractions equivalent to $\frac{4}{8}$ having lower numerator and denominator. Do you see some pattern when we write $\frac{4}{8}=\frac{2}{4}=\frac{1}{2}$?

Ankita : Um, yes. Each time we are dividing the numerator and denominator by 2 as we go from left to right.

I : Does that mean that we have to go step by step and cannot directly write $\frac{4}{8}=\frac{1}{2}$?

Ankita : Why not! If we divide both 4 and 8 straightaway by 4 we get $\frac{1}{2}$.

I : So, do you agree that **we can also write equivalent fractions by dividing the numerator and denominator by their common factor?**

Ankita : Yes Auntie.

I felt happy as Ankita looked convinced and enjoyed doing the activity with me. Next I asked her to do a few exercises like filling up the boxes in $\frac{5}{10}=\frac{\square}{2}$, $\frac{14}{56}=\frac{2}{\square}$. Ankita took no time to do them correctly. I also utilised this opportunity to explain to her that when we write $\frac{4}{8}=\frac{2}{4}=\frac{1}{2}$ then $\frac{1}{2}$ is in the lowest terms because now the numerator and denominator do not have any common factor. So we cannot divide them any further.

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Now here is a related exercise for you.

E5) What method would you use to assess whether the concept of finding equivalent fractions by the division method has got across to your students or not?

You may have noticed that at times children ask why they cannot write equivalent fractions by adding or subtracting the same number from the numerator and the denominator, like we do with multiplication and division? Children should be left to themselves to find out the answers to such problems. Ask them to write a fraction say $\frac{1}{2}$. Let them add 2 to both the numerator and denominator and check if $\frac{1}{2}=\frac{1+2}{2+2}=\frac{3}{4}$.

E6) What would you do to make children understand that $\frac{5}{6}$ is not equal to $\frac{5-2}{6-2}=\frac{3}{4}$.

Yet another problem children usually face is to understand the equivalence of $\frac{4}{4}, \frac{5}{5}$ ... to 1 and write $\frac{4}{4}=1$, or $\frac{5}{5}=1$. The following scene may help you to make your students understand it.

Example 6

Teacher : Here is an apple. I cut it into four equal parts. So these four parts put together is a whole apple. Now Renu, come here and take this one part.

Renu : Yes madam.

Teacher : What portion of apple have you got Renu?

Renu : One part.

Teacher : Out of how many parts?

Renu : One out of four parts.

Teacher : Write it on the board

Renu : $\frac{1}{4}$.

In the same way the teacher gave Renu a second piece and then a third piece and made her write $\frac{2}{4}$ and $\frac{3}{4}$ on the board i.e. the fraction for the amount of apple she got. Finally, she gave the last piece to Renu.

Teacher : Now how many parts of the apple have you got?

Renu : Four parts.

Teacher : Out of how many parts have you got 4 parts?

Renu : 4 out of 4.

Teacher : So can I write the fraction for this as $\frac{4}{4}$.

Renu : Yes madam.

Teacher : O.K. children tell me how much of the apple has Renu got till now?

Students : Full/Whole/One.

Teacher : Can I write $\frac{4}{4} = 1$

Students : Yes madam.

Teacher : In the same way we have $\frac{5}{5} = 1$, $\frac{6}{6} = 1$ ... etc.

The above activity can also be done with a piece of paper cut into four equal pieces or with 4 dice or 4 marbles.

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Once the children understand that $\frac{4}{4} = 1$, $\frac{5}{5} = 1$ .... etc. It will help them to understand that 5 one-fourth parts $\left(\frac{5}{4}\right)$ is a fraction greater than 1. If a sheet of paper is divided into 4 parts then 5 one-fourth parts would cover one whole sheet and would require the use of another sheet, and therefore is a fraction greater than one.

E7) Suggest an activity for explaining $\frac{5}{4}$ to a Class IV child.

You would agree with that a child needs to understand how to order fractions before we teach her subtraction We shall talk about it now.

