UNIT 10 GENERALISING ARITHMETIC TO ALGEBRA

Structure

10.1 INTRODUCTION

Over the past few weeks some of us did an informal survey among the people in our neighbourhoods, regarding their attitude to algebra. We found that most school-children dread mathematics in general, and algebra in particular. Many parents who spoke to us regard it a credit to confess that they had been poor at algebra while in school. When asked why they had done badly, most of them said that it was because doing algebra required them to handle complicated formulae which are difficult to remember. Many of them also wondered why algebra is taught in school, as they find it of no use.

Why do people have such a negative attitude to algebra? What can we do to improve the situation? In this unit we try and answer these questions. We look at some ways of teaching algebra that may make children algebra-friendly, and algebra child-friendly.

Children first meet algebra in the guise of 'generalised arithmetic'. In Sec.10.2 we have discussed what this means, as well as why it should be studied. We have particularly stressed what generalisation means, and which generalisations are valid.

In the next section we focus on the concept of a variable. This concept is necessary for learning and using the language of algebra, and is very difficult for children to understand. We have listed some common errors children make because they have not understood the concept and use of a variable. In this section we also suggest ways of slowly and carefully leading children towards this concept.

In Sec.10.4 we look at ways in which children are expected to use variables. They do so when they apply formulae and while solving equations. While using variables, the usual child applies certain thumb-rules mechanically, without understanding the algebra involved. We suggest some teaching strategies that may improve the situation.

Algebra, and mathematics on the whole, is about generalisation. We have discussed aspects of this in Unit 1, as well as in this unit. Therefore, you may like to quickly glance through Unit 1 before going through this unit.

Let us now look at the learning objectives of this unit.

Objectives

After reading this unit, you should be able to

- describe the skills children develop while learning algebra;
- suggest ways by which a child of Class 6 and above can learn to generalise arithmetical principles;
- identify common errors children make while dealing with variables;

- suggest strategies for communicating the concept of a variable to children;
- describe ways of helping children use variables;
- assess the effectiveness of your teaching methods.

10.2 WHY LEARN ALGEBRA?

Children first come across algebra as 'generalised arithmetic', that is, when they study arithmetical **patterns, laws** and **relationships**, try and understand them, **generalise** them and express them in a **precise** manner, using words and symbols. An instance of finding general arithmetical patterns is the following:

'Algebra' comes from the Arabic term 'al-jabr'.

When we multiply different numbers by 10, say,

$$10 \times 1 = 10$$
$$10 \times 3 = 30$$
$$10 \times 7 = 70$$
$$10 \times 12 = 120$$
$$10 \times 26 = 260$$
$$10 \times 365 = 3650$$
$$10 \times 270 = 2700$$

we can observe a pattern in the product. This pattern helps us form **a general rule** about the process of multiplication **by** 10, namely,
'To obtain the multiple of **any number** by 10, shift every digit of the number leftwards by one place and put a zero in the ones place.'

As you had read in Unit 1, the ability to generalise from specific instances, and to specialise from general relationships is an essential part of mathematical thinking. This mental ability is not restricted to mathematics. We need it often in our day-to-day existence, for example, when we build the concepts of an animal, blue, girl, round, etc.

Similarly, because of our ability to generalise from our daily experiences, we expect the sun to rise in the East and set in the West every day, we accept that human beings are mortal, and so on.

At this point, we need to think about the difference in the use of 'in general' in ordinary language and in mathematics. When we use this term **in English**, we mean **'most of the time'**, not necessarily in every case. For example, 'In general, the monsoons arrive in Kerala by the end of May' is an accepted statement, even though it is not true in certain years.

But, when we use 'in general' **in mathematics**, we mean that the statement that it is applied to **is true for all the cases for which the conditions are satisfied.** Thus, 'In general, if the rightmost digit in an integer is 5, then the integer is a multiple of 5' is acceptable mathematically because it is true **for all** integers whose rightmost digit is 5. But the statement 'In general, prime numbers are odd' is not mathematically acceptable even though it is true for all primes **except one**, that is, the prime number 2. (Can you prove this?) Similarly, the generalisation that all odd numbers are primes is not acceptable since, for example, 9 is odd but it is not a prime number.

