UNIT 6 ONES, TENS AND MORE

Structure

6.1 INTRODUCTION

We are often confronted with children not being able to deal with H T O, i.e., 'hundreds', 'tens' and 'ones' (or 'units'), with comfort, though they are supposed to have "learnt" the concept. For instance, take this friend of mine who has a fair amount of experience of interacting with students of Classes 6,7 and 8 in various schools in rural and semi-urban areas. On many occasions this interaction revolved around 'hundreds', 'tens' and 'ones'. He discovered that the children of these classes were not able to read 5-digit numerals though they were supposed to "know" such numbers. For example, the 2 in 20013 variously denoted 2 hundreds, 2 thousands, 2 lakhs and even 2 crores. It was quite obvious that they did not have a clue about the value that the position of the digits indicated. Of course, once a hint was given, many of them were able to quickly formulate the right answer.

This lack of understanding also shows up in problems of addition and subtraction. In addition sums all they know is that they must add columnwise. Why? Because "Teacher says so". And when they are given problems involving addition of 3 or 4 numerals written one below the other, with each having 3 or 4 digits, they often write the digits of the answer in the wrong columns. This leads to what we think are unusual and unlikely answers ; of course, they don't think so !

All this often frustrates the adults, who give up and end by labelling and/or scolding the children. Instead, shouldn't we ask ourselves whether we have really tried to see **why** the children don't understand? Could it be that they haven't been given enough activities to be able to clarify the concepts concerned in their minds? Or, could it even be that we, who are attempting to get the child to learn, are not clear about the concepts?

We have looked into these problems in this unit, and have some suggestions to make. We hope that this unit will make you appreciate the depth of the problems, and that **there is no single solution.**

We also hope that studying this unit will generate confidence and interest in you to try out new methods of teaching the concept of place value in the decimal system.

'Decimal' comes from the Sanskrit word 'dasha'

Objectives

After studying this unit, you should be able to

- evolve and use alternative activities to clarify the learner's conceptual understanding of ones/tens/hundreds;
- assess the learner's understanding of ones/tens/hundreds;

- differentiate between 'place value' and the system of ones/tens/.....

Let us now consider the issues in this unit one by one.

6.2 DEVELOPING AN UNDERSTANDING

A **numeral** is the written symbol that represents a number. For example, 25 and XXV are numerals representing the same number concept.

The other day I was showing the children's book '203 Cats' to my 7-year-old niece. She had recently learnt how to write large numerals in her school. She, very confidently , looked at the title and read 'Twenty-three Cats'. I asked her to look at the title again and read it to me. She again read 203 as twenty-three. Then I wrote 213 on a piece of paper, and asked her to read this. She read it correctly as two hundred and thirteen. So, why couldn't she read 203 properly? Where did the problem lie ? Could it be that she had never thought about the importance of the position of a digit in a numeral ? Or could it be a problem related to her understanding of zero ?

To answer these questions, at least partially, maybe we should go back a bit in time, and see how my niece's teacher had taught her to read and write numerals.

Example 1: In Class 1, the teacher had written down the digits 0,1,....,9 on the board. Then she made all the children recite the corresponding number names. Finally, she made them write the numerals in their home-work book, several times.

Some months later the teacher started teaching them how to represent two-digit numbers in the following way. She first reminded them of the one-digit numbers. Then she turned to the blackboard and wrote down

10
11
12
.
.
.
19

The children dutifully copied down what she'd written. Then she said the number names loudly one by one, while pointing to the corresponding numerals. Finally, she made the children write each of them down five times, saying, "Remember 1 with 0 is ten, 1 with 1 is eleven,..., 1 with 9 is nineteen."

After some more practice, the teacher was satisfied that the children **knew** the numerals from 1 to 100. A year later, in the same way they were taught how to represent the numbers from 101 to 1000. And the Class 2 teacher told them that, if she asked them to write any numeral, say hundred and fifty-two, they must first write H T O in their books (as in Fig.1). She explained that H, T and O represent hundreds, tens and ones. Since hundred and fifty-two has one hundred, five tens and two ones, the children should write it by putting 1 under H, 5 under T and 2 under 0.

