UNIT 11 DEVELOPING LANGUAGE

Structure

	Page Nos.
11.1 Introduction	27
Objectives	28
11.2 Role of Language In Explaining Mathematics	28
11.3 Word Problems	29
11.4 Learning The Language of Mathematics	31
11.5 Summary	35
11.6 Comments On Exercises	35

11.1 INTRODUCTION

If you ask any of your friends whether mathematics learning and language learning are related, they would probably say not. This is because they have yet to realise that the relation of mathematics with language is at three levels:

1) **Language of explanation:** In the class when the teacher explains mathematics (concepts, formulae, operations, procedures, propositions, etc.) she uses ordinary language to do so. In fact, every mathematical construct a child develops is presented to her through ordinary language.

   We shall discuss this aspect in Sec. 11.2.

2) **Language of problem-solving :** One of the principal aims of learning mathematics is to develop the ability to convert real-life problems into mathematical problems, solve them using known techniques, and interpret the results as meaningful solutions to the real-life problem. Children develop this ability by being exposed to appropriate word problems. Here is where language comes in again.

   We shall discuss the difficulties involved in this aspect in Sec. 11.3.

3) **Mathematics as a language:** Mathematics is itself a language with its own symbols, words and rules of syntax. It is based on a certain consistent set of assumptions, and built up from there according to the rules of logic. The understanding and application of such logic, necessary for developing mathematical thought, depends upon the level of development of ordinary language. For example, only after developing the ability to use conjunctions such as 'and', 'but', 'therefore' and 'or', are the children ready for mathematically logical statements such as "Every square is a rectangle, but every rectangle is not a square."

   Another point of interaction between the two languages is the use of certain common words. Some words from ordinary language are used in mathematics, but with a precise mathematical meaning, e.g., difference, add, multiply, power. Such words are

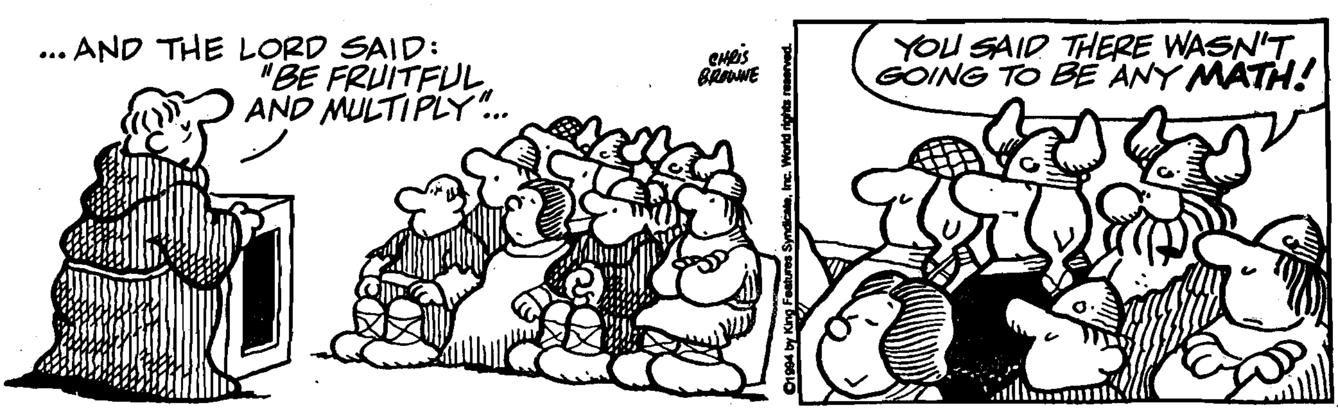

Fig. 1

often a source of misconceptions, since their meanings may conflict with those in ordinary language usage.

We shall talk of these aspects in detail in Sec. 11.4.

While discussing the different aspects of the interface between mathematics and language, we shall usually stick to examples involving numbers. But, this relationship is true for any area of mathematics. So, while going through the future blocks, please keep observing how often this happens. Such observations would help you internalise the objectives of this unit.

