UNIT 7 GOAL PROGRAMMING

Objectives

After completion of this unit, you should be able-to:

- Explain the concept of goal programming,
- Formulate business problems involving multiple goals as goal programming problems,
- Describe the simplex procedure used in modified form to solve the goal programming problem.

Structure

7.1 Introduction
7.2 Concepts of goal programming
7.3 Goal programming model formulation
7.4 Graphical method of goal programming
7.5 The simplex method of goal programming
7.6 Application areas of goal programming
7.7 Summary
7.8 Self-assessment exercises
7.9 Further Readings

7.1 INTRODUCTION

Organizational objectives vary according to the characteristics, types, philosophy of management, and particular environmental conditions of the organization. There is no single universal goal for all organizations. Profit maximization, which is regarded as the sole objective of the business firm in classical economic theory, is one of the most widely accepted goals of management. However, in today's dynamic business environment, profit maximization is not the only objective of management. In fact, business firms do on quite a few occasions place higher priorities on goals other than profit maximization. We have seen, for example, firms place great emphasis on social responsibilities, social contributions, public relations, industrial and labour relations, etc. Such goals are sought because of outside pressure or voluntary management decisions. Non economic goals exist and are gaining greater significance.

Thus, we see management has multiple conflicting objectives to achieve in the present day business scenario. This implies that the decision criteria should be multidimensional and the decision involves multiple goals. In this context, goal programming assumes greater importance as a powerful quantitative technique capable of handling multiple decision criteria.

7.2 CONCEPTS OF GOAL PROGRAMMING

Linear programming technique that has been applied quite extensively to various decision problems has a limited value for problems involving multiple goals. The difficulty in using linear programming lies in the unidimensionality of the objective function, which requires maximization or minimization of one objective function - an over simplification of reality! However, in real life situations multiple goals and subgoals must be taken into account. Various goals may be expressed in different units of measurement such as rupees, hours, tons, etc. Often multiple goals of management are in conflict or are achievable only at the expense of other goals. Furthermore these goals are incommensurable. Thus, the solution of the problem requires an establishment of a hierarchy of importance among these incompatible goals so that the low-order goals are considered only after the higher-order goals are satisfied or have reached the point beyond which no further improvement are desired. If management can provide an ordinal ranking of goals in terms of their contributions or importance to the organization and all goal constraints are in linear relationships, the problem can be solved by goal programming.

How is an objective function to be determined and expressed in algebraic form when there exist multiple, incommensurable, and conflicting goals? Goal programming provides an answer to- this. In 8oul programming, instead of trying to maximize or minimize the objective criterion directly as in linear programming, deviations between goals and what can be achieved within the given set of constraints are to be minimized.

In goal programming, the objective function contains primarily the deviational variables that represent each goal or subgoal. The deviational variable represented in two dimensions in the objective function, a positive and a negative deviation from each subgoal and/or constraint. Then, the objective function becomes the minimization of these deviations, based on the relative importance or priority assigned to them.

The solution of linear programming is limited by quantification. Unless the management can accurately quantify the relati9nship of the variables in cardinal numbers, the solution is only as good as the inputs. The distinguishing characteristic of goal programming is that it allows for ordinal solution. 'Stated differently management may be unable to specify the cost or utility of u goal or a subgoal, but often upper or lower limits may be stated for each subgoal. The manager is usually called upon to specify the priority of the desired attainment of each goal or subgoal and rank them in ordinal sequence for decision analysis. The true value of goal programming is, therefore, the solution of problems involving multiple, conflicting goals according to the manager's priority structure.

Activity 1

Describe the specialities of goal programming

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7.3 GOAL PROGRAMMING MODEL FORMULATION

It is important to consider formulation of the model before getting into the details of goal programming solution. Infact, the most difficult problem in the application of management science to decision analysis is the formulation of the model for the practical problem in question. Model formulation is the process of transforming a real-world decision problem into a management science model. While the advent of task, micro-computers the powerful problem formulation requires a great deal of efforts in terms of conceptualization.

In order to provide some experience and insight into formulating and analyzing a goal programming problem, typical examples will be provided in this unit. One key to successful application of goal programming is the ability to recognize when a problem can be solved by goal programming and to formulate the corresponding model.

Example 1

The manager of the only record shop in a town has a decision problem that involves multiple goals. The record shop employs five full-time and four part-time salesmen. The normal working hours per month for a full-time salesman and a part-time salesman are 160 hours and 80 hours respectively. According to the performance records of salesmen, the average sales has been five records per hour for full-time salesmen and two records per hour for part-time salesmen. The average hourly wage rates are Rs. 3 for full-time salesmen and Rs. 2 for part-time salesmen, Average profit from the sales of a record is Rs. 1.50.

In view of past sales record and increased enrollment at the local college, the manager feels that the sales goal for the next month should be 5500 records. Since the shop is open six days a week, overtime is often required of salesmen (not necessarily overtime but extra hours for the part-time salesmen). The manager believes that a good employer-employee relationship is an essential factor of business success.

Therefore, he feels that a stable employment level with occasional overtime jai Prognmming requirement is a better practice than an unstable employment level with no overtime. However he feels that overtime of more than 100 hours among the full-time salesmen should be avoided because of the declining sales effectiveness caused by fatigue.