A number whose factors are only 1 and itself is called a prime number. Thus, 5 is prime, but 9 is not.

So, while making a mathematical generalisation, we need to be very careful to discard it if we find even one case for which it is not true. Here's an exercise about this.

E1) List three general rules that you find in your day-to-day existence. Which of these would be acceptable according to mathematical logic?

Let us now consider the skills children learn while studying algebra. As children get more and more opportunities to generalise, in real-life and in mathematics, they gradually develop several abilities. For instance, they learn to look for common properties of the members of a set. They learn to look for general patterns and relationships, in mathematics as well as in other fields. This develops their ability to deal with abstractions. They learn to think of, not one individual member, but of the set as a whole. And, using the language of algebra helps them to be more logical, clear and precise in their way of thinking and in expressing rules and relationships.

To appreciate a major reason for studying algebra, consider the following problems / riddles.

1. A number and half of it add up to 63. What is the number? (Such problems were given to Egyptians to solve as far back as 1700 BC.)
2. A father is 30 years older than his son. 10 years ago the father's age was 4 times that of his son. How old is the father now?
3. 5 teas and 4 vadas cost Rs. 18.50. But 4 teas and 5 vadas cost Rs. 17.50. How much does a tea cost?
4. When the price of coffee increased by 20%, a man reduced his coffee consumption by 20%. Has his expense increased or decreased, and by what percent?
5. Two numbers add up to 100 and their product is 2499. What are the numbers?
6. A car travels uphill at a speed of 40km/hr and immediately returns downhill at 60 km/hr. What is the average speed of the car?

How do we solve these problems? Can we do it by trial-and-error, that is, by assuming a number to start with, and proceeding to solve the problem? Let's see if this procedure works for the first problem listed here.

Let us assume the number to be 100. According to the information in the problem, 100 and half of 100 should add up to 63. But it doesn't. So 100 doesn't work.

Let's try other numbers. In the following table we have done so, and given the sum in each case.

Assumed number	Half of the number	Sum
100	50	150
40	20	60
50	25	75
60	30	90

None of these numbers are giving us the result. We would need to go through, don't know how many numbers, before we finally get the right number, if at all.

Instead, why don't we just take a general number, call it x, and move from there? We know that the sum of x and half of x is 63, that is, $x + 1/2\ x = 63$,

that is, $(3/2)\ x = 63$,

that is, $x = 63 \times 2/3$ $\left(\text{multiplying both sides by } \frac{2}{3}\right)$

$= 42.$

How easily we got the number merely by using a general number x, instead of a series of particular ones! This is an instance of the use of algebra for solving seemingly complicated problems easily.

Yet another reason for studying algebra is FUN! Many games and puzzles in mathematics have their solution in algebra. For instance, consider this think-of-a-number game:

Think of any 2-digit number. Interchange the digits. Add the new number to the original one. Divide your answer by the sum of the digits of your original number. Now your answer will be 11! (We have discussed the way such games work in Sec.10.4.)

You may like to try the following exercises now.

E2) How would you solve Q.2 given above? How would you solve it using algebra? Which method is easier?

E3) List any other reasons you can think of for learning to generalise arithmetic.

By now, you would agree that children need to study algebra. You would also agree that they should learn it through a process that they enjoy and which keeps them interested. Unfortunately, the way most of us teach them algebra, leaves the children disinterested and with several difficulties. Let us consider these difficulties and some ways of overcoming them.

10.3 LEARNING ALGEBRA

In the previous section you read that children first meet algebra in the garb of generalised arithmetic. You would agree that to be ready for it, they need to develop their ability to recognise patterns and be able to clearly enunciate what they have discovered. For this purpose, we should give children plenty of opportunities to identify and use patterns and relationships involving numbers, right from the time they begin to learn numbers. From these patterns, they should be helped to develop the ability to make generalisations. For example, when they are learning to add numbers, they could be encouraged to discover the general fact that addition is commutative and subtraction is not (see Unit 7). Or, when children are dealing with multiplication, they may be helped to come out with the rule related to multiplication by 10 that we noted earlier.