Objectives

After reading this unit, you should be able to

- explain the role of ordinary language in mathematics learning;
- describe the difficulties children face when dealing with word problems, and develop strategies to help them overcome these difficulties;
- explain what the language of mathematics is;
- suggest ways to help children learn the language of mathematics;
- evaluate your teaching strategies.

11.2 ROLE OF LANGUAGE IN EXPLAINING MATHEMATICS

Suppose you are helping a child acquire the concept of '3'. How would you do it? Wouldn't you help her abstract '3-ness' from various sets of 3 objects, such as pencils, books, children, trees, etc.? In order to prepare the child for this abstraction and associated labelling, we need to use concrete objects, images, and so on. Alongside we need to explain everything to her in ordinary language.

In the same way, to communicate **any** mathematics to a child, we need to do it in her first language, or any other language that she understands. Further, the ordinary language used needs to be pitched at the level that the child understands. Unfortunately, many teachers and textbook writers don't think about this aspect. This is one major reason for people believing that "mathematics is difficult"

Take a glance at any mathematics textbook for primary school children. Do you feel that children of that age group would be able to understand the language in which the concept or process is explained? For instance, consider the following extract from a **Class 2** text, that is **meant for a 6-year-old.**

'The portion of an object we touch and see is called its ***surface****. Let us take a straight rod and put it on the top of a table in* ***any*** *manner we like. We see that the rod touches the surface of the table everywhere. The surface of the top of the table is a plane surface.'*

How many 6 or 7-year-olds are expected to understand this? When they don't, what kind of misconceptions do they retain? A better way to convey the concept would be to use words intuitively familiar, to the child, along with gestures. Words like 'surface' should be avoided if they are inappropriate for the level of language development of the children.

Further, we should avoid saddling young children with wordy explanations and definitions of mathematical concepts like 'surface', 'triangle', etc. It is enough, at this stage, if the children can give examples to differentiate between what is a surface and what is not.

Unless we take care of these aspects right from the beginning, the later concepts and processes will never be clear to the children. They will never be comfortable with the language of mathematics. At each stage they would be trying to grasp at some indication or some clues, to be able to at least memorise the definition or algorithm. And then, when faced with any slightly unfamiliar problem, the children are likely to make errors of various kinds, as we have mentioned in the earlier units.

Here is an exercise about what we have said so far.

E1) Analyse any chapter of the mathematics textbook of a child that you are teaching. How would you reword portions of it, in the light of what you have just read?

Let us now examine the problems that arise because of a lack of comprehension of the language used in a mathematics situation.

11.3 WORD PROBLEMS

As you have read in the units so far, we need to introduce mathematical concepts and processes to children through appropriate word problems. Such problems provide them with **motivating contexts**. Of course, while giving children word problems, we must ensure that they are simply worded and related to the children's real-life experiences.

In solving word problems, the first step that the child has to take is to understand what the problem is saying. Then she has to translate the problem from the real-life context into an appropriate mathematical expression, write it in symbols, and then choose and use an appropriate procedure to solve the mathematical problem. Following this, she has to interpret the result in the context of the real-life situation she started with.

Thus, the steps involved in dealing with a verbal problem are

1. understanding the real-life problem
2. converting it into mathematical statements
3. formulating the equivalent mathematical problem
4. solving the mathematical problem
5. interpreting the solution in the context of Step 1.

Unfortunately, most children are not exposed to word problems from the early stages on. Most teachers and textbooks go directly to abstract number problems, or use pictures without words, to convey concepts. They only come to word problems towards the end of the year. As a result, children have a lot of difficulty in dealing with such problems.

For example, the other day a teacher told us that whenever he presented children with word problems involving any of the four operations, he would hear whispers and sense a general panic in the room. ''And sometimes,'' he said, ''if you give children a word problem soon after teaching them multiplication and division, the reaction is interesting. They know that the problem has to be related to what has recently been done in class. So it has to be multiplication or division. And then they ask, ''Do we divide or multiply, Sir?''.''

