UNIT 12 FRACTIONS OF WHICH WHOLE ?

Structure **Page No.**

12.1 INTRODUCTION

Although children meet fractions in everyday life they usually have very confused ideas about the real implication of the concept. For example, in conversation they often wrongly use the word 'half' when the word 'part' should have been used. We can only have half a cake if the cake has been divided into two equal parts. But, in practice, each piece is only **approximately** half of the cake. If we buy half a kilogram of tea, we have simply bought a packet of tea. In this case, the word, 'half' is only a label to many children. Another common misconception among children regarding half is that it can be obtained only when we divide a single whole, i.e., when the whole is a single object, and not when the whole is a set of objects. Children find it difficult to divide a whole in halves. Usually, they continue to divide, not stopping at two parts. They think that one cut produces one part and two cuts produce two parts. It is also seen that children when dividing an object say, cake for instance, take out two small pieces, leaving a large part undivided. This is because of the absence of any relation between the parts and the whole. **If the child has half, he cannot conclude that the other part is also half, as this also involves relating the part to the whole.** Why do children have confused ideas? In this unit, we have suggested some teaching methods which may help learners to overcome these confusions.

Objectives

After studying this unit, you should be able to

- use several child-centred methods for teaching the concept of a fraction;
- evaluate the efficacy of the method used;
- do remedial teaching whenever necessary.

12.2 IS HALF REALLY HALF ?

Many of us would have encountered situations like the one that happened at Renu's house. Her two children, Meena aged 10 and Ravi aged 8, were having their food. They both finished but one chapati was still left. So Renu asked both of them to share the chapati and finish it. Ravi took the chapati cut a corner of it and gave it to his sister. I asked Ravi, how much has he given to his sister and he promptly replied 'half'. Then I took a biscuit, cut a corner of it and gave it to him. I again asked him how much he has got and his reply was the same, 'half'. So, for Ravi every piece of a whole was 'half'.

Sonu, here is your half a glass of milk.

This is a very common problem occurring with the children of this age group. Any piece of a whole is half for them. But why does this misconception exist? One reason for this could be that right through their childhood they have seen their mother giving them half a chapati or half a glass of milk in the same manner. But out of curiousity I visited and observed a 3rd standard class, where the teacher was starting

to teach 'Fractions', using the NCERT book. The teacher drew various figures as in Fig. 1.