Sometimes, of course, children may make false generalisations, like 'multiplication makes bigger'. This is because they may have noticed it for positive numbers (2×3 is greater than 2, etc.). But you can ask them if it is always true. You could get them to test it for various cases. Ask them what happens when one of the numbers is zero, for instance. When they realise that their generalisation is not true in this case, they should learn to alter it — maybe to 'multiplying any two non-zero numbers results in a bigger number'. Then you could ask them to try it for fractions, say $\frac{1}{2}$. And so on.

In this way you could help children to learn to be careful while generalising, as well as to alter/discard generalisations that are not valid.

Here is an exercise related to this.

E4) Give an example of a wrong mathematical generalisation made by children. How would you help them realise that it is false?

Algebra develops from a search for pattern, relationships and generalisation. The concept that helps us to study general relationships is that of **a variable**. This is some letter (say, a,b,x,y,...) that represents one or more numbers. This means that in some cases the letter may represent just one number (as in E2), and sometimes it may represent several numbers (e.g., if $x+1$ is less than 9, then x can be any number less than 8). So it can take varying values, and hence it is called a variable.

Developing an understanding of 'variable' is crucial for developing an understanding of algebra. This is where children face a major block. The statement 'Let x be a number' is extremely difficult for a child to understand. Actually, this is not the first time a child is faced with a letter that represents a number. Many textbooks for Class 4 and 5 take a sudden jump from particular cases like $2 + 3 = 3 + 2$, $4 + 5 = 5 + 4$, etc., to the general statement '$a+b = b+a$ for any two numbers a and b'. The child reacts very negatively to this, ignoring it or building a mental block against such use of letters. Therefore, when children are introduced to a variable, it isn't surprising that they rarely understand the concept. Consequently, right from the time they begin studying algebra, upto the time that they can manage to get rid of it, they gather misconceptions which lead to situations like the following:

i) The child doesn't understand that the letter represents a number. So she ignores it. For example, she says that $3x + 4$ is 7.

ii) The child thinks that a letter represents some objects. For example, when 11-year-old Rashmi was asked what p is in 7p, she said it was anything, like apples, or pencils, or books.

iii) The child thinks that the letter represents only those objects whose names start with that letter. For example, Amar insisted that the a in 3a could stand for apples, or alligators, not for bananas "because then it would have been 3b."
This notion of the child may be because of the short notation she has been using for the units earlier (e.g., 3m for 3 metres).

iv) The child thinks that a letter stands for a particular number. For example, when Saba was asked what x would be if x + 6 is less than 10, she said x = 1. When asked if there were any other possibilities, she said there weren't.

v) Most children are confused by the representations like 2a, a^3, etc. They don't differentiate between, say a^3 and 3a. Some write $5a^2$ when asked to add 3a and 2a.

vi) Many children don't know how to apply the laws governing the operations on letters. Some write $3x + x + 5x = 8x$, ignoring the numerical coefficient 1 of x. Some write $a^{m-n} = a^m - a^n$.

There are many other errors that you must have come across. You can note them down while doing the following exercise.

E5) List at least 3 other kinds of errors related to the concept and use of a variable that children make.

How can we help rid children of such misconceptions? While planning any strategy for this, we must remember to proceed slowly. We must remember that the concept of a variable is not acquired in a hurry and that the learner will go through several stages before arriving at the concept.

Before mapping out your strategy, you may like to read about what Ms. Acharya does in this context.

Example 1: Ms. Acharya has been teaching mathematics to the children of Class 6 of a municipal school for some years. Over this period, she has been carefully evaluating how well she has been able to teach the children the concept of a variable. Each time, she uses the feedback she gathers to modify her teaching strategy. At present she considers the following one "successful".

Ms. Acharya says, "Children come to me with an understanding of numbers and experiences with operating on them. I try to build on that experience to introduce them to algebra." What she does, for instance, is to bring a closed carton to the classroom. The children wonder what is in it, till she tells them that she is going to fill it with mangoes to send to her friends. Then she asks them, "How many mangoes do you think will fit into it?" Varying answers come from different corners of the classroom — 50, 40, 60,... . So she says, "OK! Now, remember whatever number you've said. If I add 5 more mangoes to that, how many will I have?" Some hesitation, and then the class erupts with 55, 45, 65, and so on. So she writes on the board the number she would now have:

(The number of mangoes) + 5.

