UNIT 15 QUALITY ASSURANCE

Objectives

After successful completion of this unit, you should be able to:

- appreciate the role of quality control, quality assurance and quality assessment
- design a single sampling plan
- draw an operating characteristic curve for a given single sampling plan
- compute the AOQL
- assess process variability
- draw up an and R Chart
- decide whether a process is in control or not.

Structure

15.1 INTRODUCTION

You all must have come across the word **quality** in different contexts. All of us look for good quality in goods and services. We all realise the fact that the major achievement of industrial revolution has been the ability to mass produce a variety of goods of uniform quality, the classic example being the automobile. You will agree that the characteristic which sets apart Japanese goods as a sups or class, is precisely :heir quality. In this unit we will describe the assessment, control and management of he quality function in an industrial organisation. We will develop a simple working definition of quality and then outline the assurance aspects of quality management. Statistical techniques have contributed substantially to the success of modern quality control. Two of the important statistical techniques, viz., Acceptance Sampling and Control, Charts will be developed in detail. Finally we will outline the role of computers in the area of quality control.

While this unit will outline methods and techniques which are useful in quality control and quality management, a successful quality improvement will depend on the skill and efficiency of the manager in using these techniques.

15.2 CONCEPT OF QUALITY

Even though we all talk of quality, it is not easily defined. Before we give a definition, it will be a good idea to give your own definition.

Activity A

In the space provided, write down your definition of quality, in your own words. Also write down how quality can be measured.

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One of the accepted definitions of quality is **fitness for use**. An equally good definition is conformance to requirements. Note that in both the definitions quality is defined relative to use, rather than as a general characteristic that may be intangible. By this simple, yet practical definition, if a product or service lives up to expectations, it is of high quality. On the other hand, extra fine finish or using materials that are far stronger than required does NOT add quality to an item unless it somehow causes the item to conform to its requirements better. To appreciate the definition of quality, try the following activity.

Activity B

Judge the quality of the set of notes you are reading based on these definitions.

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You will notice that the same item may be perceived to be of quite different quality based on individual perception of end users. You should always keep this point in mind.

15.3 COST OF QUALITY

The term cost of quality is often a misnomer. Cost of quality is a measure of the cost to the firm for a lack of quality. It is very difficult to measure and often cannot be found in account books. One has to carefully back calculate, as most of the cost elements are hidden.

Quality costs are distributed throughout the organisation. Most organisations include only the cost of quality control departments whereas the cost of inspection, and measurement carried out in production departments are often ignored. More importantly the cost of bad workmanship, wastages, rework, etc. are often not included in quality costs. Careful examination of quality costs should account for Prevention, Assessment, Control Costs and costs due to lack of control.

"Quality is free, but it is not a gift". This statement sums up the opinion that effective; permanent quality improvement is difficult to achieve, but more than pays for itself in increased productivity.

1.5.4 QUALITY MANAGEMENT

Quality assessment is an investigation of the level of quality being achieved. **Quality control** on the other hand, begins with assessment, and includes action taken to eliminate unacceptable quality. The typical quality control programme is based on periodic inspection, followed by feedback of the results and changes or adjustments whenever necessary. Quality assurance includes quality control, but it also refers to emphasis on quality in the design of products, processes and jobs and in personnel selection and training. Total quality control refers to the managerial commitment to' quality so as to include the quality aspect in every functional area of work, production, marketing, finance and personnel. It also includes behavioural science based techniques like Quality Circles, Zero Defect Programmes. Naturally, the management of quality is an extensive area of study.

Quality assurance as an idea is quite old, but a systematic inclusion of quality assurance in organisations is a twentieth century phenomenon. Statistical methods of quality control were first proposed by Shewart in 1924, in the United States. Intensive training courses in statistical control popularised by the American and Japanese industry contributed to much of the success of quality control programme. Recently the concept of Quality Circles has been a runaway success in Japanese industries. A quality circle is a group of employees whose assignment is to identify problems, formulate solutions, and present their results to management with Suggestions for implementation. It is getting increasingly popular with employees and management in India also.

15.5 QUALITY ORGANISATION

A common mistake is to view quality as the responsibility of the department that produces the goods or services. Lack of conformance, however, can be a problem of design or even advertising. Consider the manufacture of automobile transmissions. If the gears are improperly designed, the transmission will fail despite the best quality of the manufactured gears. **That Quality is Everyone's Business** must be understood by everyone in the organisation.