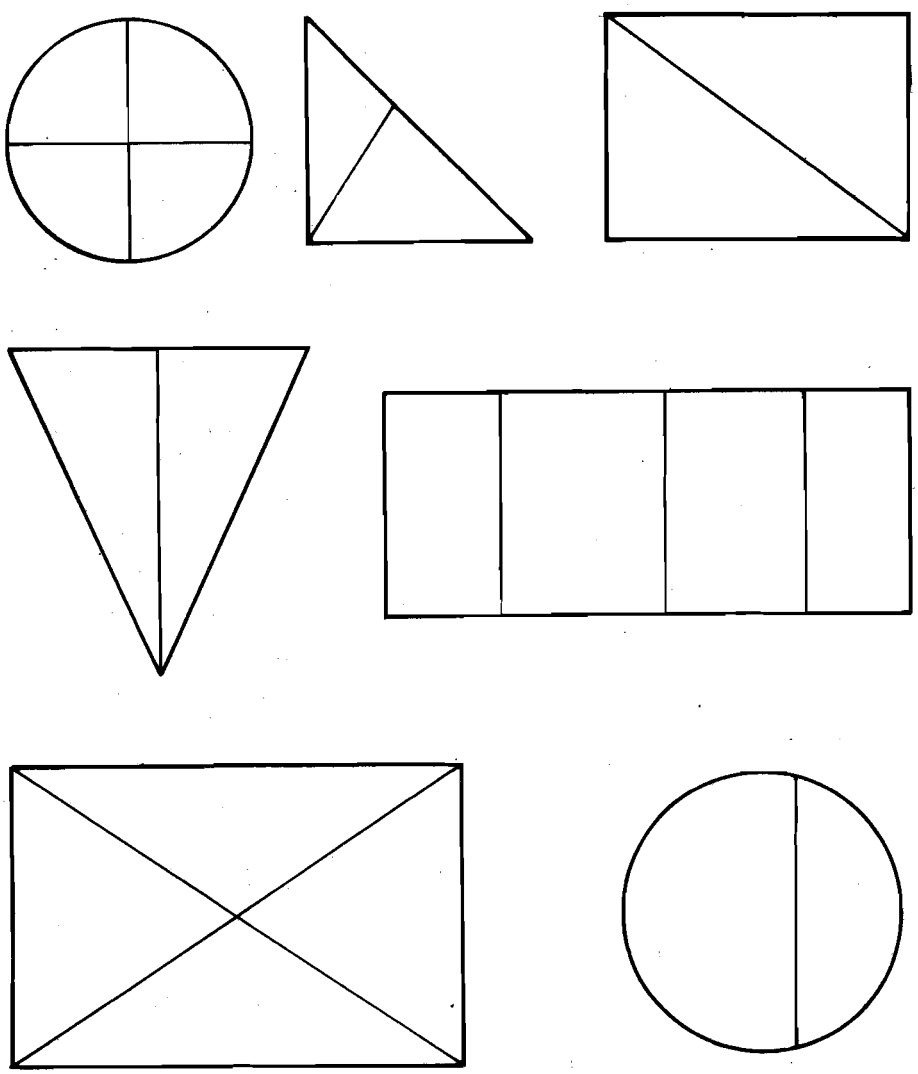

Fig.1

Then she asked the class, which of these figures were divided into two equal parts, and which were divided into four equal parts. The response of the class was quite good as most of the students came out with the correct answers.

Then she drew some more figures like those in Fig. 2.

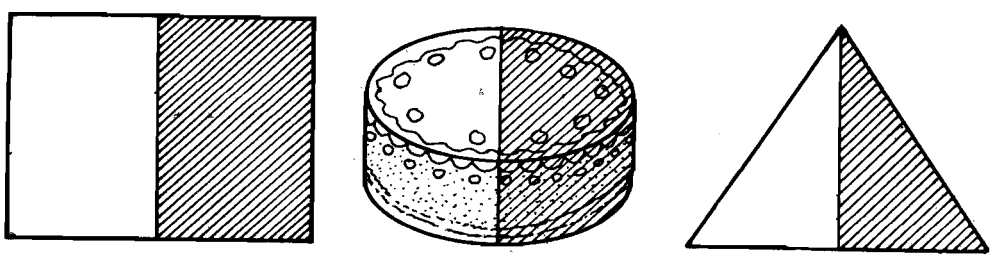

Fig.2

Each figure was divided into two equal parts. One shaded and the other unshaded. She told everyone that each of these shaded and unshaded portions were one-half of the whole and we call each of these parts as 'one-half'. Then she took one paper, folded it by bringing two opposite edges together and cut it into two equal pieces (see Fig.3).

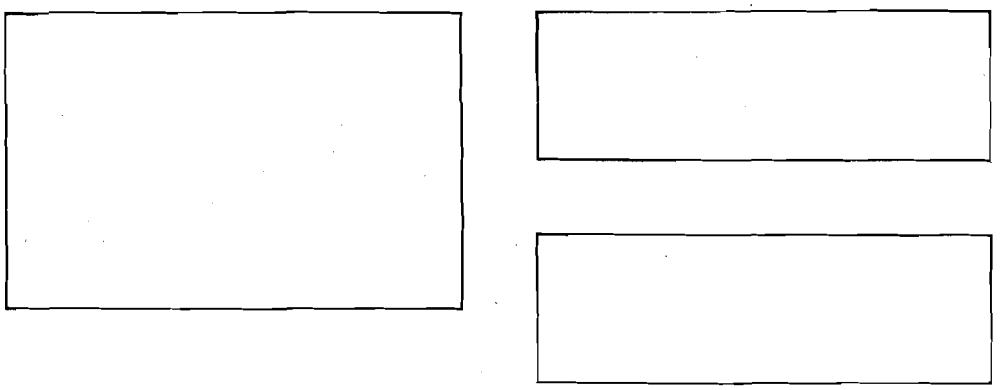

Fig.3

She called two students and gave a piece each to both of them. Then she asked the whole class how much each of them had? The entire class answered 'one-half'. Then she took both the pieces and put them one over the other to show that both the pieces were equal. In the same way she introduced the concept of one-fourth (or a quarter). Then she gave questions from the book to be done in the class and some questions for "Home work". The teacher looked quite satisfied. It appeared that she succeeded in doing whatever she intended doing. But I was a bit doubtful. I was not sure that the importance of having **two equal parts** for finding 'one-half' of any whole, really got through to the children. Also that these two equal parts can be obtained in a number of ways, i.e., it is not always necessary to have the dividing line as a straight line.

I thought I would follow up my observations with some evaluation to gauge how much communication actually took place between the teacher and the learners. One of the children, Anita, was a daughter of a good friend of mine. A couple of days later I went to Anita's place. After a bit of casual conversation, I showed Anita five figures as shown in Fig 4.