Ms. Acharya believes that arriving at and understanding this stage is very difficult for children. They struggle with this for quite a while. She encourages them to discuss their ideas about what they think is happening. There are occasional arguments. But, with patience and with repeated trials with different numbers, they accept this representation.

Then she asks them, "Now, if Chhotu (*one of the kids there*) eats 2 out of these mangoes, how many will I have?". She, and the children, discuss this and conclude that their answers of 53, 43, etc., can also be written as

(the number of mangoes) + 5 – 2,

which is, (the number of mangoes) + 3.

She points out to them the tediousness of writing the long expression 'the number of mangoes' each time, and asks them if there is a way out. She tells them that since she doesn't know exactly how many there are, she can't replace the phrase by a specific number. Could she possibly use a smaller sign for it? What if she calls the number n, and just writes n instead of the whole phrase, where **n can be any number?** They usually agree to this. So she writes n+3 on the blackboard, and simultaneously says that she has n+3 mangoes left now.

''Now, suppose I add another box of mangoes of the same size. How many will I have?'' she asks. Involving children in thinking about this, she gradually gets them to write

$$n + n + 3$$

At this stage she prepares a table:

No. of mangoes in a box	10	20	30	n
No. of mangoes in 2 boxes	20	40	60	n + n
No. of mangoes I have	23	43	63	n + n + 3

Here she may ask, ''Can we say definitely that the number of mangoes in the box is 10? or 20? or 30?'' Through examples she guides them towards seeing that it may be anything — 15, 25, 47, ..., and that for any of these values they can find out 2n + 3, the number of mangoes she finally has. Then she points out to them that the letter n that they are using for showing the number of mangoes is **a variable,** because it can be any number.

She continues in this way, trying to get the children to think about the meaning and use of a variable. To reinforce their understanding, she also does some examples like the following ones with them.

i) The relation between a child's age and her mother's age: For instance, she tells them that the mother's age is 6 years more than 5 times the child's age. Then she asks them to discuss among themselves and tell her how they would show this relationship. (If the child's age is taken as the variable x, then the mother's age, is $(5 \times x + 6)$.)

ii) The natural number immediately following a natural number: She asks them how they would use symbols to show that the natural number immediately following any natural number is got by adding 1 to it. (If n is the number, the next one is n + 1.)

iii) The number of sticks required to make triangles: She asks the children how many sticks are used for making 1 triangle? 2 triangles that don't touch each other? and so on. She writes their answers down in a table as below.

No. of triangles	1	2	3	......
No. of sticks	3	6	9	......

Then she asks them to find the relationship between the number of sticks used and the triangles that they make. Once they see the pattern, she asks them to write it down in symbols. (To make x triangles, you need $3 \times x$ sticks.)

iv) She shows them number patterns, like

$$2 \times 1 + 15 = 17$$
$$2 \times 0 + 17 = 17$$
$$2 \times 5 + 7 = 17$$
$$\vdots \quad \vdots$$

Then she asks them to give 10 different pairs of numbers that can be written in the box and circle in $2 \times \square + \bigcirc = 17$.

Ms. Acharya does several examples and exercises of this kind, off and on, through the year, with them. In this way she feels that the children learn what a variable is.

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Here's an exercise about this now.

E6) What are the strong points and the shortcomings of Ms. Acharya's method?

E7) Create 3 more examples like the questions given above where relationships are expressed in terms of a variable.

Once children have become comfortable with the idea of a variable, they can be led towards dealing with it wherever they need to. Let's see some ways of doing so.

10.4 USING VARIABLES

If you look at the school syllabus, you will find that the main use of variables in school mathematics is

i) in formulae (e.g., the perimeter of a square, area of a square, and so on),

ii) in solving equations (e.g., if $x + 3 = 10$, then $x = 7$).

Let us see how children can be made comfortable with both these uses.