H	T	O
1	5	2

Fig. 1

The teacher, then, gave them a lot of drill in writing two-and three-digit numerals.

Finally, she told the children what the place and place value of the digits in a numeral are. She did this by giving them a few examples on the board. (For example, the place values of 4, 2 and 7 in 427 are 4×100, 2×10 and 7×1, respectively.)

Then she gave them questions of the following kind as homework: how many tens are there in 251?

Most students' answer to this question was 5. One said 25.

? Why do you think these different answers came forth ?

Many of us tend to teach the concepts involved in H T O in a mechanical fashion. So, children who have been taught in this way would say that there are 5 tens in 251. We need to go further, using concrete activities, to remind them that after all, 1 hundred is 10 tens, etc. Only then could we lead them towards realising that there are 25 tens in 251.

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You may like to try an exercise now.

E1) A child who has been taught in the manner given above, when asked to read 203, reads it as twenty-three. What kind of activities would you suggest that she do so that she realises her error ?

Coming back to the present, I thought I would try to explain the numeral system to my niece the way a friend of mine did to his children. What I did was the following.

Example 2 : I gave my niece a whole heap of beads and showed her how to divide it up into sets of 10 beads each. Then I showed her how she could lay out each set of 10 beads in a line, and call it a string. After she had made some strings, I told her that with 10 strings she could make a necklace.

Fig.2: Learning to count by tens and ones.

She started making strings and necklaces with the beads, and slowly tried to form a relationship between a necklace and a string in her mind. After a bit, I asked her how many strings she would exchange for a necklace. She thought for a moment and said, "10."

Then I asked her how many necklaces she could make from 107 beads. She thought for a while, and then said, "10 strings, and 7 beads will be left." I asked, "How many necklaces does that make?" To help her answer this, I asked her to actually take 107 beads and try and make as many necklaces as possible, given the fact that a necklace meant 10 strings and each string meant 10 beads. She took the beads and ended up getting one necklace and 7 beads.

Next, I asked her how she would write that. The two of us worked out a system in which we wrote N S B — the number of necklaces was to be written below N, the number of strings below S and the number of beads below B. Under N she wrote 1 and under B she wrote 7. I asked her, "What about the number of strings?", to which she said, "There are no strings." So I asked her how she would show that. She thought for a moment, and then wrote 0 below S.

(**Note :** Children may tend to ignore writing 0 in a numeral, because they think that it denotes 'nothing', and hence it need not be written.)

Then I wrote H T O above N S B, and asked her if she agreed with that. She thought for a bit, and then said that she did because 100 beads was one necklace and 10 beads was one string. "Fine ! Now, how much is 325?" I asked her. She promptly replied "3 necklaces, 2 strings, 5 beads." "How many beads does that make?" "Three hundred and twenty-five," she said.

After some more of such questions we played the following game. I gave her 3 digits. She was supposed to use them to make as many numerals as she could, and arrange them in decreasing order. Once I felt that my niece was enjoying the game, I extended it to 4 digits. And she made all possible numerals with them, including those like 0129 with 0, 1, 2 and 9. I felt that it was very important to have her practise these ideas for a reasonable time and in a leisurely manner, without pressure. And this was a good opportunity for her to do so.

A child needs to return to a concept again and again, to be able to understand it thoroughly.

However, it is necessary to **go over the concept with the learner again and again.** This is because, even though a child may be able to do the activity at a particular point of time, there can be no guarantee that she would be able to repeat it later. Therefore, I repeated this activity, and some other activity for the same purpose, with her a week or 10 days later. In the mean time I gave her several exercises for practising the use of H T O.

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Now for some related exercises !

