UNIT 6 ASSIGNMENT PROBLEM

Objectives

After studying this unit, you should be able to :

- formulate an assignment problem
- use Hungarian method for solving an assignment problem
- choose appropriate method for balancing and solving an unbalanced assignment problem
- introduce appropriate modification when some of the assignments are infeasible
- modify the assignment problem when the objective is to maximise the objective function
- formulate and solve the crew assignment problem.

Structure

6.1 INTRODUCTION

The assignment problem in the general form can be stated as follows : Given n facilities, n jobs and the effectiveness of each facility for each job, the problem is to assign each facility to one and only one job in such a way that the measure of effectiveness is optimised (Maximised or Minimised).

Several problems of management has a structure identical with the assignment problem. A departmental head may have five people available for assignment and five jobs to fill. He may like to know which job should be assigned to which person so that-all these tasks can be accomplished in the **shortest possible time.** Likewise a truck company may have an empty truck in each of the cities 1, 2, 3, 4, 5, 6 and needs an empty truck in each of the cities 7, 8, 9, 10, 11, 12. It would like to ascertain the assignment of trucks to various cities so as to **minimise the total distance covered.** in a marketing set up by making an estimate of sales performance for different salesmen as well as for different territories one could assign a particular salesman to a particular territory with a view to **maximise overall sales.**

It may be noted that with n facilities and n jobs there are n! possible assignments. One way of finding an optimum assignment is to write all the n! possible arrangements, evaluate their total cost (in terms of the given measure of effectiveness) and select the assignment with minimum cost. The method leads to a computational problem of formidable size even when the value of n is moderate. Even for n 10 the possible number of arrangements is 3628800.

It is thus necessary to develop a suitable computation procedure to solve an assignment problem.

6.2 SOLUTION OF AN ASSIGNMENT PROBLEM

Suppose is the measure o effectiveness when ith person is assigned jth job. It is also assumed that the overall measure of effectiveness is to be **minimised** (such as total time taken to accomplish all the jobs). As in the case of transportation problem we introduce **decision variables x_{ij} with double subscripts** so that x_{ij} is the number of ith individuals assigned to jth job. Since ith person can be assigned only one job and jth job can be assigned to only one person we have

$$x_{i1} + x_{i2} + \dots\dots\dots + x_{in} = 1 \qquad i = 1, 2, \dots, n.$$

$$x_{1j} + x_{2j} + \dots\dots\dots + x_{nj} = 1 \qquad j = 1, 2, \dots, n.$$

The objective function is

$$\text{Minimise } c_{11}x_{11} + c_{12}x_{12} + \dots\dots\dots + c_{nn}x_{nn}$$

$$x_{ij} \geq 0.$$

The structure of an assignment problem is thus identical with that of a transportation problem. A Table similar to that of a transportation problem can be constructed as follows ·

Table of an Assignment Problem

Person	Job 1	...	...	n	TOTAL
1	c_{11}	c_{12}		c_{1n}	1
2	c_{21}	c_{22}		c_{2n}	1
3					.
.					.
.					.
.					
n	c_{n1}	c_{n2}		c_{nn}	1
TOTAL	1	1		1	n

The assignment problem is thus a special case of transportation problem where m = n and ai = bi = 1. It may however be easily observed that any basic feasible solution of an assignment problem contains **(2n-1) variables of which (n--1) variables are zero.** Due to this high degree of degeneracy the usual computational techniques of a transportation problem become very inefficient A separate computational device is required to solve assignment problem.

The basic result on which the solution of an assignment problem is based can be stated as follows (Mustafi, 1988) :

If a constant is added to every element of a row and/or a column of the cost matrix of an assignment problem the resulting assignment problem has the same optimum solution as the original problem and vice versa.

This result can be used in two different ways to solve the assignment problem. If in an assignment problem some cost elements are negative, we limy convert them into an equivalent assignment problem where all the cost elements are non negative by adding a suitably large constant to the elements of the relevant row or column. Next we look for a feasible solution which has **zero assignment cost** after adding suitable constants to the elements of various rows and columns. Since it has been assumed that all the cost elements are non negative, this assignment must be optimum. Based on this principle a computational technique known as **Hungarian Method** is developed which is discussed below.