13.4 ORDERING FRACTIONS

A child's understanding of the ordering of two fractions that is deciding which of the relations 'is equal to', 'is less than', or 'is greater than' holds for two fractions, requires an understanding of the ordering of unit fractions. This can be done by doing a simple activity like the following.

Fractions with numerator one are called unit fractions

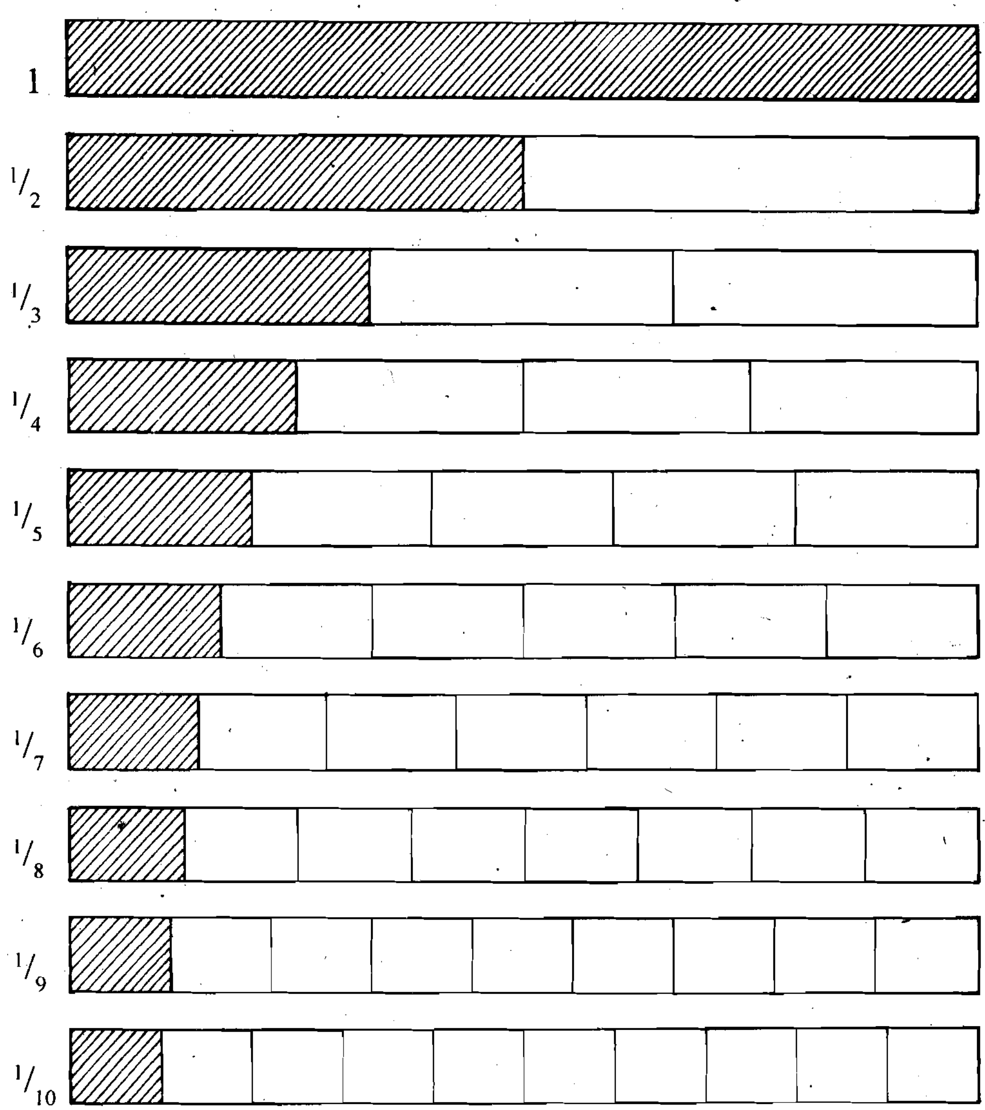

Fig.11

Example 7: Take ten strips of paper which are equal in size. Keep the first strip as it is. Divide the second strip into two equal parts, third into three equal parts,... and like this the tenth into ten equal parts. Then paste them on a card board one below the other. Shade one part of each strip. (see Fig.11)

Showing the children the different shaded portions of the same sized strips helps them to realise that $1 > \frac{1}{2} > \frac{1}{3} > .... > \frac{1}{10}$.