E2) What are the strengths and shortcomings of the methods of teaching H T O in Examples 1 and 2?

E3) a) Think of another activity for getting children to practise H T O, especially one that deals with a context consisting of 0 as one or more digits.

b) Compare your activity with the activity done by me. Which is better, and why ?

E4) Suggest a group activity to help a class of 50 children learn the use of H T O in contexts where one of the digits is zero.

So far we have considered different ways of helping children understand the decimal system of representation. What is important for us to remember is that **the concept of place value is usually understood by children only at a later stage.** This understanding builds up as they become better at applying the arithmetic operations. We also need to recognise the fact that this understanding is **not explicit,** and an explicit understanding is not even necessary. We will discuss this some more in the next section, as well as in Units 7 and 8.

6.3 PROBLEMS RELATED TO APPLYING OPERATIONS

Some of us were testing Class 4 children with addition and subtraction problems. We gave them sums that were written horizontally and those that were written vertically. Some of the addition sums involved carry-over, and some of the subtraction sums involved borrowing.

When we looked at the answer sheets of the children, we had strange observations. About the children who had produced correct answers for all the questions, we couldn't learn much. About those who had done everything or nearly everything wrong, we could only conclude that they needed to re-learn addition and subtraction using concrete materials. From the answer sheets of those who had made mistakes in some of the questions, we could learn quite a lot.

It is difficult to produce the individual answers and the insights that they were providing. But, let's look at some broad patterns that we found, which are similar to those that others have observed. These are :

i) The number of questions correctly solved was proportionately much larger where they were written vertically rather than where they were written horizontally.

ii) The questions that did not involve carry-over or borrowing had a much larger percentage of success for the children.

iii) Some children, while writing down problems or copying them from the board, shifted the places of the digits. As a result, they got unusually high answers. They were, however, completely unperturbed by this since the answers didn't seem strange to them. Therefore, the wrong answers had not got them to reflect on their method of solving the sum.

You may like to add to these observations after doing the following exercise.

E5) Set a question paper of addition and subtraction problems involving two-digit numerals, for children of Class 4 or 5 around you. Some should be written horizontally and some vertically. Ensure that the children don't talk to each other or copy from each others books.

a) Analyse the answers according to the observations mentioned above. You may like to add to those observations.

b) Look carefully at the answer sheets of children who have made mistakes. Do you see a pattern in these mistakes ?

c) Talk to the children to find out how many of the mistakes are related to lack of confidence in their grasp of H T O.

Let us now look into a problem faced by little Mukesh, who likes mathematics.

Example 3 : Mukesh was doing his homework on vertical addition, and had to compute :

$$\begin{array}{r} 5\;3 \\ +\;3\;4 \\ \hline \\ \hline \end{array} \qquad \text{and} \qquad \begin{array}{r} 6\;8 \\ +\;4\;5 \\ \hline \\ \hline \end{array}$$

He did the first one easily, just the way his teacher had taught him. He first added 3 and 4, and wrote 7 beneath 4. Then he added 5 and 3, and wrote 8 beneath 3.

The next sum was also manageable. While doing it he could be heard mumbling to himself, "8 + 5 = 9, 10, 11, 12, 13. So I write 13 here *(below 5)*. Then 6 + 4 is 7, 8, 9, 10 and so I write 10 *(below 4)*." He was satisfied with his answer, 1013. But, next day, his teacher wasn't !

His teacher tried to quickly repeat the method she had given the class on the previous day, namely, in 68+45, since 8+5 is 13, write 3 below 5, and write 1 on the right of the problem [see Fig. 3(a)]. Then, 6 + 4 is 10, plus the 1 on the side is 11 [see Fig. 3(b)], so write 11 below 4. So the answer is 113.

$$\begin{array}{r} 6\;8 \\ +\;4\;5 \\ \hline 3 \\ \hline \end{array}\;\; 1 \qquad\qquad \begin{array}{r} 6\;8 \\ +\;4\;5 \\ \hline 1\;1\;3 \\ \hline \end{array}\;\; \not{1}$$

(a) (b)

Fig. 3

Quite a few of Mukesh's classmates had made a mistake in the sum. The teacher thought it was just a matter of practice. So she gave them more exercises to do over a period of time. And then she moved on to discussing other concepts. Some students managed to cope with what she was teaching, and some fell by the wayside.