Hungarian Method : The method is listed below in the form of a series of computational steps, when the objective function is that of minimisation type.

Step 1 : Find out the cost table from the given problem. If the number of origins are not equal to the number of destinations, a dummy origin or destination must be added [For details, please refer Section 6.3]:

Step 2 : Find the smallest cost in each row of the cost table. Subtract this smallest cost element from each element in that row. Therefore, there will be atleast one zero in each row of this new table, called the first Reduced Cost Table.

Find the smallest element in, each column of the reduced cost table. Subtract this smallest cost element from each element in that column.• As a result of this, each row and column now has atleast one zero value in the second reduced cost table.

Step 3 : Determine an assignment as follows :

i) For each, row or column with a single zero value cell that has not be assigned or eliminated, box that zero value as an assigned cell.

ii) For every zero that becomes assigned, cross out all other zeros in the same row and for column.

iii) If for a row and for column there are two or more zero and one cannot be chosen by inspection, choose the assigned zero cell arbitrarily. .

iv) The above process may be continued until every zero cell is either assigned (boxed) or crossed out.

Step 4 : An optimal assignment is found, if the number of assigned cells equals the number of rows (and columns). In case you had chosen a zero cell arbitrarily, there may be an alternate optimum.

If no optimum solution is found (some rows or columns without an assignment), please go to step 5.

Step 5 Draw a set of lines equal to the number of assignments made in Step 3, covering all the zeros in the following way.

i) Mark check (V) to those rows where no assignment has been made,

ii) Examine the checked (V) rows, If any zero cell occurs in those rows, check (V) the respective columns that contain those zeros.

iii) Examine the checked (V) columns. If any assigned zero occurs in those columns, check (V) the respective rows that contain those assigned zeros.

iv) The process may be repeated until no more rows or column can be checked.

v) Draw lines through all unchecked rows and through all checked columns.

Step 6 Examine those elements that are not covered by a line. Choose the smallest of these elements and subtract this smallest from all the elements that do not have a line through them,

Add this smallest element to every element that lies at the intersection of two lines. The resulting matrix is a new revised cost tableau.

Example 1

A job shop has four men available for work on four separate jobs. Only one man can work on any one job. The cost of assigning each man to each job is given in Table 1 below. The objective is to assign men to jobs such that the total cost of assignment is a minimum.

Table 1

Jobs

	To / From	1	2	3	4
Men	A	20	25	22	28
	B	15	18	23	17
	C	19	17	21	24
	D	25	23	24	24

Solution

Step 1 : The ost tableau (Table

Step 2 : Find the first and second reduced cost tableau (Table 2 & 3).

Table 2
First Reduced Cost Tableau

Table 2
First Reduced Cost Tableau

To / From	1	2	3	4
A	0	5	2	8
B	0	3	8	2
C	2	0	4	7
D	2	0	1	1

Table 3
Second Reduced Cost Tableau

To / From	1	2	3	4
A	0	5	1	7
B	0	3	7	1
C	2	0	3	6
D	2	0	0	0

Step 3 : Determine an Assignment.

Examine row A of Table 3. You will find that it has only one zero (Al). Box this zero. Cross-out all other zeros in the boxed column. This way you can eliminate cell B1.

Now examine row C. You find that it has one zero (C2) Box this zero. Eliminate all the zeros in the boxed column. This is how cell D2 gets eliminated.

There is one zero in column 3. Therefore, D3 gets boxed and this enables us to eliminate cell D4.

Therefore, we can box (assign) or eliminate all zeros. (Refer Table 4).

Table 4
Assignment of Jobs

To / From	1	2	3	4
A	[0]	5	1	7
B	~~0~~	3	7	1
C	2	[0]	3	6
D	2	~~0~~	[0]	~~0~~

Step 4 : The solution obtained in Step 3 is not optimal. This is because we were able to make three assignments when four **were** required.

Step 5 : Cover all the zeros of Table 4 with three lines, since three assignments were made. Check (V) row B since it has no assignment. Please note that row B has a zero in column 1, therefore, we check (V) column 1. We then check (V) row A, since column 1 has an assigned zero in row A.

