UNIT 14 COMPETITIVE SITUATIONS: GAME THEORY

Objectives

After reading this unit, you should be able to :

- explain that decisions will have to be taken in a competitive environment
- describe that the outcome of one business enterprise depends on what the competitor will do
- discuss the conceptual framework on the strategies to be adopted in a competitive situation.

Structure

14.1 INTRODUCTION

Many a decision is taken in a competitive situation in which the outcome depends not on that decision alone but rather on the interaction between the decision-maker and that of a competitor. The term "games" now includes not only pleasurable activities of this kind, but also much more earnest competitive situations of war and peace. It, was not co-incidental that the classic work on the theory of games was first published during the Second World War. Many competitive situations are still too complex for the theory in its present state of development to solve. Other methods, of which war games are long-established examples and, business games of more recent origins, have been used. The availability of computers has allowed increasingly large scale operations to be represented with great realism. The theory of games has been represented with great realism. The theory of games has been developed alongside gaming techniques, and a knowledge of the concepts involved, especially the importance of the role of chance, helps to clarify the issues in many decision= making processes.

14.2 DEFINITIONS AND EXPLANATION OF SOME IMPORTANT TERMS

A competitive situation is called a game if it has the following properties;

a) There are a finite number of participants called **players.**

b) Each player has finite number of possible **course of action**.

c) A **play** occurs when each player chooses one of his courses, of action. The choices are assumed to be made simultaneously, i.e. no player knows the choice of another until he has, decided on his own.

d) Every combination of course' of action determines an outcome which results in a gain to each player. (A loss is considered a~ negative gain.)

We shall consider in this section those games which have only-two players, and where the gain of one is the loss of the' other. Such a game is called a Zero-sum two

person game. In other words, in a zero-sum game there is no "cut for the house" as in professional gambling, and no .capital is created or destroyed during the course of play.

The gains resulting from a zero sum two person game are most easily represented in the form of a matrix, known as the pay of matrix. An example of such a matrix is given below.

		Player B			
		I	**II**	**III**	**IV**
	I	6	0	-2	-5
Player A	**II**	3	2	1	3
	III	-1	1	0	4

The number of rows of the matrix correspond to the number of courses of action of player A, and the number of columns correspond to the number of course of action of player B. The elements within the matrix represent the gain to player A for each outcome of the game; thus a positive entry indicates a payment from B to A whilst negative entry denotes a payment from A to B. For example, if player A use his third course of action and player B use his first, A pays B one unit.

The **strategy** of player is the decision rule he uses to decides which course of action he should employ. This strategy may be of two kinds :

a) A pure strategy is a decision always to select the same course of action.

b) Mixed Strategy is a decision to choose atleast two of his courses of action with fixed probabilities; i.e. if a player decides to use just two courses of action with equal probability, he might spin a coin to decide which one to choose. The advantage of a mixed strategy is that an opponent is always kept guessing as to which course of action is to be selected on any particular occasion.

The **Value** of a game is the expected gain of player A if both players' use their best strategies. We define "best strategy" on the basis of the **minimax criterion of optimality**. This states that if a player lists the worst possible outcome of all his potential strategies, he will choose that strategy which correspond to the best of the worst outcomes. The implication, of this criterion is that the opponent is an extremely shrewd player who will ensure that, whatever our strategy, our gain is kept to a minimum.

Example 1: The following is the pay-off matrix in dollars for the two persons zero-sum game.

		Player B		
		I	**II**	**III**
	I	-2	5	-3
Player A	**II**	1	3	5
	III	-3	-7	11

Using Minirnax criterion, find the best strategy for each player.

Solution : If player h chooses,first course of action, the worst that can happen is that he loses $I; if he chooses second course of action the worst that can happen to him is that he loses $11. If player B chooses Minimax criteria (minimising the Maximum loss), the best thing for him would be to use first course of action.

Similarly if player A chooses first course of action, the maximum he would lose is $3. If he chooses action II the worst that can happen is that he wins $1; and if he chooses action 11I, the maximum he would lose is S7. Thus° the player A would minimise his maximum losses by choosing action ll.'

The solution of a game involves finding

i) The best strategies for both players.

ii) The value of the game.

It is a feature of the solution that the situation where both players use their best strategies is stable, in the sense that neither player can increase his gain by deviating from his initial strategy once he becomes aware of his opponents.