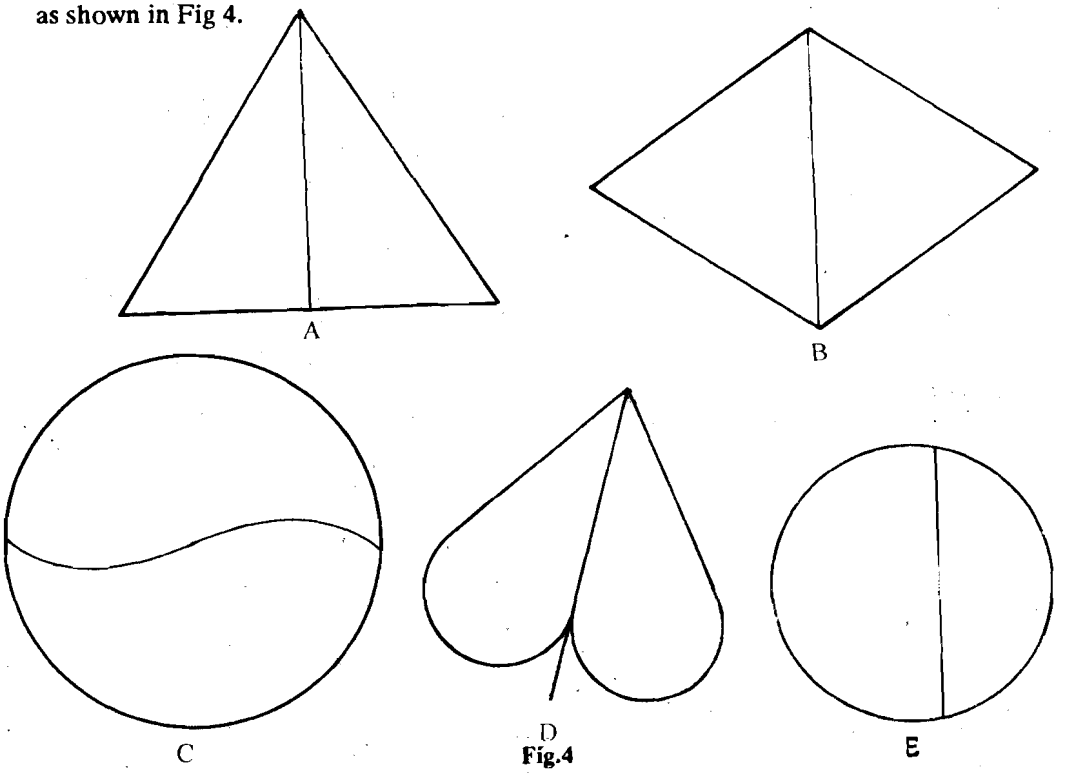

Fig.4

Example 1

I : Anita, tell me which of the following figures are divided into two equal parts (equal halves).

Anita : Fig. A B and E are divided into two equal parts.
Fig. C is not and um.... D is also divided into two equal parts.

I : Are you very sure that C is not divided into two equal parts and E is divided into equal parts?

Anita : Yes Auntie, I am very sure.

I : O.K., let's take a paper cutting in the shape of C. I will cut it along the curved line drawn. So now we have these two cuttings (see Fig.5).

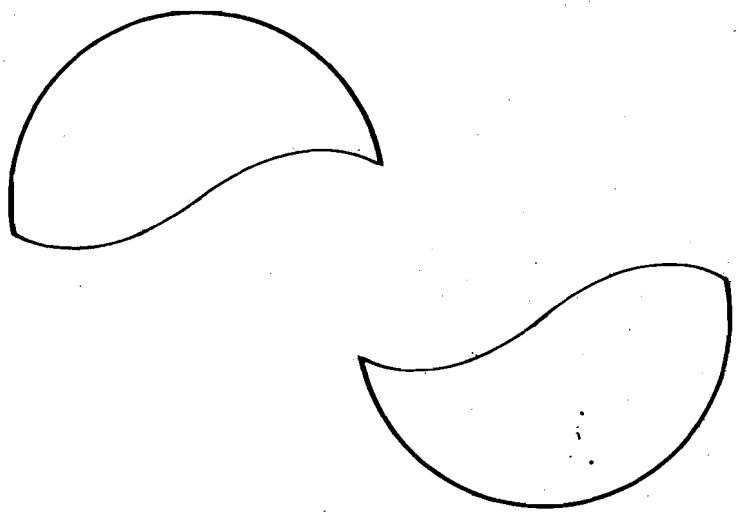

Fig.5

Now, what happens if you put one cutting over another?

Does the first cutting cover the second one completely?

Anita : Yes Auntie.

I : That means the two cuttings are equal.

Anita : Oh! yes, even these two parts are of the same size. Now I know C is also divided into two equal parts.

I felt happy, as Anita herself came out with the answer. Then I took up Fig. E with her. Although difference in the two parts in E was not too obvious but, I could convince her with the help of a paper cutting that the two parts are not equal. I took up two more figures like the one shown in Fig. 6 with her. But now Anita

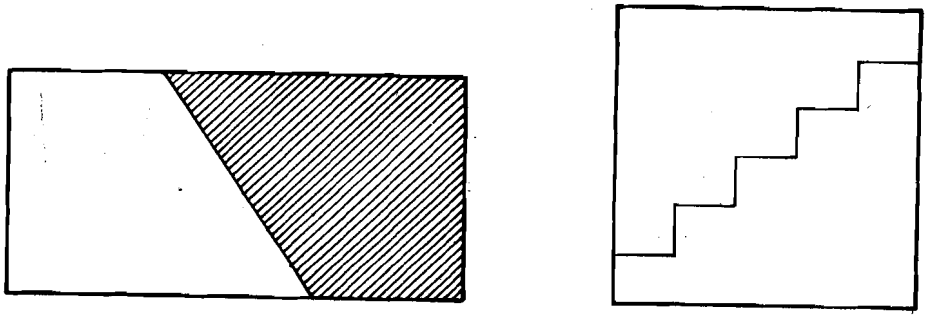

Fig.6

looked very confident and took no time in giving me the correct answers.

