UNIT 9 NEGATIVE NUMBERS

Structure

9.1 INTRODUCTION

Some months ago I was talking to a group of around 20 children from Classes 7 to 10. Among other things, I asked them what a negative number is. Most of them reacted like Yuvraj did. He thought about my question for some time, and then said, "Oh, you mean integers? Ya, they are....um....points on that side of zero." He is clearly referring to their representation on the number line. When asked where these numbers are used, after some thought he said, "In profit and loss." "How? Give me an example." He couldn't.

Why are Yuvraj and most children unclear about what a negative number is, and about operations on them? Apart from indifferent methods of teaching them, the children don't see the need for learning such numbers. May be the teachers also don't realise that such numbers are used extensively in the sciences, commerce and mathematics. As our society becomes more technologically advanced, learning of mathematical concepts and processes, including negative numbers, becomes more and more important. Hence, the need for children to study and understand negative numbers and how to apply the basic operations on them.

Historically, the need for negative numbers was felt by mathematicians when they confronted situations where a large number had to be subtracted from a smaller one, for instance, for solving equations like $4x + 10 = 2$. Diophantus, the Greek mathematician (around 275 AD) called such equations 'absurd', since the solution could only be $x = -2$, an absurd number for him. To make such solutions meaningful, they felt it necessary to create these new numbers and give meaning to them. This gave rise to negative numbers and operations involving negative numbers. Brahmagupta (approximately 630 AD) was the first Indian mathematician to record the use of such numbers, and clearly state the rules for applying the operations on them.

Ever since negative numbers have been created, people have felt a need to use some sign for differentiating between positive and negative numbers. For instance, the ancient Greeks wrote $\uparrow 3$ to depict 'minus 3'. Other symbols for this have been m.3, $\overline{m}\,3$, $\tilde{m}\,3$, ③, $\dot{3}$. Of course, nowadays, the common way to denote 'minus 3' is -3. But, whatever way we write it, it does need a special symbol to distinguish it from $+3$. How can we help children realise this fact and become familiar with negative numbers? In this unit we look at some activities for this purpose.

In Sec. 9.2 we discuss different ways in which we can introduce the concept of negative numbers to children. In Sec. 9.3 we consider teaching strategies for communicating the rules regarding the addition and subtraction of **signed numbers,** that is, positive and negative numbers.

Throughout the unit you will find discussions of common errors committed by children while handling negative numbers, and some difficulties experienced by them in this regard. You will find some suggestions for helping children overcome these difficulties. We hope that you will evolve many more methods for the same purpose.

Objectives

After reading this unit, you should be able to evolve effective ways of

- conveying the meaning of negative numbers to children;
- teaching children how to add and subtract signed numbers;
- evaluating your teaching strategies.

9.2 WHAT IS A NEGATIVE NUMBER?

One of my friends who teaches children finds it very difficult to teach 'negative numbers'. She says, "We can introduce children to positive numbers through concrete objects by giving them different sizes of sets which they can see. But how do we present negative numbers to them?"

The usual textbook introduces negative numbers to children in a very abrupt and abstract manner. It mentions the need for being able to, say, take 10 away from 6, as a reason for extending the whole number system by adding on negative numbers. Then it goes on to define negative numbers as additive inverses of positive numbers. For example, it says that (–3) is that number which, when added to 3, gives zero. Not surprisingly, children find this introduction abstract, and end up by not being able to understand the concept at all.

Matters become a little better when, at the next step, the textbook shows the visual representation of negative numbers along the number line. Children see this as something less abstract, and therefore, easier to understand. And then, forever after, a negative number is nothing but a point on the number line for them. They have no idea what it signifies.

How can we help children understand what a negative number is? One way is to expose children to situations which show up the need for using negative numbers. For instance, you can tell them stories involving credit and debit (or profit and loss) like the following one in Khushi Khushi (Class 5).

This is the story of a shopkeeper who gave out his goods on loan. The shopkeeper, Satbir, often forgot who had bought what and for how much. He devised a way to help himself remember — a line on the account book for each customer. Whenever a customer bought anything, Satbir put a mark on the customer's line to show the amount the customer owed him. For example, Manku bought material worth Rs. 8 on loan, and Jaggu bought material worth Rs. 3, also on loan. So their records were as in Fig. 1.

