UNIT 1 OPERATIONS RESEARCH-AN OVERVIEW

Objectives

After studying this unit, you should be able to:

- Explain the meaning, purpose, and limitations of Operations Research
- Describe the historical background of Operations Research
- Discuss the approach and tools of Operations Research
- Describe the delicate relationship between Operations Research Specialist and Manager.

Structure

1.1 INTRODUCTION

Operations Research (O.R.) is a relatively a new discipline. Its content and boundaries are not yet fixed. Therefore, proposing a formal definition of the term OPERATIONS RESEARCH (Or OPERATIONAL RESEARCH as some people call it) is a difficult task. Operations Research begins when some mathematical and quantitative techniques are used to substantiate the decision being taken. We make decisions in our everyday life without even noticing them. Decision making is one of the main activities of a manager. In simple situations decisions are taken simply by common sense, sound judgment and expertise without using any mathematics.

But here the decision we are concerned with are rather complex and heavily loaded with responsibility. Examples of such decision are finding the appropriate product mix when there are large number of products with different profit contributions and productional requirement or planning public transportation network in a town having its own layout of factories, apartment blocks etc. Certainly in such situations also decisions may well be arrived at intuitively from experience and common sense, yet they are more judicious if backed up by mathematical reasoning. The search of a decision may also be done by trial and error but such, a search may be cumbersome and costly. Preparative calculations may avoid long and costly research. Doing preparative calculations is the purpose of operations research. Operation Research does mathematical scoring of consequences of a decision with the aim of optimizing the use of time, efforts and resources and avoiding blunders.

The tools of operations research are not from any one discipline, rather Mathematics, Statistics, Economics, Engineering, Psychology, etc. have contributed to this newer discipline of knowledge. Operations research takes tools from subjects like mathematics, statistics, economics, engineering, psychology etc. and combines them to make a new body of knowledge for decision making. Today, it has become a professional discipline that deals with the application of scientific methods for decision making, and especially to the allocation of scarce resources.

O.R. worker tries to find order in apparent chaos by identifying the structure in complex situations. He develops the understanding of how the components of organizations interact, so as to explain and predict the effects of actions taken on

these components. Much of this work is done with the help of analytical and numerical techniques and by developing and manipulating mathematical and computer models of organizational systems. Such systems may be composed of people, machine and procedures. Complete information about such a system may not be available. A purpose of O.R. is to provide a rational basis for making decisions in the absence of complete informations.

Operations Research can also be treated as science devoted to describing, understanding and predicting the behaviour of systems, particularly man-machine systems. Thus operations research workers are engaged in three classical aspects of science:

a) Describing the behaviour of systems.
b) Analyzing this behaviour by constructing appropriate models.
c) Using these models to predict future behaviour, that is, the effects that will be produced by changes in the systems or in the methods of operations.

The operating systems studied by operations research workers arise in a wide variety of practical, military, industry, and governmental environment. Thus, the results of their research frequently make important contributions to solutions of problems of choice, policy, and planning that arise in these environments. It is to be noted that operations research workers in an organization are not the decision makers themselves. They merely present their findings to the executives in-charge of operations, who is supposed to make decisions. Operations research function is a staff function. Operations research is assistance to executives in improving the operations under their control.

What particularly distinguishes operations research from other research and engineering is its emphasis on analysis of operations as a whole. It is an interdisciplinary approach that provided useful solutions to problems of military operations in World War II, and has since been found equally successful in non-military operations. Most present-day business applications are primarily concerned with mathematical and statistical analysis of the results of possible alternative actions. Often using techniques especially designed or refined for business problems, operations researchers have been able to provide remarkable and diverse benefits to companies. Some such benefits are improved inventory and reorder policies, minimum cost production schedules, optimum location and size of warehouses, and guidance in sales and advertising policies. The basic pattern applied is clarification of various courses of action open, estimation of the outcome to be expected from each, and evaluation of these in terms of the overall goal desired.

