UNIT 10 INVENTORY SYSTEMS AND MODELLING

Objectives

After going through this unit, you should be able to:

- understand various concepts used in inventory management models;
- classify various types of inventory related costs;
- identify various factors affecting inventory;
- develop simple inventory models;
- differentiate between deterministic and probabilistic inventory models; and
- apply simple inventory models in practical situations.

Structure

10.1 INTRODUCTION

Inventory may be defined as 'usable but idle' resource. If the resource is some tangible item such as materials, then it is termed as stock. Thus, inventory is a stock of items on hand at a given time. There are three kinds of inventory stocks.

a) Raw materials used in production.

b) Semi-finished goods part way through production process (or work in progress).

c) Finished goods awaiting sale.

In general, the problem of inventory is, if too much of stock is kept then it amounts to wasted cost. On the other hand, if too little stock is kept then it leads lost sales. Therefore, the ideal situation is to have stock at desired levels. The inventory control or management is a technique maintaining stock at desired levels. Like any stock, inventory has inflow and outflow. Stock inflow is from new production or receipt of material from suppliers. Stock outflow is finished goods to customers or use of materials in the production process.

The demand for inventory could be:

a) **External or independent demand:** Where the usage is determined outside the firm, as with finished goods inventory demanded by customers. In this case, with many different customers, demand may be fairly constant.

b) **Internal or dependent demand:** Where the usage is derived from firm's rate of production of finished product, as with demand for materials and work-in-progress. In this case, the demand may be constant or variable depending on production scheduling.

In order to implement an optimal inventory policy, managers must answer two questions:

a) Reorder point – when to order?

b) Order quantity – how many units to order?

Inventory Costs: An inventory system may be defined as one in which the following three types of costs are significant:

1) **Ordering Costs (C_o)**: These are the costs of purchasing or producing items for inventory replenishment. This is generally called ordering cost. The cost elements may include administrative costs of preparing requisitions, placing orders or setting up manufacturing runs. This cost is usually assumed to be independent of the quantity ordered and therefore is a fixed cost of placing order or setting up production run cost function. It is expressed as Rs./order.

2) **Holding or Carrying Costs (C_h):** This is the cost of owning inventory and using facilities for storing and safeguarding inventory stock through time. The cost elements here may include:

   a) Storage costs which includes maintenance and operating cost of storage facilities; inventory taxes and insurance; expected losses due to casualty, deterioration or obsolescence.

   b) Opportunity cost of funds tied up in the value of inventory and the facilities needed for managing it.

The inventory carrying (holding) cost is often expressed as a percent of inventory value per year.

3) **Shortage or Stockout Penalty Costs (C_s):** These are the costs of running out of an item in inventory. These costs depend on how the company handles shortages. The cost elements here may include:

   a) Lost sales cost elements include shortages that cause customers to buy elsewhere. The cost is the lost sales plus possible loss of the customer forever.

   b) Backorder cost elements include the firm backlogs shortages and satisfies the unmet demand at the beginning of next period. The costs include lost demand, penalty cost, loss of good will, and urgent replenishment costs. This is generally expressed as Rs./item short/unit time.

10.2 INVENTORY MODELLING

In the earlier section you learned that one of the objective of inventory management is to decide how much to order. This is determined by the Economic Order Quantity (EOQ) by minimizing the total inventory cost using the following cost model:

The answer to the second objective of 'when to order' depends on the type of inventory system with which you are dealing with. There is a certain time needed to process an order by the customer and the supplier. The time between placing an order and its arrival is known as lead-time. The lead-time tells us, when to place an order during each stock cycle. The level of stock that is enough to last the lead-time is known as reorder level or reorder point.

The inventory models can be further classified as either deterministic (where demand and lead-time are known) or probabilistic (where demand and/or lead-time are having probability distribution). For building an inventory model, you should determine the following:

a) Is demand for the item deterministic or probabilistic?

b) Is there a time lag between procurement and receipt?

c) Is monitoring of inventory level continuous or periodic?

d) Does firm backlog orders when inventory shortage occurs?

e) Does the inventory cycle (use, replenish) continue indefinitely or is there a terminal cycle?

f) Does inventory lose value over time from deterioration, theft or obsolescence?