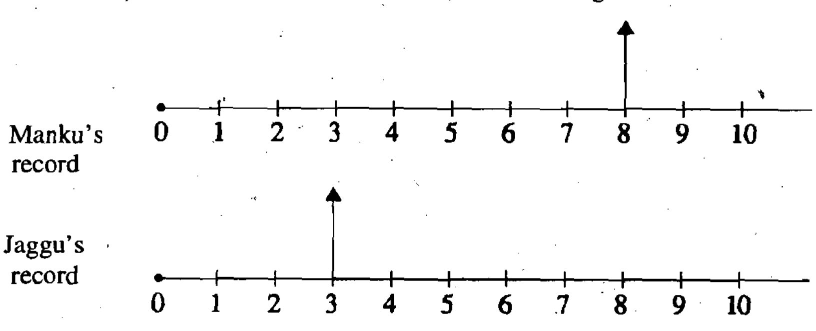


Fig. 1

A few days later Manku cleared his account and Jaggu bought more material worth Rs.2 on loan. Their lines were altered accordingly by Satbir, as in Fig. 2.

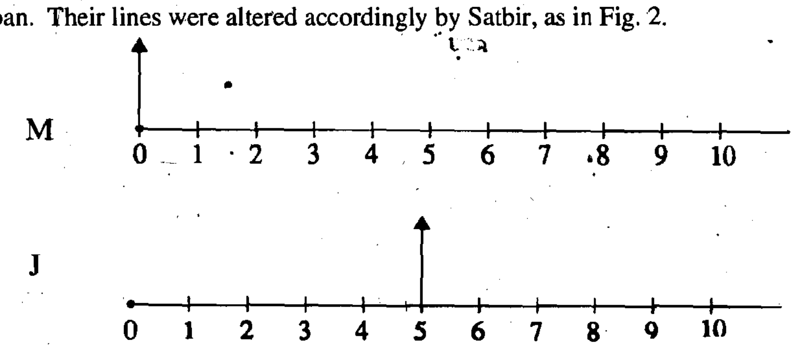


Fig. 2

The next day Manku bought material for Rs. 3 and gave Satbir a five-rupee note. Satbir did not have the change. "Doesn't matter," Manku said, "I will take the balance tomorrow." Now Satbir was in a fix. How should he record this?

He thought, "When I have to take money, I put a mark to the right of zero. But now that I have to give money, where do I put the mark?" After thinking deeply, he extended Jaggu's line, and made a mark on it. (*At this point you could extend the line to the left of zero and ask them: Where do you think Satbir put the mark? After a pause, you could continue.*) He decided to show the amount he has to pay back by moving two steps to the left of zero (see Fig. 3).

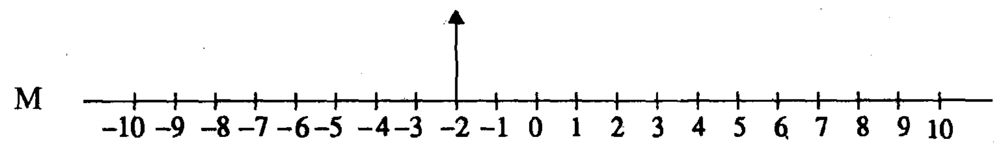
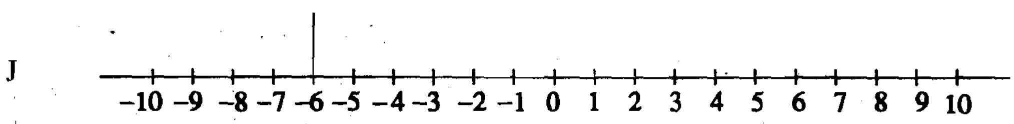


Fig. 3

Now he could present his account with Manku and Jaggu by saying that he had to take (−2) rupees from Manku and +5 rupees from Jaggu. Notice that, to keep track of whether he had to **give** money or **get** money, he placed a sign before the amount.

After such a narration you could give them problems like 'If Jaggu's mark is as shown below, and he buys material worth Rs. 3, where will the arrow shift to? Now would he have to give money or take money from Satbir?'