As stated earlier defining O.R. is a difficult task. Salient aspects related to definition stressed by various experts on the subject are as follows:

a) Pocock stresses that O.R. is an applied science: he states "OR is scientific methodology-analytical, experimental, quantitative-which by assessing the overall implication of various alternative courses of action in a management system, provides an improved basis for management decisions."

b) Morse and Kimball have stressed the quantitative approach of O.R. and have described it as "a scientific method of providing executive departments with a quantitative basis for decisions regarding the operations under their control."

c) Miller and Starr see O.R. as applied decision theory. They state, "O.R. is applied decision theory. It uses any scientific, mathematical or logical means to attempt to cope with the problems that confront the executive, when he tries to achieve a thorough-going rationality in dealing with his decision problem."

d) Saaty considers O.R. as tool of improving the quality of answers to problems. He says, "O.R. is the art of giving bad answers to problems which otherwise have worse answers.

All these definitions put together enable us to know what -O.R. is, and what it does.

Activity 1

Define Operations Research.

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1.2 HISTORY

O.R. has its beginning in World War II. The term, operations research, was coined by McClosky and Trefthen in 1940 in U.K. British scientists set up the first field installations of radars during the battle and observed air operations. Their analysis of these led to suggestions that greatly improved and increased the effectiveness of British fighters, and contributed to successful British defence. Operations research was then extended to antisubmarine warfare and to all phases of military, navel, and air operations, both in Britain and the United States, and was incorporated in the post-war military establishments of both the countries.

The effectiveness of operations research in military spread interest in it to other governmental departments and industry. In the U.S.A. the National Research Council formed a committee on operations research in 1951, and the first book on the subject "Methods of Operations Research", by Morse and Kimball, was published. In 1952 the Operations Research Society of America came into being. Success of O.R. in military attracted the attention of industrial managers who were seeking solutions to their complex problems.

Today, almost every large organisation or corporation in affluent nations has staff applying operations research, and in government the use of operations research has, spread from military to widely varied departments at all levels. This general acceptance to O.R. has come as managers have learned the advantage of the scientific approach on which O.R. is based. Availability of faster and flexible computing facilities and the number of qualified O.R. professionals enhanced the acceptance and popularity of the subject. The growth of O.R. has not been limited to the U.S.A. and the U.K. It has reached to many countries of the world: Indicative of this is that the International Federation of Operations Research Societies, founded in 1959, now comprises member societies from many countries of the world.

India was one of the first few countries who started using O.R. In 1949, first O.R. unit was established in Regional Research Laboratory at Hyderabad. At about same time another group was set up in Defence Science Laboratory to solve the problems of stores, purchase and planning. In 1953, O.R. unit was established in Indian Statistical Institute, Calcutta, with the aim of using O.R. methods in national planning and survey. O.R. Society of India was formed in 1955. The society is one of the first members of International Federation of O.R. societies. The society started publishing OPSEARCH, a learned journal on the subject in 1963. Today O.R. is a popular subject in management institutes and schools of mathematics and is gaining currency in industrial establishments.

1.3 APPROACH, TECHNIQUES AND TOOLS

An O.R. team (in some companies this may be single individual) consists of trained researchers utilizing the skills and tools of applicable sciences, without any preconceived ideas of what the correct solution should be. The first step is to formulate the problem, after which a mathematical model is generally constructed to represent the system being studied. Mathematical models or conceptual models may be in the form of equations or formulae developed to relate important factors or variables of the operations under study. The model then can be tested and operated upon mathematically to determine the effects of changing the values of the variables. Optimization of some criterion may be purpose of such an exercise.

Optimization means achieving the best-maximum or minimum value of the criterion. Sub-optimization occurs either when some criterion subordinate to the overall criterion is optimized or when the overall criterion is optimized over only a portion of the total of all possible alternative actions. A criterion is a measure by which results

can be evaluated. It measures the effectiveness of the operations under study in terms of the ultimate objective of the operations, and may be net profit, return on investment, cost or some other thing appropriate to a particular study.