10.3 DETERMINISTIC INVENTORY MODELS

In this section, you will learn some simple inventory models with deterministic demand and lead time.

10.3.1 Economic Order Quantity (EOQ) Model with no Shortages

This is simplest of the inventory models. Here, the items enter inventory in batches at specified intervals, usedup gradually, and then replenished. It is popularly known as EOQ Model or 'Wilson's lot size formula'. The following are the assumptions made in this model:

a) Constant, deterministic, and uniform demand for a period,

b) No shortage costs involved,

c) The process is continuous, infinitely recurring inventory cycles, no quantity discounts or backlogging of shortages.

Let us use the following symbols in developing the EOQ model with no shortages.

C_o = Ordering cost or set-up cost per order (Rs. per order)

C_h = Carrying cost per unit per time period (Rs. per inventory unit per unit time)

t = Order cycle (length of time between two successive orders)

Q = Number of units ordered per order/produced per batch

D = Demand (the quantity used per year)

C = Cost per unit of purchase or manufacture

Q* = Economic order quantity or optimal number of units per order to minimize the total cost

TC = Total annual inventory cost

L = Lead time

With these assumptions and symbols, the inventory level against time is shown in Figure 10.1.

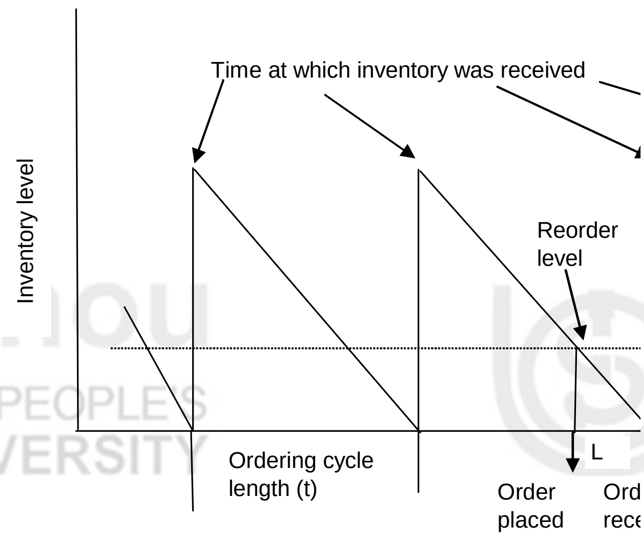

Figure 10.1: Basic Inventory Model

An order size Q units is placed and received instantaneously when the inventory level is zero. The stock is depleted at uniform rate D. The order cycle is,

The number of orders per year =Annual demand ÷ Order size =

Since the minimum level of stock is zero and maximum level of stock is Q, the

average inventory level (average number of units carried) is

Since there is no shortage cost in this model, the total inventory cost per year is,

$$\begin{bmatrix} \text{Total} \\ \text{related} \\ \text{cost (TC)} \end{bmatrix} = \begin{bmatrix} \text{Purchase} \\ \text{cost per} \\ \text{year (PC)} \end{bmatrix} + \begin{bmatrix} \text{Ordering} \\ \text{cost per} \\ \text{year (OC)} \end{bmatrix} + \begin{bmatrix} \text{Carrying} \\ \text{cost per} \\ \text{year (CC)} \end{bmatrix}$$

$$= \left[\begin{pmatrix} price \\ per \\ unit \end{pmatrix} \begin{pmatrix} No.\ of\ units \\ purchased \\ per\ year \end{pmatrix} \right] + \left[\begin{pmatrix} Cost\ of \\ ordering \\ per\ order \end{pmatrix} \begin{pmatrix} No.\ of\ orders \\ placed \\ per\ year \end{pmatrix} \right]$$

(1)

For an optimum inventory policy or EOQ, we minimize the total cost. For this we apply first order and second order derivatives. Note that here CD is a fixed cost and

and are variable costs and are dependent on the order size Q.