E1) Create another story for explaining negative numbers to children.

By such stories as the one above, the children are also introduced to the representation of negative numbers on the number line.

There are many other contexts in which we can introduce negative numbers to children, and help them learn to place such numbers in increasing or decreasing order. Let's see what Ms. Inderjeet Kaur has to say about this.

Example 1: Inderjeet has worked with various groups of science and mathematics middle-school teachers. With them, she has been able to try out some ways of teaching children about negative numbers and their properties. When asked to spell out her ideas on this matter, she said, "When I teach children any concept, I try to relate it to objects and situations that they are familiar with. For example, there are several situations in their daily life that I can link with negative numbers — climbing stairs, profit and loss, the past and the future, measuring temperatures, and even marking exam responses."

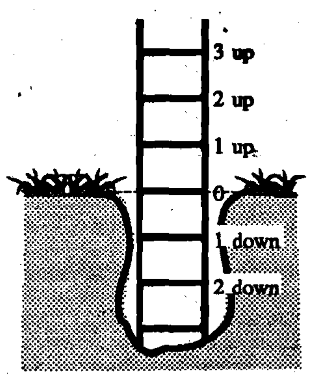


Fig. 4

When we asked her how she related the climbing of stairs to negative numbers, she told us, "I show them signed numbers as movement in opposite directions — climbing up and climbing down. For this purpose I use the example of a ladder standing in a pit. The ground level is 0, one rung up the ladder is 1 up, two rungs up is 2 up, and so on (see Fig. 4); and then 1 rung below the ground level (into the pit) is 1 down, 2 rungs below is 2 down, and so on. Then I ask them to do several problems like 'If you go up 2 steps and then go up by 5 steps more, at what position would you be?', 'If you go up by 5 steps, and come down by 5 steps, at what position will you be?' and 'If you go down by 3 steps, at what position will you be?'. Once the students have enough practice with the up and down notation, I just drop the word 'up' and replace 'down' by a '−' sign. I make it clear that the sign '−' is placed before the 'down' numbers just to differentiate them from the 'up' numbers.

With this ladder model, the children also realise that −15 is further away from 0 than −5 is, and that it requires 8 steps upwards to go from −5 to 3. Of course, I give them several problems to do, which slowly help them realise these properties of negative numbers, and their relationship with positive numbers.

It is not easy for children to understand that, say, – 8 is further away from zero than – 7 is. If you ask a child of Class 6 which is more, – 8 or –2, she would say – 8. This response is to be expected because of her familiarity with positive numbers. Therefore, we need to look for ways of helping her understand that, say, –2 is greater than –8, and the ladder really helps in this.''

We need to reinforce a child's understanding of a concept through applications across disciplines.

We asked Inderjeet why she didn't use the horizontal number line approach. To this, she pointed out that the up-down representation is easier for the children to comprehend rather than the left-right one. She said that this could be because while explaining the left-right model, children get confused about the positive and negative directions, because what is to her left may or may not be to their left, depending on which way they are facing the line. ''But, once the children get used to the vertical strip, I rotate it so that it coincides with the usual number-line representation. Then, we do exercises of identifying the negative numbers along this number line, till the children become comfortable with it. I take any opportunity, inside or outside the classroom to reinforce the concept,'' said Inderjeet.

Next, she tried to explain her use of the past and the future. She told us, ''In this case, I represent 'today' by 0 on a time line (see Fig. 5), 'tomorrow' by 1, 'the day after' by 2, and so on. Then I ask them how I should represent 'yesterday'. With some hints and discussion, the children usually agree to represent it by 1 mark before zero. But then I ask them how they would differentiate between the 1 on the right of 0 and the 1 on the left of 0. Shouldn't there be some mark that shows the difference between yesterday and tomorrow? They think about it, and usually agree. This is when I suggest –1. Then we extend this to –2 for 'the day before yesterday', and so on. In this way, they begin to see today as zero, the past in terms of negative numbers and the future in terms of positive numbers.