Please note that no other rows or columns can be checked. You may draw three lines through unchecked rows C & D ad column 1, the checked column. This is shown in Table 5.

Table 5

To / From	1	2	3	4
A	[0]	5	1	7
B	X	3	7	1
C	2	[0]	3	6
D	2	X	[0]	X

Step 6 : Develop the new revised tableau. Examine those elements that are not covered by a line in Table 5. Take the smallest element. This is 1(one) in our case.

By subtracting 1 from the uncovered cells and adding 1 to elements (C_l & D_1) that lie at the intersection of two lines, we get the new revised cost tableau as given in Table 6 below.

Table 6

New Revised Cost Tableau

To / From	1	2	3	4
A	0	4	0	6
B	0	2	6	0
C	3	0	3	6
D	3	0	0	0

Step 7 : Go to Step 3 and repeat the procedure until you arrive at an optimal assignment.

Step 8 : Determine an assignment.

By examining each of the four rows in Table 6, we find that it is only row C which has got only one columns have two zeros. Choose a zero arbitrarily, say Al and box this cell. Thus cell A3 and B1 get eliminated. Therefore, row B(B4) and column 3 (D4) has one zero. These are boxed and cell D4 is eliminated. Thus all zeros are either boxed or eliminated in Table 7.

Table 7

An Optimal Assignment

To / From	1	2	3	4
A	[0]	4	X	6
B	X	2	6	[0]
C	3	[0]	3	6
D	3	X	[0]	X

Since the number of assignments equal the number of rows (columns), the assignment in Table 7 is optimal. The total cost of this assignment is 78.

Remember that we had chosen a zero cell arbitrarily, an alternate optimum solution exists and is given by A3, B1, C2 and D4. You may please verify it yourself.

Activity 1

A tourist car rental firm has one car in each of the five depots D_1, D_2, D_3, D_4, D_5 and a customer in each of the five cities C_1, C2, C3, C4, C_5. The distances in Kilometers between the depots and the cities are given in the following matrix. How should be cars be assigned to the customers so as to minimise the total distance covered?

Depots	Cities				
	C_1	C_2	C_3	C_4	C_5
D_1	140	115	120 -	30	35
D_2	110	100	90	30	15
D_3	155	90	135	60	50
D_4	170	140	150	60	60
D_5	180	155	165	90	85

…………………………………………………………………………………………
…………………………………………………………………………………………
…………………………………………………………………………………………
…………………………………………………………………………………………
…………………………………………………………………………………………...

6.3 UNBALANCED ASSIGNMENT PROBLEM

The number of persons to be assigned and the number of jobs were assumed to be the same in the previous section. Such an assignment problem is known as a **balanced assignment problem.** If the number of persons is different from the number of jobs the assignment problem is said to be **unbalanced.** If the number of jobs is less than the number of persons, some of them cannot be assigned any job. We introduce **one or more dummy jobs of** zero duration to make the assignment problem balanced. This balanced problem can be solved using the method developed in the previous section. The persons to whom the dummy jobs are assigned are left out of assignment. Likewise, if the number of persons is less than the number of jobs we introduce **one or more dummy persons** with duration time zero to make the assignment problem balanced. We solve this balanced assignment problem and the jobs assigned to the dummy persons are left out. Two examples have been presented to illustrate the solution of unbalanced assignment problems.

Example 2

Solve the following unbalanced assignment problem of minimising total time for performing all the jobs.

Operator	Job				
	1	2	3	4	5
1	6	2	5	2	6
2	2	5	8	7	7
3	7	8	6	9	8
4	6	2	3	4	5
5	9	3	8	9	7
6	4	7	4	6	8

Solution

Since the number of jobs is less than the number of operators we introduce a dummy job with duration zero. The revised assignment problem is given below.