Also, quality begins at the product concept stage and extends throughout the development, production, delivery and use of an item. Causes of poor quality can occur anywhere in the organisation, from top management to the shop floor worker, in accounting, production, sales, service or any other functional area of management (including.the quality control function itself!). Some quality problems have routes outside the organisation, such as defective supplies from a vendor or incorrect specifications from a customer.

To achieve success, a major commitment to quality must be made by top management, and it must be visible to all the employees. Major quality problems often cross departmental lines, so barriers to system-wise actions must be removed. Quality improvement must be established as a positive effort rather than blaming the assignment. In general, training is tale key to the success of quality control. Keeping these principles in mind, several alternatives exist for organising the quality control function.

Activity C

Ascertain from your organisation how the quality control function is set up. It may be a good idea to draw up an organisation chart.

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15.6 ACCEPTANCE SAMPLING

One of the powerful statistical techniques of quality control is Acceptance Sampling. This technique is generally used in those situations where items are inspected in batches, generally known as lots. For example, you may receive a shipment of 10,000 electric bulbs and you may have to decide whether to accept the shipment or return it back to the supplier. The acceptability will depend on the acceptable quality of the lot, which in turn depends on the use and the price you are willing to pay for this quality. Suppose you decide to accept if the average fraction defective is less than 5 per cent. Then to ascertain the actual quality you may decide to inspect each acid every bulb. Such a strategy of 100 per cent inspection, however, may often be expensive and impractical. In such cases a more intelligent way is to use the concept of **Sampling Inspection**.

Activity D

Think of a common situation where 100 per cent inspection is

a) Impractical

b) Impossible

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The idea of sampling inspection is to inspect only a small portion of the lot and **infer** the quality of the lot, based on the quality of the sample. Acceptance is based on the inference made from the sample and hence the.technique is known as Acceptance Sampling. Typically a lot is specified by its size (N) and the fraction (f) of defectives that are expected to be present (at the most) in the lot. The principles of statistics are used in the inference process.

Interestingly the concept of acceptance sampling is no different from the strategy adopted by a typical housewife who decides whether or not a pot-ful of rice is cooked by inspecting just a spoonful of grains.

Two things must be kept in mind. In order that sampling inspection might work, the sample must be **representative** of the lot. Typically this is ensured by choosing the sample at random so that every portion of the lot has equal representation in the sample. Such a sampling is known as **Random Sampling**. Second, a sample is only representative and not identical (in characteristic) with the lot. In the inference process, therefore, a few good lots will be rejected and a few bad lots will be accepted. We can control such **sampling errors**, but they cannot be eliminated. In fact in the design of **sampling plans** we will ensure that the errors are kept below certain acceptable levels.

Sampling Plans

We will first consider a **single sampling plan** in which accept/reject decisions are based on the results of a single sample of n items from the lot of n items. Each of the n sample items is inspected and categorised as either **acceptable** or **defective**. Such a plan is known as **Sampling by Attributes** (we will not discuss **Sampling by Variables** in this unit. The interested reader may refer to the references cited at the end). If the number of defective items in the sample exceeds a pre-specified cut off level, c, the entire batch is rejected. (Depending on costs, a rejected lot may be scrapped, 100 per cent inspected or returned back to the manufacturer). Since a finding of c or fewer defective items in the sample implies accepting the batch, c is often referred to as the **acceptance level**. A Sampling Plan is specified by the values of n and c.

The sampling plan is supposed to separate good lots from bad lots. As mentioned earlier there are bound to be sampling errors. We will now study the probabilities of such error graphically, using an **Operating Characteristic Curve**.

The Operating Characteristic Curve

It is useful to have a simple picture that allows us to compare sampling plans as to how they will react to different lots with unknown, varying fraction defective. Such a comparison is provided by the, operating characteristic curve (OCC) which displays the probability of accepting a lot with any fraction defective.