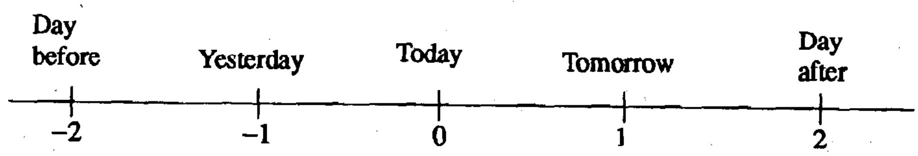


Fig. 5: A time line

Earlier, Inderjeet had mentioned another instance of the practical use of negative numbers which children would find interesting, that is, in multiple-choice tests. When we asked her about this, she said, ''People who take such tests often like to try and guess the answers of questions they don't know. In order to balance this kind of guessing, negative marks are allotted for every wrong answer, i.e., a certain number of marks are deducted. For example, one system is:

2 marks for every correct answer,
0 for questions not attempted,
–1 for questions answered incorrectly.

I explain this background to the children. Then I plot the movement of marks along the number line for a particular case. For instance, I show them the performance of a child in 10 questions in a multiple-choice test with a marking scheme as suggested.

1.	√	5.	√	9.	√
2.	×	6.	×	10.	√
3.	not answered	7.	√		
4.	√	8.	×		

Then I tell them how we could show the way the marks move along a number line. I tell them to start from the point 0 on the line. Then, for each question answered correctly, the marks move two steps to the right; if it is wrongly answered, the marks move one step in the opposite direction; and if it is not answered, we don't move at all. So, since the child has done the first question correctly, we first move 2 places to the right. The second answer is wrong, so we go back by one step, that is, to –1. The third question gives her 0 marks, so there is no change in the marks. From here on I leave it to them to carry on, asking them to clearly write at which point the child's marks are after the fourth answer, the fifth answer, and so on. At the same time, I ask them to mark the position of the child's marks on the line after each answer. They should be able to tell me where the child's marks ended up eventually."

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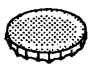
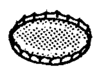
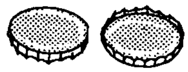
This discussion with Inderjeet helped us to think about different ways of helping children understand negative numbers. All the methods she suggested had used the extended number line as a device for familiarising children with such numbers. Are there other ways of achieving the same teaching objective? You can think about this while doing the following exercise.

E2) From what has been discussed above, how would you introduce negative numbers with some context to children? (Give a new context from the ones presented above.)

Inderjeet, in Example 1, hadn't used any concrete objects for communicating the concept and properties of negative numbers to children. As you know, one of the best ways that children learn is through the use of activities built around such objects. Can we think of some such activities or games? Here are a few examples.

- Take 10 soda bottle tops that are identical. These bottle tops can be placed downwards () or upwards (). We attach the value +1 to the tops in the first position, and –1 to those in the second position. Now, the game is that each player throws these 10 tops in a single move after shaking them vigorously. After the tops land she removes all pairs like () (i.e., +1 and –1). Then she counts the remaining tops. If, say, 4 pairs are formed and 2 tops facing downwards are left, she gets +2 points. If 3 pairs are formed and 4 tops are left facing upwards, then her points are –4. The first player to get 10 points wins the game.

  This game could be played by a few children, or could be turned into a group game. What is useful about this game is that the bottle top being downward-facing or upward-facing presents children with a **concrete, though symbolic, representation of positive and negative numbers.** It helps to reinforce the idea that, say, –5 is that number which nullifies +5.

- For this game each child has 12 small holes dug in the ground in two equal rows of 6 each, along with a bowl. In the beginning, the situation for each child is as given in Fig. 6, with one marble filling each hole, and an empty bowl. This is the zero situation. There is also a bank of marbles which all the children can draw from.