The manager has set the following goals:

1) The first goal is to achieve a sales goal of 5500 records for the next month.
2) The second goal is to limit the overtime of full-time salesmen to 100 hours.
3) The third goal is to provide job security to salesmen. The manager feels that full utilization of employees' regular working hours (no layoffs) is an important factor for a good employer-employee relationship. However, he is twice as concerned with the full utilization of full-time salesmen as with the full utilization of part-time salesmen.
4) The last goal is to minimize the sum of overtime for both full-time and part-time salesmen. The manager desires to assign differential weights to the minimization of overtime according to the net marginal profit ratio between the full-time and part-time salesmen.

Based on the problem stated above, the following constraints can be formulated.

1. Sales Goal

Achievement of the sales goal, which is set at 5500, is a function of total working hours of the full-time and part-time salesmen and their productivity (sales per hour) rates.

$$5x_1 + 2x_2 + d_1^- - d_1^+ = 5500$$

where x_1 = Total full-time salesmen hours/month

x_2 = Total part-time salesmen hours/month

d_1^- = Underachievement of sales goal

d_1^+ = Overachievement of sales goal

5 = Sales per hour for full-time salesmen

2 = Sales per hour for part-time salesmen

5500 = Sales goal for month

2. Regular Working Hours

Salesmen hours are determined by the regular working hours for each type of salesman and the number of full-tune and part-time salesmen employed. With the full-time salesmen the total regular working hours per month will be 5 x 160 = 800. For part-time salesmen, the total salesmen hours/month will be 4 x 80 = 320. Thus we have:

$$x_1 + d_2^- - d_2^+ = 800$$

$$x_2 + d_3^- - d_3^+ = 320$$

where d_2^- = Underutilization of the total regular full-time salesmen hours per month

d_2^+ = Overtime given to full-time salesmen/month

d_3^- = Underutilization of the total regular part-time salesmen hours/month

d_3^+ = Extra working hours given to part-time salesmen

3. Overtime

In this record shop example, the manager's second goal is to limit the overtime of full-time salesmen to 100 hours. we do not have a deviational variable to minimize in order to achieve this goal in the above formulated constraints . Therefore, we must introduce a new constraint. Note that the manager listed the minimization of overtime for full-time salesmen as a part of the fourth goal in essence , he has two separate goals regarding the overtime work of full-time salesmen. To limit the overtime for full-time salesmen as a part of the fourth goal, In essence,, he has two separate goals regarding the overtime work of full-time salesmen. To limit the

$$d_2^+ + d_{21}^- - d_{21}^+ = 100$$

where d_2^+ = Actual overtime of full-time salesmen

d_{21}^- = Difference between the actual overtime of full-time salesmen and desired 100 hours of overtime

d_{21}^+ = Overtime in excess of desired 100 hours.

We introduced both the negative and positive deviation from the allowed 100 hours of overtime because the actual overtime can be less than, equal to, or even more than 100 hours. Now, we have a deviational variable to minimize to achieve the second goal, i.e., d^+_{21}.

The goal programming model for the above problem is thus fomulated below:

Minimize $Z = P_1d_1^- + P_2d_{21}^+ + 2P_3d_2^- + P_3d_3^- + 3P_4d_3^+ + P_4d_2^+$

Subject to

$$5x_1 + 2x_2 + d_1^- - d_1^+ = 5500$$
$$x_1 + d_2^- - d_2^+ = 800$$
$$x_2 + d_3^- - d_3^+ = 320$$
$$d_2^+ + d_{21}^- - d_{21}^+ = 100$$
$$x_1, x_2, d_1^-, d_2^-, d_3^-, d_{21}^-, d_1^+, d_2^+, d_3^+, d_{21}^+ \geq 0.$$

In the above model, the differential weight of 3 is assigned to d3 at the P_4 level on the basis of net marginal profit ratio per hour between the full-time and part-time salesmen. The productivity ratio (sales per hour) between the full-time and part-time salesmen is 5 to 2, while the hourly wage rate for overtime is Rs. 4.50 and Rs. 2.00. (We have assumed here that a full-time saleman will charge 50% of the normal wage rate/hour as extra charges for overtime, i.e., Rs. 3 + Rs. 1.50 = 4.50). The marginal profit per hour of overtime is Rs. 3 hour the full-time salesmen and Re. 1 for the part-time salesmen. The relative cost of an hour of overtime for the part-time salesmen is three times that of the full-time salesmen. Therefore, $3P_4$ is assigned to d3 where as P_4 is assigned to d2.

Example 2

A textile company produces two types of materials A and B. The material A is produced according to direct orders from furniture manufacturers. The material B, is distributed to retail fabric stores. The average production rates for the material A and B are identical at 1000 metres/hour. By running two shifts the operational capacity of the plant is 80 hours per week.

The marketing department reports that the maximum estimated sales for the following week is 70000 metres of material A and 45000 metres of material B. According to the accounting department the profit from a metre of material A is Rs. 2.50 and from a metre of material B is Rs. 1.50.