Activity 1

Give a comprehensive explanation of the term "game theory"..

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Activity 2

Define with the help of an example, the following

i) Players

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ii) Pay-off matrix

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iii) Outcome

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Activity 3

Define the following;

1) Strategy

2) Pure strategy

3) Mixed strategy

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14.3 SADDLE POINTS

The, simplest type of game is one where the best strategies for both players are pure strategies. This is the case if, and only if, the pay-off matrix contains a saddle point. **A saddle point** is an element of the--n e4Tix which is both the smallest element in its row and the largest element in its column. We shall see why pure strategies are the best strategies in this case by reference to Example 2.

Activity 4

For the pay-off matrix given in Example 1, determine the saddle point.

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Example 2 : The pay-off matrix for a two person, zero-sum game 'is given below. Fine the best strategy for each player and the value of the game.

Player B

		I	II	III	IV	V
	I	-2	0	0	5	3
Player	II	3	2	1	2	2
	III	-4	-3	0	-2	6
	IV	5	3	-4	2	-6

Solution: On examining each of his pure strategies Player A notes that 'the' worst outcome if he plays his course of action I throughout occurs when B also plays I and the resulting gain to A is -2. Similarly the worst outcomes when A plays II, III and IV are 1, -4 and -6 respectively. Player A, therefore, can guarantee a gain of at least I by using his courses of action II throughout.

Looking now at the game from the point of view of player B; his worst outcomes, taking each pure strategy in turn, are 5, 3; 1, 5, and 6 (N.B. As the entries in the matrix represent gains to A, the larger the entry the more unfavourable the result is to B) Player B can guarantee a loss of at most 1 by using his strategy III throughout.

If A can guarantee a minimum gain of. 1, B cannot have a better strategy than one which precludes A from gaining more than 1. Similarly, if B can guarantee a maximum loss of 1, A's best strategy is that which ensures that B loses no less than l

The solution is, therefore;

Player A uses his 'course of action 11 throughout.

Player B uses his course of action III throughout.

The value of the game is l

The solution is stable, in that when player A realises that B is playing 111 throughout he cannot afford to change his own strategy, 1 being the largest element in column 111. For similar reasons B has no incentive to change his initial strategy when he discovers how A is playing. These choices are completely ‘spy roof ‘ in the sense that neither of the player can profit from knowledge of the other's choice..

It will be noticed from Example I that a saddle point occurs when the element which_ is the largest of the row minima equals the element which is the smallest of the column maxima. The solution of such a game lies in player A choosing consistently that course of-action corresponding to the row through the saddle point and player B choosing always that course of action corresponding to the column through the saddle point. The element at the saddle point determines the value of the game.

Example 3 : Determine the solution of the following game

Player B

		I	**II**	**III**
	I	-2	15	-2
Player A	**II**	-5	-6	-4
	III	-5	20	-8

Solution : The row minima, corresponding to player A's courses of action 1, 11 and 111, are -2, -6, and -8 respectively. The column maxima, corresponding to B's course of action are -2,10 and -2. The largest of the row minima and the smallest of the column maxima are both equal to -2. However, there are two elements whose value is -2 and hence the game has two saddle points. The solution to such a game is not unique:

Player A uses his course of action 1 throughout.

Player B may use either of his courses of action 1 or Ill throughout, or a nixed strategy which uses any combination of these courses of action.

14.4 DOMINANCE

We can sometimes reduce the size of a game's pay-off matrix by eliminating a course of action which is so inferior to another as never to be used. Such a course of action is said to be dominated by the other. The concept of dominance is especially useful for the evaluation of two person zero-sum games where a saddle point does not exist.

Example 4: Determine the solution of the game whose pay-off matrix is given below:

Player B

		I	**II**	**III**
	I	-4	-6	3 -
Player A	**II**	-3	-3	6
	III	2	-3	4

Solution: This game has no saddle point. Consider, however, player B's first and third courses of action. From his point of view his first courses of action is superior to his third, regardless of A's strategy, the elements in column 1 being smaller, element for element, than those in element III. Player B has no incentive to use his third course of action which is dominated by his first: The effective pay-off matrix is reduced to the one below:

Player B

		I	**II**
	I	-4	6
Player A	**II**	-3	-3
	III	2	-3'

By similar reasoning, player A's third course of action dominates his second, for although his second and third courses of action are equivalent if B chooses II, his third course of action is clearly superior if B chooses I: Player A has no incentive to choose II and game reduce to the following:

This can be solved by the method of the previous example yielding the solution, A plays (1/3, 0, 2/3), B plays (3/5, 2/5, 0) and the value of the game is zero.