Operations research uses any suitable techniques or tools available. These frequently include common mathematical or statistical procedures, cost analysis, or electronic computation. However, O.R. analysts have given special impetus to the development and use of the techniques like, linear program, waiting line theory, game theory, inventory control models and simulation. In addition, some other common tools are non-linear programming, integer programming, dynamic programming, sequencing theory, Markov process, network scheduling-PERT and CPM, symbolic logic, information theory and utility/value theory. The list, of course, is not exhaustive. Detailed discussion on some of these techniques will be presented in appropriate units. But brief explanation of these term will not be out of place here.

1) Linear Programming

Linear Programming is basically a constrained optimization technique which tries to optimize some criterion within some constraints. It consists of an objective function which is some measure of effectiveness like profit, loss or return on investment and several boundary conditions putting restriction on the use of resources. Objective function and boundary conditions are linear in nature. There are methods available to solve a linear programming problem.

2) Waiting Line Theory or Queuing Theory

Waiting line theory or queuing theory deals with the situation in which queue is formed. Customers waiting for service, machines waiting for repairmen and aircraft's waiting for landing strips are some of the situations in which queue is formed. If we assume that there are costs associated with waiting in line, and if there are costs of adding more channels (service facilities), we want to minimize the sum of costs of waiting and the costs of providing service facilities. Waiting line theory helps to make calculations like expected number of people in the queue, expected waiting time in the queue, expected idle time for the server, etc. These calculations then-can be used to determine the desirable number of service facilities.

3) Game Theory

Game theory is used for decision making under conflicting situations where there are one or more opponents. Opponents, in game theory, are called players. The motives of the players are dichotomized. The success of one player tends to be at the cost of others and hence they are in conflict. Game theory models a conflict situation and helps us to improve the decision process by formulating appropriate strategy.

4) Inventory Control Models

When to buy, how much to buy and how much to keep in stores are some of the questions which production managers, purchase managers and material managers address themselves to. Inventory _ control models provide rational answer to these question in different situation of supply and demand for different kind of materials. Inventory control models help managers to decide reordering time, reordering level and optimal ordering quantity. The approach is to prepare a mathematical model of the situation that expresses total inventory costs in terms of demand, size of order, possible over or under stocking and other relevant factors and then to determine optimal order size, optimal order level etc. using calculus or some other technique.

5) Simulation

Simulation is basically data generation technique. Sometimes it is very risky, cumbersome or time consuming to conduct real study or experiment to know more about a situation or problem. Analytical methods also cannot be used in all cases. Sometimes due to large number of variables or large number of interrelationships among the variables and the complexity of relationships, it is not possible to develop an analytical model representing the situation. Even if model building is possible, solving the model may not be possible. In such situations simulation is used. It is to be noted that simulation does not solve a problem; it only generates information or data needed for decision problem solving or decision making. Thus, simulation is a

data generation technique and is used when actual experimentation is not feasible, analytical model building or solution of model is not possible.

6) Non-Linear Programming

Non-linear programming methods may be used when either the objective function or some of the constraints are not linear in nature. Non-linearity may be introduced by such factors as discount on price of purchase of large quantities and graduated income tax etc. Linear relationships may be employed to approximate non-linear conditions, but the approximation becomes poorer as the range is extended. Non-linear methods may be used to determine the approximate area in which a solution lies and linear methods may then be used to obtain a more exact solution.

7) Integer Programming

Integer programming method may be used when one or more of the variables can take only integral values. Examples are the number of trucks in a fleet, the number of generators in a powerhouse and so on. Approximate solutions can be obtained without using integer programming methods, but the approximation generally becomes poorer as the numbers become smaller. There are techniques .to obtain solution of integer programming problems.

8) Dynamic Programming

Dynamic programming is a method of analyzing multistage decision processes, in which each elementary decision is dependent upon those preceding it as well as upon external factors. It drastically reduces the computational efforts otherwise necessary to analyze results of all possible combinations of elementary decisions.

9) Sequencing Theory

Sequencing theory is related to Waiting Line Theory. It is applicable when the facilities are fixed, but the order of servicing may be controlled. The scheduling of service or the sequencing of jobs is done to minimize the relevant costs.

10) Markov Process

Markov process for decision making is used in situations where various states are defined. The probability of going from one state to another is known and depends on the present state and is independent of how we have arrived at that state. Theory of Markov process helps us to calculate long run probability of being in a particular state (steady state probability). This steady state probability is used in decision making.