This activity for showing $1 > \frac{1}{2} > \frac{1}{3} > .... > \frac{1}{10}$ can also be tried using the number line. It may act as a visual aid for the learners for representing fractions. You can test it by trying it with your students.

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E8) Using a number line show that $1 > \frac{1}{2} > \frac{1}{3} > .... > \frac{1}{10}$ in your class. How do your students react to it? Do they find it difficult to grasp the representation of fractions on a number line or does it turn out to be an effective visual aid for them?

Understanding the important relationship between the size and number of parts into which a whole is divided is very important in determining the order of fractions. If a child has a clear understanding about it then it becomes easier for him/her to order unit fractions, fractions with the same denominator and fractions with the same numerator. But sometimes children interpret this relationship between the size and number of parts wrongly. This reminds me of the experience I had with my 8-year old niece Ankita.

Once, I was talking to her about ordering of fractions.

Ankita looked very confident about ordering unit fractions. She took no time in saying that $\frac{1}{8}$ is less than $\frac{1}{5}$. She explained the way she understood it that if she shares a cake with 7 other children her share would be less than what she would get if she shares it with 4 other children. She could very easily order fractions like $\frac{1}{3}, \frac{1}{6}, \frac{1}{11}, \frac{1}{20}$. But I was surprised when she said $\frac{7}{8}$ is less than $\frac{4}{8}$ because seven is more parts and as the parts become more in number they become smaller in size. I said let's see. I took two paper strips of the same size and divided each into 8 equal parts (see Fig.12). Ankita helped me in doing this.

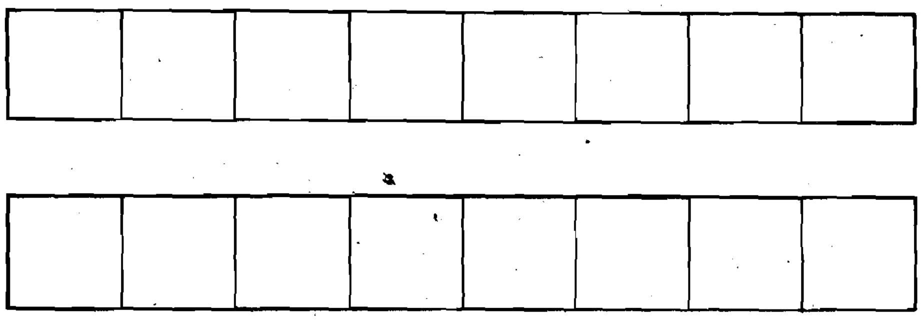

Fig.12

Example 8

I : Ankita, now take the first strip and colour 4 parts out of 8 red.

Ankita : O.K. Auntie. Here it is (Fig.13)

Fig.13

I : What fraction is this red part?

Ankita : $\frac{4}{8}$.

I : O.K! Now take the second strip and colour 7 parts out of 8 grey.

Ankita : Here it is (Fig.14)

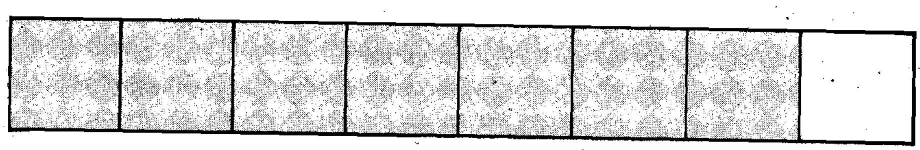

Fig.14

I : What fraction is this grey part?

Ankita : $\frac{7}{8}$.

I : Which is more the red shaded part or the grey shaded part?