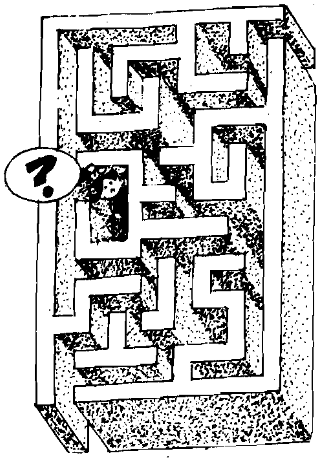

Fig. 2: For a child a word problem is like a maze.

If you interact with children of Class 4 or 5, more often than not you will find that they show a total lack of confidence when faced with word problems. To see this, consider a typical set of 10 or 11-year-old children who have been given some directly stated problems and some word problems to do concerning the four arithmetic operations. You would find certain common patterns. Children first attempt those problems that are written in proper algorithmic form. Then they begin whispering and attempting to confer. And then a large majority of them give up.

The few that attempt the word problems begin very warily and tentatively. The fear and lack of confidence is visible. (This is quite in contrast with the confidence with which they solve the directly stated problems involving numerals and algorithms.) They try various guesses. Some of them mostly start by **adding** some numbers. After that they look around and try to peep into others' copybooks to see whether they have done the same or not.

Some children look at the problem for key words, hoping that these would give them a clue to the operation to be performed. The reasoning may be: ''If the word 'divide' or 'share' occurs, it has to be division. If the problem has 'total number' written somewhere, it could be addition. If it has 'how many more' or 'how many left' written, it has to be subtraction. And if the word 'times' or 'rate' occurs, then the chances are that it is multiplication.''

- There are other hints they look for too, for instance, the size of the numbers involved. Regarding this, their reasoning appears to be: "If the numbers are small, they have probably to be multiplied; if one number is large and the other small, try division; if, on dividing, you get a remainder, then you have made a mistake and some other operation needs to be tried."

There are many other such markers that are used by children to help guess the operation to be performed, and on which numbers they are to be performed. Why do children feel the need to resort to such markers?

Children probably tackle word problems in this way because of the kind of emphasis we place on what we consider 'learning'. For many of us, the main thing to be learnt in arithmetic is how to carry out the operations correctly and speedily. The pressure of the textbook and the curriculum also pushes teachers towards making children deal with larger and larger numbers, and with more and more complex algorithms and rules. And all this is done without giving the children time to think, understand and absorb the mathematics involved.

Many of us add to the complication by providing children with efficient strategies for guessing (without understanding!) which operation to perform. We also provide them with shortcuts and algorithms to follow. These methods produce the required answer for familiar problems alright. But the children have no clue as to what is being done and why. And then they aren't able to solve even the same problem if it is worded differently.

Unfortunately, we are reluctant to spend adequate time on having children practise the operations with concrete objects and in familiar contexts. We don't allow them enough practise with solving problems that involve small numbers, and for which they don't need to use the algorithms. (For example, problems like 'Anna had 7 marbles. She gave 3 to her friend Ajeet. How many marbles was Anna left with?'.) As you know from the previous units, such exposure is necessary for children to be able to deal with problems. From simple problems involving one operation, they could gradually be exposed to word problems having many steps. In this way they would even be able to deal with problems like 'Ashok's mother gave him Rs. 50/–. He bought 2 dozen oranges at Rs. 10 a dozen and 3 kg potatoes at Rs. 6 per kg. How much money should he bring back?'.

Why don't you try an exercise now?

E2) List any other reasons for children not being comfortable with word problems.

While answering E2, you may have pointed out that a major difficulty that children have when faced with problems set in real-life contexts is the **interpretation of the answer.** In most cases, the numbers in the answer have been produced by a process that the children don't understand. Therefore, they are often unable to relate the numbers to a real-life situation. Because of this, they may come out with ridiculous answers.

For example, eight 10-year-old children were given the problem, '149 children have to go home in a jeep. Each jeep trip can take 12 children, at the most. How many trips would the jeep need to make?'. To solve it they divided 149 by 12.