The management of the company decides that a stable employment level is a primary goal for the firm. Therefore, whenever there is demand exceeding normal production capacity, the management simply expands production capacity by providing overtime. However, management feels that overtime operation of the plant of more than 10 hours per week should be avoided because of the accelerating costs. The management has the following goals in the order of importance:

1) The first goal into avoid any underutilization of production capacity (i.e., to maintain stable employment at normal capacity).
2) The second goal is to limit the overtime operation of the plant to 10 hours.
3) The third goal is to achieve the sales goals of 70000 metres of material A and 45000 metres of material B.
4) The last goal is to minimize the overtime operation of the plant as much as possible.

Formulate this as a goal programming problem to help the management for the best decision.

Production Capacity

The production capacity is limited to 80 hours at present by running two- shifts. However, since overtime of the plant is allowed to a certain extent, the constraint may be written as:

$$x_1 + x_2 + d_1^- - d_1^+ = 80$$

where x_1 = Number of hours used for producing material A

x_2 = Number of hours used for producing material B

d_1^- = Underutilization of production capacity as set at 80 hours of operation

d_1^+ = Overutilization of normal production capacity beyond 80 hours.

Sales Constraints

In this case, the maximum sales for material A and material B are set at 70,060 and 45,000 metres, respectively. Hence, it is assumed that overachievements of sales beyond the maximum limits are impossible. Then, the sales constraints will be (as before, k, and x_2 are expressed in thousands):

$$X_1 + d_2^- = 70$$

$$X_2 + d_3^- = 45$$

where d_2^- = underachievement of sales goal of material A

d_3^- = underachievement of sales goal of material B.

Overtime Operation Constraint

From the case itself, only production and sales constraints can be formulated. However, the analysis of goals indicates that overtime operation of the plant is to be minimized to 10 hours or less. To solve the problem by goal programming, we need a deviational variable that represents the overtime operation of the plant beyond 10 hours. By minimizing this deviational variable to zero we can achieve the goal. Since there is no such deviational variable in the three constraints presented above, we must create a new constraint.

The overtime operation of the plant d_1+, should be limited to 10 hours or less. However, it may not be possible to limit the overtime operation to 10 hours or less in order to meet higher order goals. Therefore, d+ can be smaller than, equal to or even greater than 10 hours. By introducing some new deviational variables, a constraint regarding overtime can be expressed as:

$$d^+_1 + d^-_{12} - d^+_{12,} = 10$$

where d^-_{12} = negative deviation of overtime operation from 10 hours.

d^+_{12} =overtime operation beyond 10 hours.

One metre of material A gives a profit of Rs.,2.50 and one metre of B gives a profit of Pa. 110, Since the production rate is same for both A an d B namely 1000 metres per hours, the hourly profits of A and B are in the ratio of 2.50: 1.50 or 5 : 3. Hence it is appropriate to assign these as differential weights in goal 3. The differential weights imply that management is relatively more concerned with the achievement of the sales goal for material A than that for material B.

Now, the model can be formulated as:

$$\text{Minimize } Z = P_1 d_1^- + P_2 d_4^+ + 5P_3 d_2^- + 3P_3 d_3^- + P_4 d_1^+$$

Subject to

$$x_1 + x_2 + d_1^- - d_1^+ = 80$$

$$X_1 + d_2^- = 70$$

$$X_2 + d_3^- = 45$$

$$d_1^+ + d_{12}^- - d_{12}^+ = 10$$

$$x_1, x_2, d_1^-, d_2^-, d_3^-, d_1^+, d_{12}^-, d_{12}^+ \geq 0.$$

Example 3

XYZ computer company produces three different types of computers: Epic, Galaxie, and Utopia. The production of all computers is conducted in a complex and modern assembly line. The production of an Epic requires five hours in the assembly line, a Galaxie requires 8 hours, and a Utopia requires 12 hours, The normal operation line of the assembly are 170 hours per month. The marketing and accounting departments have estimated that profits per unit for the three types of computers are Rs. 100000 for the Epic, Rs. 144000 for the Galaxie, and Rs. ,252000 for the Utopia. The marketing department further reports that the demand is such that the firm can expect to sell all the computers it produces in the month.

The president of the firm has established the following goals according to their importance:

1) Avoid underutilization of capacity in terms of regular hours of operation of the assembly line.
2) Meet the demand of the north eastern sales district for five Epics, five Galaxies, and eight Utopias (differential weights should be assigned according to the net profit ratios among the three types of computers).
3) Limit overtime operation of the assembly line to 20 hours.
4) Meet the sales goal for each type of computer: Epic, 10; Galaxie, 12; and Utopia, 10 (again assign weights according to the relative profit function for each computer).
5) Minimize the total overtime operation of the assembly line.

With the practice and experience gained from the previous two examples, we can easily set up a goal programming model for the above problem.