Why do you think Anita thought C had not been divided in half.

Anita's problem about E was not unique. It is usually seen that unless it becomes too obvious, as in Stage IV, Fig.7, a child would continue saying upto Stage III, that the

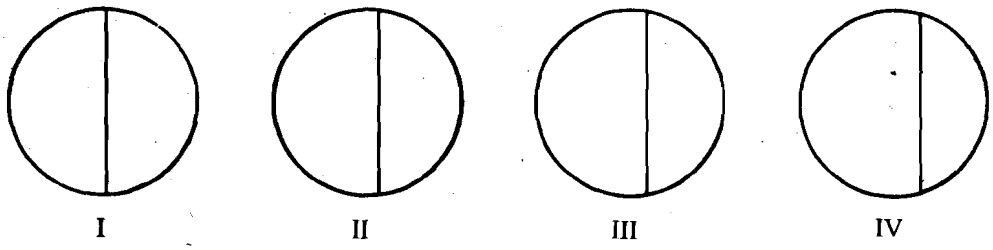

Fig.7

two parts are equal. This shows that while introducing 'one-half' of a whole in the class, stress should be given on getting two equal parts and it should be shown to the children when actually two parts are equal. In this connection here we suggest two simple activities which a teacher can take up in the class.

Two parts are equal if one put over the other covers it completely.

Example 2 : Take one big piece of paper and 3-4 smaller pieces of papers. Ask children to find which pieces are halves of the bigger paper.

Example 3 : Take a ribbon. Ask two boys in the class to share it between themselves so that each of them get half a ribbon. Once they have done it you can make them check whether the two pieces are equal.

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Also, a teacher needs to involve the learners in some more activities like dividing different shapes as circle, square or a triangle into two equal parts in 3 or 4 ways. Children should not only experience dividing a square, triangle, rectangle etc. into two equal parts. They should also realise that if there are ten cars in a race and five fail to finish then **half the number** of cars have failed to finish.

To start with, you could introduce situations where equal sharing is possible. For example, if two children share 10 sweets equally, each has one-half of the sweets, so one-half of 10 equals 5. Ask the children to put out a set of 6 or 8 or 12 bottle tops, chalk pieces or, other objects and ask them to remove half of them. How many objects have they? Discuss it with them.

E1) List two other activities to check that the importance of having two equal parts for finding half of any whole has really got through to children.

The concept of $\frac{1}{4}, \frac{1}{3}, \frac{1}{6}$, etc. can be introduced in the same manner.

Here are some suggestions for activities, that you may find useful.

Example 4 : Take a paper cutting of any of the shapes in Fig. 8.

Fold it by bringing two opposite edges together. Press it at the fold. Now unfold the paper. You will find that it has been divided by a line into two equal parts i.e., each part when laid one on top of the other, covers the other one completely. Each part is one half of the whole. Repeat the activity by folding the paper once again the same

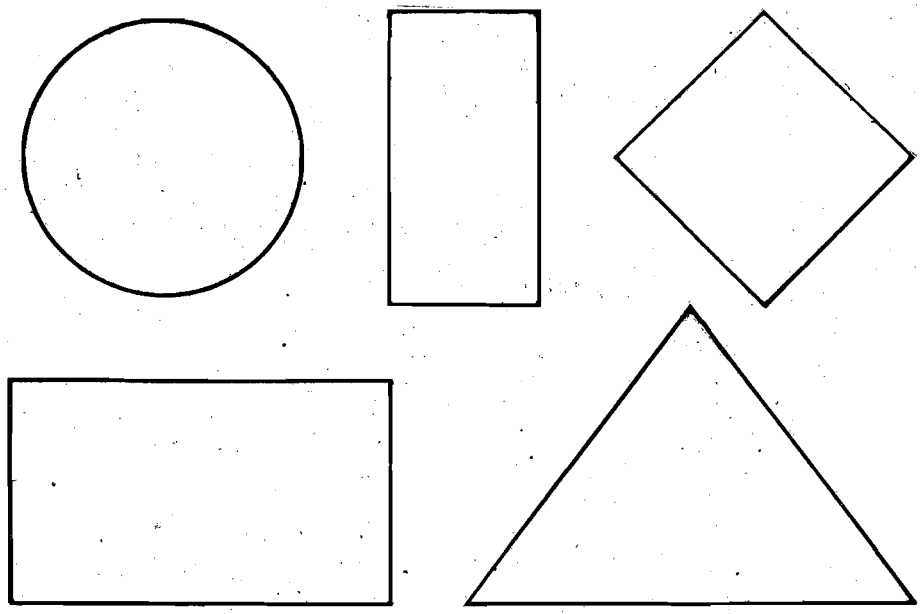
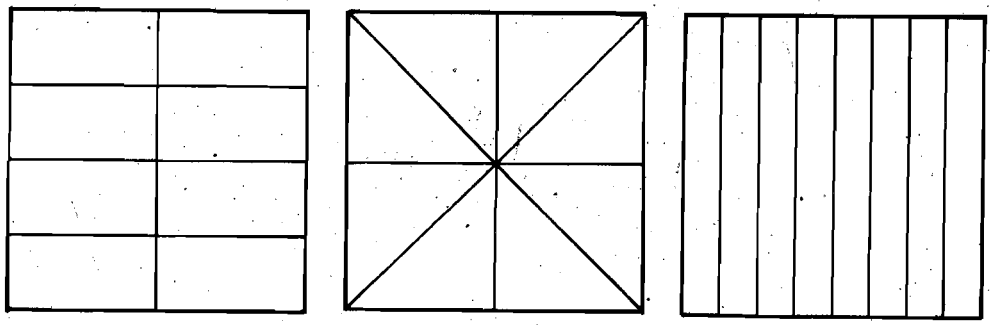


