BLOCK 4: INTRODUCTION TO RELATIVISTIC QUANTUM MECHANICS (OER)

In you course MPH-004 and Blocks 1 to 3 of MPH-008 you have studied the fundamental concepts of quantum mechanics and the mathematical framework that is required for applying quantum mechanics to physical systems, including approximation methods which deal with physical problems for which analytic solutions to the Schrödinger equation are out of reach. All of this falls under the broad classification of "**Non-Relativistic** " quantum mechanics except when you calculated the perturbation correction to the hydrogen atom energy levels due to relativistic effectts in Unit 5, however that was in the framework of time-independent perturbation theory. Most of you are also familiar with "Special Relativity" from your undergraduate courses and from Block 4 of MPH-007 in the context of Electrodynamics.

Quantum mechanics and special relativity were two of the most successful theories of the twentieth century and it seems natural to expect that quantum mechanics should also

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annihilation effects to understand such reactions.

In this Block you will study "**Relativistic**" quantum mechanics at an introductory level. As the name suggests, you will see how special relativity can be incorporated into quantum mechanics. You will study the relativistic wave equations like the Klein-Gordon equation for scalar particles and the Dirac equation (1928) for particles with spin and the new ideas (of the existence of anti particles) that come from these equations. You will study their application to the Hydrogen atom problem and calculate the relativistic corrections to the energy levels of the atom. You will also learn about the semi-classical Bohr-Sommerfeld quantization and its relativistic generalization. These equations do not incorporate particle creation and annihilation which forms a part of Quantum Electrodynamics, which was invented later in the 1930s and is beyond the scope of this syllabus.

This portion of your course MPH-008 is being offered through Open Educational Resources (OERs) in the form of recorded lectures by an expert in this field . You are familiar with studying from OERs from your MPH-005 course of the first semester. We will provide you the sequence of lectures, which you will be able to access from the given YouTube link.

The **syllabus** for this Block is:

Klein Gordon equation; Dirac's relativistic wave equation, position probability density, expectation values, Dirac matrices, plane wave solutions of the Dirac equation; the spin of the Dirac particle; significance of negative energy states and antiparticles.

This syllabus is covered in the first five lectures (Lectures 1 to 5) of the following NPTEL course :

Relativistic Quantum Mechanics By Prof. Apoorva D Patel

The course is available at the following link:

https://nptel.ac.in/courses/115108074

and you may watch the video lectures on the above link. You may also watch the lectures on You Tube at the following links:

Lecture 1: https://www.youtube.com/watch?v=77A9xWz_ugo

Lecture 2: https://www.youtube.com/watch?v=M6wGpiU3xvg

Lecture 3: https://www.youtube.com/watch?v=nSj6x9wEbF8

Lecture 4: https://www.youtube.com/watch?v=rwCDvLJclLY

Lecture 5: https://www.youtube.com/watch?v=2d2wP6MSiqM

Please click on these links to view the video lectures.

Before you start viewing the lectures please ensure that you are familiar with what you have studied in quantum mechanics so far, as well as with special relativity. You must also be familiar with tensors which you have learnt in MPH-001. The topics are mathematically intensive, so please keep a pen and paper at hand while watching these lectures and work out the steps of the derivations yourself.

We are sure that you will enjoy studying this course. If you face any difficulty while studying this course, you can consult your Counselor at the study centre or send an e-mail to us at mscph@ignou.ac.in with a cc to the course coordinators.

We now give you a brief introduction to the material covered in these lectures .

In Part 1 : Introduction to Relativistic Quantum Mechanics and the Klein Gordon Equation which is covered in Lecture 1 there is a brief overview of relativistic quantum mechanics and its realm of application. You will learn the notations for different quantities before moving on to the first of the relativistic wave equations, which is the Klein- Gordon equation. To derive the relativistic wave equation for a free particle, Klein Gordon equation, the standard prescription for transformation from classical to quantum is used , where the Hamiltonian is replaced by the corresponding operator $\hat{H} = i\hbar\frac{\partial}{\partial t}$ and the momentum by $\hat{p} = -i\hbar\nabla$ in the Hamiltonian for a relativistic free particle: $H = \sqrt{p^2c^2 + m^2c^4}$. This results in a second order differential equation in space and time which has as its solutions monochromatic, plane waves with the added complexity of allowing for both positive and negative energy values, thereby introducing the fundamental notion of an antiparticle. You will derive a charge conservation equation from the Klein-Gordon equation similar to the continuity equation for the probability current that you derived in Unit 3 of MPH-004 from the Schrödinger equation.