1. Normal Operation Capacity of the Assembly Line .

The normal capacity of the assembly line for the month is 170 hours. With this capacity, the firm produces three types of computers. The total operation hours required to produce the computers will simply be a function of production rate (in number of hours) for a unit of each type of computer. Now, we can formulate the normal operation capacity of the assembly line as:

$$5x_1 + 8x_2 + 12x_3 + d_1^- - d_1^+ = 170$$

where x_1 = number of Epic computers

x_2 = number of Galaxie computers

x_3 = number of Utopia computers

d_1^- = underutilization of normal operation hours of the assembly line

d_1^+ = overtime operation of the assembly line

2. Sales Constraint

First, the firm has outstanding orders from the north-eastern sales district as follows:

$$x_1 + d_2^- - d_2^+ = 5$$
$$x_2 + d_3^- - d_3^+ = 5$$
$$x_3 + d_4^- - d_4^+ = 8$$

The firm also has sales goal for the month, which are:

$$x_1 + d_5^- - d_5^+ = 10$$
$$x_2 + d_6^- - d_6^+ = 12$$
$$x_3 + d_7^- - d_7^+ = 10$$

3. Overtime Operation of the Assembly Line

We have already mentioned that frequently we have to introduce new constraints in order to define deviational variables that we must minimize to achieve certain goals. The president listed the minimization of -overtime operation of the assembly line to 20 hours. As we do not have a deviational- variable to minimize in order to achieve this goal, we introduce this following constraints:

$$d_1^+ + d_{11}^- - d_{11}^+ = 20$$

where d_{11}^- = the difference between the actual overtime operation of the assembly line and the allowed hours of overtime operation.

d_{11}^+ = overtime operation of the plant in excess of 20 hours.

Now, the above problem can be formulated as a goal programming model.

$$\text{Min } Z = p_1 d_1^- + 20\, p_2 d_2^- + 18\, p_2 d_3^- + 21\, p_2 d_4^- + p_3 d_{11}^+ + 20\, p_4 d_5^- + 18\, p_4 d_6^- + 21\, p_4 d_7^- + p_5 d_1^+$$

Subject to:

$$5x_1 + 8x_2 + 12x_3 + d_1^- - d_1^+ = 170$$
$$x_1 + d_2^- - d_2^+ = 5$$
$$x_2 + d_3^- - d_3^+ = 5$$
$$x_3 + d_4^- - d_4^+ = 8$$
$$x_1 + d_5^- - d_5^+ = 10$$
$$x_2 + d_6^- - d_6^+ = 12$$
$$x_3 + d_7^- - d_7^+ = 10$$
$$d_1^+ + d_{11}^- - d_{11}^+ = 20$$
$$x_1, x_2, x_3, d_1^-, d_2^-, d_3^-, d_4^-, d_5^-, d_6^-, d_7^-, d_{11}^-, d_1^+, d_2^+, d_3^+, d_4^+, d_5^+, d_6^+, d_7^+, d_{11}^+ \geq 0.$$

In the objective function of the above model, we note differential weights assigned to the second and fourth priority factors. We remember that the criterion used for the weights was the net profit ratio among the three types of computers. Here, we have an assumption that the cost of operating the assembly line is proportional to which computer the line is producing. The net profit ratio, therefore, will simply be determined by dividing the profit by operation hours required to produce each type of computer. For the Epic, the profit is Rs. 100,000 per unit and it requires five hours of assembly operation. Hence, the profit per hour of assembly operation for Epic is Rs. 20,000. Similarly, profits per hour of assembly operation for the Galaxie and Utopia will be Rs. 18,000 and Rs. 21,006 respectively. The differential weights are based on these figures.

Activity 2

In company XYZ, the management specifies overtime to be restricted to a maximum of 25 hours. Write an appropriate goal constraint incorporating the deviational variables.

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Activity 3

Company XYZ, produces two products. The maximum sales potential for product 1 and product 2 are 30 units and 40 units respectively. Write the goal constraints for achieving the sales goal by incorporating the deviational variables.

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Activity 4

In example 3 under formulation, if the hourly profits are changed into hourly costs for the products, how will the objective function change? Write the changed objective function.

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Steps of Goal Programming Model Formulation

Thus far we have illustrated model formation of goal programming problems with relatively simple examples. The steps we have taken in the model formulation can be briefly summarized as follows:

1. Define Variables and Constants

The first step id model formation is the definition of'c4oice variables and the right hand side constants. The right hand side -constants may bi either available resources or specified goal levels. It requires a careful analysis of the problem in order to identify all relevant variables that have some effect on the set of goals stated by the decision maker.

2. Formulate Constraints

Through an analysis of the relationships among choice variables and their relationships to the goal , a set of constraints should be formulated .A constraint may be either a system constraint that define the relationship between choice variables and the goals . it should be remembered that if there is no deviational variable to minimize in order to achieve in a certain goal , a new constraint must be created . Also , if further refinement of goals and priorities is required , it may be facilitated by decomposing certain deviational variables.

3. Develop the Objective Function

Through the analysis of the decision marker's goal structure ,the objective function must be developed . First , the preemptive priority factors should be assigned to certain deviational variables that are relevant to goal attainment . Second , if necessary differential weights must be assigned to deviational variables at the same priority level . It is imperative that goals at the same priority level be commensurable.

7.4 GRAPHICAL METHOD OF GOAL PROGRAMMING

The graphical method of solving goal programming problem is quite similar to the graphical method of linear programming. In linear programming, the method is used to maximize or minimize an objective function with one goal, whereas in goal programming, it is used to minimize the total deviation from a set of multiple goals. The deviation from the goal with the highest priority is minimized to the fullest extent possible before deviation from the next goal is minimized. The graphical method is illustrated with the help of the following example.