Fig.8

way and the paper is divided into four quarters. Continuing in this way you could familiarise them with $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}$, etc.

Note that the square and rectangle can be folded into eighths in different ways, e.g. see Fig.9.

Square



Rectangle

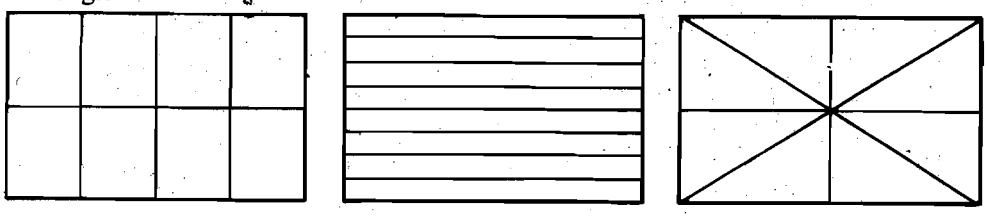


Fig.9

Children find this activity easy and interesting to do. But they may not find it easy to extend this activity to show 1/3. In this case you can help the students in folding the paper into 3 equal parts and then show $\frac{1}{3}, \frac{1}{6}, \ldots$ etc.

Let's take the case of Ankit who is an active student of mathematics. Ankit, came to ask me some of his doubts in mathematics. Ankit studies in the fourth standard and likes to do mathematics. While talking to him I discovered that they had just finished doing fractions in their class. So I thought of asking him to solve a problem. I asked him to write the fraction of a whole corresponding to the shaded portion in each of the three figures I drew (see Fig.10).

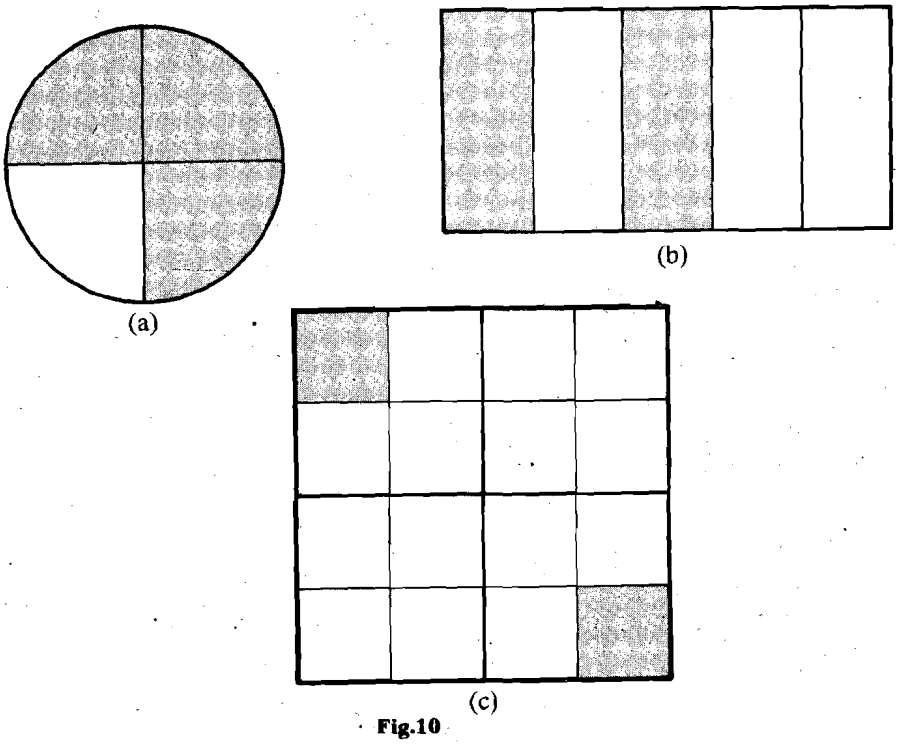


Fig.10

His answer to the problem was (a) $\frac{3}{4}$, (b) $\frac{2}{5}$, (c) $\frac{1}{4}, \frac{1}{4}$.