All, except one, gave the answer, '12 trips, and 5 children are left behind'. Only one child produced the answer '13 trips'. On asking her why, she said that the 5 children which are left behind would need another trip.

When an older group of children was asked the same problem, many came up with the answer 12.4 . When questioned further about interpreting their answer, some of them rounded it off to 12, because, as they said, ".4 is less than .5." Therefore, according to them, 149 children would require 12 trips and 151 children would need 13 trips (since 12.6 approximates to 13).

Do you agree with what we have said so far? Why don't you try the following exercise to see if our observations are valid?

E3) Think of some division problems based on real-life situations that a 10-year-old around you would relate to. Ask some 10 or 11-year-olds the questions, and note down their responses. How appropriate were their answers?

As you have seen, children face difficulties at many levels when dealing with word problems. The major obstacle seems to be the inability to mathematically represent a real-life problem, and vice versa. To do this a child should be able to meaningfully link the ordinary language in the problem with mathematical terms and symbols. A mediator may be necessary to establish such a relation. Such a mediator could be a concrete or a pictorial representation of the problem, explanatory gestures that the adult makes, etc. For instance, if the problem is 'how many stones would you have if you combined 2 sets of 5 stones each?', the child could use actual pebbles to get the answer.

Another strategy that may help children cope with word problems is the following one suggested by Homi, a teacher trainer.

Example 1: Homi has been trying ways of helping children of Class 4 and above deal with word problems. He believes that one of the methods that serves this purpose is to create word problems along with the children. For example, he asks them, "Munni is 5 years old, and her mother is 29 years old. How can you relate their ages?" A child may come out with $29 - 5 = 24$. Another may come out with $6 \times 5 = 29 + 1$, and so on. Then he takes any one of the relationships, say, $29 - 5 = 24$, and asks them to give a corresponding 'word' relationship. At this point he needs to give them hints like 'what was 29 in the problem?', 'what was 5 over there?', and so on. After some time, Homi and the children come out with the expression,

mother's age − Munni's age = 24 years.

Then he encourages them to do it for, say, $6 \times 5 = 29 + 1$, or other relationships that they have worked out. Suppose they come out with

6 × Munni's age = mother's age + 1 year.

Around this he asks them to build a word problem. He says that, with some help, they come out with problems like 'Munni's mother is 29 years old. One year from now her age will be 6 times Munni's present age. How old is Munni?'. Then he, and the children, solve the problem they have created by reversing the steps they used to create the problem.

According to Homi, the children find such an instructional process very interesting. It also helps them see the relationship between the different steps involved in solving a word problem.

——————— × ———————

You may like to try an exercise now.

E4) "A major reason for children feeling unsure of how to solve word problems is that a single mathematical expression can have several verbal mathematical analogues." Explain this statement by taking the expression '$4 + 3 = 7$' as an example.

Let us now look into the third level that we had mentioned in the introduction.

11.4 LEARNING THE LANGUAGE OF MATHEMATICS

What is the language of mathematics? Like any language it is made up of concepts, terminology, symbols, algorithms, and syntax which is peculiar to it. Children can only acquire this language by using it, that is, by speaking in it, writing in it and listening to it. Throughout the earlier units we have stressed the importance of talking mathematics with children, encouraging them to talk about the activity they are doing, building on their understanding by discussing mathematics with them. This kind of interaction is what shapes their understanding, builds their mathematical language and thought.

Of course, when younger children try and explain what they are doing, it may not be consistent or logical. This is because, to explain properly, the child needs to reflect on what she has done. And reflection is a higher level cognitive ability. Nevertheless, it is important that children begin with putting together explanations. Doing so will give them opportunities to develop the ability to put together the various mathematical processes involved, organise them and put them into words. This will go a long way in developing their **understanding of and liking for mathematics.**

Unfortunately, most teachers don't have the kind of interaction with their pupils that we have suggested above. For example, in Class 1, nearly 30 hours are set apart in the curriculum for teaching the concept of numbers and numerals from 1 to 9. And how is this teaching done in the usual mathematics class? The teacher picks up some object a bag, an umbrella, a pen, or any easily available object in the classroom and calls it 'one'. She then writes its numeral, i.e., 1, on the blackboard, which the pupils dutifully copy down. This much is done in 2 or 3 minutes and the teacher is satisfied that she has taught the concept 'one' to the children. In the same way she presents them with several other number names and symbols, and expects them to absorb all this new spoken and written language. She is quite surprised when it is suggested that the children may be quite confused by this 'explosion' of terminology.