Operator	Job					
	1	2	3	4	5	6 (dummy)
1	6	2	5	2	6	0
2	2	5	8	7	7	0
3	7	8	6	9	8	0
4	6	2	3	4	5	0
5	9	3	8	9	7	0
6	4	7	4	6	8	0

Using the Hungarian Method described in the previous section the assignment leading to minimum cost is : Operator 1 to job 4, Operator 2 to job 1, Operator 3 to dummy 6, Operator 4 to job 5, Operator 5 to job 2, Operator 6 to job 3. The total minimum completion time is 16. Operator 3, therefore, can not be assigned.

Example 3

The personnel manager of a company wants to assign Mr. X, Mr. Y and Mr. Z to regional offices Delhi, Bombay, Calcutta and Madras. The cost of relocation (in Rupees) of the three officers at the four regional offices are given below :

Officer	Office			
	Delhi	Bombay	Calcutta	Madras
Mr. X	16000	22000	24000	20000
Mr. Y	10000	32000	26000	16000
Mr. Z	10000	20000	46000	30000

Solution

Since there are fewer persons than offices we introduce a dummy person with a relocation cost zero. The revised Assignment Problem is given below.

Officer	Office			
	Delhi	Bombay	Calcutta	Madras
Mr. X	16000	22000	24000	20000
Mr. Y	10000	32000	26000	16000
Mr. Z	10000	20000	46000	30000
Dummy	0	0	0	0

Using Hungarian Method of solution, the minimum relocation cost is given by Dummy (no one) to Calcutta, Mr. X to Bombay, Mr. Y to Madras and Mr. Z to Delhi. The cost of relocation for this assignment is Rs. 48000.

Activity 2

Work out the various steps of the solution of the **example 2**

…………………………………………………………………………………………
…………………………………………………………………………………………

Activity 3

Work out the various steps of the Solution of the **example 3**

…………………………………………………………………………………………
…………………………………………………………………………………………
…………………………………………………………………………………………

6.4 PROBLEM WITH SOME INFEASIBLE ASSIGNMENTS

It is sometimes possible that a particular person is incapable of doing certain work or a specific job cannot be performed on a particular machine. The solution of the assignment problem should take into account these restrictions so that the **infeasible**

assignments can be avoided. This can be achieved by **assigning a very high cost** to the cells where assignments are prohibited. The method of solution is illustrated using the following example.

Example 4

A metal shop has five jobs to be done and has five machines to do them. The cost matrix gives the cost of processing each job on any machine. Because of specific job requirement and machine specifications certain jobs cannot be done on certain machines. These have been shown by X in the cost matrix. The assignment of jobs to machines must be done on a one to one basis. The objective is to assign the jobs to the machines so as to minimize the total cost within the restrictions mentioned above.

Machines	Jobs				
	1	2	3	4	5
1	80	40	X	70	40
2	X	80	60	40	40
3	70	X	60	80	70
4	70	80	30	50	X
5	40	40	50	X	80

Because certain jobs cannot be done on certain machines we assign a high cost (Say 500) to these cells and modify the cost matrix before solution. The revised assignment problem is given below

Machines	Jobs				
	1	2	3	4	5
1	80	40	500	70	40
2	500	80	60	40	40
3	70	500	60	80	70
4	70	80	30	50	500
5	40	40	50	500	80

Solving this revised assignment problem we allot Job 1 to Machine 3, Job 2 to Machine 5, Job 3 to Machine 4, Job 4 to Machine 2 and Job 5 to Machine 1. The minimum cost of assignment is 220.

Activity 4

Give the step by step solution of the example given in this section

…………………………………………………………………………………………
…………………………………………………………………………………………
…………………………………………………………………………………………
…………………………………………………………………………………………
…………………………………………………………………………………………

6.5 MAXIMISATION IN AN ASSIGNMENT PROBLEM

There are problems where certain facilities have to be assigned to a number of jobs so as to **maximise** the overall performance of the assignment. The problem can be converted into a minimisation problem and Hungarian Method can be used for its solution. Using notations as in Section 6.2 this problem can be formulated as

$$x_{i1} + x_{i2} + \dots \dots \dots \dots + x_{in} = 1 \qquad i = 1, 2, \dots, n.$$

$$x_{1j} + x_{2j} + \dots \dots \dots \dots + x_{nj} = 1 \qquad j = 1, 2, \dots, n.$$