E2) a) What do you think Ankit's problem was?

b) Is this a very common problem with children?

c) How would you help him clarify his concepts?

E3) How would you help a child show $\frac{1}{3}, \frac{1}{6}$ by paper folding?

E4) Can you suggest an activity to help a child get the idea of $\frac{1}{5}$.

E5) How will you evaluate the success of your method of teaching fractions in your class?

What you may have realised from the above observation is that relating a part to a whole is another point of confusion that children face. We will address this problem in the following section.

12.3 WHOLE AND PART

The obstacle for understanding fractions is that it cannot be thought of as a separate, independent entity. **Fractions have meaning only in relation to the whole to which**

they apply. To recognize a fraction of something, we need a concept of the whole. You would agree with me when I say that three quarters (three-fourths) may be the expression of a single whole being divided into four equal parts, of which three have been chosen; or it may be the expression of the fact that there were four members/objects in a set out of which three members/objects have been taken; or, somewhat differently, it may imply that three whole have been divided into four equal parts. While these aspects lead to the same numerical result, you should bear in mind that they are different concepts and children need guidance in relating to them.

After talking to few teachers and parents it is felt that when children are given a single whole it is easy for them to understand the concept of 'part' and 'whole'. But when they are asked to find the part of a whole consisting of a collection of objects, say ten cars or five chairs, then they find it difficult to write the part of this whole. We could overcome such problems if we always refer to the whole to which any fraction applies. We should not talk about a quarter, but about a quarter of an apple, a quarter of a metre, or a quarter of twelve, and so on. Also, to start with we can introduce this concept restricting our discussion to a single whole.

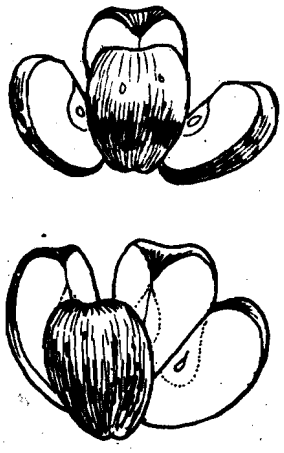


Fig. 11

In this regard some activities may be undertaken by the teacher in the class. Here is an activity, that was observed in a class, which I would like to share with you. Consider the following situation.

Example 5

Teacher : Here is an apple cut into four equal pieces/parts, When I put all these four parts together (showing it), I get the whole of the apple. (see Fig. 11)

Teacher : Mohan, come here and take one piece. What part of an apple have you got?

Mohan : One

Teacher : Out of how many parts?

Mohan : One out of four parts.

Teacher : Can you tell me what fraction of an apple have you got?

Mohan : Yes, madam, I have one-fourth of an apple.

Teacher : O.K., now take this second piece also. How much have you got now?

Mohan : I have got two pieces now.

Teacher : Out of how many pieces?

Mohan : I have two out of four.

Teacher : You have two out of four pieces. Now can you say this in terms of halves?

Mohan : Yes, Madam, I have got half of an apple now.

Teacher : Take one more piece and tell me the part of an apple you have got

Mohan : Now, I have three pieces out of four.

Teacher : That means you have three-fourths of an apple now.

Mohan : Yes, Madam.

Teacher : Mohan, now has three parts of an apple out of four parts, and these four parts, when put together make one whole apple. So this is how we obtain three-fourths of a whole. The whole in this case being an apple.

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The teacher then involved all the children in various activities with different objects like piece of paper, piece of bread, biscuit, chapati etc., whatever she could gather in her class. She showed $\frac{1}{2}, \frac{1}{4}, \frac{3}{4}$, to all the children with the help of these objects. In between she asked them a few ques ons. After she made them answer all the questions she looked quite satisfied.

As we have mentioned earlier children have problems in relating a part to a whole when the whole is a collection of objects. It is therefore necessary that we acquaint a learner with different possible whole. Once we feel that children are confident about a part-whole relationship with respect to a 'whole' consisting of a single object we can extend our discussion to other types of 'whole'.

Let's take up once again the problem of three-fourth of a whole and see how many other types of 'whole' we can acquaint a learner with.

1. A whole consisting of more than one but fewer than four objects

Three cakes to be shared equally by four children.