11) Network Scheduling-PERT AND CPM

Network scheduling is technique used to plan, schedule and monitor large projects. Such large projects are very common in the field of construction, maintenance, computer system installation, research and development designs etc. The technique aims at minimizing trouble spots, such as, delays, interruptions and production bottlenecks by identifying critical factors and coordinating various parts of overall job. The whole projector job is diagrammatically represented with the help of network made of arrows representing different activities and interrelationships among them. Such a representation is used for identifying critical activities and critical path. Two basic techniques in network scheduling are Program Evaluation and Review Technique (PERT) and Critical Path Method (CPM). CPM is used when time taken by activities in the project are known for sure and PERT is used. when activities time is not known for sure-only probabilistic estimate of time is available.

12) Symbolic Logic

Symbolic logic deals with substituting symbols for words, classes of things, or functional systems. It incorporates rules, algebra of logic and propositions. There have been only limited attempts to apply this technique to business problems; however it has had extensive application in the design of computing machinery.

13) Information Theory

Information theory is-an analytical process transferred from the electrical communications field to operations research. It seeks to evaluate the effectiveness of information flow within a given system. Despite its application mainly to

communications networks, it has had an indirect influence in simulating the examination of business organizational structures with a view to improving information or communication flow.

14) Utility/Value Theory

Utility/Value theory deals with assigning numerical significance to the worth of alternative choices. To date this has been only a concept, and is in the stage of elementary model formulation and experimentation. When developed this may be very helpful in the decision making process in assessing the worth of various possible outcomes.

Activity 2

Describe some of the tools of Operations Research.

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1.4 RELATIONSHIP BETWEEN O.R. SPECIALIST AND MANAGER.

The role of the O.R. specialist is to help the manager make better decisions. Decision making is a key responsibility of managers. Managers now need and in the future will need Q.R. specialists because the size of organizations is increasing to the point where they are becoming unmanageable by traditional means and methods. Even without this problem, managers face enormously complicated and fast changing situations. Creative and workable solutions to these problems require the cooperative involvement of O.R. specialists and managers. Decision makers move towards O.R. $^{a}p^{proach}$ when:

a) They see the problem as complex involving many variables and relationships.

b) They don't think they can develop a solution without O.R. methods.

c) They view the problem as repetitive, thus quantitative solution which is repeated will save the time and money.

d) They initially feel that the data in the problem are numeric.

e) They know of similar situations in which decisions have been improved by using O.R. techniques.

f) They had personal experience of applying O.R. methods to solve problems.

g) The decision environment lends itself to a specification of goals.

Skills in qualitative analysis are inherent in the manager and generally increase with experience. Skills in quantitative analysis can be acquired by study of mathematical tools such as those discussed above. Using these tools, "managers can improve their decision making effectiveness. They can compare and combine the qualitative and quantitative information at their disposal and thus make the best possible decisions.

Activity 3

Comment on the relationship between O.R. specialist and manager.

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1.5 TYPICAL APPLICATIONS OF O.R.

Although the complete list of O.R. techniques and their applications would fill volumes itself, the following is an abbreviated set of applications to show how widely these techniques are used today:

1) Accounting:

Cash flow planning. Credit policy analysis.

Planning of delinquent account strategy.

2) Construction:

Allocation of resources to projects.

Determination of proper work force. Deployment of work force.

Project scheduling, monitoring and control.

3) Facilities Planning:

Factory size and location decision.

Hospital planning.

International logistics systems design.

Estimation of number of facilities required.

Transportation loading and unloading.

Warehouse location decision.

4) Finance:

Dividend policy making. Investment analysis. Portfolio analysis.

5) Manufacturing:

Inventory control.

Projection marketing balance.

Production scheduling. Production smoothing.

6) Marketing:

Advertising budget allocation.

Product introduction timing.

Selection of product mix.

7) Organizational behaviour:

Personnel justification/planning.

Scheduling of training programs.

Skills balancing.

Recruitment of Employees.