10.4.1 Use In Formulae

A few days ago, I attended a workshop at a Mathematics Centre. During the talk and discussion on ways of initiating children into applying formulae, what came up was the following.

Children don't need to be in Class 5 or 6 before they are exposed to formulae. Even children of Class 3 can learn to create and use formulae. For this we could begin with examples related to squares. We could ask children to draw squares of varying lengths along the lines given on arithmetic paper (see Fig. 1). Then we could ask them to find the corresponding perimeters. We could ask them to form a chart, like the one below.

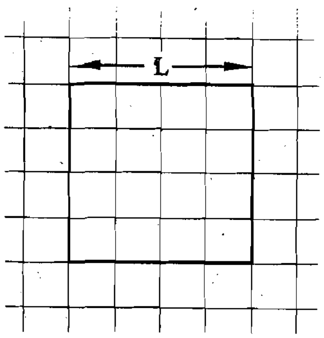


Fig. 1: A square of length 4 units on arithmetic paper.

Length of a square	1 unit	2 units	5 units	10 units
Perimeter of the square	4 units	8 units	20 units	40 units

The children should be asked to look at the results and try and come out with a rule relating the length and the perimeter. We could guide them through discussions to obtain a rule. If a child comes out with a wrong rule, we would need to discuss it in detail, including why it is wrong. Again, through discussions we could lead them to the rule:

The perimeter of a square of length L is 4L.

We could, similarly, ask them to build a table and find the formula for the area of a square in terms of the number of squares of the arithmetic paper.

Once children are comfortable with formulae involving one variable, we could introduce them to those for finding the perimeter and area of rectangles. In this case the formulae involve two variables, an idea which children find difficult to understand. To help them, we could have discussions about why the length of a side of a rectangle is a variable. Once they accept this, they could denote the length by L, say. Then we could ask, "Now, is the breadth also a variable? Can I take it to be L? Why not? What happens if I do?" Let them conclude that if the length and breadth are represented by the same letter, they would be equal and the rectangle would be a square — not a general rectangle.

We could let them draw different rectangles on arithmetic paper to help them conclude that the breadth is also a variable, but a different one — the value it takes does not depend on the value of L. This is an important step, and children should be given enough time and examples to realise this.

When the children get used to the idea of two independent variables in the formula, we could ask them to draw a chart as below. And then, as in the case of a square, we could help them arrive at the formulae for the perimeter and area.

Length	Breadth	Perimeter	Area
2	3	10	6
4	5	18	20
3	7	20	21
.	.	.	.
.	.	.	.
.	.	.	.
L	b	$2 \times (L + b)$	$L \times b$

You may like to try an exercise now.

E8) Do you agree with the method suggested above? If not, what changes would you make in it?

Formulae are not only related to geometry. We can also generalise relationships between numbers and write them as formulae. For instance, you can ask the children to give any two numbers, say 2 and 3. Then you can ask them to see the relationship between 2 + 3 and 3 + 2. You can do this with several pairs of numbers. Then you could lead them towards algebraically showing that the sum of any two numbers remains unchanged if their order is changed, that is, you can lead them to

$$a + b = b + a \text{ (the commutative rule for addition)}$$

Fig. 2: Is adding water to milk a commutative action?

You could then ask them to re-check whether the formula is true by taking various values of a and b. For instance, is it true when a = 11 and b = 13? when a = 7 and b = 37? and so on.

To facilitate children's understanding of substituting numbers for variables in algebraic expressions, you could use the 'cannon'. For instance, to explain to them that, not only is x a variable, but so is 3x + 2, you could use the '3x + 2 cannon' (see Fig.3). Depending on the value of x you feed into the cannon, the shell will be fired to a distance of 3x + 2. So, if you feed 5 into the cannon, the shell will go to a distance $3 \times 5 + 2 = 17$.