Therefore the Dominance Principle can be summarised as:

1) If all elements in a column are greater than or equal to the corresponding elements in another column, then that column is dominated and can be deleted from the matrix.

2) Similarly; if all elements in a row are less than or equal to the corresponding elements in another row, then that row is dominated and can be deleted from the matrix.

Activity 5

Explain briefly the importance of the principle of "dominance"

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14.5 MIXED STRATEGIES: GAMES WITHOUT SADDLE POINTS

A stable solution can only exist in terms of pure strategies when the pay-off matrix has a saddle point. If there is no saddle point for a pay-off matrix, the problem can be solved by using the concept of mixed strategies. Here, the problem becomes one of determining probabilities with which each course of action should be selected. A mixed strategy game can be solved by following methods :

(1) Algebraic Method

(2) Calculus Method

(3) Linear Programming Method

Algebraic Method: Consider the two person zero-sum game with the following pay-off matrix:

		Player B	
		I	**II**
Player A	**I**	a	b
	II	c	d

If this game is to have no saddle point the two largest elements of the matrix must constitute one of the diagonals. We here assume this and therefore both players use mixed strategies: Our task is to determine the probabilities with which both players choose their course of action.

Let Player A chooses action I with probability p and therefore chooses action II with a probability 1-p. ln short, A's strategy is written as (p, 1-p). Suppose the player B, select strategy I. Then expected gain to Player A for this game is given by

$$ap + c(1-p) \quad \ldots\ldots\ldots\ldots\ldots\ldots(1)$$

On the other hand, if Player B selects strategy II, then Player A's expected gain is

$$bp + d(1-p) \quad \ldots\ldots\ldots\ldots\ldots\ldots(2)$$

However, for Player A to be indifferent to which strategy Player B selects, the optimal plan for Player A requires that its expected gain to be equal for each of Player B's possible strategies. Thus equating two equations, we obtain:

$$ap + c(1-p) = bp + (1-p)$$
$$p(a-b) + c - cp = d - dp$$
$$p(a-b) + p(d-c) = d-c$$
$$p\{a + d - (b+c)\} = d-c$$
$$P = \frac{d-c}{a+d-(b+c)} \quad -(3)$$

Similarly if Player B selects strategy I and II with a probability of q and 1-q respectively. The expected loss to B when Player A adopts strategies I and II respectively are

$$aq + b(1-q) \quad \text{———} (4)$$
and
$$cq + d(1-q) \quad \text{———} (5)$$

By equating expected losses of Player B, regardless of what Player A would choose, we have

$$aq + b(1-q) = cq + (1-q)$$
$$aq - cq = d - dq - b + bq$$
$$q(a-c) = q(b-d) + d - b$$
$$q\{a - c - b + d\} = d - b$$
$$q\{a + d - (b + c)\} = d - b$$
$$q = \frac{d-b}{a + d - (b + c)} \quad \text{———— (6)}$$

The value of game V is found by substituting the value of p in one of the expressions for the expected gain of A.

eg $V = ap + c(1-p)$

which on substitution and rearrangement, becomes

$$V = \frac{ad - bc}{a + d - (b + c)} \quad \text{———— (7)}$$

The solution of the game is :

A play's (p, 1 − p) where $p = \dfrac{d-c}{a + d - (b + c)}$

B play's (q, 1 − q) where $q = \dfrac{d-b}{a + d - (b + c)}$

and value of the game V is $= \dfrac{ad - bc}{a + d - (b + c)}$

It should be remembered again that the above analysis assumes that the game has no saddle point. If the game has a saddle point the above result does not hold and the solution of a game with a saddle point is obtained by the simpler method of Example 2.

Example 5: Consider the game of matching coins, Two players, A and B, each put down a coin. If coins match, i.e. both are head or both are tails, A gets rewarded otherwise B. However, matching on heads gives a double premium. Obtain the best strategies for both players and the value of the game.