Ankita : Grey is more

And before I could ask her any other question she herself came out with the answer. By now she was enjoying herself and wanted me to do some more activities of this kind. So I showed her $\frac{4}{4}$ and $\frac{5}{4}$ with the help of a paper. In between I asked her a few questions like which of the two $\frac{4}{4}$ and $\frac{5}{4}$ is more? Why is it so? and she responded well.

Now she was very clear that $\frac{4}{13}$ is less than $\frac{4}{8}$. But to decide which of $\frac{7}{8}$ or $\frac{12}{13}$ is less was still a problem for her. I thought I must get the answer from her and proceeded as follows:

I : Ankita, do you have any problem in comparing numbers like $\frac{7}{8}$ and $\frac{11}{8}$, or $\frac{3}{7}$ and $\frac{4}{7}$ etc.?

Ankita : No auntie, it's very simple. I don't see any problem here.

I : Why do you think it's easier to compare $\frac{7}{8}$ and $\frac{11}{8}$ than comparing $\frac{7}{8}$ and $\frac{12}{13}$?

Ankita : Here it's easier because the numbers have common denominator and so we have to only bother about the numerators.

I : Fine! But can't you change $\frac{7}{8}$ and $\frac{12}{13}$ to equivalent fractions with the same denominator?

After little bit of thinking

Ankita : Yes I can do that.

She did some calculations and came out with the answer.

Ankita : The two numbers equivalent to $\frac{7}{8}$ and $\frac{12}{13}$ are $\frac{91}{104}$ and $\frac{96}{104}$.

I : Can you decide now which is greater $\frac{7}{8}$ or $\frac{12}{13}$?

Ankita : (With a smile on her face) Yes $\frac{12}{13}$ is bigger.

Ankita looked thrilled and happy. She really enjoyed doing such an exercise and looked confident. I also felt happy,

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Once you feel that children have clearly understood the concept of ordering fractions through experience with physical and diagrammatic models they could be introduced to a general algorithm—usually called a **common denominator algorithm** to determine the order of fractions. According to this **for comparing two or more fractions change them to equivalent fractions with the same denominator. Then the fraction with the greatest numerator is the largest.**

E9) Which is bigger $\frac{12}{13}$ or $\frac{7}{8}$? How will you make a child understand this without applying 'a common denominator algorithm'.

We will now talk about a common problem that children usually face, namely, converting improper fractions to mixed fractions.

13.5 IMPROPER FRACTIONS

Primary school teachers usually complain that children while dividing an improper fraction to convert it into a mixed fraction usually write the remainder instead of quotient as the whole number part of the mixed fraction. I happened to notice the same thing last Sunday when I visited a friend's house. Her 9 year old daughter Mini was busy doing her homework. I just glanced at her note book. She was doing sums on fractions converting improper fractions to mixed fractions. She divided 7 by 4 as

$$\begin{array}{r} 1 \\ 4\overline{)7} \\ \underline{4} \\ \underline{3} \end{array}$$

and wrote $\frac{7}{4} = 3\frac{1}{4}$. I quietly watched her. She again wrote $\frac{11}{6} = \begin{array}{r} 1 \\ 6\overline{)11} \\ \underline{6} \\ \underline{5} \end{array} = 5\frac{1}{6}$.

Fractions with numerator greater than denominator are called improper fractions.

Mixed fractions are sum of whole numbers and a proper fraction.

While talking to her it appeared that she had some vague idea of what her teacher might have told her about the conversion of improper fractions into mixed fractions and was doing sums mechanically without having in mind any visual picture of what she was doing. I tried to make it clear to her and proceeded as follows

Example 9

I took 5 circular paper cuttings each divided into four equal parts (see Fig.15)

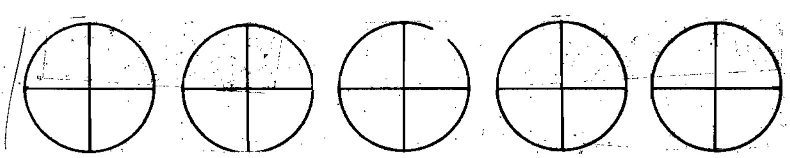
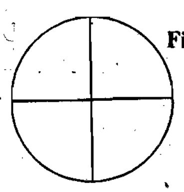
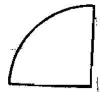


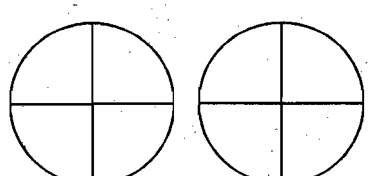
Fig.15

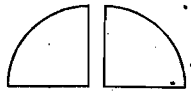
I : What is $\frac{1}{4}$ of 

Mini : It is one part out of 4.