8) Purchasing:

Materials transfer. Optimal buying. Optimal reordering.

9) Research and Development:

Control of R & D projects. Product introduction planning.

A similar list can be prepared for any major field of human endeavour. Military activities alone would cover an entire book.

Activity 4

Identify some areas of application of O.R. technique in your organization.

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1.6 PHASES AND PROCESSES OF O.R. STUDY

O.R. is a logical and systematic approach to provide a rational basis for decision making. The phases and processes of O.R. study must also be quite logical and systematic. There are six important steps in O.R. study, but it is not necessary that in all the studies each and every step is invariably present. These steps are arranged in following logical order.

Step I : Observe the Problem Environment

Step I in the process of O.R. study is observing the problem environment. The activities that constitute this step are visits, conferences, observations, research etc. With the help of such activities, the O.R. scientist gets sufficient information and support to proceed and is better prepared to formulate the problem.

Step II : Analyze and Define the Problem

Step II is analyzing and defining the problem. In this step not only the problem is defined, but also uses, objectives and limitations of the study are stressed in the light of the problem. The end results of this step are clear grasp of need for a solution and understanding of its nature.

Step III : Develop a Model

Step III is to construct a model. A model is representation of some real or abstract situation. O.R. models are basically mathematical models representing systems, processes or environment in form of equations, relationships or formulae. The activities in this step defining interrelationships among variables, formulating equations, using known O.R. models or searching suitable alternate models. The proposed model may be field tested and modified in order to work under stated environmental constraints. A model may also be modified if the management is not satisfied with the answer that it gives.

Step IV : **Select Appropriate Data Input**

Garbage in and garbage out is a famous saying. No model will work appropriately if data input is not appropriate. Hence, taping right kind of data is a vital step in O.R. process. Important activities in this step are analyzing internal-external data and facts, collecting opinions and using computer data banks. The purpose of this step is to have sufficient input to operate and test the model.

Step V : Provide a Solution and test Reasonableness

Step V in O.R. process is to get a solution with the help of model and data input. Such a solution is not implemented immediately. First the solution is used to test the model and to find limitations if any. If the solution is not reasonable or if the model is not behaving properly, updating and modification of the model is considered at this stage. The end result of this step is solution that is desirable and supports current organizational objective.

Step VI : Implement the Solution

Implementation of the solutions obtained in previous step is the last step of O.R. process. In O.R. the decision making is scientific but implementation of decision involves so many behavioural issues. Therefore, the implementing authority has to resolve the behavioural issues. He has to sell the idea of use of O.R. not only to the workers but also to the superiors. Distance between management and O.R. scientist may offer a lot of resistance. The gap between one who provides a solution and one who wishes to use it should be eliminated. To achieve this O.R. scientist as well as

management should play a positive role. A properly implemented solution obtained through O.R. techniques results in improved working and wins the management support.

A brief summary of steps, process activities and process output is presented below:

Process Steps	Process Activities	Process Output
Step I Observe the Problem environment	Visits Conferences, Observations Research	Sufficient information and support to proceed
Step II Analyze and define the problem	Define uses Define objectives Define limitations	Clear grasp of need for and nature of solution requested
Step III Develop a Model	Define interrelationships Formulate equations Use known O.R. model Search alternate model	Model that works under stated environmental constraints
Step IV Select Appropriate data input	Analyze internal-external data Analyze facts Collect opinions Use computer data banks	Sufficient inputs to operate and test model
Step V Provide a solution and test its reasonableness	Test the model Find limitations Update the model	Solution that supports current organizational objectives
Step VI Implement the Solution	Resolve behavioural issues Sell the idea and give explanations Get management involved	Improved working and Management support for long run operation of model

1.7 LIMITATIONS OF OPERATIONS RESEARCH

Operations Research has certain limitations. However, these limitations are mostly related to the problems of model building and the time and money factors involved in its application rather than its practical utility. Some of them are as follows :

1) **Magnitude of Computations:** O.R. tries to find out optimal solution taking into account all the factors. In the modern society these factors are enormous and expressing them in quantity and establishing relationships among these require voluminous calculations which can only be handled by machines.