Therefore, for total cost to be minimum, applying the first order derivative of TC with respect to Q and setting equation (1) as zero,

(2)

or (3)

or $\begin{pmatrix}\text{Annual}\\\text{ordering}\\\text{cost}\end{pmatrix} = \begin{pmatrix}\text{Annual}\\\text{carrying}\\\text{cost}\end{pmatrix}$ (4)

From equation (3), we get

EOQ = Q* = (5)

Applying the second order derivative of TC with respect to Q on equation (2),

Since is greater than zero, Q* is a minimum.

Therefore, we interpret that:

a) Firm orders or manufactures "optimal economic order quantity" Q* units of inventory items per batch.

b) Optimal inventory cycle length $t^* = \dfrac{\textit{Optimal Order Quantity}}{\textit{Annual Demand}} = \dfrac{Q^*}{D} = \sqrt{\dfrac{2C_o}{C_h D}}$ years (or periods).

c) The total number of orders per year (N) is the reciprocal of the cycle period (t^*). That is, there are / = batches (orders) per year. Therefore, $N = \dfrac{D}{Q^*} = \sqrt{\dfrac{C_h D}{2C_o}}$

d) When carrying cost C_h rises, optimal Q* and t* fall.

e) When fixed procurement cost CD rises, 1/t* = optimal number of batches per year falls.

f) Optimal results are independent of variable procurement cost per unit C

g) The CD term in TC, while a cost of doing business, is not really an inventory cost.

h) The reorder level = (Lead time in years)x(Demand rate per year)=LD.

i) From equation (1), the variable part of the total cost is

Example 1

A juice store sells 5200 cases of juice /year. Each case costs Rs.200 whole-sale. There is a Rs. 1000 juice delivery charge. The store's working capital could be invested at 10%/year. The owner pays 10% of inventory value in theft insurance and taxes. No other store costs are related to inventory. Find the optimal inventory policy? If the lead time is 15 days, find reorder level.

Solution

For this problem:

Time period = 1 year

C_o = Rs.1000/delivery

C = Rs.200/case

D = 5200 cases/year

C_h = 20% of value/case = 0.2C = Rs. 40 per case per year

L= 15 days =

$$Q^* = \quad = \sqrt{\frac{2\times1000\times5200}{40}} = \quad = \text{Approximately 510 cases/delivery}$$

t* = = 0.098 years (˜ 36 days) between deliveries

N = = 10 deliveries/year

Therefore, TC of inventory system: = 200 (5200) +

1000(5200)/510 + 40(255) ˜ Rs.10,60,396/year

Kindly note of Rs.10,60,396/year, CD = Rs.10,40,000 is a fixed cost and independent of the inventory policy. Therefore, the total variable cost = Rs.10,60,396–Rs.10,40,000 = Rs.20,396.

Reorder level = L×D = $\frac{15}{365}\times 5{,}200$ = ≈ 214 units.

10.3.2 Economic Order Quantity (EOQ) Model with Shortages

In this model, stock-outs are permitted which implies that shortage cost is finite or it is not large. All the assumptions of the model given in section 10.3.1 hold good here also. Also, in addition to the symbols given in section 10.3.1, the following symbols are used in this model.

C_s = Shortage cost (or back order cost) per unit per year

I = Inventory level

S = Shortage level

Q = Order size (Q = I+S)

t_1 = Portion of the cycle period for inventory carrying

t_2 = Time of the stock-out

t = Cycle period ($t = t_1+t_2$)

$$\begin{pmatrix} Total \\ variable \\ \cos t \end{pmatrix} = \begin{pmatrix} Annual \\ ordering \\ \cos t \end{pmatrix} + \begin{pmatrix} Annual \\ carrying \\ \cos t \end{pmatrix} + \begin{pmatrix} Annual \\ shortage \\ \cos t \end{pmatrix}$$

$$= \frac{C_o D}{Q} + \frac{I^2 C_h}{2Q} + \frac{(Q-I)^2 C_s}{2Q} \qquad (1)$$

The problem here is to choose Q and I in order to minimize equation (1) above.