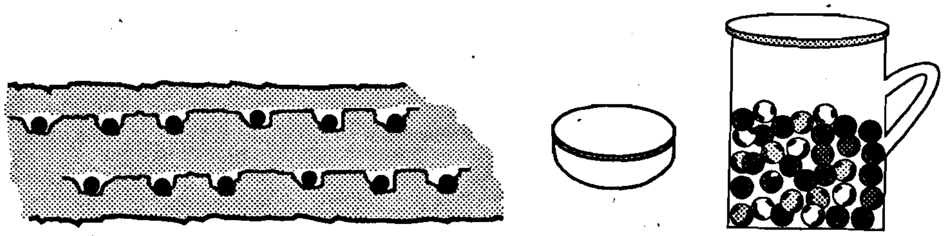


Fig. 6

Now, a child, say Manju, throws the dice and picks up as many marbles from the bank as the number on the dice, and puts them in her bowl. For instance, if she throws a 3, then she has 12 marbles in the holes, plus 3 marbles in the bowl. In this way, each player throws the dice once, picks up marbles and puts them in her bowl. After one round, Manju has her second throw. This is the 'return' throw. Whatever number she throws, she has to put back that many marbles in the bank. For instance, if the dice shows 2, she picks out 2 marbles from the 3 in her bowl, and returns them to the bank. But what if she throws a 5? To return 5, she has to take the 3 marbles in her bowl and 2 marbles from those that are filling the holes (see Fig. 7).

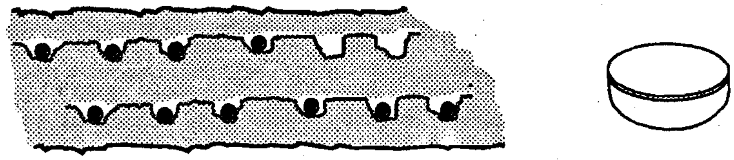


Fig. 7: A concrete representation of –2.

She is, thus, 2 marbles short of the zero position, that is, she needs two marbles to reach zero. So she has got –2 marbles. The other children play this round similarly. The third round would again be a 'take from the bank' round, and so on.

Throughout the game, **it is important that the children explain the points they have got,** and why. For instance, Manju could say, "I have to give 5. So I give these (*the ones in the bowl*) and 2 from here (*the holes*). Now I need 2 to fill the holes." This will help children to clarify the concepts involved and become familiar with the related language. The game can also be played in groups.

- Another game is with a set of 51 cards, numbered from, say, –25 to +25. Four children can play with one such pack. To play the game, one of them shuffles the cards and deals out 10 cards each to all the players. The rest of the cards are placed in a pile, face down, in the centre. To win the game, a child must make sequences of length 3 or more like –23, –22, –21, or –1, 0, 1, 2, or 3, 4 5, with all the cards in her hand. Each time a child is allowed to draw a card from the central heap, and discard one, face up, as in 'rummy'.

Why don't you do these exercises now?

E3) Which properties of negative numbers do children understand better through playing the games listed above?

E4) What other dice games can you think of to help children practise dealing with negative numbers?

We have discussed some ways of helping children relate to negative numbers. Once they get used to such numbers, their problems don't end. They really feel at sea when they have to apply the operations on **signed numbers, that is, both positive and negative numbers**. Let us see how we can improve matters in this context.

9.3 OPERATIONS ON SIGNED NUMBERS

A Class 8 child, when asked to solve (–2) – (–5), wrote –3. When asked how he had got the answer, he said, "There are 3 minuses, minus minus plus, and one minus left. Then, 5 – 2 is 3. So the answer is –3."

The set of **integers** consists of positive numbers, negative numbers and zero.

Such errors are, unfortunately, very common. Actually, this is not surprising, considering the way we usually give the rules for applying the operations to children, without any explanation about why they work. Let us look at some strategies that may help to improve the situation. We will first look at some ways of communicating to children the rule for adding integers.

9.3.1 Addition

Any addition involving whole numbers can be, very naturally, linked to concrete objects. But, while adding negative numbers, such natural contexts are not available. This is also the reason for children not being able to understand negative numbers, as you read in Sec. 9.2. Can the solutions we suggested there work here too? Could we modify the activities suggested there to explain addition of integers to them? The following example may be of help in this matter.

Example 2: 12-year-old Kutty was having a difficult time with negative numbers. His mother consulted some teacher-trainers about this matter. One of them explained the activity with bottle tops to her. When she tried it out with Kutty, it seemed to work. At least he got the idea that '+1 and –1 cancel each other'. She wrote this down for him as (+1) + (–1) = 0. He accepted this fact. So, she asked him what (+2) + (–2) would be. He used the bottle tops, and came up with the answer 0. (+3) + (–3)? 0. In this way, he gradually realised that

(+n) + (–n) = 0 for any natural number n.