Expected Learning Outcomes

After you have heard this lecture, you should be able to:

- ❖ derive the Klein-Gordon equation from the relativistic energy-momentum relation for the free particle;
- ❖ explain how relativistic theory predicts the existence of antiparticles ;
- ❖ derive the charge conservation equation from the Klein Gordon equation ;

In **Part 2 : Particles and Antiparticles and the Two Component Framework** which is covered in Lecture 2 **,** you will first study in detail the charge conservation equation derived from the Klein Gordon equation in Lecture 1 and interpret the different quantities appearing in the equation like the density (ρ) and the current $(\vec{j})$. You will see how the intepretation of these quantities is different from the interpretation of the corresponding quantities that you have studied in non-relativistic quantum mechanics in Unit 3 of MPH-004. It turns out that in the relativistic case ρ is actually the charge density which can either be positive or negative with $E > 0$ corresponding to $\rho > 0$, and $E < 0$ corresponding to $\rho < 0$. In the next part of the lecture you learn how negative energy solutions are essential if you wish to localise a particle in position space to a region smaller than the Compton wavelength. The Klein Gordon equation applies to charged scalar particles. In the next part of the lecture you learn the two-component framework in which the Klein –Gordon equation is reformulated by separating the particle and anti-particle degrees of freedom. (Note that the Pauli matrices $\sigma_1, \sigma_2, \sigma_3$ are the matrices $\sigma_x, \sigma_y, \sigma_z$ that you have studied in Unit 14 of MPH-004) . You will study the effect of electromagnetic coupling and see how this confirms the opposite charge of particles and anti-particles.

Expected Learning Outcomes

After you have heard this lecture, you should be able to:

- ❖ write the charge conservation equation in terms of four vectors and interpret the variables in this equation;
- ❖ explain why negative energy solutions of the Klein-Gordon equation are necessary ;
- ❖ be able to reformulate the Klein-Gordon equation in the two component framework and transform it to two equations with explicitly positive and negative energies; and
- ❖ explain the effect of electromagnetc coupling.

In **Part 3 : Coupling to Electromagnetic Field and the Coulomb** field covered in Lecture 3, you learn more about the effect of electromagnetic coupling in the Klein –Gordon equation and learn how to solve the Klein Gordon equation for a Coulomb field and derive the expression for the hydrogen atom energy levels using the Klein-Gordon equation. You will see how this solution is different from the solution you obtained in Unit 9 of MPH-004. You will also see why the solution does not give the correct result for the fine-structure splitting of the energy levels. Recollect that you have worked out the fine structure splitting in Unit 5, using time-independent perturbation theory by adding a relativistic correction to the Hamiltonian. There is also is a brief introduction to the principle of semi-classical quantization or Bohr-Sommerfeld quantization which connects classical and quantum mechanics, through the action angle principle and the notion of adiabatic invariance, which you have studied in Unit 10 of MPH-006.

Expected Learning Outcomes

After you have heard this lecture, you should be able to:

- ❖ solve the Klein Gordon equation for a Coulomb field;
- ❖ derive the expression for the energy levels and compare it to the results derived from Schrödinger equation;
- ❖ explain why the solution could not explain the fine-structure splitting of the energy levels; and
- ❖ explain the Bohr-Sommerfeld quantization principle.

In **Part 4 : Bohr-Sommerfeld semi-classical solution of the Coulomb problem** covered in Lecture 4, you learn how to apply Bohr-Sommerfeld quantization to the Hydrogen atom problem and analyze the results. You will also study the relativistic generalization of these results and see how these results compare with the results you had from the Klein –Gordon equation. The next topic covered in this lecure is the relativistic wave equation of Dirac. You learn about the Dirac matrices and their properties. Please work out all the mathematical steps carefully while viewing the lecture.

Expected Learning Outcomes

After you have heard this lecture, you should be able to:

- ❖ apply the Bohr-Sommerfeld quantization and derive the expression for the energy levels for the Hydrogen atom;
- ❖ carry out the relativistic generalization of the results and compare with the results obtained from solving the Klein-Gordon equation;
- ❖ explain why the relativistic generalization of the semi-classical quantization results work inspite of spin angular momentum being excluded from the calculations;
- ❖ write the Dirac equation: and
- ❖ state and apply the properties of Dirac matrices.

In **Part-5: Dirac matrices, Covariant form of the Dirac equation covered in Lecture 5,** you learn in detail about the Dirac equation. You first learn to write the covariant form of the Dirac equation and you are introduced to the gamma matrices and their properties. The wavefunction in the Dirac equation is a multicomponent object , called a spinor, which you have studied about in Unit 14 of MPH-004. You will learn about the Dirac basis and the interpretation of the four components of the Dirac equation in terms of particles/antiparticles and spin degrees of freedom. You derive the current conservation equation from the Dirac equation and understand the significance of the various terms in the equation. You will learn the form for the velocity operator in the Dirac equation and derive the equation of motion for the orbital angular momentum. You will understand how this leads to the notion of the intrinsic angular momentum or “spin “angular momentum of a particle. You will derive the equation of motion for the spin angular momentum and see how total angular momentum is conserved. In the remaining part of the lecture you will write the solutions of the Dirac equation for a free particle and obtain the positive and negative energy solutions.

Expected Learning Outcomes

After you have heard this lecture, you should be able to:

- write the covariant form of the Dirac equation;
- define and apply the properties gamma matrices;
- derive the current conservation equation from the Dirac equation and explain the various terms in the equation;
- obtain the form for the velocity operator in the Dirac equation;
- derive the equation of motion for the orbital and spin angular momentum;
- show that the total angular momentum is conserved;
- write the solutions of the Dirac equation for a free particle and obtain the positive and negative energy solutions.

Further Readings:

1. **Relativistic Quantum Mechanics** by James D. Bjorken and Sidney D. Drell**,** *McGraw-Hill Book Company*(1964).
2. **Advanced Quantum Mechanics** by J. J. Sakurai, *Pearson Education* (1967).

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