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The following exercise is about the situation given above.

E6) According to you, what are the reasons for Mukesh's, and the other children's, inability to solve the sums correctly ?

If you look closely at the children's problem in Example 3, you may feel that they faced it because of their poor understanding of what the ones/tens/hundreds places denote. My neighbour's child, who is in Class 3, also seemed to have the same problem. I tried a method with her, which I shall share with you now.

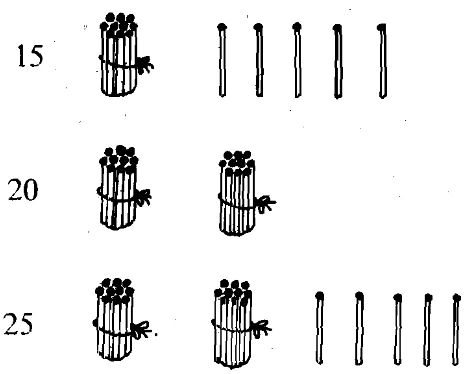
Example 4 : I had just come back from a very interesting talk arranged by a Mathematics Centre, it was aimed at parents of primary school-going children. They had talked about, and demonstrated, various methods for enhancing the understanding of place value. I thought of trying out what was discussed and 7-year-old Radha, my neighbour's daughter, gave me an opportunity to do so.

She wanted me to help her with some sums. I asked her if she was willing to do it my way, which she agreed to, with some hesitation. So I asked her to help me add 15, 25 and 10. She said, "Let me write it down." Then she proceeded to do it as in Fig. 4. I asked her to tell me how she got that answer. She said "First I added the numbers in the ones place, 5 + 0 + 5 = 10, and wrote it down. Then I added the numbers in the tens column, which is 4. So the answer is 410."

15
10
25
———
410
———

Fig. 4

I didn't make any negative comments, but I did the following activity with her. I gave her a heap of matchsticks, and asked her to make some bundles of ten matchsticks each from the heap. Then I asked her to give me 5 matchsticks, which she did. When I asked her for 10 sticks, she counted them out and gave them to me. Gradually she was able to understand that giving one bundle meant giving 10 sticks, and that whenever we have 10 sticks we could made a bundle with them. Soon, for any number asked, she gave the required number of sticks and bundles :



I asked her, "How much do you get when you add up all these bundles and sticks ?" She counted all her bundles and sticks, and said, "45".

Next, I asked her to give me 35 sticks. She gave me 3 bundles and 5 sticks. Then I asked her to give me 60 sticks. She pondered for a moment, and then gave me six bundles.

Then I asked her to play a game with me. I also called her older sister, and Mukesh. We sat down in a ring after collecting a lot of stones, a dice, about 20 cards and 10 coloured beads. I said each of us would have a piece of paper on which we would keep a record of how the game proceeds. The record consisted of 3 columns : one for the stones, one for the cards and one for the beads.

The game is played as follows :
Each player throws the dice by turn, and picks up as many stones from the

ground as the number shown on the dice. Then the person writes down the number of stones in the stones column. When her turn comes round again, she throws the dice and picks up as many stones as the number that the dice shows this time. If the number of stones with her becomes more than 10, she has to put 10 stones back into the main pile and pick up one card instead. Whenever that happens the person should alter her record, to write the number of cards and stones available with her now. Whosoever gets 5 cards first wins the game.

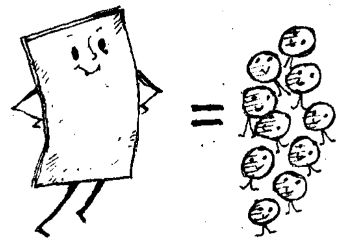


Fig. 5 : One card equals ten stones.