I : Is it  ?

Mini : Yes

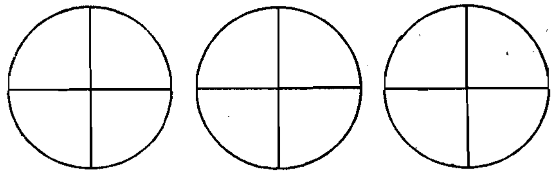
I : O.K.! Now tell me what is $\frac{1}{4}$ of 

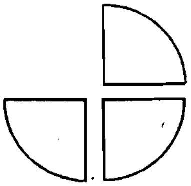
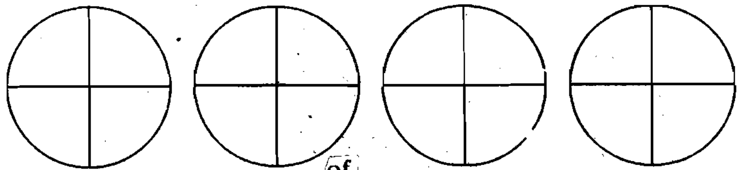
Mini : It's .

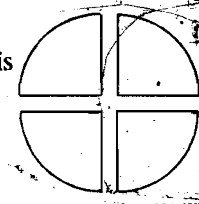
I : What fraction does it represent?

Mini : It's 2/4.

I : Good. So it's two one-fourths. In the same way can we say

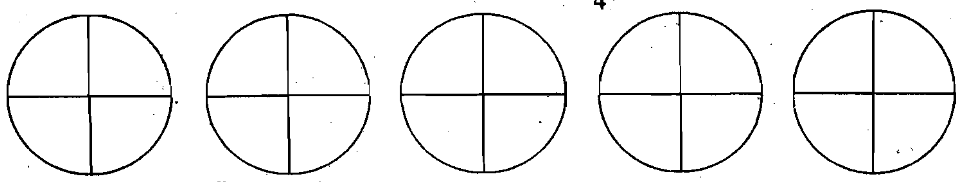
$\frac{1}{4}$ of  i.e. three one-fourths

 $= \frac{3}{4}$ and $\frac{1}{4}$ of 

or four one-fourth is 

Mini : Yes auntie.

I : **O.K.**! Now can you tell me what is $\frac{1}{4}$,

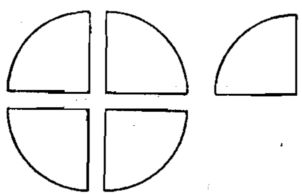


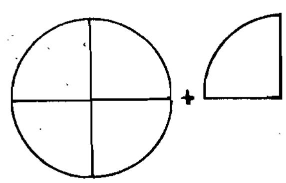
or five one-fourths

Mini looked a bit confused so I thought of helping her

I : Mini isn't $\frac{5}{4}$ same as $\frac{1}{4}$ of 5.

Mini : Amm ... Oh Yes! Now I can tell you what $\frac{1}{4}$ of 5 is. It's



I : Isn't it same as  $= 1 + \frac{1}{4}$.

Mini : Yes auntie they are same.

I : O.K. This means that $\frac{5}{4}$ is 1 whole and $\frac{1}{4}$ more. We write it together as $1\frac{1}{4}$.