2) **Non-Quantifiable Factors:** O.R. provides solution only when all elements related to a problem can be quantified. All relevant variables do not lend themselves to quantification. Factors which cannot be quantified, find no place in O.R. Models in O.R. do not take into account qualitative factors or emotional factors which may be quite important.

3) **Distance between Manager and Operations Researcher: O.R.** being specialist's job requires a mathematician or a statistician, who might not be aware of the business problems. Similarly, a manager fails to understand the complex working of O.R. Thus there is a gap between the two. Management itself may offer a lot of resistance due to conventional thinking.

4) **Money and Time Costs:** When the basic data are subjected to frequent changes, incorporating them into the O.R. models is a costly affair. Moreover, a fairly good solution at present may be more desirable than a prefect O.R. solution available after sometime..

5) **Implementation:** Implementation of decisions is a delicate task. It must take into account the complexities of human relations and behaviour. Sometimes resistance is offered only due to psychological factors.

1.8 SUMMARY

O.R. is a relatively new academic discipline. It has its origin in World War II and soon became very popular throughout the world. O.R. is successfully used not only in military affairs but also in industry, government and business. India is one of the first few countries of the world who started using O.R.

Defining O.R. is a difficult task as its boundaries and content are not yet fixed. O.R. can be regarded as use of mathematical and quantitative, techniques to substantiate the decision being taken. O.R. takes tools from subjects like mathematics, statistics, engineering, economics, psychology etc. and uses them to score the consequences of possible alternative actions. Today it has become a professional discipline that deals with the application of scientific methods to decision making-especially to the allocation of scarce resources.

Developing appropriate mathematical models for situations, processes, systems or environment is basic essence of O.R. study. The model then can be tested, and operated on to determine the effects of changing the values of variables with particular reference to optimization of some criterion. Linear programming, non-linear programming, dynamic programming, game theory, Markov process, waiting line theory and simulation are some of the models and techniques that are used in O.R. There is role for O.R. in almost all areas of business decisions.

Operations Research has certain limitations also. These limitations are mostly related to the problems of model building and time and money factor involved in its applications rather than its practical utility. Magnitude of computation 'involved, lack of consideration for non-quantifiable factors and psychological issues involved in implementation are some of the limitations of O.R. Specialists in O.R. are not the decision makers themselves; they only provide rational basis for decision making to executives. There is a gap between who provides a solution and who wishes to use it. Due to this gap, management sometimes offers resistance to the use of O.R. The gap between O.R. scientists and managers is diminishing fast. O.R. techniques are gaining acceptance and respect day-by-day as they improve manager's decision making effectiveness.

1.9 KEY WORDS

Criterion is a measure by which results can be evaluated.

Dynamic Programming is a method to **analyze** multistage decision process.

Game Theory is a technique for decision making in situations of conflict.

Information Theory is a theory to evaluate the effectiveness of information flow within a system.

Integer Programming is a technique to ensure only best integral values of variables in programming problem

Inventory Control Models are models used to express total inventory cost to calculate optimal ordering quantity and ordering time to minimize total inventory cost.

Linear Programming is a constrained optimization technique to optimize a linear measure of effectiveness (objective function) under linear constraints.

Non-Linear Programming is a constrained optimization technique with a non-linear measure of effectiveness or constraints or both.

Optimization is achieving maximum or minimum (the best) value of some effectiveness criterion.

Sub-optimization is said to occur either when some criterion subordinate to the overall criterion is optimized or when the overall criterion is optimized over only a portion of the total of all possible alternative actions.

Waiting Line Theory deals with situations in which queues are formed: This is used to make calculations like expected number of items in the queue, expected waiting time, expected idle time, etc. which in turn are used to take decision regarding appropriate number of service channels.

1.10 SELF ASSESSMENT EXERCISE

1) Define Operations Research.
2) Discuss the limitations of operation research.
3) Enumerate, with brief description, some of the techniques of O.R.
4) Comment on the relationship between managers and O.R. specialists.
5) Describe the various steps involved in O.R. study.

1.11 FURTHER READINGS

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