The optimization results are:

$$EOQ = Q^* = \sqrt{\left(\left(\frac{2C_o D}{C_h}\right)\left(\frac{C_s + C_h}{C_s}\right)\right)}$$

$$\text{Inventory level} = I^* = \sqrt{\left(\left(\frac{2C_o D}{C_h}\right)\left(\frac{C_s}{C_s + C_h}\right)\right)}$$

Shortage level = $Q^* - I^*$

Cycle period = t^* =

No. of orders/year =

Example 2

Suppose in example 1, the store allows backlogging juice shortages and the shortage cost is Rs.100/case per year. Find the optimum inventory policy.

Solution

We have,

Time period = 1 year

C_o = Rs.1000/delivery

C = Rs.200/case

D = 5200 cases/year

C_h = 20% of value/case = 0.2C = Rs. 40 per case per year

C_s = Rs.100/case per year.

The optimal policy is,

$$EOQ = Q^* = \sqrt{\left(\frac{2C_oD}{C_h}\right)\left(\frac{C_s + C_h}{C_s}\right)} = \sqrt{\left(\frac{2\times 1000\times 5200}{40}\right)\left(\frac{100+40}{100}\right)}$$

$$= \sqrt{2{,}60{,}000\times 1.4} \text{ = 603 cases per order}$$

$$\text{Inventory level} = I^* = \sqrt{\left(\frac{2C_oD}{C_h}\right)\left(\frac{C_s}{C_s + C_h}\right)} \text{ = 431 cases}$$

Shortage level = $Q^* - I^*$ = 603-431=172 cases

Cycle period = t* = = 0.116 years or 42 days

No. of orders/year = = 8.6 or say 9 deliveries per year

Total variable cost = = Rs.17,238

10.3.3 Economic Production Lot Size Model (EPLS)

The economic production lot size model is associated with a manufacturing environment. Here no shortages allowed. The EPLS realistically shows that inventory gradually built over a period of time because production and consumption go side by side where production rate is higher than consumption rate. In this model also all the assumptions made in EOQ model discussed in section 10.3.1 hold good. Here the order size is taken as production size p units per day, daily demand rate (demand¸working days) d units per working day, and p > d. When stock of inventory reaches zero, new production begun. When stock of units on hand reaches maximum, production stops until inventory hits zero. The following are the symbols used in the model.

C_o = fixed set-up costs per production run

C_h = holding cost per unit per year

p = daily production rate (capacity/working day)

Q = number of units produced per run

D = demand per year (month, etc.)

d = daily demand rate (demand/working day)

C = cost per unit of manufacture

t_1 = length of production run in working days

t = length of inventory cycle (years or time periods)

I = level of inventory

During each production run, Q units are produced in t_1 days.

Therefore, $Q = pt_1 \Rightarrow$ production time = t_1 =

Cycle time = t =

During production run, inventory is growing by (p – d) units per day.

Maximum inventory level =

Minimum inventory level = 0

Average inventory carried = I =

$$= \frac{(p-d)\frac{Q}{p} + 0}{2} = \frac{(p-d)Q}{2p}$$

$$Q^* = \sqrt{\frac{2C_oD}{\frac{C_h(p-d)}{p}}} = \sqrt{\left(\left(\frac{2C_oD}{C_h}\right)\left(\frac{p}{p-d}\right)\right)}$$

$$t^* = \frac{Q^*}{D}$$

Example 3

A company sells 20,000 cases of detergent per year. Production set up costs Rs.5,000 a run. Manufacturing costs are Rs.250 per case. The company has production capacity of 50,000 cases per year. The company is open for 300 days per year for business. Annual inventory carrying costs is 20% of inventory value. Find optimum inventory policy.