In the same way she helped him realise that **(–n) + (+n) = 0** for any natural number **n.**

Now, she decided to move to problems like (–3) + (–2). He couldn't do it. She asked him to show –3 with the tops, which he did.
"Now show me –2 with some other tops," she said, and he did.
"Good! Now combine them. What do you get?" He did so, and said 5.
"Are you sure? See which side the tops are facing," she reminded him.
"Oh! –5," he corrected himself.

She asked him to write down what he had done, using mathematical symbols. With some help, he wrote (–3) + (–2) = –5.

She did some more examples of this kind, and asked him to simultaneously write down his solutions. Then, by looking at the list

$$(-3) + (-2) = -5$$
$$(-1) + (-3) = -4$$
$$(-2) + (-3) = -5$$
$$\vdots \quad \vdots \quad \vdots$$

and with some hints, he began to see a pattern. From this pattern he came out with the rule for adding two negative numbers, namely, "First I add just the numbers and then put the minus sign before it." She asked him to explain what he meant with an example. So Kutty showed her that, to get (–10) + (–5), he would first add 10 and 5, get 15, and then put a minus sign before it to get the answer, –15.

In this way, he had himself come to the rule, which we can algebraically write as **(– m) + (– n) = – (m + n),** where m and n are any positive numbers.

A couple of days later, Kutty's mother repeated this activity with him. And then, she asked him to add negative numbers without using the bottle tops, which he did.

Now she decided to bring in the number line, a device which his class teacher had used for introducing negative numbers. She modified it to a number strip (see Fig. 8), with integers from –50 to +50 written on it. Since Kutty had grown comfortable with the "bottle tops" representation of positive and negative integers, she tried to relate it to the numbers on the strip.

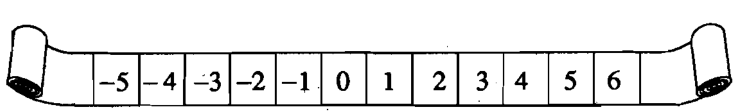


Fig. 8: A number strip

For this she gave him a button. Then she explained that if he had 3 downward-facing tops, that is, +3, he should place the button on 3 in the strip, i.e., move 3 steps right from 0. And then, if he was given 4 more such tops, how many would he have? "So, where would you place the button?" "On 7," he said. "So, you will move 4 steps more to the right on the strip, won't you?"

She, similarly, helped him relate (–2) + (–3) to moving 3 steps leftwards from –2, and therefore, reaching –5. Through a few more examples of this kind, she drew his attention to the fact that when he added positive numbers, he moved rightwards, and when he added negative numbers he moved leftwards.

"Now use the strip and tell me what 2 + (–2) is," his mother said. He knew that the first 2 meant that he needed to first move his button two steps to the right from zero. So he did that Then he hesitated. His mother said, "Without using the strip tell me what 2 + (–2) is." Without any hesitation he said 0. "Good! So, how will you get to 0 from where you are now on the number strip?" "By moving back 2 steps," he said. "Yes, isn't that what –2 is? For –2 you have to move 2 steps to the left, remember?" In this way, she gradually helped him understand that, for instance, 2 + (–3) = –1, and so on.

Kutty's mother would repeat the activity with him off and on over a period of time, till she felt that he had learnt how to add any two signed numbers.

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Here are some exercises for you now.

E5) Evolve an activity using the outer cover and inner box of matchboxes for helping children learn how to add a positive number and a negative number.

E6) Create a game through which children can practise adding signed numbers. What properties of signed numbers would they learn by playing this game? Try it with some 11–year–olds, and see whether your teaching objective is achieved.

Let us now look at ways of helping children understand a related operation.

9.3.2 Subtraction

The other day I heard a teacher speaking at a workshop meant for gathering common errors that children make. Regarding subtraction of negative numbers, she found that children commonly made the following kinds of mistakes:

$$19 - (-11) = 8,$$

$$19 - (-11) = -8,$$

$$-19 - 11 = 30,$$

$$-11 - (-19) = -8$$

Why do children commit such errors? We can say that, for instance, the first kind of error is committed because the child has ignored the sign in front of the negative number. Thus, she has treated the question as one of subtracting counting numbers. But the overriding reason is that the child has not understood the process of subtracting signed numbers. This is what needs to be remedied. How can we do this?