Somehow the children manage to cope with such methods of teaching, with the help of adults around them. (Of course, a large number of children, especially those from rural areas, do not have access to any adult who can help them.) Because of this, the children end up with several wrong notions about mathematical concepts, processes and skills. One of the outstanding examples of this is their understanding (or lack of it) of the algorithms for applying the arithmetical operations.

Quite a few children can perform computational procedures correctly by rote, without understanding the mathematics involved. For example, when some of us interacted with the children of Class 7 of some schools in Madhya Pradesh, we found that most of them could not read 5-digit numerals correctly. But several of them could correctly carry out long multiplications involving 4 or 5-digit numerals.

This failure to understand the basis of an algorithm is often due to an inadequate understanding of why we write numerals the way we do. Because of this, children make errors like the following:

When Jamuna was asked to apply the algorithm to add 20 and 1, she wrote $\begin{array}{r} 20 \\ +\ 1 \\ \hline 30 \end{array}$. But, when she did the same sum orally (informally), she got 21.

Thus, while many children know what hundreds, tens and ones are and what adding one, ten or hundred to a number means, they are not able to relate it to what is happening in the written algorithms. They don't understand the steps and procedures suggested in the algorithm. This is also illustrated by the following example.

When 14-year-old Ajay was asked to solve 312 – 47, he wrote

$$\begin{array}{rccc} & 3 & 1 & 2 \\ - & 1\not{0} & 5\not{4} & 7 \\ \hline & 2 & 6 & 5 \end{array}$$

When he was asked to explain why he had changed 4 into 5, he said, "Because 2 is less than 7. So I take one away. Then I have to put one back below." "But, why do you add it to the bottom?" he was asked. A little defensively, he replied, "But, this isn't wrong! If I add 265 and 47, I get 312."

Evidently this child knows how to apply the method, and he knows the meaning of the operation. But he is not able to link the two.

Here's an exercise about what we have just discussed.

E5) In your experience what, if any, is the relationship between children knowing how to use an algorithm and their being able to explain the concepts or processes involved? If a child is not able to explain why an algorithm works, how do you handle the situation?

In order to help children understand the various words related to a mathematical concept, we can present the words to them through stories that are interesting. (Examples of such a story are in Unit 9, and other units.) But, to understand the abstract concepts without a context is difficult for children. The following example illustrates this.

4-year-old Shiela had been exposed to numbers and small additions with concrete objects in her play school. One day, her teacher asked her, "How many is two and one?" Shiela couldn't reply. After a long pause the teacher continued:

Teacher: *Well, how many stones is two stones and one stone?*
Shiela: *Three stones.*
Teacher: *So how many is two and one?*
Shiela *(hesitantly, questioning) : Four?*
Teacher: *What is one stone and one more stone?*
Shiela: *Two stones.*
Teacher: *How many is one and one?*
Shiela: *One, maybe.*

There is no connection between the questions with stones and questions with abstract numbers for Shiela. She may be thinking,
"Well I don't know this one. But it has to be different from the other stone one, and so here goes."

This example suggests that helping children to move from what they have experienced to more generalised situations is tricky. They have to understand the language, and the logic of the particular case.

Once a child becomes familiar with some mathematical language, she would be in a position to appreciate its power as a means of communication which is concise and unambiguous. As she uses this language, she would see in what ways it differs from the language of daily use in its conciseness, precision and syntax. This difference led the famous German philosopher Goethe to remark that "Mathematicians are a sort of Frenchman. Whenever you say anything to them, they translate it into their own language, and right away it is something else."