Solution

Time period = 1 year

C_o = Rs.5000/run

C = Rs.250/case

D = 20,000 cases/year or d = 20,000¸300 = 67 cases/day

C_h = 20% of Rs.250 = Rs.50 per unit per year = Rs. 0.14 per unit per day

P = 50,000/cases per year or p = 50,000¸300 = 167 cases per day

The optimal policy is,

$$Q^* = \sqrt{\left(\left(\frac{2 \times 5000 \times 67}{0.14}\right)\left(\frac{167}{167-67}\right)\right)} = 2827 \text{ cases per production run}$$

= = 2827÷167 = 17 production days per cycle or run

$$t^* = \frac{Q^*}{D} = 2827 \div 20000 = 0.14 \text{ years per cycle (or 42 business days per cycle)}$$

= 1÷0.14 = 7.14 cycles per year

Total variable cost = = Rs.35491 per year

10.3.4 Economic Order Quantity (EOQ) Model with Price Breaks

When the items are bought in bulk by the firm, the supplier usually offers some discount in price. Here no shortages allowed. This model is the same as the EOQ model discussed in section 10.3.1 except that the inventory item may be purchased at a discount if the size of the order exceeds a given limit.

Whenever, the quantity discounts are available, the price C may vary according to;

Where $C_1 > C_2 \ldots > C_j > \ldots > C_n$

C_j = price per unit for the j^{th} lot size

Purchasing cost per unit time of the respective lot sizes are DC_1, DC_2, …DC_j,…,DC_n

The total cost of the respective lot sizes are:

$\mathbf{C}_h = iC_j$,

Where i = percentage of change for j = 1,2,…,j,…,n

Also, since price varies with size of the purchase, the fixed cost DC_j cannot be ignored for minimizing the total cost. The functions DC_1, DC_2, …DC_j,…,DC_n are illustrated in Figure 10.2. The bold faced part of the respective curves show feasible part of the total cost.

Figure 10.2 Quantity Discounts

The steps for determining EOQ are :

Step 1: Find the EOQ for lowest price

If , the optimum order quantity is . If , go to step 2.

Step 2: Find the EOQ for the next lowest price

If , then compare the total cost TC_{n-1} for purchasing quantity q_n and select last cost purchase quantity. If , go to step 3

Step 3: Find the EOQ for the next lowest price

If $Q^*_{n-2} \geq q_{n-2}$, then compare the total cost TC_{n-1} for purchasing quantity q_n and select last cost purchase quantity. If , continue in this manner until

Example 4

The demand for a particular component used in manufacturing TV sets is 3000 units per annum. Ordering cost is Rs. 500, inventory carrying charge is 25% of the purchse price per year. The supplier has offered the following purchase prices:

Rs. 10 for purchasing $Q_1 < 500$

Rs. 9 for purchasing $500=Q_2<1000$

Rs. 8 for purchasing $Q_3>1000$

Find the optimum purchase quantity.

Solution

Step 1: $= Q_3^* = \sqrt{\frac{2\times500\times3000}{0.25x8}}$ = 1225 units

Since, , that is 1225?1000, the optimum order quantity is 1225 units.

10.4 PROBABILISTIC INVENTORY MODELS

In section 10.3 we have discussed simple deterministic inventory models where the demand and lead time are known. But in actual business life we may not be knowing the actual demand and lead time. In such situations we may have to go for probabilistic inventory models The probability distribution of future demand is usually determined from the past experience. In such situations, we minimize the expected costs rather than actual costs. The expected costs are calculated by multiplying the actual cost for a particular situation with probability of occurrence of that situation. We assume that the demand D is a random variable with probability distribution p(D). The demand pattern may be either discreet probability distribution or continuous probability distribution. Also, the probabilistic inventory models can be single period or multi period models. In section 10.4.1, we will discuss simple single period discreet probabilistic inventory models. In section 10.4.2, we will discuss simple single period continuous probabilistic inventory models. Section 10.4.3 covers multi period probabilistic inventory models.