Solution: The pay-off matrix is as follows

		Player B	
		I (Heads)	II (Tails)
Player A	1(Heads)	2	-1
	II(Tails)	-1	1

It may be noted that the game has no saddle point. Therefore, the best strategy for both players are mixed strategies. Let players A's strategy. be (p, p) and player Bs be (q, 1 - q). Using equations (3), (6) and 7 we obtain:

$$p = \frac{1 - (-1)}{(2 + 1 - (-1 - 1)} = \frac{2}{5}$$

$$q = \frac{1 - (-1)}{5} = \frac{2}{5}$$

$$v = \frac{2 \times 1 - (-1)(-1)}{5} = \frac{1}{5}$$

Both the players should play heads 2/5 of the time. The game is unfair to B as he will loss on average 1/5 per ply.

Example 6: Two breakfast food manufacturing firms A and B are competing for an increased market snare. To improve its market snare, both the firms decide to launch the following strategies:

Al, B1	=	Give Coupons
A2, B2	=	Decrease price
A3B3	=	Maintain present strategy
A4, B4	=	Increase advertising.

The e pay-off matrix, shown in the following table describes the increase in market share for firm A and decrease in market share for firm B.

Firm B

		B1	**B2**	**B3**	**B4**
	A1	35	65	25	5
Firm A	**A2**	30	20	15	0
	A3	40	50	0	10
	A4	55	60	10	15

Determine the optimal strategies for each firm and the value of the game.

Solution: You may apply the rule of dominance to obtain the reduced pay-off matrix as

Firm B

		B3	**B4**
Firm A	**A1**	25	5
	II	10	15

Please note that there is no saddle point for, the reduced pay- off matrix given above. Therefore, both firms will use mixed strategies to obtain optimal solution. Let firm A's strategy by (p, 1-p) and firm Bs be (q, 1-q). Using equations (3), (6) and (7) we obtain

$$p = \frac{15 - 10}{25 + 15 - (10 + 5)} = \frac{5}{25} = \frac{1}{5}$$

Therefore Firm A will choose strategy A1 with probability of 1/5 and strategy A4 with a probability of 4/5.

$$q = \frac{15 - 5}{25 + 15 - (10 + 5)} = \frac{10}{25} = \frac{2}{5}$$

Therefore Firm B will choose strategy B3 and B4 with a probability of 0.4 and 0.6 respectively.

The value of the game V is given by :

$$v = \frac{(25)(15) - (10)(5)}{25 + 15 - (10 + 5)}$$

$$= \frac{375 - 50}{25}$$

$$= \frac{325}{25}$$

$$= 13$$

2 Calculus Method: In this method, instead of equating the two expected values, the expected value for a given player is maximised Let us consider the same pay-off matrix (given below) was used for Algebraic Method.

		Player B	
		I	II
Player A	I	a	b
	II	c	d

Suppose player A selects strategy I with probability p and therefore chooses strategy II with probability 1-p. Similarly, the player B chooses strategy 1 with probability q and therefore chooses strategy II with probability 1 - q.

$$\bar{E}(p,q) = apq + c(1-p)q + bp(1-q) + d(1-p)(1-q)$$

For maximising the expectation, we should have

$$\frac{\partial E}{\partial p} = 0 \text{ and } \frac{\partial E}{\partial q} = 0$$

$$\frac{\partial E}{\partial P} = aq - cq + b - bq - d(1-q) = 0$$

$$= aq - cq + b - bq - d + dq = 0$$

$$= q(a + d - c - b) = d - b$$

$$\therefore \quad q = \frac{d-b}{a+d-(b+c)}$$

This is identical to the expression we obtained in equation (6) of the Algebraic Method.

Also, we will have

$$\frac{\partial E}{\partial q} = ap + c - cp - bp - d + dp = 0$$

$$= p(a - c - b + d) = d - c$$

$$\therefore \quad p = \frac{d-c}{a+d-(b+c)}$$

This is the same as obtained in equation (3) of the Algebraic Method.