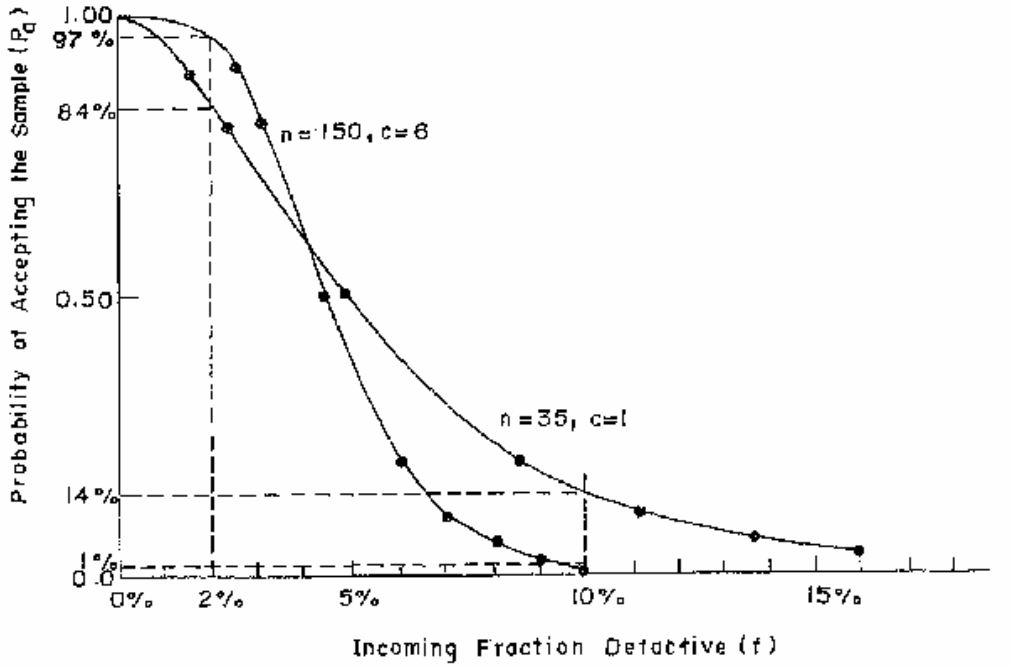

Figure 1: Operating Characteristics Curve

Figure I shows OCC for two single sampling plans A and B with n = 35, c = 1 and n = 150, c = 6, respectively. For example, suppose that a lot with F = 10 per cent defectives is considered to be a bad lot and" a lot with f = 2 per cent defectives is considered to be a good lot. From Figure I; it is clear that sampling plan A would stand a 14 per cent chance of accepting a bad lot. The same unfortunate error can occur with the sampling plan B, with larger sample size also, but the probability of error is much smaller. In fact it is only 1 per cent. The sampling plan B is also better at not rejecting good lots (f = 2 per cent). Sampling plan A has 16 per cent chance of rejecting a good lot whereas sampling plan B has only 3 per cent chance of rejecting a good lot.

It is not surprising that a larger sample does a better job of discriminating between good and bad lots. It has more information. However, the price for increased accuracy is higher inspection costs. The design of a sampling plan has to optimally trade off cost with discrimination.

The values of the ordinates of the Operating Characteristic Curve are determined from the Poisson Distribution. The actual details can be found in the advanced texts listed in the reference.

At this moment, pause for a while and check for yourself whether you have understood OCC. Do the following activity.

Activity E

a) How will the OCC change shape as
   i) n is increased, keeping c constant.
   ii) c is increased, keeping n constant.

b) What will be the limiting shape of the OCC.

c) Can you interpret your answers to (a) and (b).

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Design of Single Sampling Plan

You have to design a sampling plan (n, c) that has an OCC that meets certain pre-specified requirements. Generally the design is based on the following criteria that are related to the probability of making either of the following errors: accepting a bad lot 03) and rejecting a good lot (a). The criteria are established subjectively and ultimately should reflect the cost of accepting a bad lot or rejecting a good lot. Needless to say before a and (3 values can be specified, one has to decide what is a good lot and what is bad lot. Invariably this is done by specifying the lower/upper limits of fraction defective (f), as illustrated below:

AQL (Acceptable Quality Level) the fraction defective (f) that the user considers acceptable. Thus if a batch were known to have a fraction defective equal to AQL, it should not be rejected.

LTPD (Lot Tolerance Per cent Defective) the fraction defective that defines a bad lot or one that should be rejected. Of course AQL must be less than LTPD.

Producers Risk (a) the largest allowable probability of rejecting a good lot (due to statistical error). Note that a good lot has fraction defective less than or equal to AQL (generally 5 per cent).