Example

A manufacturing firm produces two types of product - A and B. According to past experience, production of either Product A or Product B requires an average of one hour in the plant.. The plant has a normal production capacity of 400 hours a month. The marketing department of the firm reports that because of limited market, the maximum number of Product A and Product B that can be sold in a month are 240 and 300 respectively. The net profit from the sale of Product A and Product B are Rs. 800 and lbs. 400 respectively. The manager of the firm has set the following goals arranged in the order of importance (preemptive priority factors).

P_1 : He wants to avoid any underutilization of normal production capacity.

P_2 : He wants to sell maximum possible units of Product A and Product B. Since the net profit from the sale of Product A is twice the amount from that of Product B, the manager has twice, as much desire to achieve sales for Product A as for Product B.

P3: He wants to minimize the overtime operation of the plant as much as possible.

Solution

Let x_1 and x_2 be the numbers of units of Product A and Product B to be produced respectively. Since overtime operation is allowed, the plant capacity constraint can be expressed as:

$$x_1. + x_2 + d_1^- - d_1^+ = 400$$

where d_1^- is the underutilization of production capacity and d_t is the overtime operation of the normal production capacity. Since the sales goals given are the maximum possible sales volume, positive deviations will not appear in the sales constraints. The sales constraints care be expressed as:

$$x_1 + d_2^- = 240$$

$$x_2 + d_3^- = 300$$

where d_2^- = underachievement of sales goal for product A

d_3^- = underachievement of sales goal for product B.

Now, the complete model can be written as:

Minimize $Z = P_1 d_1^- + 2P_2 d_2^- + P_2 d_3^- + P_3 d_1^+$

Subject to

$$x_1 + x_2 + d_1^- - d_1^+ = 400$$
$$x_1 + d_2^- = 240$$
$$x_2 + d_3^- = 300$$
$$x_1, x_2, d_1^-, d_2^-, d_3^-, d_1^+ \geq 0.$$

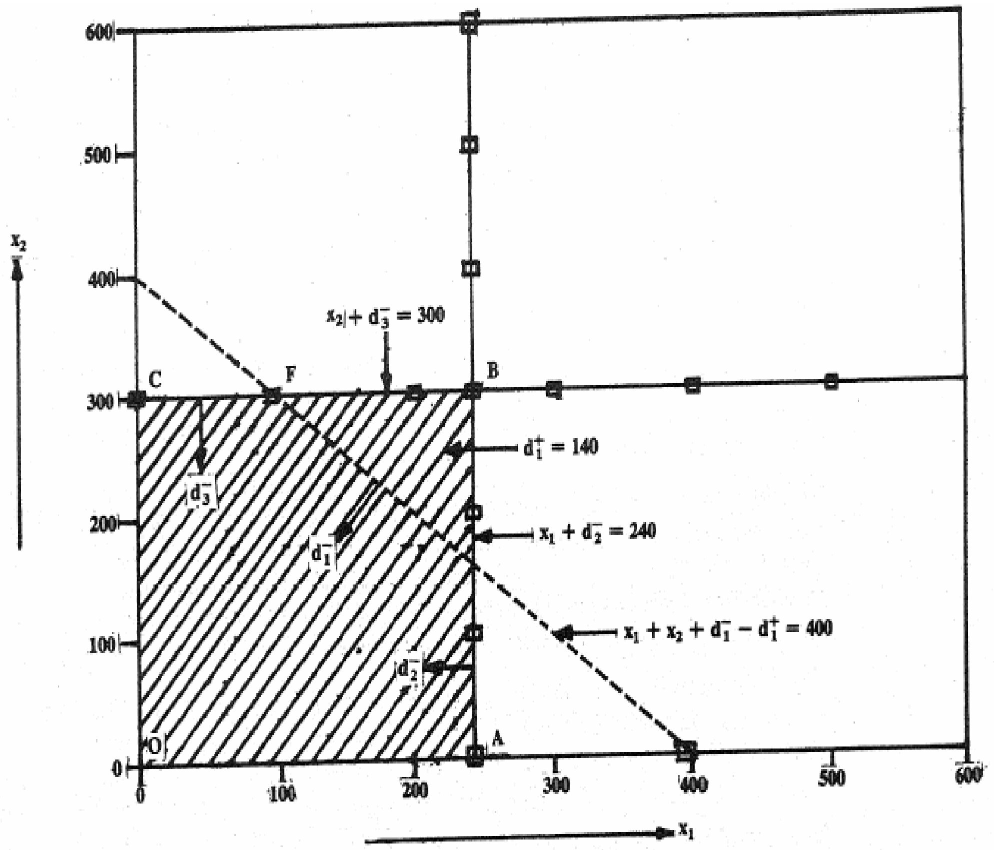

Figure 7.1 : Goal Constraints and Graphical Solution

The first step in solving the problem by graphical method is to plot all the goal constraints on a graph as displayed in figure 7.1 since the underutilization and overutilization of the plant capacity is permissible, the deviational variable $d_1^-$and d_1^+ , are indicated by arrows in figure 7.1 . Similarly d_2^- and d_3^- , are also denoted by arrows in the diagram. Once all the goal constraints have been plotted on the graph the feasible region is confined to the shaded portion OABC.