All of us enjoyed the game. After we had played it 3 or 4 times, and everybody except me had managed to win, I proposed that we extend the game.

In the extended game, we could collect 10 or more cards. And anyone getting 10 cards, would have to put all of them in the middle and pick up a bead instead. Whoever got 3 beads first would win.

This game became very long drawn out, and nobody seemed to be winning. Suddenly Mukesh said, "It will take a lot of time because each bead needs 10 cards and each card needs 10 stones. That means each bead needs 100 stones." I said, "Well we could play this game with two dice together and decide the winner." When we played with two dice the elder sister and I had to often help the youngsters fill up their chart. And soon, they tired of it.

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The following exercises may help you to look more closely at the activities done above.

E7) Why did the two dice game become more difficult?

E8) Do you find the activities in Example 4 helpful as a means of understanding ones, tens and their relationship? If so, in what way? If not, write down an activity, different from the ones given in this unit so far, which would help children understand ones/ tens/hundreds.

What is the **underlying principle** involved in the activities given in Example 4? Is it not the principle of exchange ? That is, the principle that 'one of these is ten of those', and so on, as we move leftwards from one place to the next in a numeral ? As you have seen, **a child needs to thoroughly understand this principle to be able to handle the four arithmetic operations, correctly and easily.** In fact, we can gauge whether a child has understood the idea by the ease with which she can handle the hundreds/tens/ones in the following cases of mental arithmetic.

We'll also discuss this in Units 7 and 8

i) Mentally adding 1, 10 or 100 (or its multiples) to any given number, especially examples containing the digit 9 (for example, 93 + 10).

ii) Mentally subtracting 1, 10 or 100 (or its multiples) from any given number, especially examples containing the digit 0 (for example, 804 – 10).

Here is an example of a mental activity which might help children to get used to mental addition or subtraction. This activity can be done by a large number of children, divided into 3 or 4 groups, or by a single child.

On a blackboard write headings like the following :

SUBTRACT 1   ADD 2   SUBTRACT 100   ADD 20

Then, below each of them one can write down some numerals, and ask the different groups of children to continue the rest of the column. For example, you could write 404, 184, 901, and 150 below each of the above headings. Then the groups can give you the answers to 404 – 1, 184 + 2, 901 – 100, 150 + 20. The group that answers correctly first can get a point. The game can continue in this way.

Similarly, if the children are working individually, you can give each child a worksheet with these kind of headings. They could be asked to fill 8 steps, say, in each column in the sheet in a stipulated time, by performing mental operations. So, for example, if the column headed 'Subtract 100' starts with 801, a child would finish up writing : 801, 701, 601, 501,......1.

This activity can be adapted to any situation we want. For instance, with 11-year-olds, or older children, the worksheet can be

SUBTRACT 1paise	ADD 2p.	SUBTRACT Re1/-	ADD 20p.
Rs. 404	Rs. 184	Rs. 9.01	Rs. 1.50

Another, more interesting, form of the activity for 9 or 10-year-olds can be using the grid in Fig. 6 (a). The instruction for moving across and down the grid can be any combination of ADD 1, SUBTRACT 1, ADD 10, SUBTRACT 10, ADD 100, SUBTRACT 5, and so on. A starting numeral is written in the top left corner and a finishing numeral in the bottom right corner.

ADD 10 →

SUBTRACT 2 ↓

15				
				47

(a)

ADD 10 →

SUBTRACT 2 ↓

15	25	35	45	55
13	23	33	43	53
11	21	31	41	51
9	19	29	39	49
7	17	27	37	47

(b)

Fig. 6

Children can work individually in completing these grids, ensuring that they obey the given instruction for all movements left to right along the rows or down the columns.