Now when we divide 5 by 4 as

$$4\overline{)5}\begin{array}{l}\ \ 1\\ \ \ 4\\ \hline \ \ 1\end{array}$$

then the quotient gives the whole number part of the mixed fraction and $\frac{\text{remainder}}{\text{divisor}}$ gives the proper fraction part of it. So we have $\frac{5}{4}$ = Quotient $\frac{\text{remainder}}{\text{divisor}} = 1\frac{1}{4}$.

Now what do you have to say about your number $\frac{7}{4}$? Mini just looked at her notebook and smiled.

Mini : Now I am clear about it. According to what you have just told me $\frac{7}{4}$ is 1 whole and $\frac{3}{4}$ more. So it's $1\frac{3}{4}$.

Mini understood her mistake. She looked very confident as she answered a few more problems of the same type correctly.

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And now here is an exercise for you.

E10) What was Mini's problem?

E11) Could you suggest any other method which you might have followed to correct Mini's mistake?

13.6 SUMMARY

In this unit we have tried to bring the following points.

1) It is usually noticed that children find it difficult to name the fractions and symbolise them. Understanding the notation by which fractions are symbolised is a problem for them. Any confusion in a child's mind about the procedure by which a fraction is named should be cleared before proceeding with the topic. It is only after children become familiar with fraction names such as a 'half', 'third', 'quarter' etc. and the meaning of these names in relation to various wholes they should be introduced to symbols for fractions. It should be clear to them that when we draw a line under '2' and write '3' beneath the line as $\frac{2}{3}$ then the denominator of the fraction $\frac{2}{3}$ tells us that the whole has been divided into three equal parts and the numerator tells us that we are looking at two of these parts. They should be given a lot of practice in translating fraction names to fraction symbols. Various activities can be taken up in the class by the teacher in this regard.

2) The numerator-denominator notation for fraction also makes it possible to denote the same fraction in infinitely many ways. As a group activity children can be asked to find as many numbers as they can for a particular fraction. After they have done such activities they may be introduced the concept of equivalent fractions. Paper folding activity and other similar activities for different categories of whole help children understand why more numbers are equivalent. Once the children have got enough experience of finding the fractions equivalent to a given fraction let them observe the pattern followed in each case and discover the rule for writing equivalent fractions.

3) Once children become familiar with the rule for writing equivalent fraction enough practice should be given to them for obtaining an equivalent fraction by dividing the numerator and the denominator by a common factor as it has been observed that children find it difficult to apply.

4) Sometimes children interpret the relationship between the size and number of parts into which a whole is divided wrongly. This also affects their understanding of ordering fractions. Paper shading activity can help them clear their doubts about deciding which number is greater. Teacher can start with comparing unit fractions and then go to fractions with common numerator or denominator. This way they can themselves evolve the pattern

for comparing other fractions like $\frac{5}{6}$ and $\frac{7}{8}$, $\frac{3}{4}$ and $\frac{6}{7}$ etc. by generating a general algorithm called a common denominator algorithm.

5) Problem of converting an improper fraction into mixed fraction which commonly occurs among the children aged 8-10 years can be taken care by doing activities with the concrete objects and actually showing what $\frac{7}{4}$ (or, seven one-fourths or, $\frac{1}{4}$ of 7) means. This will give them visual picture of what they are doing and help them in understanding, the concept.

13.7 COMMENTS ON EXERCISES

E1) The illustration in Fig.3 and the explanation given by Ankita is correct. Children need a great deal of time and discussion to focus on the correct aspect. Many examples of naming fractions are required to develop a sense of labelling fractions. A teacher in a class may divide the children into groups and each group be asked to record each fraction of the whole piece of paper in as many different ways as possible. Each different way of recording it earns them an extra point. A team with maximum points is the winner. Such activities will help them build up experience of naming fractions without their knowing that they have gained background experience of the equivalence of fractions.

E2) In this regard once I happened to see a teacher doing an activity in the class which I would like to share with you. Teacher divided the children in a class into 4-5 groups. Then she drew a figure on the board (See Fig. 16).