To obtain the value of game, you may substitute the values of p, 1-p, q and 1-q in the expression for E (p, q),

3 Linear Programming Method: The linear programming technique can be of help in solving mixed strategy games of dimensions greater than (2 x.2) size. To explain the procedure, however, we will use Two-persons zero - sum game with the following pay-off matrix:

		Player B	
		I	II
Player A	I	a	b
	II	c	d

Let Player A chooses strategies I and II with probabilities p1 and p2 = 1-pa respectively. Similarly Player 0 chooses strategies I and II with probabilities q1 and q2respectively. Let V devotes the value of the game.

As we know that the objective of Player A is to maximises expected gain. This can be achieved by maximising the value of the game (V), i.e., it might gain more than V if Player B adopts a poor strategy. Therefore, the expected gain of Player A will be as follows

$ap_1 + cp_2 \geq V$ (If Player B adopts strategy 1)

$bp_1 + dp_2 \geq V$ (if Player B adopts strategy 2)

$p_1 + p_2 = 1$

$p_1, p_2 \geq 0$

By dividing the above throughout by V, we obtain

$$\frac{ap_1}{V} + \frac{cp_2}{V} \geq 1$$

$$\frac{bp_1}{V} + \frac{dp_2}{V} \geq 1$$

$$\frac{p_1}{V} + \frac{p_2}{V} = \frac{1}{V}$$

Define $x_1 = \frac{p_1}{V}$ and $x_2 = \frac{p_2}{V}$

Therefore, we have

$$ax_1 + cx_2 \geq 1$$

$$bx_1 + dx_2 \geq 1$$

$$x_1 + x_2 = \frac{1}{V}$$

As Player A wants to maximise V, it is equivalent to minimising $\frac{1}{V}$

Therefore, the linear programming problem can be written as

Maximize Z = V or Minimize $W = \frac{1}{V} = x_1 + x_2$

Subject to the following constraints

$$ax_1 + cx_2 \geq 1 \qquad \text{———— (A)}$$

$$bx_1 + dx_2 \geq 1$$

$$x_1, x_2 \geq 0$$

The objective of Player B is to minimise its expected losses.

This can be reduced by minimising the value of V i.e., it might lose less than V if player A adopts a poor strategy. Hence the expected loss for Player B will be as follows:

$aq_1 + bq_2 \leq V$ (if Player A adopts strategy 1)

$cq_1 + dq_2 \leq V$ (if Player A adopts strategy 2)

and

$q_1 + q_2 = 1$

$q_1, q_2 \leq 0$

Dividing the above throughout by V, we obtain

$$\frac{aq_1}{V} + \frac{bq_2}{V} \leq 1$$

$$\frac{cq_1}{V} + \frac{dq_2}{V} \leq 1$$

$$\frac{q_1}{V} + \frac{q_2}{V} = \frac{1}{V}$$

Define $\frac{q_1}{V} = y_1$ and $\frac{q_2}{V} = y_2$

Therefore we have,

$$ay_1 + by_2 \leq 1$$

$$cy_1 + dy_2 = 1$$

$$y_1 + y_2 = \frac{1}{V}$$

As minimising V is equivalent to maximising 1 ,the resulting linear programming problem can now be given as :

Minimize V or Maximise $\frac{1}{V} = y_1 + y_2$

Subject to the constraints

$$ay_1 + by_2 \leq 1$$
$$cy_1 + dy_2 \leq 1$$
$$y_1, y_2 \geq 0$$

Please note that problem (A) is the dual of problem (B). Therefore, solution of the dual problem can be obtained from the optimal simplex table of primal.

Activity 6

For the problem given in Example 6, determine the optimal strategies for each Firm and the value of the game using Linear Programming Method.

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14.6 2 x n GAMES

Games where one player has only two course of action whilst the other has more than two are called ‘ 2 x n ‘games. It tan be shown that; if such games do nail have saddle points, there Oasts, for the players with n courses of action, a combination of just two of these which provides his best strategy. The first part of the solution of such games involves identifying this within which lies the solution of the larger game.

Example 7: Determine which course of action Player B will not use in the following game. Obtain the best strategies for both players and the value of the game.

		Player B		
		I	**II**	**III**
Player A	**I**	-3	-1	7
	II	4	1	-2

Solution: Since there is no saddle point, and since no course of action dominates any other, we consider each 2x2 sub-game in turn and find its value. As it is player B who has to make the decision of which course of action to choose, he will select the pair of courses of action corresponding to the 2x'2 sub-game whose value is least.