While children are learning the use of various symbols, they may come across difficulties like the following:

i) Distinguishing between expressions like 32 and 3/2, or 2x and x^2.

ii) Multiple ways of expressing '=' in words. For example, 8 – 2 = 6 reads 'from 8 take away 2, **which leaves** 6', and 6 + 2 = 8 reads '6, added to 2 **makes** 8'.

iii) A single mathematical fact can be presented in various ways. For example, 8 – 2 = 6 can also be interpreted as 6 + 2 = 8; 7 = 3 + 4 is the same as 3 + 4 = 7. (Note that this kind of commutativity may not appear very unusual to us. But children find it very confusing. They also absorb the second form of the equation (i.e., 3 + 4 = 7) much more easily than the first.)

iv) Graduating from sentences like 5 + 3 = 8 to sentences like 5 + 3 = 3 + 5. (Children have to go beyond viewing equations as sentences to viewing them as number relationships.)

Do you agree with our list of difficulties? They have been gathered from several interactions with children, some of which are:

- Suhani, given ☐ = 4 + 5 , counts on her fingers and writes: 9 = 4 + 5, but reads, '5 plus 4 equals 9'. She also says, "4 + 5 = 9 is better than 9 = 4 + 5 because I'm used to having it (i.e., = 9) on that side."
- When 6-year-old Fatima was given 3 + 2 = 2 + 3, she said, "You forgot to put the 5." Then she wrote 3 + 2 = 5 and 2 + 3 = 5.
- Mohan, aged 7 years, accepts 3 + 2 = 2 + 3 "because they both have the same numbers. Only 2 + 3 is backwards."
- Another child, Disha, would not accept 3 + 2 = 2 + 3 at all. She changed it to 3 + 2 + 2 + 3 = 10.

These examples show that while we are looking at children's attempts to solve problems, we need to think about the mathematical language in which the question has been stated. When we present children with problems and / or solutions, we need to do so in different ways. Otherwise the children may believe, for instance, that the '=' sign is only valid if one or more operation signs come before it. They may even believe that the answer must come after the 'is equal to' sign.

Do you think about this aspect of learning when teaching children mathematics? Reflect on this while doing the following exercise.

E6) How, and at what stage, do you begin introducing mathematical symbols and sentences in the class ? After reading what we have just said, what changes would you like to make in it?

Once children have understood how to handle mathematical 'sentences' which involve one operation, their problems may still not be over. More difficulties arise in reading mathematical expressions that have more than one operation. In this case the equation has to be read correctly and operations performed in a certain order. The number of symbols also increases because of brackets being introduced. Regarding this, let us consider the following example.

9-year-old Gul had been taught about the use of brackets in mathematical expressions. To gauge her understanding of these symbols, she was asked to solve $5 + 2 \times 3 =$ —.

Interviewer (I): *What is the value of the left side?*
Gul (G): *21*
I: *How do you get that?*
G: *$5 + 2$ is 7 and 7×3 is 21.*
When asked to insert brackets in the expression, she put them round the first two numbers, that is, $(5 + 2) \times 3$, which retained the left-to-right order she had followed anyway.
When she was asked if she could put the brackets anywhere else, she put them round the whole expression, i.e., $(5 + 2 \times 3)$. Her usage was consistent with her explanation:
I: *Why do you put in brackets?*
G: *To show which one you do first.*

This example reinforces the need for being very patient with children while introducing them to new symbols / terminology / processes. We need to give them many opportunities to use the new written language, to talk about what they are using, and its purpose.

Here's an exercise for you now.

E7) How do you help children understand the purpose of brackets, and how to use them?

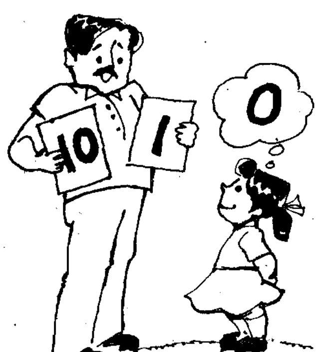

Fig. 3: "What is the 'difference' between 10 and 1?"