The next step is to analyse each goal in the objective function in search of the optimal solution in the feasible region. The first goal is completely achieved at line EF in the reduced feasible region EBF. Now, the manager would like to move to priority level 2, where the differential weight for product A is two times that of B (assigned in the ratio of the profits). The sales goal of 'product A is completely achieved at line EB in the feasible region E3F. The sales goal of product B is fully met at line FB in the feasible region EBF. So, we see that the first two goals are completely achieved at point B in the feasible region EBF. The third goal on overtime operation cannot

The best way to explain the simplex method of goal programming is through an example. So, let us take example 2 under goal programming model formulation (Refer 7.3).

be achieved without dispensing with the first two goals in the priority order . The optimal solutions to the problem is given as under:

$$x_1 = 240,\ x_2 = 300,\ d_1^- = 0,$$
$$d_2^- = 0,\ d_3^- = 0,\ d_1^+ = 140$$

7.5 THE SIMPLEX METHOD OF GOAL PROGRAMMING

The best way to explain the simplex method of goal programming is through an example. So, let us take example 2 under goal programming model formulation (Refer 7.3)

$$\text{Minimize } Z = P_1 d_1^- + P_2 d_4^+ + 5P_3 d_2^- + 3P_3 d_3^- + P_4 d_1^+$$

Subject to

$$x_1 + x_2 + d_1^- - d_1^+ = 80$$
$$X_1 + d_2^- = 70$$
$$X_2 + d_3^- = 45$$
$$d_1^+ + d_{12}^- - d_{12}^+ = 10$$
$$x_1, x_2, d_1^-, d_2^-, d_3^-, d_1^+, d_{12}^-, d_{12}^+ \geq 0.$$

Before the solution by the simplex method is presented for the goal programming problem, a few points to be observed are given below:

First, in goal programming the purpose is to minimize the unattained portion of goal as much as possible. This is achieved by minimizing the deviational variables through the use of certain preemptive priority factors and differential weights.' Since there is no profit maximization or cost minimization in the objective function, the preemptive factors and differential weights represent q values.

Second, it should be remembered that preemptive priority factors are oridinal weights and they are not commensurable. Consequently, Z_j or (Z_j - C_j) cannot be expressed by a single row as in linear programming .Rather the simplex criterion becomes a matrix of (m x n) size, where m represents the number of preemptive priority factors and n is the number of variables (inclusive of decision and deviational variables).

Third, since the simplex criterion Z_j - C_j is expressed-as a matrix rather than a row, a new procedure must be devised for identifying the key column. Again since $P_j >> P_{j+1}$ the selection procedure of the column must be initiated from P_j and move gradually to the lower priority levels.

The solution to this problem by simplex method is explained below: Table 5.3

Cj			0	0	P_1	$5P_3$	$3P_3$	0	P_4	P_2
C_B	X_B	b	x_1	x_2	d_1^-	d_2^-	d_3^-	d_{12}^-	d_1^+	d_{12}^+
P_t	d_1^-	80	1.	1	1	0	0	0	-1	0
$5P_3$	d_2^-	70	1	0	0	1	0	0	0	0
$3P_3$	d_3^-	45	0	1	0	0	1	0	0	0
0	d_{12}^-	10	0	0	0	0	0	1	1	-1
	P_4	0	0	0	0	0	0	0	-1	0
Zj-Cj	P_3	485	5	3	0	0	0	0	0	0
	P_2	0	0	0	0	0	0	0	0	-1
	P_1	80	1	1	0	0	0	0'	-1	0

The initial simplex table is presented in Table 5.3. The first four rows of the table are set up in the same way as for L.P. problem, with the coefficients of the associated variables placed in the appropriated entries. Below the thick line which separates the constraints from the objective function, there are four rows and each row stands for a priority goal level.

Note that the optimal criterion (Z, - Q is a 4 x 8 matrix because there are four priority factors and eight variables (two decision .variables and six deviational variables) in the model. Preemptive priority goals are written in basic variables column X_B below the thick line from the lowest at the top to the highest at the bottom.

It should be apparent that the selection of the key column is based on the per unit contribution rate of each variable in achieving the goals. When the first goal is completely attained, then the key column selection criterion moves on to the second goal, and so on. This is why the preemptive priority factors are listed from the lowest to the highest so that. the key column can be easily identified at the bottom of the table. To make the simplex table relatively simple, the Z matrix is omitted.

In goal programming the Z_i values'($P_4 = 0$, $P_3 = 485$, $P_2 = 0$ and $P_l = 80$) in the resources column (X_B) represents the unattained portion of each goal.

The key column, would be determined by selecting the largest positive element in $Z_j - C_j$ row at the P_l level as there exists an unattained portion of this highest goal ($P_i = 80$). There are two identical positive Z_j - C; values in the X_l and X_2 columns. In order to break this tie, we check the next flower priority levels. Since at the priority 3, the largest element is 5 in a row, therefore x_l becomes the key column. The values of $Z_j - C_j$ are computed as follows:

$Z_j - C_j$ = (Elements in C_B column)* (corresponding elements in X columns) -.C_J (priority factors assigned to deviational variables).