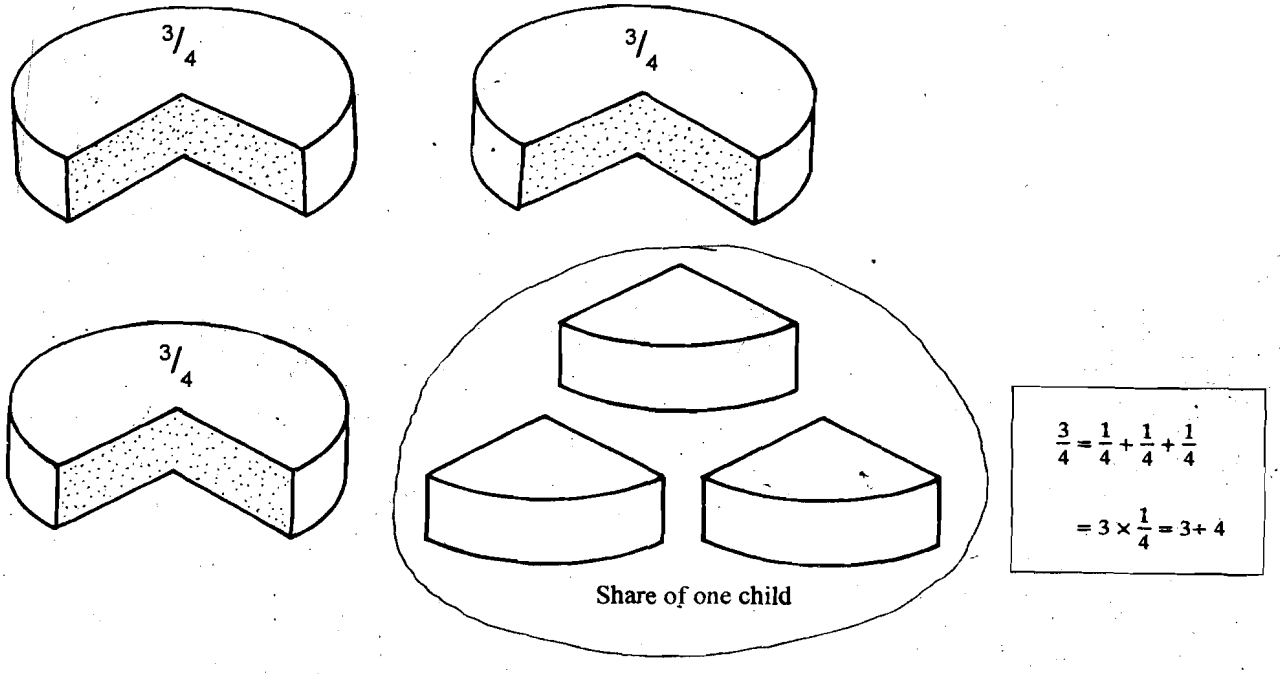


Fig:12

2. **A whole consisting of exactly four objects**

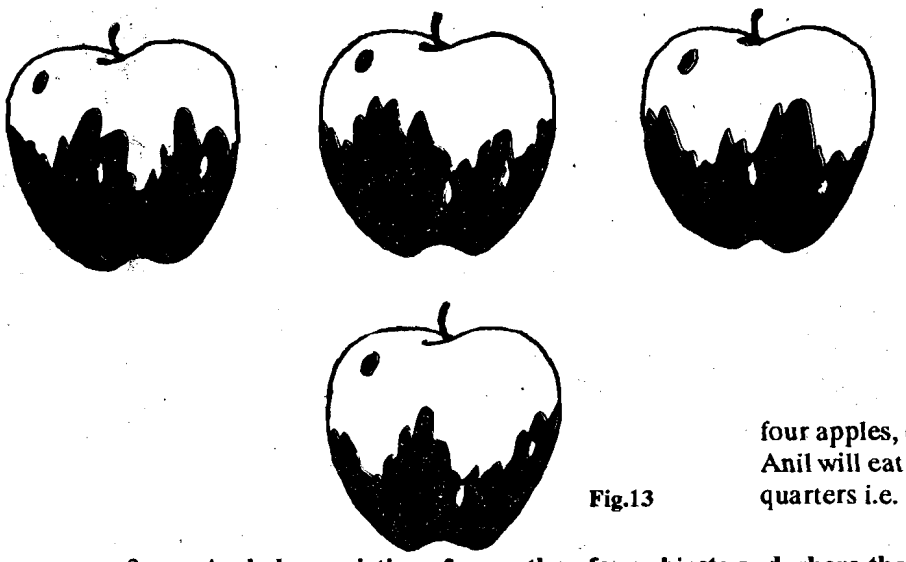
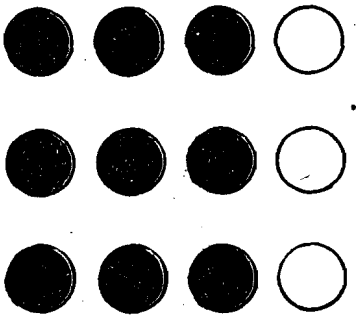


Fig.13

four apples, of which Anil will eat three quarters i.e. three

3. **A whole consisting of more than four objects and where the total is exactly divisible by four**



Twelve marbles, three quarter of which are black i.e., nine are black.

Fig.14

Teacher can take up activities in the class with each of the 'whole' mentioned above. Here we give one for you. Let's look at the following situation.

Example 6 : Teacher brought some biscuits and toffees in the class and started interacting with the children. She called one boy Mohan near her.