Consumers Risk ((3) the largest allowable probability of accepting a bad lot (due to statistical error). Note that a bad lot as fraction defective greater than or equal to LTPD (generally 10 per cent).

Example 1

Consider a manufacturing situation with the following values:

AQL = 0.02, a = 0.05
LTPD = 0.10, 13 = 0.01

From Figure I you can verify that sampling plan A (n = 35, c = 1) has a probability of acceptance of 84 per cent for a fraction defective of 0.02 (AQL). In other words this plan has a 16 per cent chance of rejecting a good lot. Similarly, it has a probability of 0.14 of rejecting a bad lot with f = 0.10 (LTPD). Since both the values are higher than the allowed values of 0.05 and 0.01, respectively, this sampling plan is not acceptable. Only larger values of n can yield better discriminating power.

The sampling plan B(n = 150, c = 6) has the probability of accepting a good lot by 97 per cent and probability of rejecting a bad lot by 1 per cent. In other words it has a α value of 0.03 and $_R$ value of 0.01. This discriminating power is even more than what is needed. The plan is acceptable but it may be possible to get the required discrimination with smaller sample size and in turn with a lower inspection cost.

One way to decide the optimal **sampling** plan is to search through several sampling plans with n values between 35 and 150 and select the one that matches a and (3 values more closely. An easier way is to use **Thorndike Chart** (Table 1). This chart can be used for

a) Plotting OCC, and
b) Designing a Sampling Plan.

We will illustrate the design of sampling plan using this chart. Before we move to this topic, ensure that Plan A does not meet the requirement and Plan B meets more than the requirements, by following the arguments given earlier.

To design the sampling plan follow the instruction at the bottom of the chart. Note that we have to read off 13 = 0.01 and (1- a) = 0.95 rows only. We first need to find a c value for which $\mu p/\mu_a$ LTPD/AQL. For this problem LTPD/AQL = 0.10/0.02 = 5. Starting with c=0, we read off μp=4.6052 and $\mu_{«}$=0.0513 and so p /μ_as 5 is not satisfied. Continuing with C=1, 2, 3, 4, 5 we find that for C=6, $\mu a/\mu_{«}$=14.5706/3.2853.5. Hence we choose c=6.

To get the value of n, the limits are $n_\alpha=\mu_\beta$/LTPDL and $n_\alpha=\mu_\alpha$/AQL. Reading off the table we get.

n_β=14.5706/0.10 = 146
n_α = 3.2853/0.02 =165

Table 1

Table 1

Thorndike Chart for Single Sampling Plans

		Acceptance Number C										
		0	1	2	3	4	5	6	7	8	9	10
Acceptance Probability	P_a	$\mu = nf$ = expected number defective in the sample										
	0.010	4.6052	6.6383	8.4059	10.0450	11.6046	13.1085	14.5706	16.0000	17.4027	18.7831	20.1447
β rows, entries	0.025	3.6889	5.5716	7.2247	8.7672	10.2416	11.6683	13.0595	14.4227	15.7632	17.0848	18.3904
denoted μ_β	0.050	2.9957	4.7439	6.2958	7.7537	9.1535	10.5130	11.8424	13.1481	14.4346	15.7052	16.9622
	0.100	2.3026	3.8897	5.3223	6.6808	7.9936	9.2747	10.5321	11.7709	12.9947	14.2060	15.4066
	0.200	1.6094	2.9943	4.2790	5.5150	6.7210	7.9060	9.0754	10.2325	11.3798	12.5188	13.6507
	0.500	0.6931	1.6783	2.6741	3.6721	4.6709	5.6702	6.6696	7.6692	8.6690	9.6687	10.6685
	0.800	0.2231	0.8244	1.5350	2.2968	3.0895	3.9037	4.7337	5.5761	6.4285	7.2892	8.1570
$(1-\alpha)$ rows,	0.900	0.1054	0.5318	1.1021	1.7448	2.4326	3.1519	3.8948	4.6561	5.4325	6.2213	7.0208
entries denoted $\mu_{1-\alpha}$	0.950	0.0513	0.3554	0.8177	1.3663	1.9701	2.6130	3.2853	3.9808	4.6952	5.4254	6.1690
	0.975	0.0253	0.2422	0.6187	1.0899	1.6235	2.2019	2.8144	3.4538	4.1154	4.7954	5.4912
	0.990	0.0101	0.1486	0.4360	0.8233	1.2791	1.7853	2.3302	2.9061	3.5075	4.1302	4.7712