The objective function is

$$\text{Maximise } p_{11}x_{11} + p_{12}x_{12} + \dots \dots \dots \dots + p_{nn}x_{nn}$$

Where p_{ij} is the performance corresponding to the assignment of the ith facility to the jth job. The assignment where the objective function $p_{11}x_{11} + \ldots\ldots\ldots + p_{nn}nn\ x_{nn}$ is **maximised** is the assignment where $-p_{11}x_{11}\ -p_{12}x_{12}\ - \ldots\ldots\ldots -p_{nn}x_{nn}$ is **minimised.** Furthermore, the assignment which minimises $nP - p_{11}x_{11} - p_{12}x_{12} \ldots\ldots\ldots -p_{nn}x_{nn}$ also minimise $-p_{11}x_{11} -p_{12}x_{12} - \ldots\ldots\ldots\ -p_{nn}x_{nn}$ where P is a constant. Suppose we take P as the **maximum** of all the performance indices p_{ij}. Then

$$P = Px_{11} + Px_{12} + \ldots\ldots\ldots\ldots\ldots\ldots + px_{nn}.$$

$$nP - p_{11}x_{11} - p_{12}x_{12} - \ldots\ldots\ldots\ldots\ldots\ldots - p_{nn}x_{nn}.$$

$$= (P - p_{11})x_{11} - (P - p_{12})x_{12} + \ldots\ldots\ldots\ldots\ldots\ldots + (P - p_{nn})x_{nn}.$$

Hence the maximisation of performance is identical with minimisation of cost, where the cost corresponding to the assignment of ith facility to the jth job is given by $P - p_{ij}$. Thus we have to construct a new Table of Assignment costs with "$P - p_{ij}$" values and apply the standard method of solution. The procedure is illustrated with the help of the following example.

Example 5

Five different machines can do any of the required five jobs with different profits resulting from each assignment as shown below :

Job	Machine A	B	C	D	E
1	30	37	40	28	40
2	40	24	27	21	36
3	40	32	33	30	35
4	25	38	40	36	36
5	29	62	41	34	39

Find out the maximum profit possible through optimal assignment.

Solution

The highest profit in the Assignment Table is 62. We subtract each profit value from 62. The revised assignment Table is given below :

Job	Machine A	B	C	D	E
1	32	25	22	34	22
2	22	38	35	41	26
3	22	30	29	32	27
4	37	24	22	26	26
5	33	0	21	28	23

We now apply Hungarian Method to obtain the minimum cost assignment of the revised problem. The solution is to assign Job 1 to Machine C, Job 2 to Machine E, Job 3 to Machine A, Job 4 to Machine D and Job 5 to Machine B. The maximum profit through this assignment is 214.

Activity 5

Verify the solution of the assignment problem presented in Examples 5.

………………………………………………………………………………………………
………………………………………………………………………………………………
………………………………………………………………………………………………
………………………………………………………………………………………………
………………………………………………………………………………………………
………………………………………………………………………………………………
………………………………………………………………………………………………

6.6 CREW ASSIGNMENT PROBLEM

The method of solution discussed in this unit can be utilised to plan the assignment of crew members in different locations by a transport company. The technique is illustrated with the help of the following example

Example 6

A trip from Madras to Bangalore takes six hours by bus. A typical time table of the bus service in both directions is given below :

Department from Madras	Route Number	Arrival at Bangalore	Arrival at Madras	Route Number	Departure from Bangalore
06.00	a	12.00	11.30	1	05.30
07.30	b	13.30	15.00	2	09.00
11.30	c	17.30	21.00	3	15.00
19.00	d	01.00	00.30	4	18.30
00.30	e	06.30	06.00	5	00.00

The cost of providing this service by the transport company depends upon the time spent by the bus crew (driver and conductor) away from their places in addition to service times. There are five crews. There is a constraint that every crew should be provided with more than 4 hours of rest before the return trip again and should not wait for more than 24 hours for the return trip. The company has residential facilities for the crew of Madras as well as at Bangalore. Find Which line of service be connected with which other line so as to reduce the waiting time to the minimum.