Considering each 2x2 sub-game we obtain the following values:

		Player B	
		I	**II**
Player A	**I**	-3	-1
	II	4	1

This game has a saddle point, shown in bold type above , and its

Value V1 = 1

(b)

		Player B	
		I	**II**
Player A	**I**	-3	7
	II	4	2

$$\text{The value} \quad V_2 = \frac{(-3)(-2) - 7*4}{(-3-2) - (7+4)} \qquad 1\frac{3}{8}$$

		Player B	
		I	**II**
Player A	**I**	-3	7
	II	4	2

$$\text{The value} \quad V_3 = \frac{(-1)(-2) - 7*1}{(-1-2) - (7+1)} = \frac{5}{11}$$

The2x2 sub-game with the lowest value is (c) and hence the solution to this game provides the solution to the larger game. Solving (c) by the method of Example 5, we find that A chooses I with probability 3/11, whilst B chooses II with probability 9/ 11. Thus the solution to the larger game is A plays (3/ 11, 8/ 11), B plays (0, 9/11, 2/ 11), and the value of the game is 5111.

The above method of solution is feasible manually provided "n" is small. It becomes laborious as "n" gets larger.

There are three 2x2 sub-games in a 2A game, 6 in 2x4, 10 in a 2x5, etc. when n is large; we can resort to computer either by writing simple programmes or by using a standard package.

14.7 EXPLOITING AN OPPONENT'S MISTAKES

We have so far considered the best strategies for both players in a two person game. The definition of "best strategy" implies that the game is being played against a rational opponent whose object is to maximise his own gain. It is possible for a player A to take advantage of the knowledge that player B is not using his best strategy.

Consider the following game of matching coins. Two players, A and B, each put down a coin. 11' the coins match, i.e. both are heads or both are tails, .A collects them both otherwise B collect them both. The pay-off matrix for this game is given below:

		Player B	
		I (Heads)	II (Tails)
	1(Heads)	2	-1
Player A	II(Tails)	-1	1

Player A's strategy of (Iq2) is optimal against a shrewd opponent in that it protects him against loss. II', however, B is observed to play "heads" more frequently than "tails", A can increase his gain by also playing "heads" more frequently than "tails". If, for example, B plays "heads" twice as often as "tails", i.e. a strategy of (2/3, 1/3) A increases his gain by also choosing for his strategy, (2/3, 1/3),

The gain to A under these circumstances being

2/3 (1/3) + 1/3 (-1/3), i.e. 1/9. It can be shown that if the opponent's strategy is known in advance, a player achieves his maximum gain by employing a pure strategy. Thus in this example, A could increase his gain to one-third by playing heads throughout. Unfortunately, if A did this, B would almost certainly notice and be -led to modify his own strategy. The problem for A in practice is to exploit B's blunders without showing him the error of his ways.

In the foregoing discussion, the concept of a number .of mixed strategy implies that a game is to be played a number of times. What of games that are played only onces? It is true that the implication has been that games are repeatable. The reason for this is that in a mixed strategy it is intuitively easier to imagine the probabilities of the various choices as the frequencies with which each course of action would be chosen if the game were repeated many times. The meaning of the term "expected gain" is also more readily understood under this assumption. In fact, the results hold whether a game is played only once or repeated many times. If a game is played only once and the solution suggests a mixed strategy for a particular player, this implies that he must not in any event, disclose in advance his proposed course of action. By using a chance device the player ensures that he chooses a course of action that his opponent cannot anticipate. The probabilities given by his mixed strategy determine the characteristics of the chance device he should use.

Activity 7

In managerial economics, you have studied price determination under duopoly. Can you apply game theory in this problem if you are one of the duopolist.(Assume you can know in advance the strategy of the opponent.)

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Example 8: A has two ammunition stores, one of which is twice as valuable as the other. B is an attacker who can destroy an undefended store but he can only attack one of them. A can successfully defend only one of them. A learns that B is about to attack one of the stores but does not know which. What should he do?

Solution: The value of the smaller store is 1, the value of the larger store is 2.

If both stores survive, A loses nothing.

If only the larger survives, A loses 1.

If only the smaller survives; A loses 2.