1 To plot an OC curve for a given sample plan (n,c): (a) Find the column for your c value. (b) Divide each number in that column by n. The results are the f values for the horizontal axis. (c) The P. values, for the vertical axis, are in the far left column,

2 To find a single sampling plan:
a) Find c for which $\mu pl\mu_x$ LTPD/AQL.
b) Then choose any n between $n_\beta = \mu_\beta$/LTPD and $n_\alpha = \mu_\alpha$/AQL

3 To find the acceptance probability for a given n,c, and f: (a) Multiply (n) (f). (b) In the appropriate c column, find values above"and below nf. (c) In the P. column read upper and lower limits for P. in two rows from step (b) (interpolate, if you wish).

Hence sampling plans with size in the range of 146 to 165 will satisfy the requirement. The exact values of a and 13 for any sampling plan can be determined using Thorndike Chart again. The exercise at the end of the unit will give you an opportunity to design many other sampling plans and decide the exact values of Consumer's Risk and Producer's Risk.

Average Outgoing Quality

The inspection process rejects lots with high fraction defectives. After rejection either you may stop, or you may continue the inspection of all the items in the rejected lot and all defective items are replaced with good items. Such a policy is known as **Rectifying Inspection.**

In rectifying inspection, all outgoing lots consists of N items either accepted ones or rejected ones. Suppose a lot has incoming fraction defective f. If it is accepted (N - n) items remain uninspected. We, therefore, expect f (N n) defectives in the accepted lots (assuming that the defectives found in the sample are replaced with good ones). In contrast, if it is rejected and hence (100 per cent rectified and inspected) there are no defectives. Thus if P_a is the probability that the sampling plan will accept the lot,

$$\text{Outgoing fraction defective} = \frac{(Pa)(f)(N-n)+(1-pa)(0)}{N}$$

$$= (Pa)(f)\left(\frac{N-n}{N}\right) \approx (Pa)(f)$$

A plot of outgoing fraction defective against incoming fraction defective (f) is generally called the **Average Outgoing Quality** (AOQ) curve. Figure II shows the curve for sampling plan B (n = 150, C = 6) of the earlier example. This curve has a surprising property that, as f increases, there comes a point at which the outgoing fraction defective actually begins to improve. The reason being that the sampling plan rejects most bad lots and they are rectified through 100 per cent inspection. The most critical incoming fraction defective f gives the worst outgoing quality. On the average, the value of that critical f is not important but the corresponding outgoing fraction defective generally known as average outgoing quality limit (AOQL) is extremely useful. No matter what the incoming fraction defective is, the long-run average outgoing fraction defective will not be worse than AOQL.

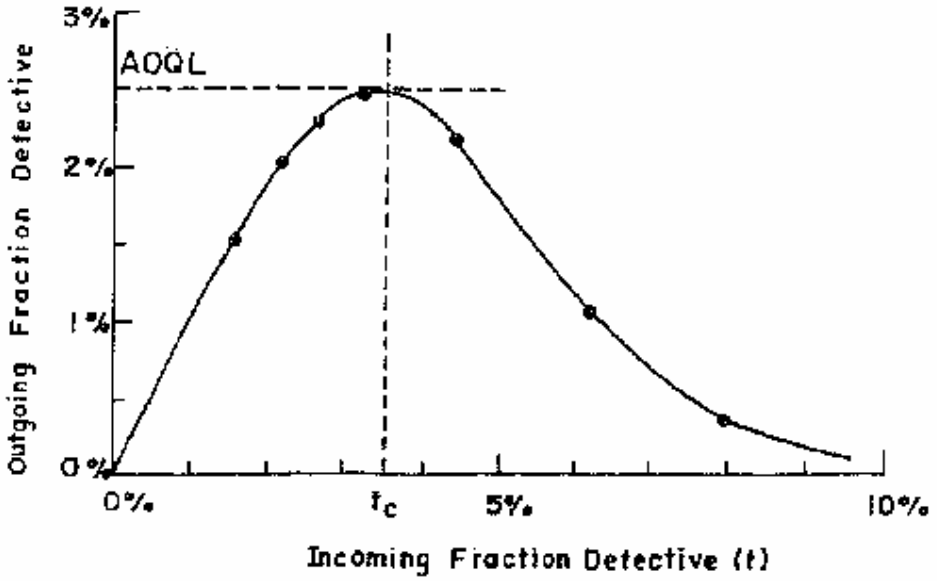

Figure II: AOQ Curve for Sampling Plan B(N = 10,000)