10.4.1 Single Period Discreet Probabilistic Demand Model

You may understand that perishable goods, spare parts and other seasonal goods requires one time purchase only. The demand for such items should be either discreet or continuous. In such cases, since the purchases are made only once, the lead time factor do not apply in these models. The following symbols are used in this model:

D = Demand for an item in units (assumed to be a random variable); $D = d_j$ with probabilities $p(d_j)$, j = 1,2,3,…n

P(D) = Cumulative probabiility

Q = The number of units stocked (or purchased)

C = Unit cost price

C_h = Carrying cost for the entire period

C_s = Shortage cost

S = Selling price per unit

V = Salvage value

C_1 = Cost of over stocking (or cost of over ordering) = $C+C_h-V$

C_2 = Cost of under stocking (or cost of under ordering) = $S-C-(C_h/2)+C_s$

Since, the demand $D = d_j$, j = 1,2,3,… is a random variable, $p(d_j)$ denotes the probability of demand such that, $p(d_1)+ p(d_j)+ p(d_j)+\ldots = 1$.

If $d_j < Q$, the item is over stocked and incur a marginal over stocking cost C_1 per unit of excess inventory.

If $d_j > Q$, the item is under stocked and incur a marginal under stocking penalty cost of C_2 per unit shortage.

To find the optimal quantity Q^*, we minimize the expected inventory cost. For this we use marginal analysis.

Let E[C(Q)] = expected inventory cost if Q units are ordered

E[C(Q+1)] = expected inventory cost if Q+1 units are ordered

MC = Marginal cost of ordering one extra unit

E(MC) = E[C(Q+1)] – E[C(Q+)] = expected marginal cost of ordering one extra unit

Then, optional order quantity Q^* = the smallest Q for which E(MC) = 0

If $D \leq Q$, increasing Q by one unit will increase overstock by a unit, so MC $=c_1$. This occurs with probability P(Q).

If D = Q+1, increasing Q by one unit will decrease shortage by a unit, so MC = $-c_2$. This occurs with probability 1-P(Q).

Therefore, $E(MC) = c_1P(Q) - c_2(1\text{-}P(Q))$

$$= (c_1 + c_2)P(Q) - c_2 = 0$$

Thus,

The optimal demand Q^* is the smallest Q for which $P(Q^*) = P(D)$

Example 5

A news paper seller buys Q news papers (in multiples of 50) at a price of Rs 2 each and sells them at a price of Rs. 2.50. He can return unsold papers for Rs. 1.25 refund. The probability distribution of demand is shown below.

Demand	Probability
100	0.30
150	0.20
200	0.30
250	0.15
300	0.05

How many papers should be ordered.

Solution

Demand (d_j)	Probability ($p(d_j)$)	Cumulative probability P(D)
100	0.30	0.30
150	0.20	0.50
200	0.30	0.80
250	0.15	0.95
300	0.05	1.00

Marginal cost of overstock = C_1 = Rs. 2 – Rs. 1.25 = Rs. 0.75

Marginal cost of overstock = C_2 = Rs. 2.50 – Rs. 2 = Rs. 0.50

For $Q^* = 150$, $P(150) = 0.50 > 0.40$, but $P(100) = 0.30 < 0.40$

Therefore, the optimal quantity is 150 papers.

10.4.2 Single Period Continuous Probabilistic Demand Model

This model is similar to that discussed in 10.4.2. The only difference is that in this model we assume the demand D is a continuous random variable with probability density function f(d) and cumulative probability density function F(D). The optimal

quantity Q^* is found where

Example 6

A training institute reserves Q rooms in a nearby hotel for its trainees at Rs.500 per room. If the demand is greater than Q then rooms can be obtained at Rs.900 each. The demand is normally distributed. $D \sim N(\mu_d\, \sigma^2_d)$ where $\mu_d = 5{,}000$ and $\sigma_d = 2000$. How many rooms should be reserved now?