Since both parties have only two possible courses of action, A's pay-off matrix is as follows:

		B	
		Attack smaller store	Attack larger store
A	Defend smaller store	0	-2
	Defend Larger store	-1	0

Before analysing this problem as a game, let us consider how A and B might othe4ise reason in the real situation. A would argue that he cannot afford to take the risk of losing the larger ammunition store and therefore he must defend this one. -if B is shrewd, however, he would anticipate A's reasoning and attack the smaller .store with a certain gain of 1. Thus A's "logic" would result in a gain of- 1.

The problem may be solved by the methods outlined earlier. A's best strategy is the mixed strategy (4/3, 2/3) and the value of the game is -2 3. In order to make his decision, therefore, A should roll a die. He should defend the smaller store if a 1 or a 2 turns up; otherwise he should defend the larger one. This strategy leads to more favourable expected outcome than the "logic" mentioned above.

Having illustrated how a "one-off" decision should be reached, we know that decision in pratice are not made in the above manner. An executive would probably not inspire confidence in his ability if it were known that he made his decision by selecting a number at random; and what manager would ascribe to the toss of a chin the credit for a decision which proved successful---or catastrophic? There is an important difference between games and many of the decisions that have to be made in practice.

By definition, in a game nothing is known of an opponent's strategy, save that he is an extremely shrewd player. In a decision-making situation this is not generally so. Even where there is an component the decision is often based on some knowledge of his likely strategy.

14.8 SUMMARY

At the outset, a very comprehensive introduction to the theory of games has been provided with a clear conceptual framework and definition of important terms used in games. The pay-off matrix has been explained through an example.

The next point in discussion has been the concept of saddle point-an element of the matrix which is both the smallest element in its row and the largest element in its column. We have also mentioned that saddle point occurs in the case of pure strategies adopted by both the -players. Examples have worked out to illustrate the method.

Mixed strategies have been covered as situations where the saddle points do not exist. The concept is well brought out through an example. The algebraic solution giving the mixed strategies for the players, and the value of the game has been derived so as to enable you to use the formul4s for a given problem.

The principle of dominance and its -importance in reducing the size of a game's pay-off matrix has been dealt with through an example.

2xn game has been the next topic of discussion. In this, one player has only two options while the other has more than two courses of action. The solution of such games involves identifying combinations of 2x2 sub-games within which lies the solution of the larger game. One numerical example has been worked out to bring the essence.

Exploiting the opponent's mistake is very crucial-in business involving competition where the rival firms can thrive on their opponent not using his best strategy. The relevant aspects have been succinctly explained through an example.

14.9 SELF-ASSESSMENT EXERCISE S

1. Solve the following zero-sum two-person games. Obtain the best strategies for both players and the value of the games:

		Player B	
		I	**II**
Player A	**I**	1	0
	II	0	2

		Player B				
		I	II	III	IV	V
	I	4	0	1	7	-1
	II	0	-3	-5	-7	5
Player A	III	3	2	3	4	3
	IV	-6	1	-1	0	5
	V	0	0	6	0	0

Determine the optimal strategies for each firm and the value of the game.

2. Two players A and B match the coins. If the coins match, then A wins. Otherwise he loses. Determine the optimum strategies for the players and the value of the game.

3. "Game theory provides a systematic quantitative approach for analysing competitive situations in which the competitors make use of logical process and techniques in order to determine an optimal strategy for winning." Comment.

4. Describe the strengths and limitations of game theory.

5. Two companies A and B are competing for the same product. Their different strategies are given in the following pay-off matrix:

		B1	B2	B3
Company A	A1	2	-2	3
	A2	-3	5	-1

Determine the best strategies and find the value of the game.

5. Reduce each of the following games by using the rule of dominance and then solve the reduced game by any of the method you have studied:

(a)

	B1	B2	B3
A1	3	8	5
A2	6	2	7
A3	4	5	6

(b)

	B1	B2	B3	B4	B5
A1	8	7	6	-1	2
A2	12	10	12	0	4
A3	14	6	8	14	16

7 The following is the pay-off table for a zero-sum two-person game, with the pay-off being the amounts player B losses to player A

	Player B: I	II
Player A	4	-3
	-2	2

Determine the best strategies and find the value of the game.

14.10 FURTHER READINGS

Sasieni, M., Yaspan, A., and Friedman, L. (1959)-Operations Research-*Methods and Problems.* John Wiley & Sons, Inc., New York. .

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