You do not have to determine AOQL by plotting the AOQ. One can use the following AOQL factor table. To get the AOQL simply use the formula,

$$\text{AOQL}=(\text{AOQL factor})\left(\frac{1}{n}-\frac{1}{N}\right)$$

Table 2
AOQL Factor

Acceptance Number (C)	AOQL factor (Y)
0	0.3679
1	0.8400
2	1.3711
3	1.9424
4	2.5435
5	3.1682
6	3.8120
7	4.4720
8	5.1457
9	5.8314
10	6.5277

Convince yourself that you understand the mechanism of determining AOQL by doing the following exercise.

Activity F

a) Using OCC for sampling plan B draw the AOQ curve over the range of 0-10 per cent fraction defective and determine AOQL.

b) Verify your answer to (a) using AOQL factor in Table 2 and the formula.

Many managers prefer to use the AOQL as a criterion for designing a sampling plan rather than trying to decide on values of AQL, LTPD, a and 13. Dodge and Romig (1959) present tables that are designed for this purpose. In fact these tables meet the requirement of a specified AOQL and minimise the expected number of items inspected per lot.

The OCC approach and AOQL based approach are but two of the many other approaches that can be used to design a sampling plan. Choice among them is a match of personal experience, the exact situation and the objectives of the organisation. Regardless of the approach, all sampling plans have both an OC curve and an AOQ curve, so the principles discussed in this unit can be used to evaluate any sampling plan.

Double and Multiple Sampling

Extensions of the single sampling plans to double and multiple sampling plans are also available. In a double sampling plan; after the first n_i samples have been inspected there are three choices depending on the number of defectives found:

1 reject the lot

2 accept the lot, and

3 draw a second sample of n_2 items.

If choice (3) is made the final accept/reject decision is made on combined sample of $n_l + n_2$ items. A multiple sampling plan operates in the same way, but with more than two samples. Double and multiple sampling plans reduce inspection costs because many accept/reject decisions are made based on the first sample which is smaller than that of the single sampling plan. However, single sampling plan is more common and easy to use. Details of multiple sampling plan are found in references.

15.7 PROCESS CONTROL

Variability

All products and services have a certain amount of natural variability because of variations in the input as well as imperfections in the process. For example, different quality of raw materials could have been used and different quantities of chemicals could have been used in the process. This **process variability** may be measured by the process standard deviation v, which indicates how much the products will vary even if the process is in control.

Products have to meet **specified tolerances** imposed by their intended use. Accordingly the natural variability must be substantially smaller than the specified tolerance. This -is explained in Figure .III in which the central line is the desired average of the process and the dashed lines are the `3-Sigma limits' representing the natural process variability.