Solution

Marginal cost of over stock c_1 = Rs.500

Marginal cost of under stock c_2 = Rs.900 – Rs.500 = Rs.400

$$P(D \le Q^*) \quad = \quad P(Z \le \frac{Q^* - 5000}{2000}) \quad = \quad 0.4444$$

Using the standard normal table

Q* = – 0.14(2000)+5000 = 4720 rooms

10.4.3 Multi-period Probabilistic Inventory Models

In single period inventory models, demand is a significant factor and the lead time do not have any role to play. However, in multi period inventory models both demand and inventory play significant role in the inventory policy decision process. The variations in demand and/or lead time pose risks in taking right decisions. We try to absorb the risks by carrying inventories called safety stocks or buffer stocks. In this Unit we will consider reorder level with variable demand.

A constant buffer stock is maintained throughout the planning period. The size of the buffer stock is determined such that the probability of running out of stock during lead time does not exceed a prescribed value. The following symbols are used in this model:

U = Random variable representing demand during lead time

L = Lead time

= Average daily demand

μ_L = Average demand during lead time

σ_L = Standard deviation of the demand during lead time

σ_d = Standard deviation of daily demand

B = Buffer stock (or safety stock)

Z = Number of standard deviations needed for a specified confidence interval

R = Reorder level

The assumption of the model is that the demand U during the lead time L is normally distributed with mean μ_L and standard deviation σ_L. Symbolically $N(\mu_L, \sigma_L)$.

Figure 10.3 shows buffer stock (B), lead time (L), average demand during lead time (μ_L), reorder level ($B+\mu_L$), EOQ(Q*) and the maximum level of inventory (B+Q*).

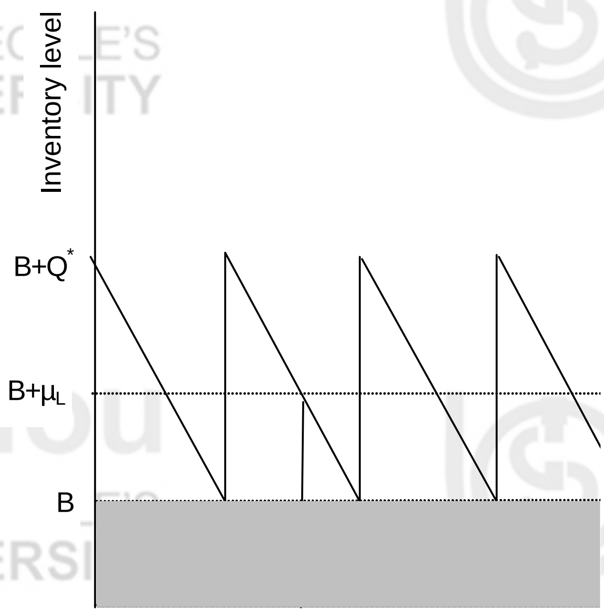

Figure 10.3: Relationship between buffer stock, lead time reorder level, and EOQ

Average demand during lead time = μ_L =

Buffer stock = B = Z

Standard deviation of demand during lead time = σ_L =

Therefore, the reorder level is

R* = Average lead time demand + Buffer stock

= μ_L +B

= + Z

EOQ = Q* =

Maximum inventory level = B+Q*

Average inventory level = $B+(Q^*/2)$

Example 7

The daily demand for an item is normally distributed with mean 30 units and standard deviation 5 units. Lead time is 10 days. Find the reorder point with 95% service level and 5% stock out probability. Also, find the buffer stock.

Solution

D = 30 units

D_d = 5 units

L = 10 days

Z = 1.65 (for a 95% service level)