E9) Try the two activities detailed above with a few children around you. Evaluate whether they really helped to improve the children's performance of mental arithmetic.

Another activity which may help children to move from concrete materials towards an abstract understanding of HTO is playing with the number chart (see Fig. 7).

There are several kinds of activities possible with this chart. For example, a child can be asked to select any number in the chart. Then she could find out whether the number is larger than or smaller than the number just above/below it. After this she could be asked to subtract the smaller number from the larger one and find the difference between them.

From the chosen number, the child can be asked to find out how much the numbers in all these directions differ from the selected number. She can, further, be asked to discover patterns like in which direction the numbers increase/decrease, in which direction the difference is in tens and why, etc.

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

Fig. 7 : A 10×10 number chart

Can we use this chart to help children understand carry-over and borrowing? Rani's example may help you answer this question.

7-year-old Rani, who was working with the number chart, was given the task of adding 37 and 26. She first located 37 in the chart, and then wondered what to do next. To help her, the teacher asked her if she could break up 26 into two numbers.

"Yes, 13 and 13", said Rani.

"OK, now break up 13," said the teacher.

"10 and 3".

"So if 13 is 10 plus 3, in the same way what is 26?"

"20 plus 6", she said, after a pause.

"OK, so you have 37, and you want to add 26. So, can you do it by first adding 20, and then adding 6?", the teacher asked. She agreed.

"So how do you move to a number that is 26 larger than 37?"

She moved downwards by 2 squares, and then to the right by 6. It was while moving rightwards, when she reached the edge and had to move down and leftwards to the other edge, that she began to understand the meaning of carry-over in this context. From 57 she went right till she came to 59 and then went to 60, situated on the left edge of the next row, and then counted on to 63.

What Rani was really learning was how to split numbers into tens and ones and add, so as to understand what is involved in carrying-over. This splitting should be introduced to children through groups of objects first, as discussed in Example 4. Gradually they can move towards adding two-digit numbers, writing them in tens and ones. For example,

34	→	3 tens and 4 ones
+28	→	2 tens and 8 ones
		5 tens and 12 ones
	=	5 tens and 1 ten and 2 ones
	=	6 tens and 2 ones
	=	62

Doing these sums initially in this way helps children see why they carry over numbers to the next column.

You may like to devise some related activities now.

E10) Give an activity using simple equipment, which teachers in a classroom can use to develop the understanding of H T O While designing it, what points did you keep in mind?

E11) Design a game where the winning strategy involves the process of learning H T O.

So far we have been discussing how we can help children understand the concepts involved in the representation of numbers in the decimal system. Some books and people use the term 'place value' in this context. But how many of us realise what this concept really is? In the next section we look at it.

6.4 WHAT IS PLACE VALUE ?

(This section is only for your consumption, and not meant to be passed on to your learners.)

Children of elementary school are not likely to understand the general concept of 'place value'.

You may have realised that in the decimal system the numerals are represented briefly by only 10 symbols, which we call **digits**. The brevity is achieved because of the simple device used as a basis for the symbolic representation, namely, that it depends completely on the **place** (or position) in which each digit is written in relation to other digits. For example, ten, hundred and thousand are represented by the same digits, but placed in different positions.

This is possible because we assign different values to different places. These values are the place values (or position values). For example, in the decimal system the values associated with the columns going from right to left, are 1 (i.e., 10^0), 10 (i.e., 10^1), 100 (i.e., 10^2), and so on. Therefore, the place value of 4 in 1420 would be 4×100, i.e., 400.