1) As the service time is constant for• each line it does not appear directly in the computation. If the entire crew resides at Madras then the waiting times in hours at Bangalore for different route connections are given in the following Table :

Table 1

Route	1	2	3	4	5
a	17.5	21	M	6.5	12
b	16	19.5	M	5	10.5
c	12	15.5	21.5	M	6.5
d	4.5	8	4	17.5	23
e	23	M	8.5	12	17.5

If Route a is combined with Route 1, the crew after arriving at Bangalore at 12 Noon start in at 5.30 next morning. Thus the waiting time is 17.5 hours. **Some of the assign meats are infeasible.** Route 3 leaves Bangalore at 15.00 hours. Thus the crew of Route a reaching Bangalore at 12 Noon are unable to take the minimum stipulated rest of four hours if they are asked to leave by Route 3. Hence a3 is an infeasible assignment. **Its cost is thus M, a large positive number.**

2) Similarly, if the crew are assumed to reside at Bangalore then the waiting times of the crew in hours at Madras for different route combinations are given in

Table 2

Route	1	2	3	4	5
a	18.5	15	9	5.5	M
b	20	16.5	10.5	7	M
c	M	20.5	14.5	11	5.5
d	7.5	M	22	18.5	13
e	13	9.5	M	M	18.5

3) As the crew can be asked to reside either at Madras or at Bangalore, minimum

waiting time from the above operation can be computed for different route combination by choosing the **minimum of the two waiting tins. This** is presented in Table 5. The asterisk marked waiting times indicates that the crew are based at Madras, otherwise they are based at Bangalore.

Table 3

Route	1	2	3	4	5
a	17.5*	15	9	5.5	12*
b	16*	16.5	10.5	5*	10.5*
c	12*	15.5*	14.5	11	5.5
d	4.5*	8*	14*	17.5*	13
e	13	9.5	8.5*	12*	17.5*

4) We now use Hungarian Method to solve assignment problem in Table 3.

Solution

Step .1 : The cost tableau in terms of waning time (Table 3)

Step 2 : Find the first and second reduced cost tableau (Tables 4 and 5).

Table 4

Fir st Reduced Cost Tableau

Route	1	2	3	4	5
a	12.00	9.5	3.5	0	6.5
b	11.00	11.5	5.5	0	5.5
c	6.5	10.00	9.0	5.5	0
d	0	3.5	9.5	13.0	8.5
e	4.5	1.0	0	3.5	9.0

Table 5

Second Reduced Cost Tableau

Route	1	2	3	4	5
a	12.00	8.5	3.5	0	6.5
b	11.00	10.5	5.5	0	5.5
c	6.5	9.0	9.0	5.5	0
d	0	2.5	9.5	13.0	8.5
e	4.5	0	0	3.5	9.0

Step 3 : Determine an Assignment

Examine row a of Table 5. You will find that it has Only one zero (a4). Box this zero. Cross out all other zeros in the boxed column. This way you can eliminate cell M.

Now examine row C. It has only one zero (C5). Box this zero. There is no other zero in the boxed column.

Now examine row d. It has only one zero (d1). Box this zero. There is no other zero in the boxed column.

Now examine row e. It has two zero (e2 and e3), We box one of the two zeros arbitrarily. Suppose we box e2. There is no other zero in the boxed column. However, the other zero in the same row i.e. e3 gets crossed out,

This way, all the zeros of Table 5 get boxed or eliminated (Refer Table 6).

Table 6

Route	1	2	3	4	5
a	12.00	8.5	3.5	[0]	6.5
b	11.00	10.5	5.5	~~0~~	5.5
c	6.5	9.0	9.0	5.5	[0]
d	[0]	2.5	9.5	13.0	8.5
e	4.5	[0]	~~0~~	3.5	9.0

Step 4 : The solution obtained in Step 3 is not optimal, as the number of assignments are less than the number of rows or columns.

Step 5 : Cover all zeros of Table 6 with four lines, since four assignments were made.

Check (√) row b since no assignment was made in this row. Note that row b has a zero in column 4, therefore, we check column 4. We then check (√) row a, since column 4 has an assigned zero in row a.