Let us first see what the different steps are in learning to subtract signed numbers. They are

i) subtraction of a counting number (for example, 4 – 10, 10 – 4 or –10 – 4),

ii) –(–n) = n, where n is any positive number (for example, –(–5) = 5),

iii) subtracting a negative number (for example, 5 – (– 6) or – 5 – (– 6)).

What activities could we create for communicating each of these steps to children? Well, they could be helped to grasp the first stage using the number line, which they would have got used to by now. If they understand that, for instance, subtracting 4 implies moving leftwards 4 steps from wherever one is on the line at that moment, they would be able to understand how the first kind of subtraction works.

Let us now see some ways of helping them grasp the other steps given above. For this, you may find Mary's strategy interesting.

Example 3: Mary has been teaching middle-school level mathematics for some years. During this period she has tried out several methods for communicating the "difficult" topics to children. Her method for teaching them negative numbers seems to work, as most of the children she teaches feel quite comfortable with such numbers and operations on them.

Mary's strategy is something like that of Kutty's mother (in Example 2). She starts with concrete representations. And then, after a certain stage, she uses the number line/strip to help them understand new concepts or processes related to numbers. For instance, to help children understand that – (–n) = n, she uses the up-down model (see Fig. 4). She asks them what the opposite of, say, 4 up is. They reply,
"4 down."
"And of 4 down?"
"4 up."
"So, if we write 4 for '4 up' and – 4 for '4 down', what is the opposite of 4?" she asks. The children usually say – 4.

Now she asks them the opposite of several positive numbers, writing them down in a column as they tell her:

Opposite of 10 is –10

Opposite of 3 is –3

Opposite of 5 is –5, and so on.

From this list she asks them to conclude the rule for getting the 'opposite' of a number. After a bit of discussion, they usually conclude that, to get the opposite of a number, one places a minus sign before it.

Now, she helps them extend this rule to negative numbers — "So,what will the opposite of –5 be?" Again there is some discussion in the class, and then the children come to the consensus that the opposite of –5 is –(–5). She writes this down on the blackboard. Now, she reminds them, "–5 is '5 down'. So, what will its opposite be?" They say '5 up'. "We also write that as 5, don't we?" she asks. In this way, with more examples, they realise, and accept, that –(–5) = 5, –(–2) = 2, and so on.

Now, she uses this fact to help children understand what happens when we subtract negative numbers. She asks them, "Since –(–5) = 5, what is 7 – (–5)?" She finds that they usually see the link and come out with the answer, 12.

In the same way, the children learn to solve, say, –5–(–7), using what they have learnt earlier, namely, –(–n) = n.

At each stage, Mary gives the children several examples and exercises to do on their own. She finds that, with enough practice, the children don't need to take recourse to the number line. They absorb the general rule for subtraction, and apply the operation easily.

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Now, you can think of other methods while doing the following exercise.

E7) Evolve a method, using the time line (Fig. 5) to teach children subtraction of signed numbers. What are the stages in which you would unfold the rule to them?

Once children learn to add and subtract signed numbers, they still have difficulty with multiplication and division. We shall not discuss these operations here. We hope that you will be able to devise activities and exercises to help children learn these operations too.

Let us now summarise the points that we have discussed in this unit.

9.4 SUMMARY

In this unit we have covered the following issues:

1) some reasons for children finding it difficult to understand what a negative number is.

2) ways of communicating the meaning and properties of negative numbers to children through activities involving concrete and iconic representation of signed numbers.

3) various activities which could help children understand how to add and subtract signed numbers.

9.5 COMMENTS ON EXERCISES

E2) Another context with which children of Class 6 are familiar is the measurement of temperature. In their Science classes, children learn that 0°C is the melting point of ice. Via the TV, radio or newspapers many of them hear that extremely cold places have temperatures like 2° below zero, or -2°. We can tell them how the temperature keeps falling further and further below zero as the place gets colder. So 15° below zero (or -15°) is colder than -10°, and so on. We can show this on a temperature line (see Fig. 9). To the left of 0 are the temperatures below zero, and to its right are temperatures above zero. The points are written on the line after discussions with the children, and some guidance from the adult. This helps them to see that **as the number becomes larger below zero, we move further away from zero.** Once children get used to this idea, through discussions and examples, we could help them realise that as we move towards the right along the line, the temperatures are increasing. After some practice, we could ask them to look at the line and say which is greater, –15 or –5. They can usually answer this.