Reorder level = R =   + Z   = 30(10) + (1.65) (5) (   ) = 326.1 units

Buffer stock = Z   = (1.65) (5) (   ) = 26.1 units

Example 8

For an item the daily demand is distributed normal with mean 100 and standard deviation 10. Lead time is 15 days. The cost of placing an order is Rs.9 and carrying costs are 20% of the unit price. The unit price is given as Rs.1.50. Back orders are allowed but there is no stock out cost. A 95% service level is desired. Find the following:

a) Buffer stock

b) Reorder level

c) Average inventory level

d) Maximum inventory level

Solution

D = 100 units daily × 365 days = 36,500 units

C_o = Rs.9

C_h = 0.2 × 1.50 = 0.3

 = 100

σ_d = 10

L = 15 days

Z = 1.65(for 95% service level)

a) Buffer stock = Z   =1.65 × 10 ×   = 64 units

b) Recorder level =   +Z   = 100 × 15 + 64 = 1564 units

c) Average inventory level =   = 64 + (1480/2) = 804 units.

d) Maximum inventory level = B + Q* = 64 + 1480 = 1544 units

10.5 SUMMARY

In this unit, you have learned the introductory concepts used in the inventory models. Various factors determine the inventory policy decisions. Primarily, there are two factors which decide the deterministic or probability nature of the models. These are demand and lead time. Some of the other factors that determine the inventory are cost, demand, lead time, shortages. Depending on the situation, you have to choose the right model. In this Unit, only simple inventory models are used. The understanding of these models will provide you a basic understanding of inventory model.

10.6 SELF ASSESSMENT EXERCISES

1) Define inventory. What are the two sources of demand for inventory?

2) What are the cost elements in ordering costs, carrying costs, and shortage costs?

3) An item is produced at the rate of 100 units per day and is consumed at the rate of 50 units per day. The set up cost is Rs.200 per production run and carrying cost in stock is Rs.400 per unit per year. Find the economic lot size per run, number of runs per year, and the total related cost?

4) A stationery shop supplies 5000 reams of paper a year. He charges Rs.1.50 per ream plus a fixed Rs.50 for delivery. A ream of paper loses about Rs.0.45 in value each year. The annual fire insurance premium per ream is Rs.0.05. Find the store's optimal paper inventory policy.

5) In problem 4 given above, suppose the stationery shop adopts a policy of backlogging un-filled demand for paper, at a cost of Rs.0.25 per ream per year. What is the store's optimal inventory policy?

6) A company sells 1,40,000 bolts per year. It produces the bolts in its own factory in discreet production runs. The production set up costs are Rs.1500 per run and production costs are Rs.104 per bolt. The company has production capacity of 3,50,000 bolts per year and is open for business 300 days a year. Annual inventory holding cost is 16% of inventory value. Find the optimal policy.

7) For an item annual demand is 1000 units, set up cost is Rs.20, and carrying cost is 2% per month of purchase price. The minimum stock is 200 units. Purchase price per unit is Rs.2 per order below 2000 units and Rs.1 if the order is 2000 or more. For the above data find optimum inventory policy.

8) A textile firm sells 10,000 meters of a particular brand of cloth per year. The daily production rate is 150 meters. Production set up costs Rs.150 per run. Weaving costs are Rs.0.75 per meter. The firm open for business 311 days per year. Find the optimal inventory policy.

9) A mobile phone shop offers the following price discounts on a particular brand phone:

Quantity	**Price**
1-49	Rs.1400
50-89	Rs.1100
90+	Rs.900

The demand is 200 units. The ordering cost is Rs.2500. The inventory carrying charges are Rs.190 per unit per year. Find the optimum purchase quantity.

10) An item is bought at Rs.50 each unit and sold at Rs.75 each unit. If shortage is there then there will be a goodwill cost of Rs.15 each unit. If the item is not sold then there will be a salvage value of Rs.10. Carrying cost during the period is estimated to be 20% of the cost price. The probability distribution for the demand is given below:

Units Stocked	2	3	4	5	6
Probability of Demand	0.30	0.25	0.20	0.15	0.10

Find the optimum number of units that can be stocked.

11) Daily demand for an item is normally distributed with mean 80 units and standard deviation of 12 units. Lead-time is 4 days. The cost of placing an order is Rs.16 and annual carrying costs are 25% of the unit price. The unit price is Rs.1.75. Find the optimum inventory policy for 90% service level.

10.7 REFERENCES AND SUGGESTED FURTHER READINGS

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