Table 7

Route	1	2	3	4	5
a	12.00	8.5	3.5	[0]	6.5
b	11.00	10.5	5.5	~~0~~	5.5
c	~~6.5~~	~~9.0~~	~~9.0~~	~~5.5~~	~~[0]~~
d	~~[0]~~	~~2.5~~	~~9.5~~	~~13.0~~	~~8.5~~
e	~~4.5~~	~~[0]~~	~~0~~	~~3.5~~	~~9.0~~

Step 6 : Develop a new revised tableau.

Examine those elements that are not covered by a line in Table 7. Take the smallest element. That is 3.5 in our case. By subtracting 3.5 from the uncovered cells and adding 3.5 to elements (C4, d4 and e4) that lie at the intersection of two lines, we get the new revised tableau as given in Table 8 below.

Table 8

New Revised Tableau

Route	1	2	3	4	5
a	8.5	5.0	0	0	3.0
b	7.5	7.0	2.0	0	2
c	6.5	9.0	9.0	9.0	0
d	0	2.5	9.5	16.5	8.5
e	4.5	0	0	7.0	9.0

Step 7 : Go to Step 3 and repeat the procedure until an optimal assignment is arrived at.

Step 8 : Determine an Assignment

By examing each of the rows in Table 8, we find that rows b, c and d have only one zero. Therefore, we box b4, C5 and d1. Since b4 is boxed, a4 gets eliminated.

We note column 2 has only one zero in row e. Therefore, we box e2 and eliminate e3.

Now, we can box a3. We note that all zeros are either boxed or eliminated in Table 9.

Table 9

An Optimal Assignment

Route	1	2	3	4	5
a	8.5	5.0	[0]	~~0~~	3.0
b	7.5	7.0	2.0	[0]	2.0
c	6.5	9.0	9.0	9.0	[0]
d	[0]	2.5	9.5	16.5	8.5
e	4.5	[0]	~~0~~	7.0	9.0

Since the number of assignments equal the number of rows (columns), the assignment in Table 9 is optimal.

Therefore the routes to be paired to achieve the minimum waiting time are a - 3, b - 3, c 5, d - 1 and e - 2. Referring back to Table 3 we can now obtain the waiting times of these assignments as well as the, residence of the crew. Thus is indicated in Table 10.

Table 10

Routes to be Paired	Residence of the Crew	Waiting time
a – 3	Bangalore	9
b – 4	Madras	5
c – 5	Bangalore	5 – 5
d – 1	Madras	4 – 5
e – 2	Bangalore	9 – 5

The minimum total waiting time is thus 33.5 hours.

The minimum total waiting time is thus 33.5 hours.

6.6 SUMMARY

The Assignment Problem considers the allocation of a number of jobs to a number of persons so that the total completion time is **minimized.** If the number of persons is-the same as the number of jobs, the assignment problem is said to be **balanced.** If the number of jobs is different from the number of persons the assignment problem is said to be **unbalanced.** An unbalanced assignment problem can be converted into a balanced assignment problem by introducing a dummy person or a dummy job with completion time zero.

Although an assignment problem can be formulated as a linear programming problem, it is solved by a special method known as **Hungarian Method** because of its special structure. If the times of completion or the costs corresponding to every assignment is written down in a matrix form, it is referred to as a **Cost matrix.** The Hungarian Method ,is based on the principle that if a constant is added to every element of a row and/or a column of cost matrix the optimum solution of the resulting assignment problem is the same as the original problem and vice versa. The original cost matrix can be reduced to another cost matrix by adding constants to the elements of rows. and columns where **the total cost or the total completion time of an assignment is zero.** Since the optimum solution remains unchanged after this reduction this assignment is also the optimum solution of the original problem.

Various ramifications of the assignment problem are possible. If a person is unable to carry out a particular job the corresponding-cost **or completion time is taken as very large** which automatically prevents such an assignment. If the objective is to maximise a performance index through assignment, Hungarian Method can be applied to a revised cost matrix obtained from the original cost matrix. The method of solution can also be utilised for **allocating crew members** to various stations of a transport organisation.