For example

$Z_1 - C_1 = P_1 * 1 + 5P_3 * 1 + 3P_3 * 0 + 0 * 1 = P_1 + 5P_3$ for column x_1

Similarly,

$\dot{Z}_2 - C_2 = P_1 + 3P_3$ for column x_2

$Z_3 - C_3 = Z_4 - C_4 = Z_5 - C_5 = Z_6 - C_6 = 0$

for columns $d_1^-, d_2^-, d_3^-, d_{12}^-$

$Z_7 - C_7 = -P_1 - P_4$ for d_1^+ column.

$Z_B - C_B = -P_2$ for d_4^+ column.

Since P_I, P_{2i} P_3 and P_4 are not commensurable, we must list their coefficients separately in their rows in the simplex criterion ($Z_j - C_j$) as shown in. Table 5.3.

The key row is determined by selecting the minimum positive or zero value when values in the resources column (X_B)) are divided by the coefficients in the key column. In this problem the key row refers to d_2^- row as shown in Table 5:3. If there is a tie, then select a row that has the variable with the highest priority factor. Coefficient lying at the intersection of key row and key columns is called key element and in Table 5.3 the key element is circled.

By utilizing the usual simplex procedure, The table 5.3 is revised to obtained Table 5.4.

Again, Table 5.4 does not give the optimal solution as the resources column (X_B) indicates unattained portions of goals. Proceeding in the usual manner, Table 5.4 suggests that an improved solution can be obtained if negative deviational variable d1 is driven out and decision variable x_2 enters into the solution. The new improved solution is shown in Table 5.5.

The solution in Table 5.5 indicates that production of 70,000 metres of material A and 10,000 metres of material B is sufficient to achieve the first, second and fourth goals and the value of $d_3 = 35$ suggest that 35,000 metres of material is not achieved.

It is also observed that all the elements in P_l and P_2 are either zero or negative which indicates that the first two priorities are achieved. Therefore, to improve the solution, the selection of the key column is done at P_3 level. Since the only positive element 3 occurs at P_3 level which lies in d column. Thus d_1 enters into the solution and d-is driven out as shown in Table 5.5. Finally, Table 5.6 presents the optimal solution. The solution is optimal in the sense that it enables -the decision maker to attain his goals as closely as possible within the given decision environment and priority structure.

Table 5.4

C_j			0	0	P_1	$5P_3$	$3P_3$	0	P_4	P_2
C_B	X_B	b	x_1	x_2	d_1^-	d_2^-	d_3^-	d_{12}^-	d_1^+	d_{12}^+
P_1	d_1^-	10	0	(1)	1	−1	0	0	−1	0
0	x_1	70	1	0	0	1	0	0	0	0
$3P_3$	d_3^-	45	0	1	0	0	1	0	0	0
0	d_{12}^-	10	0	0	0	0	0	1	1	−1
	P_4	0	0	0	0	0	0	0	−1	0
Z_j-C_j	P_3	135	0	3	0	−5	0	0	0	0
	P_2	0	0	0	0	0	0	0	0	−1
	P_1	10	0	1	0	−1	0	0	−1	0

Table 5.5

C_j			0	0	P_1	$5P_3$	$3P_3$	0	P_4	P_2
C_B	X_B	b	x_1	x_2	d_1^-	d_2^-	d_3^-	d_{12}^-	d_1^+	d_{12}^+
0	x_2	10	0	1	1	−1	0	0	−1	0
0	x_1	70	1	0	0	1	0	0	0	0
$3P_3$	d_3^-	35	0	0	−1	1	1	0	1	0
0	d_{12}^-	10	0	0	0	0	0	1	(1)	−1
	P_4	0	0	0	0	0	0	0	−1	0
Z_j-C_j	P_3	105	0	0	−3	−2	0	0	3	0
	P_2	0	0	0	0	0	0	0	0	−1
	P_1	0	0	0	−1	0	0	0	0	0

The optimal solution is $x_1 = 70$, $x_2 = 20$, $d_1^+ = 10$, $d_3^- = 25$. In other words, the company should produce 70,000 meters of material A and 20,000 meters of material B with 10 hours of overtime of the plants, resulting in 25,000 meters of underachievement in the sales goal of material B. With this solution the president of the firm is able to attain his two most important goals completely, and the next twc goals as completely as possible under the given constraints.

In Table 5.6, since the third goal is not completely attained, there is a positive value in $Z_j - C_j$ at the P_3 level. We find it (3) in the d_{12}^+ column. Obviously, we can attain the third goal to a greater extent if we introduce d_{12}^+ in the solution. We find, however, a negative value (−1) at the higher priority level, i.e., at the P_2 level. This implies that if we introduce d_{12}^+, we would improve achievement of the third goal at the expense of achieving the second goal. Thus, we cannot introduce d_{12}^+ into the solution. The same logic applies to the d_{12}^- column, where we find a positive value at the P_4 level. Therefore, the rule is that if there is a positive element at a lower priority level in $Z_j - C_j$, the variable in that column cannot be introduced into the solution as long as there is a negative element at a higher priority level.