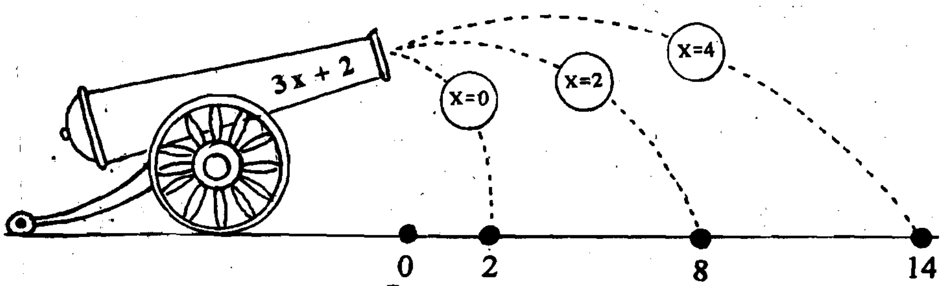


Fig. 3: Finding particular values of an algebraic expression

The important point that must be reinforced with examples here is that if a rule is true for several values, it may still not be true for all values. **Even if the rule is false for a single case, it is not an acceptable rule.**

Now, an exercise.

E9) How would you help children arrive at the formula that relates millimetres to centimetres?

Let us now see how we can help children deal with equations.

10.4.2 Use In Equations

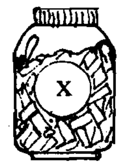
Once children have understood what a variable is, they can be exposed to situations in which they are required to solve equations. By the time children reach Class 6, they have plenty of experience of using '='. For instance, they know that 3 + 2 = 5, etc. You could ask them to fill the blanks in 3 + __ = 7 and 3 + __ ≠ 5. Alongside these activities, you could talk about some choices that make the statement true, and some that make it false.

≠ denotes 'not equal to'

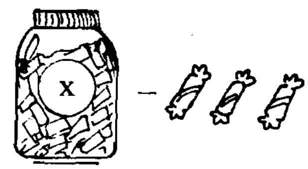
You could then progress to activities which involve creating equations with variables. For instance, you could play 'think-of-a-number' games with the child — ask her to think of a number, subtract 3 from it, multiply the result by 2 and add 8 to this. Now the child should tell you what her result is. From this you can tell her the number she had originally thought of. For example, if the result is 12, the child's number is 5.

The child may be wonderstruck by the 'magic' you've performed. Now, you can tell her how you did it, by taking her through the following chart.

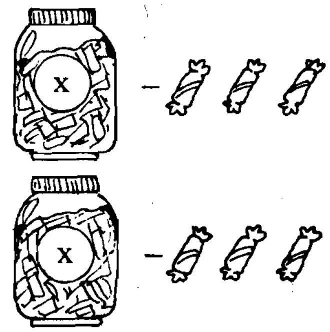
1. Think of a number



2. Subtract 3

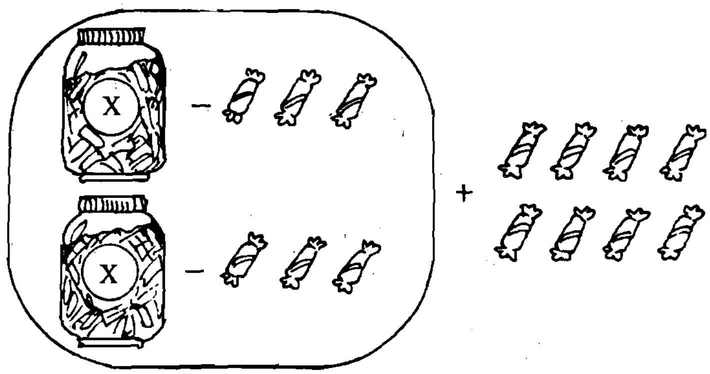


3. Multiply the number you get by 2



$2 \times (x - 3)$

4. Add 8



$2 \times (x - 3) + 8$

So, the result the child has got is

$$2 \times (x - 3) + 8 = 12.$$

You can tell her that this is an example of an **equation** with a variable in it. Here you may need to emphasise that an equation is any expression that involves the 'equal to' sign. For instance, $3x + 5 = 7$, or $3 + 2 = 5$, or $2(x - 3) + 8 = 12$. We must remember **not to refer** to one side of the equation as 'the problem', and the other side as 'the result'. Doing so creates confusion in the child's mind regarding the use of =.