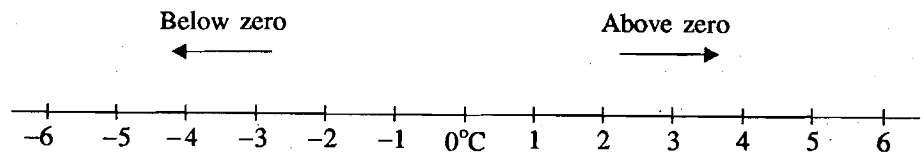

Fig. 9: A temperature line.

Of course, this should not be a one-time activity. We need to keep coming back to this number line and asking them such questions whenever we get an opportunity, even in the Science or sports periods.

E3) For instance, that –10 is further away from 0 than –2 is, and that moving, say +2 from –10, is a movement towards zero.

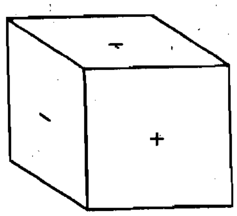

Fig. 10: A 'plus-minus' dice.

E4) Take 2 dice — one the usual, and one with three + and three – signs marked on it, as in Fig.10. In each throw the child has to see what she has obtained on the two dice. If the usual dice shows 4 and the other dice shows +, she has + 4. If the other dice shows –, she has – 4. Accordingly she moves 4 places rightwards or leftwards on the number strip (see Fig. 8). Each time a child moves, encourage her to express what she is doing and why. This activity can be turned into an individual game or a group game.

E5) Let the outside covers denote +1 each, and inner box –1 each. Then, 1 inner box and 1 outer cover cancel each other out, that is, a complete (empty) matchbox denotes 0. So, for example, if a child wants to add 3 and –5, she would first take 3 outside covers and 5 inner boxes. Then cancel as many out as possible, i.e., 3 pairs. She's left with 2 inner boxes, i.e., –2. So, $3 + (-5) = -2$.

This sum should be solved several times, using different objects, like the number line, pens and their tops, etc. This will help her absorb the fact that $3 + (-5) = -2$.

These experiences can gradually be reduced to using only the number line, till the rule is absorbed by the child.

E6) This game is similar to 'snakes and ladders'. Two players (or 2 groups) may take part in the game. Each one has a differently coloured button (or plastic counter) with her. They keep their buttons at the 0 position on the number strip like the one in Fig.8. To start with, Player A shakes the vessel of tokens thoroughly, closes her eyes and takes out a token. Suppose she gets a token with +8 written on it, she can move her red button along the strip to the right to the cell +8. Then it is B's turn. B shakes the vessel and takes out –5 (say). She has to move her green button leftwards to the –5 cell. Then A's turn comes again. Each time one may change the position of her button as follows:

a) if the token shows a positive number, move as many cells rightwards.

b) if the number is negative, move the button as many cells leftwards.

A player whose button touches –50 is out of the game. The one whose button touches 50 first wins the game. Any number of children can play together as long as each one has a different coloured button.

E7) This is, of course, merely a version of the number line. Using it, in the context of time, we could get children to practise the operation. For example, we can ask a child 'Suppose your examinations started 3 days ago, and they will last for 9 days in all. How many days after today will the exams end?', or '2 days ago I went to my grandmother's place. 5 days before that my holidays began. So, how many days ago did my holidays begin?'. Let her use the time line to solve these problems.

Other such examples can be made using a larger time line covering a greater period. You could discuss various events in history and the year of their occurrence with them. You could then ask which event precedes which. You could also ask children teasers that clarify ideas like '3000 BC is further back in time than 2000 BC'. They could be presented with exercises like

1. Raju is now 10 years old. Which year was he born in?
2. My grandmother died in 1995 at the age of 103. Which year was she born in?
3. Pythagoras died in 597 BC at the age of 85. When was he born?
4. Euclid was born in 325 BC and lived for 55 years. When did he die?