Teacher : Now here are four biscuits and I give three biscuits to Mohan, then how much has he got?

Students : He has got three biscuits.

Teacher : Out of how many?

Students : Out of four.

Teacher : Mohan, can you write on the board the fraction of biscuits you have got.

Mohan : Yes, madam. (He wrote $\frac{3}{4}$ on the board.)

Teacher : So, can we say that he has three-fourths of biscuits.

Students : Yes, Madam.

Teacher : So three-fourths in this case is three biscuits out of four biscuits. Can you tell me what is our whole in this case?

After a bit of silence in the class.

One boy : It's biscuits.

Teacher : How many biscuits?

Boy : Four biscuits.

Teacher : Good! So, our whole here is a set of objects namely, four biscuits.

Teacher again took five toffees and gave two toffees to Renu. Then she did similar exercise with the children. She asked them many questions and made them say that the 'Whole' in this case is five toffees taken together. Then she did few more exercises with them and gave them some problems to solve.

This way students can be exposed to different wholes.

E6) Similarly, suggest one activity each for other types of wholes mentioned above.

We now end this unit by giving a summary of what we have discussed in it.

12.4 SUMMARY

In this unit we have tried to bring the following points.

1) Children aged between 8 to 10 years usually have very confused ideas about 'half'. Every piece of a whole is half for them. They do not realise the importance of having two equal parts for finding 'one-half' of any whole. Also, for many of them the dividing line has necessarily got to be a straight line. Such misconceptions could be avoided to a great extent if children are given various activities in the class with the help of different concrete objects easily available around them. They can be involved in dividing different shapes such as circle, square or a triangle etc. into two equal parts in 3 or 4 ways. Once they divide a thing into halves they can be asked to check if both the parts are equal.

2) Relating a part to a whole is another major problem with the children of this age group. They do not realise that fractions have meaning only in relation to the whole to which they apply. To overcome such problems we should always refer to the whole to which any fraction applies. It is also necessary that we acquaint a learner with different possible wholes by taking up various activities in the class.

12.5 COMMENTS ON EXERCISES

E1) i) Give a piece of bread, or a chapati to a child and ask her to take half of it. After she does it let her check that both the halves are equal.

ii) Children can be given shapes of flat clay or paper and asked to divide them up between two, either by cutting, folding, or drawing.

E2) a) Ankit viewed at the four squares (with darker boundaries) within the bigger square as four wholes. He looked at them separately and answered. He could not visualise the four squares as part of the same whole i.e. the bigger square. It is a problem of relating part to a whole.

b) Yes, part-whole view of a fraction is a very common problem with children of this age group.

c) His concepts can be clarified by undertaking paper folding activities, or using various concrete objects like marbles, biscuits, toffees etc. for performing activities.

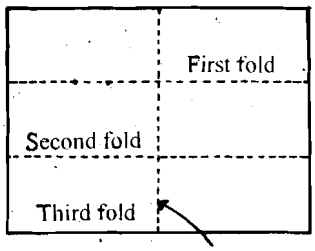


Fig.15

E3) You can take a rectangular piece of paper and fold it into three equal folds. Press it at the folds and then unfold the paper to show $\frac{1}{3}$. Repeating the same activity and folding the paper once again $\frac{1}{6}$ can be shown. (see Fig.15)

E4) Children usually find it difficult to divide a circle into five equal parts. So you can draw a circle or a rectangle, divide it into five equal parts as in Fig.16. Now shade one part out of these five parts and ask the child the fractional representation of it, or you may ask him to shade $\frac{1}{5}$ of it. You can also show it by paper folding. You can even take 10 sweets ask the child to distribute them equally among 5 friends. Now let the child say how much everybody got.

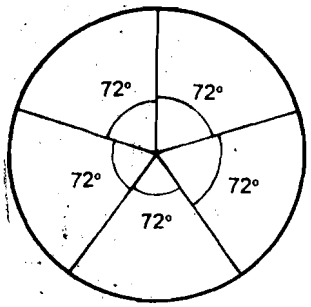


Fig.16

E5) You can first do paper sheet exercise with all the children in the class. Divide them into three groups. Ask each group to fold the paper into two equal parts, unfold and shade what corresponds to $\frac{1}{2}$. They should also be able to say that they have got $\frac{1}{2}$ of the whole. Then you may go to three parts which is much more difficult than folding into two parts. After this you may also give each group 12 marbles/toffees and ask each group to divide them into two, three or four equal parts.

E6) i) Take a dozen of bananas/oranges, three of which are rotten. Ask children to separate rotten bananas/oranges from the good ones. Now ask them the fractional representation of the remaining bananas/oranges.

ii) Take 3 chapatis and ask 4 boys to share them equally. They can do it in two ways

a) Three boys can take 1 chapati each and then each of them can give $\frac{1}{4}$ of a chapati to the fourth boy.

b) They can divide each of 3 chapatis into 4 equal parts and then share them equally.

After they have shared the chapatis equally they can be asked to write what fraction have they got.