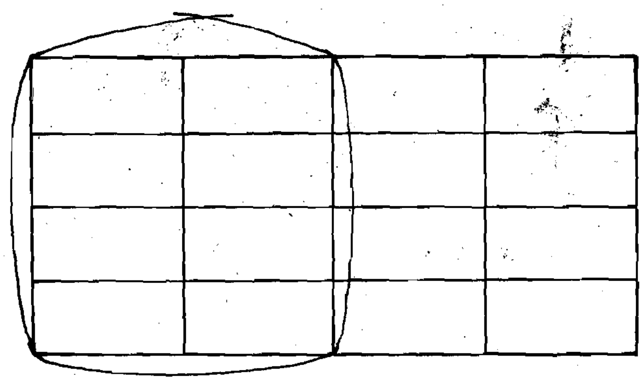

Fig.16

She wanted children to draw it on the paper and tell everything they can about the ringed area. Children worked in groups and recorded their results. They also explained to one another why it had been given that particular name. It is called a 'half' because there are two pieces like it on the whole sheet. It is called 'four-eighths' because two rectangles are called 'one-eighth' and there are four pieces of that size. Like this the teacher made them give reason for every answer they had written like it's two-quarters or eight-sixteenths or four-sixteenths etc. Children wrote out the names in word as well as in standard fraction notation because each different name and each different way of recording it gave them an extra point. Through this activity teacher could judge the amount of experience children had about the fraction-naming concept.

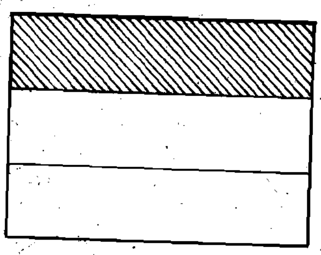

Fig. 17

E3) For showing fractions equivalent to $\frac{1}{3}$ take a rectangular paper and fold it into 3 equal parts. Press it along the sides and then unfold it. Paper is divided into 3 equal parts (see Fig.17). Fold it once again and then unfold it. It is divided into 6 equal parts (see Fig.18). Repeat the process once again the paper will be divided into 12 equal parts as shown in Fig.19. Now by colouring $\frac{1}{3}$ of Fig.17, $\frac{2}{6}$ of Fig.18, and $\frac{4}{12}$ of Fig.19, you can show that $\frac{1}{3} = \frac{2}{6} = \frac{4}{12}$.

Again using Fig.17, 18 and 19 you can show that $\frac{2}{3}$ is equivalent to $\frac{4}{6}$ and $\frac{8}{12}$.

Fractions equivalent to $\frac{1}{4}$ can also be shown by the paper folding activity. This time you have to start with folding the paper into 4 equal parts.

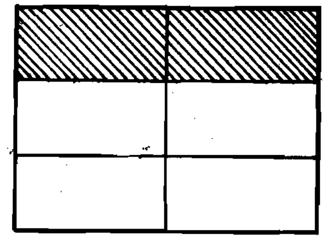

Fig. 18

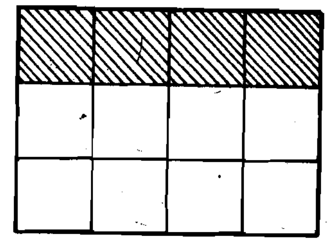

Fig. 19

E4) To find out the utility of having such activities, I talked to few teacher. Some of them were of the opinion that such activities do help children to have a better understanding of the concept of equivalence. They feel that these activities help them see and realise why $\frac{1}{3}$ is equivalent to $\frac{2}{6}$ is equivalent to $\frac{4}{12}$. It help them eliminate all the errors in their thinking and fill up the gaps in their understanding. Specialisation over a large number of cases help children in the identification of an underlying pattern. But you may not agree with them, so you can try these activities in your class and write your views about it.

E5) One way of assessing is to do exercises in the class where you can ask children to write fractions equivalent to a given fraction with lower numerator and denominator. Then going around you can check their responses. In between you can ask them to check their solution by paper folding also.