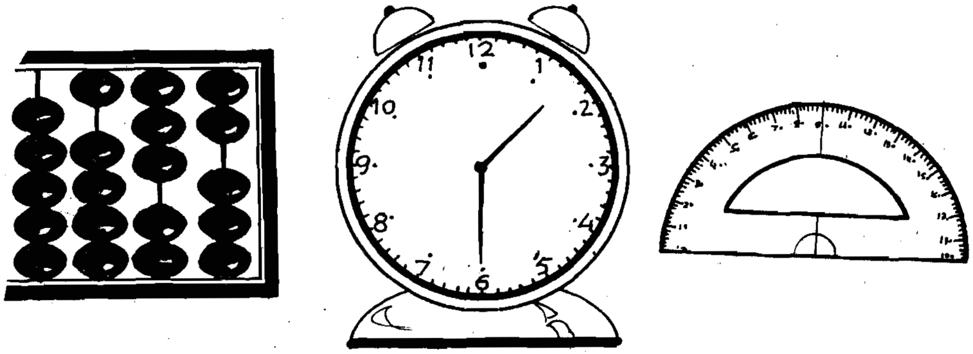
Can you guess what the place value of 4 in 1420 is, if we are working in the numeral system whose base is 7 and its powers? In this case the values of the places, going from right to left, are 1, 7^1, 7^2, 7^3, ..., i.e., 1, 7, 49, 343, and so on. So the place value of 4 would be 4×7^2, i.e., 196 (in the decimal system!).

And, how would we represent ten in the **binary system,** that is, the numeral system whose base is 2 and its powers ? In this system instead of ones, tens, hundreds, etc., we would have ones, twos, fours, eights, and so on. Since $10 = (8 \times 1) + 2 = (8 \times 1) + (4 \times 0) + (2 \times 1) + (1 \times 0)$, its representation would be 1010.

The binary system has gained in importance because it is used in computers. The quinary system (base 5) is used in the Chinese abacus. The duodecimal system (base 12) is used to count things in dozens. The sexagesimal system (base 60) is used to express time or angles (see Fig. 8).

You may be able to appreciate the utility of 'place value' when you consider other systems of numerals, like the Roman system. The symbolic representation in this is from left to right, starting with the symbol representing the largest number. As the numbers get larger and larger, their representation becomes more and more cumbersome. For example, the Roman numeral for the number 337 is C C C X X X VII.

The numerals formed by using the base 10 system (the decimal system) are called Hindu-Arabic numerals. This, and other interesting bits of information,



(a) (b) (c)

Fig. 8 : (a) Portion of abacus, showing $0 \times 125 + 1 \times 25 + 3 \times 5 + 2 \times 1 = 42$, (b) clocks make use of base 60, (c) protractor.

can be found in "Mathematics and Mathematicians (Vol. 1 and 2)" by Dedron and Itard, which is available at your study centre.

Now for an exercise which may help to familiarise you with other number systems.

E12) What are the representations of the numbers 5, 16, 25 and 32 in the binary and the quinary systems, and why ?

With this discussion on place value, we come to the end of this unit. We shall return to some of the problems discussed in it in the next two units. But, for now Let us take a quick look at what we have covered here.

6.5 SUMMARY

In this unit we have discussed some problems faced by children that are related to their not being able to grasp the concept of ones/tens/...... In particular, we have

1) looked at activities of grouping that can help them develop an understanding of the concept.

2) considered activities in which children need to deal with numerals in which at least one digit is zero.

3) considered examples of problems that show up when children are tackling operations of arithmetic.

4) discussed some methods that can help children overcome the problems in (3).

5) explained what 'place' and 'place value' mean, in general. **This explanation is for you,** the teacher. Please do not expect your learners to understand it till a much later stage, maybe in Class 8 or 9.

6.6 COMMENTS ON EXERCISES

E1) She needs to practise activities like the ones suggested in Example 2.

E2) In Example 1, the children were made to learn the numerals by rote, and without understanding. They were just given fixed rules to be followed. So, this method has all the negative points associated with 'blindly following a rule'. In Example 2, the teacher is encouraging the student to use concrete things to learn the concepts and is attempting to allow the child to develop the concept at her own pace via a route that she would find interesting and understandable.

Compare these two methods and point out the kind of gaps likely to be left in the child's understanding after these methods have been applied. Also point out how the gaps can be filled.