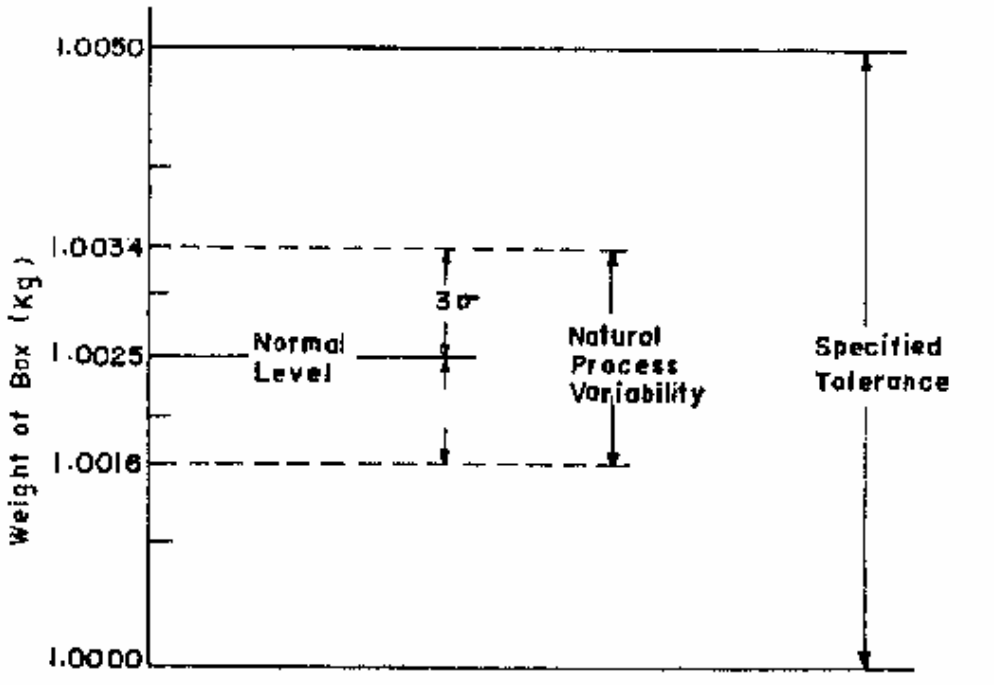

Figure III: Process Variability

It can be shown that variations of more than 3a from the process average are very unlikely. In fact it is about 0.25 per cent if the process follows the normal distribution and definitely less than 5% for most processes. The solid lines represent the tolerances specified by the intended use of the product.

Within the specified tolerances, a certain amount of process variability is to be expected. However, It is the goal of the **statistical process contro**l to determine when the process variability is getting out of hand; so that corrective action can be taken, preferably before the required tolerances are violated. This is generally achieved by a **Control Chart**

Control Chart

In order to provide rapid feedback to an ongoing production process, methods somewhat different from acceptance sampling are appropriate. Samples are taken as soon as they are available, rather than waiting for the completion of a lot. This affords the opportunity to detect unplanned changes in the process, shortly after they occur and take.a quick action, such as adjusting the machine.

The most common device used for this purpose is Shewart Control Chart introduced in 1931. The **control chart** is a visual display of the result of an inspection process incorporating carefully derived limits to indicate unusual behaviour. A control chart can be based on categorical information or actual measurement. Accordingly, they are called **control chart for variables** and **control chart for attributes**. Since control chart for variables are more commonly used and more powerful, we will describe them first.

The control chart is based on the idea that the average of a sample of several items will tend to cancel out the normal process variability, so that undesirable changes in the process will be more visible. We will illustrate the idea through an example.

X and R Charts

Consider XYZ Company that uses an automatic machine to fill 1kg. boxes of sugar. The tolerances are specified as 1.000 kg. on the lower side (legal requirement) and 1.005 kg. on the high side (no point,wasting sugar). Since the spread is only 0.005 they selected a machine that has a natural process variability of = 0.0003. The three sigma limits of the machine therefore are 3(.0003) = .0009 kg. above and below the mean. The spread is .0018 which is narrower than the specified tolerance of .005. They adjust the machine to fill boxes with an average 1.0025 kg. half way between the tolerance limits.

An Chart (Average Control Chart) was set-up to detect when the machine goes out" of control. In order to reduce the natural process variability, samples of n = 5 boxes were weighed, and the average weight per box, X, was recorded for each sample. Figure IV shows the control chart used for this machine and the four points plotted on the chart represent the values from four samples (a total of twenty 1 kg. boxes). We shall examine the details of the chart.