Another point that we must stress is that **for any value of x, the expressions on both sides of '=' must have the same value.** You could ask the child to put different values of x in the equation you have, and check if it is satisfied by them. For instance, she can try $x = 1$, $x = 3$, $x = 5$, $x = 2$, and so on. As she can see, the equation holds for $x = 5$. For another value of x, say $x = 3$, she would get $8 = 12$, which she knows is not true.

You could now move towards helping the child solve the equation. The first step at this stage is that she must realise that whatever operation is applied to one side of the equation must also be applied to the other side. You could help her see this by using equations that involve only numbers. For instance, adding 3(say) to only one side of $3 + 2 = 5$ will make the equation false.

At this stage you could shift to solving of equations. You could start by taking an equation like $x + 3 = 8$. She should consider how to get from $x + 3$ to x. You could give her some hints — what happens if you subtract 1 from either side? What happens if you subtract 3? etc.

Then you could move gradually towards equations like $2 \times x = 6$, $2 \times x + 3 = 7$, and $2 \times (x - 3) + 8 = 12$.

(**Note** that we are writing $2 \times x$ instead of 2x. The reason is that, to avoid any confusion in the child's mind, it is best to write the multiplication sign in the initial stages — till she gets used to operations on variables. Then we could use 2x and $2 \times x$ interchangeably for some time to ensure that she understands what 2x means.)

When you reach the equation $2 \times (x - 3) + 8 = 12$, which you created with her, you could tell her that to solve the equation you could reverse the steps that you used to create it. Since the last step was 'add 8', now she could subtract 8 from both sides of $2 \times (x - 3) + 8 = 12$ to get

$$2 \times (x - 3) = 4.$$

Next, she could divide both sides by 2, to get

$$x - 3 = 2.$$

Finally, she could add 3 to both sides, to get $x = 5$, which is the answer.

This kind of activity needs to be done over several days. It must also be followed up with exercises through the year, including those that would **encourage children to create their own think-of-a-number games.**

Here's an exercise now.

E10) Try out a 'think-of-a-number' game with a 10-year-old (or older) child. Ensure that the equation you get involves a variable and at least two operations. How would you help the child solve it?

With this we come to the end of this unit on ways of helping children develop a sense of algebra. Let us take a brief look at what we have done in it.

10.5 SUMMARY

In this unit we have covered the following points.

1) Children's exposure to algebra is in the form of generalisation of number patterns and arithmetical relationships.
2) Ways in which children can develop their ability to make mathematically acceptable generalisations.
3) Learning algebra helps the child to develop the ability to recognise patterns, to find common properties in a set of objects, to generalise these properties and relationships further, and to express all this in a concise and clear manner.
4) The concept that helps to express this generalisation is that of a variable.
5) Ways of communicating the meaning of a variable to children.
6) Ways of helping children learn to develop and use formulae.
7) Ways of helping children learn to create and solve equations.

10.6 COMMENTS ON EXERCISES

E1) 'Night follows day', for example.

E2) To do it algebraically, we can frame equations and solve them — an easy and quick solution. If the son is x years old, then the father is (x+30) years old. Therefore, 10 years ago, their ages were (x–10) and (x+20), respectively. We know that
$x+20 = 4(x-10)$
that is, $3x = 60$
that is, $x = 20$.
Therefore, the father is now 50 years old.

E3) For example, it helps build the ability for logical step-by-step thinking. It helps the child get a feel for the nature of mathematics.

E4) Give an example from your own experience. (For instance, have you come across their belief that the larger the perimeter of a figure, the larger its area?) Did you give the child counter-examples to prove that her generalisation was wrong?

E5) i) $2x(3y + 6) = 2x \times 3y + 6$
ii) 4xy, when $x = 7$ and $y = 5$, is 475.
iii) $(x + 8)^2 = x^2 + 64$.
There are many others that you would find while interacting with children.

E6) Some strong points, according to us, are that it is gradual, informal, intuition-based, student-friendly, and it would lead to long-term retention. A major plus point is that it emphasises concept-building and understanding, which is the exact opposite of rote-learning.

E9) One of the common errors regarding this formula is to write the rule as $10\,m = c$ instead of $10\,c = m$, where m and c denote the number of millimetres and centimetres, respectively.