E3) a) A friend of mine, an earnest teacher, effusively told me the following game he had thought of :
Divide the children into small groups and give each group two dice each. In each group the children should play turn by turn. Each child has to construct the largest number he/she can, by using the number appearing on one dice as the 'tens' digit and the other as 'ones'. Whichever child makes the largest number wins.

One of my colleagues sitting with me got very excited and said we could extend this practice in many different ways. The teacher and I said, "How?"

She said, "First of all, instead of using dice, I could make a thin tin or cardboard disc with numbers from 0 to 9. Make a pointer from a cardboard or plastic strip by cutting it in the shape of an arrow, whose length is the radius of the disc (see Fig. 9). Then make a hole through the centre of the disc, and put a stick or nail through it and the blunt end of the arrow. The arrow is hit hard with the finger and rotated. The number nearest to the place where the arrow points when it stops is noted. The child rotates the disc twice in each turn and has to make the largest number with the two digits she gets. (You can allow children as many options as they are familiar with to construct the numbers including multiplication, addition, etc., apart from tens and ones. The exercise could also be altered to have them construct the smallest number.)

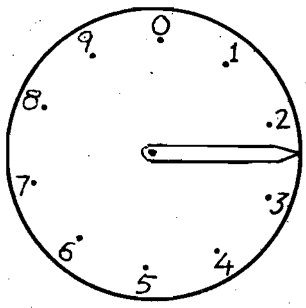


Fig. 9

If one does not have dice or a disc, the game can be played by the teacher giving 5 pairs of numbers to the groups. For each pair, the learners have to make the biggest number possible, or from the two pairs get the highest sum or lowest difference or even the highest difference.

b) Compare your activity with mine in terms of the nature of materials used, complication in instructions, actual practice, level of enjoyment, thinking and feedback available to each child, etc.

E4) The group activity could be with all the children treated as one group, or in smaller groups of 5 or 10 each. Note that a **group activity is an activity that the children do together.** It is not an activity that is given to the entire class which each child does in her own notebook in her own way. You could think of blackboard games, in which the whole class can be involved. Or, the class could be divided into groups of 5 each, some doing the card and pebbles activity and some doing the one with beads, and so on.

E5) While making the paper, include problems that are simple and those that are less simple. Make a table of the number of correct and incorrect answers. **Then talk to the children** and try and understand why they solved the problems the way they did.

E6) One reason is teaching of columnwise addition without their helping the child to understand what 'carry-over' really means. Secondly, repeating the same method and the same logic over and over again is not likely to help. Alternative methods need to be explored. Maybe, activities using concrete objects would be useful.

You can think of several other reasons.

E7) It could be that it required more patience as well as greater logical ability.

E8) Acquiring an ability to handle hundreds, tens and ones involves many subprocesses. It includes learning how to group, how grouping helps to deal with large numbers, using the formal and abstract representation of numbers. Which of these, and other, aspects does the activity with matchsticks strengthen? And what about the game with dice ?

E10) An activity to be done with a child or children needs to be properly chosen in the sense that :

i) it should be appropriate for the number of children involved.

ii) the children should be able to do it without much effort and with a lot of fun.

iii) if more than one group is doing it, then it should be flexible enough to be adaptable to different abilities.

iv) the students should really experience the concept that they are trying to learn.

v) it should keep the children actively engaged in the process of learning.

There are other points that you can add to this list.

E11) For example, each group draws 3 cards, each with a digit on it. Then each person in the group attempts to make the largest and the smallest 3-digit number with these digits.

You could also have 2 groups of children, each creating questions about HTO that have to be answered by the other group.

E12) In the binary system 5 would be represented as 101 (i.e., $1 \times 2^2 + 0 \times 2^1 + 1 \times 1$).

In the quinary system 5 would be represented as 10 (i.e., $1 \times 5^1 + 0 \times 5^0$).

You can write the numerals for the others similarly.