Table 5.6

C_j			0	0	P_1	$5P_3$	$3P_3$	0	P_4	P_2
C_B	X_B	b	x_1	x_2	d_1^-	d_2^-	d_3^-	d_{12}^-	d_1^+	d_{12}^+
0	x	20	0	1	1	−1	0	1	0	−1
0	x_1	70	1	0	0	1	0	0	0	0
$3P_3$	d_3^-	25	0	0	−1	1	1	−1	0	1
P_4	d_1^+	10	0	0	0	0	0	1	1	−1
	P_4	10	0	0	0	0	0	1	0	−1
Z_j-C_j	P_3	75	0	0	−3	−2	0	−3	0	3
	P_2	0	0	0	0	0	0	0	0	−1
	P_1	0	0	1	−1	0	0	0	0	0

Activity 5

What are the difference in the application of simplex procedure between goal programming and linear programming?

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7.6 APPLICATION AREA OF GOAL PROGRAMMING

The salient feature of goal programming is its capability to handle managerial problems that involve multiple incompatible goals according to their importance. If management is capable of establishing ordinal importance of goals in a linear decision system, the goal programming model provides management with the opportunity to analyse the soundness of their goal structure. In general, a goal programming model performs three types of analysis: (1) it-determines the input requirements to achieve a set of goals; (2) it determines the degree of attainment of defined goals with given resources; and (3) it provides the optimum solution under the varying inputs and goal structures. The goal programming approach to be taken should be carefully examined by the decision maker before he employs the technique. The most important advantage of goal programming is its great flexibility, which allows model simulation with numerous variation of constraints and goal priorities.

Every quantitative technique has some limitations. Some of these are inherent to all quantitative tools and some are attributed to the particular characteristics of technique. The most important limitation, of goal programming belongs to the first category. The goal programming model simply provides the best solution under the given set of constraints and priority structure. Therefore, if the decision maker's goal priorities are not on accordance with the organization objectives, the solution will not be the global optimum for the organization. For an effective application of goal programming - and for the matter of all mathematical techniques -, a clear understanding of the assumptions and limitations of the technique is a prerequisite. The application of goal programming for managerial decision analysis forces the detision maker to think of goals and constraints in terms of their importance to the organization.

It may be mentioned in the passing that the limitations of linear programming technique in terms of assumptions, namely proportionality, additivity, divisibility, and deterministic are attributable to goal programming also.

Goal programming can be applied to almost unlimited managerial decision areas. In the area of marketing, it can be applied to media planning and product mix decisions. In finance, it can be applied to portfolio selection, capital budgeting, and financial planning. In production, it can be applied to aggregate production planning and scheduling: In the academic field, it can be used in assigning faculty teaching schedules and for university admissions planning. In HRD area, it can be used for manpower planning. In public systems area, it can be applied to transportation systems and medical care planning. It maybe pointed out that this list is not an exhaustive one but only indicative of the typical potential areas in which goal programming can be effectively applied. -

7.7 SUMMARY

In this unit, at the outset, we have provided a brief introduction to goal programming as a powerful tool to tackle multiple and incompatible goals of any enterprise some of which may be non-economic in nature.

The next point in discussion has been on the concepts of goal programming. The distinction between goal programming and linear programming have been brought out. The distinguishing features of goal programming revolve around its ability to use the ordinal principle of preemptive priority structure of the goals of management which may be incommensurable.

The formulation of goal programming models with its steps have been covered with three typical and comprehensive examples. You would be in a position to conceptualize complex business problems after carefully going through these examples.

In the graphical method of solving the goal programming problem, one problem was formulated and solved graphically for a meaningful appreciation.

The modified simplex method employed for solving goal programming problem with its fine distinctions over linear programming situation has been elaborately dealt with. One practical example has been solved step by step to reach the final solution using the simple algorithm.

Then the application areas of goal programming in management problems have been briefly covered.

7.8 SELF-ASSESSMENT EXERCISES

1) What is goal programming? State clearly its assumptions.
2) Identify the major differences between linear programming and goal programming.
3) Explain the following terms:
   i) Deviational variables
   ii) Preemptive priority structure
   iii) Differential weights
   iv) Cardinal value and ordinal value
4) Identify the important areas where goal programming can be effectively applied.
5) An office equipment manufacturer produces two kinds of products, chairs and lamps. Production of either a chair or lamp requires 1 hour of production capacity in the plant. The plant has a maximum production capacity of 110 hours per week. Because of the limited sales capacity, the maximum number of chairs and lamps that can be sold are 6 and 8 per week, respectively. The gross margin from the sale of a chair is Rs. 80 and Rs. 40 for a lamp.

   The plant manager has set the following goals arranged in the order of importance:

   1) He wants to avoid any underutilization of production capacity.
   2) He wants to sell as many chairs and lamps as possible. Since the gross margin from the sale of chair is set at twice the amount of profit from a lamp, he has twice as much desire to achieve the sales goal for chairs as for lamps.
   3) He wants to minimize the overtime operation of the plant as much as possible.

   Formulate this as a goal programming problem and then solve both by graphical and simplex method.
6) The production manager faces the problem of job allocation among three of hit teams. The processing rate of the three teams are 5, 6, and 8 units per hour respectively. The normal working hours for each team are 8 hours per day. The production manager has the following goals for the next day in order of priority:
   1) The manager wants to avoid any underachievement of production level, which is set at 180 units of product.
   2) Any overtime operation of team 2 beyond 2 hours and team 3 beyond3 hours should be avoided.
   3) Minimize the sum of overtime.

Formulate this as a goal programming problem and solve the same by simplex method.

7.8 FURTHER RKABINGS

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