Another problem related to the first language of the children is that they sometimes find words used in a mathematics context that are taken from everyday language, like add, difference, etc. If these words have the same meaning in mathematics, children don't face a problem. But if these words are used in a different way in mathematics, children may have difficulties, as the following example shows.

Given the problem "What is the **difference** between 11 and 6?", a child responded to it by saying that 11 has two numbers. This was, of course, considered wrong by the teacher. Another child said that 11 is straight while 6 is curved. This was again considered incorrect by the teacher. Clearly the word "difference" was being interpreted in different (!) ways by the teacher and the child.

Another situation at the elementary level may arise when children are asked to add, say, 4 to 1. This may be in the sense of addition or it may be in the sense of placing 4 in front of 1 or behind 1. In each case the child would get a different answer, each of which could be considered valid by some!

Here is a related exercise.

Another situation at the elementary level may arise when children are asked to add, say, 4 to 1. This may be in the sense of addition or it may be in the sense of placing 4 in front of 1 or behind 1. In each case the child would get a different answer, each of which could be considered valid by some!

Here is a related exercise.

E8) List some more examples where the meanings the child picks up and those intended by the teacher are different.

Before ending this section, let us reiterate the main points in it. The language of mathematics is concise, precise and logical. Because of these properties, it is powerful. Children need to learn to use it and to appreciate its power. It is important that children develop the ability to read mathematical expressions and equations, and derive information from them. They should realise that an equation is a concise form of expressing a situation, a problem or a phenomenon. To help children enjoy learning this language we must **introduce it gradually,** at appropriate stages, through plenty of practice. The children must be encouraged to **talk and write in the language of mathematics.** In this way the children would internalise it and recognise its power.

Let us now summarise what you've read in this unit.

11.5 SUMMARY

We have made the following points in this unit.

1) The use of language can affect the learning of the concepts of mathematics by children, as it helps them fix the concepts in their minds.
2) While solving verbal problems, reading books on mathematics or articulating what they have understood, children need to use their everyday language as well as mathematical language.
3) How children react to word problems, and some reasons for such reactions.
4) How to help children cope with word problems.
5) Being able to apply algorithms does not mean learning has really taken place.
6) Shortcuts and easy tips to remember algorithms for some specific context and period may be very dangerous. They may induce children to make incorrect generalisations and inculcate wrong concepts.
7) We need to introduce mathematical language to children gradually, with enough practice, at appropriate stages, in a manner that draws them more and more deeply into the language.

11.6 COMMENTS ON EXERCISES

E2) a) Several different situations are represented by the same mathematical formulation.

b) A single situation can have various representations as word problems.

c) The word problems are not presented in an algorithmic manner, and therefore require time to comprehend.

There could be other reasons that you may come across while interacting with children.

E3) For instance, you could construct problems involving distributing 4 cards each to a group of players, or finding the number of benches required for a given class, etc.

E4) '4 + 3 = 7' can be expressed in at least the following ten different ways:

1. Four and three makes seven.

2. Four and three is seven.
3. Four and three add up to seven.
4. The total of four and three is seven.
5. The sum of four and three is seven.
6. Seven is three more than four.
7. Four is three less than seven.
8. Three added to four is seven.
9. Four plus three is seven.
10. Four plus three is equal to seven.

E5) One thing the examples show is that children, and many adults, who solve problems quickly and correctly are often not aware of the mathematics involved. Knowing the rules and procedures does not imply an ability to form a linkage between the various algorithms and the meaning of the operations they are meant for. We, as teachers, need to be aware of this fact.

E6) Think about how you introduce mathematical symbols and sentences, taking into account the fact that the child would need to construct the meaning of symbols and their connections with each other.

E7) You can write about what needs to be done for providing the children with opportunities to acquire capability in understanding the symbolic language of mathematics. What experiences would help them understand mathematical equations and expressions which use brackets?

E8) For example, a child may think that $4(-5) = -1$, ignoring the brackets because she has not understood what they mean.
You can think of a few other examples.