E6) This can be done by paper folding or shading. Ask children to take two sheets of paper of the same size. Let them divide one into 6 equal parts and another into 4 equal parts. Ask them to shade 5 parts out of 6 of the first sheet green, and 3 parts out of 4 of the second sheet red. Now let them compare the two coloured regions and decide whether $\frac{5}{6}$ is equal to $\frac{3}{4}$ or not. This exercise can also be done by taking two sets of 12 marbles each. Divide one set into 6 groups of 2 each and see how many marbles are there in 5 such groups. Then divide second set of 12 marbles into 4 groups of 3 each and see how many marbles are there in 3 such groups. You can now easily show that $\frac{5}{6}$ is not equal to $\frac{3}{4}$.

E7) Simple activity of paper folding can be done to show $\frac{5}{4}$. Various steps involved are

Step 1 : Fold one region into 4 parts.

Step 2 : Point to a region that is $\frac{1}{4}$ of the whole. Can you shade 5 one-fourth parts? No! We need to fold another region in order to show 5 one-fourth parts.

Step 3 : Fold another region. Cut 1 part out of four from this region and paste it along the first region (the whole of it). Now you can show $\frac{5}{4}$.

E8) While showing that $1 > \frac{1}{2} > \frac{1}{3} > ..... > \frac{1}{10}$ on a number line be careful that each time you draw a number line and take a unit length on it, this length should be same. Also when further dividing this unit length into required number of intervals they should be of equal length so that children realise that $\frac{1}{8} < \frac{1}{7}$ or $\frac{1}{7} < \frac{1}{6}$ etc. Some teachers feel that using the number line to integrate the concept of a fraction with the concept of a number is a good and effective method of teaching.

E9) $\frac{12}{13}$ is bigger than $\frac{7}{8}$ can also be shown using a number line.

You can show (see Fig.20) that

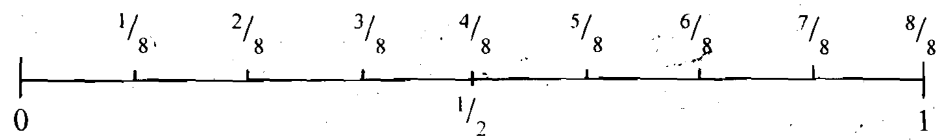

Fig.20

$\frac{8}{8}$ is 1 whole and $\frac{7}{8}$ is 1 part i.e. $\frac{1}{8}$ away (less than) from 1 whole. Similarly $\frac{13}{13}$ is whole and $\frac{12}{13}$ is 1 part i.e. $\frac{1}{13}$ away from 1 whole (see Fig. 21)

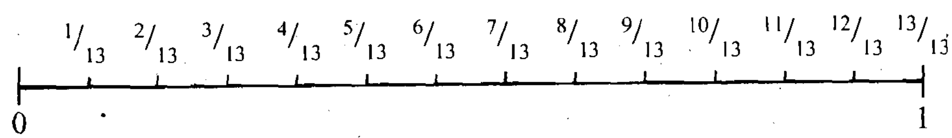

Fig.21

Now which is smaller $\frac{1}{13}$ or $\frac{1}{8}$? Let the child come out with the answer and then decide which number is bigger $\frac{12}{13}$ or $\frac{7}{8}$. If a child finds it difficult then you can help him/her by asking a few more questions in between.

E10) It appeared that Mini never got in opportunity to experience what $\frac{7}{4}$ is with the help of concrete objects, or paper shading/folding activity. She only knew these numbers $\frac{7}{4}$ or, $\frac{11}{6}$ as improper fractions, which were to be changed to mixed fraction notation and for which her teacher had told her some method.

E11) In order to correct Mini's mistake it was necessary to make her realise that $\frac{7}{4}$ means taking away 7 parts out of 4 parts and for that we require 1 full whole and another whole from which we need to take away 3 parts out of 4. She should be made to realise that $\frac{7}{4}$ = 1 whole and $\frac{3}{4}$ more. We can do this by doing a paper shading activity also. We can show that

$$\frac{7}{4} = 1 + \frac{3}{4} = 1\frac{3}{4} = \text{Quotient}\frac{\text{remainder}}{\text{divisor}}$$