6.8 KEY WORDS

Assignment Problem is a special type of linear programming problem where the objective is to minimise the cost or time of completing a number of jobs by a number of persons.

Balanced Assignment Problem is an assignment problem where the number of persons is equal to the number of jobs.

Unbalanced Assignment Problem is an assignment problem where the number of persons is not equal to the number of jobs.

Hungarian Method is a technique of solving assignment problems.

A Dummy Job is an imaginary job with cost or time zero introduced to make an unbalanced assignment problem balanced

An Infeasible Assignment occurs in the cell (i, j) of the assignment cost matrix if ith person is unable to perform jth job.

6.9 SELF-ASSESSMENT EXERCISES

1) Six contractors have submitted tenders to take up six projects advertised. It is noted that one contractor can be assigned one job as otherwise the time for completion and the quality of workmanship will be affected. The estimates of cost in thousand rupees given by each of them are indicated' below :

Contractor	Project					
	1	2	3	4	5	6
A	41	72	39	52	25	51
B	22	29	49	65	81	50
C	27	39	60	51	32	32
D	45	50	48	52	37	43
E	29	40	39	26	30	33
F	82	40	40	60	51	30

Find out the assignment such that the total cost of completing the six projects is minimum. What is the minimum cost?

2) A freight terminal can accommodate six trucks simultaneously. There is cost of sorting and transferring of loads associated with parking of each truck on each of the six sports. On a certain day, four trucks are to be simultaneously parked at the terminal. -The cost matrix. is given below.

Spot	Truck			
	1	2	3	4
7	3	6	2	6
8	7	1	4	4
9	3	8	5	8
10	6	4	3	7
11	5	2	4	3
12	5	7	6	2

Find out the assignment which minimises the total cost of parking.

3) A sales manager has to assign salesman to four territories. He has four candidates of varying experience and capabilities and assesses the possible profit in suitable units for each salesman in each territory as given below :

Saleman	Territories			
	T_1	T_2	T_3	T_4
S_1	25	27	28	37
S_2	28	34	29	40
S_3	35	24	32	33
S_4	24	32	25	28

Find an assignment that maximises the profit.

4) An airline that operates seven days a week has a time table shown below. Crews 86 must have minimum rest of six hours between flights. Obtain the pairing of flights

that minimises waiting time away from the city. For any given pairing the crew will be based at the city that results in the smaller waiting time.

Flight No.	Delhi Departure	Calcutta Arrival	Flight No.	Calcutta Departure	Delhi Arrival
1	7.00 AM	9.00 AM	101	9.00 AM	11.00 AM
2	9.00 AM	11.00 AM	102	10.00 AM	12 Noon
3	1.30 PM	3.30 PM	103	3.30 PM	5.30 PM
4	7.30 PM	9.30 PM	104	8.00 PM	10.00 PM

For each pair also mention the city where the crew should be based.

6.10 ANSWERS

Activity 1

$D_1 - C_4$, $D_2 - C_3$, $D_3 - C_2$, $D_4 - C_5$, $D_5 - C_1$.

Minimum distance 450 km.

Self-assessment Exercises

1) A – 5, B – 2, C – 1, D – 3, E – 4, F – 6

   Minimum cost 185.

2) Truck 1 – Spot 9, truck 2 – spot 8, truck 3 – spot 7, truck 4 – spot 12, spots 10 and 11 are vacant. Minimum cost is 8.

3) $S_1 - T_1$, $S_2 - T_4$, $S_3 - T_3$, $S_4 - T_2$.

   Maximum Profit 139.

4)

Flights	Station
3 – 101	Delhi
4 – 102	Calcutta
1 – 103	Delhi
2 – 104	Delhi

Total waiting time 40 hours 30 minutes.

6.11 FURTHER READINGS

Cooper, L and D. Steinberg, 1974. *Methods and Applications of Linear Programmings,* Saunders, Philadelphia : USA.

Mustafi, C.K. 1988. *Operations Research Methods and Practice,* Wiley Eastern Limited : New Delhi.

Sasieni, M.A. Yaspan and L. Friedman 1959. *Operations Research Methods and Problems,* J. Wiley and Sons, New York : USA.