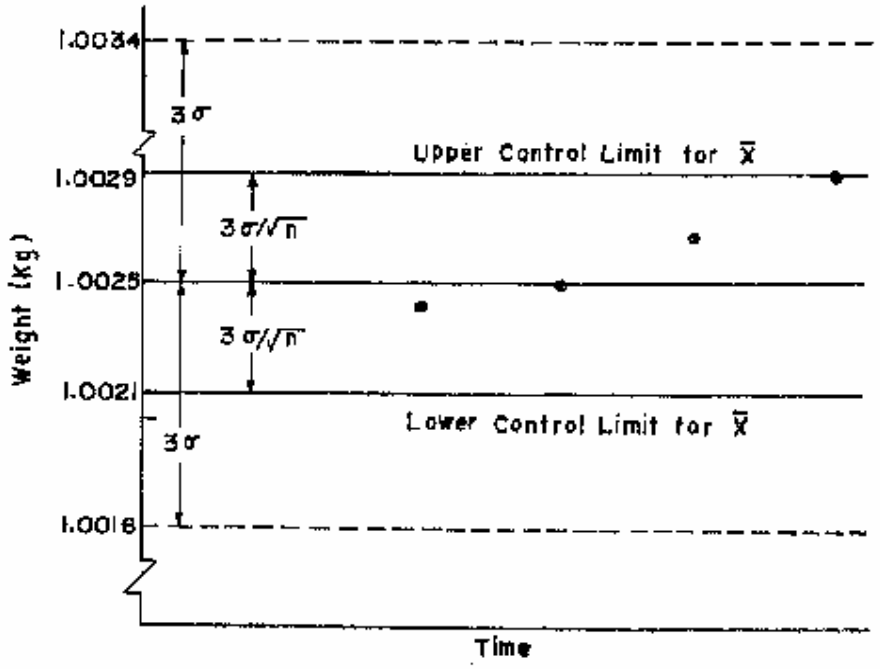

Figure IV: X̄ Chart

There is an apparent trend in X. The samples seem to be getting progressively heavier. However, appearances notwithstanding, the trend in the Figure IV may be X - due to random fluctuations, It is for this reason that we must incorporate the concept of statistical significance in our discussion. The standard deviation of the sample average is expressed through the formula,

$\sigma X = \sigma/\sqrt{n}$

Therefore, the averages of n = 5 boxes of sugar should have a standard deviation of $\sigma X=0.0003/\sqrt{5} = 0.000134 = 0.0004$. The control limits in Figure IV represent 3 sigma limits and are therefore (3) (0.000134) = 0.0004 above and below the intended average of 1.0025. If a sample average falls outside these control limits the deviation from the process average is **statistically significant.**

The fourth sample X is on the control limit and therefore there is a strong statistical evidence that more sugar is put into the boxes than what was intended and the machine needs adjustment.

The R Chart (Range Control Chart) is also used to control the processes. The range of a sample is the largest value minus the smallest. An R chart is appropriate if process sometimes goes out of control in such a way that there is inconsistency in the values, but no shift in the mean value of the process. For example, a worker who is basically good might produce an inconsistent set of sizes (of some manufactured product) when he is fatigued. The R chart used to plot the data of XYZ Company appears in Figure IV.

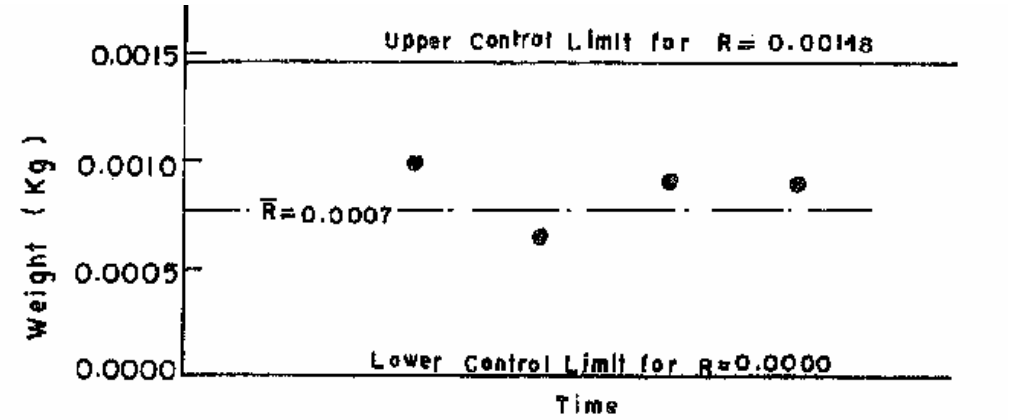

Figure V: R Chart

'The control limits for R chart are determined differently. There is no need for lower control limit which is generally (for n-6 it is zero) zero. Table 3 is used to set the upper control limit for an R chart and factor D_2 from the table is multiplied by the ' process standard deviation, cr. In the case of XYZ Company data, D_2=4.918 (for a sample size of 5) and hence upper control limit=(0.0003) (4.918)=0.00148. As can be seen in Figure IV no statistically significant shifts are present.

Table 3
R Chart Factor

Sample Size	D_2
2	3.686
3	4.358
4	4.698
5	4.918
6	5.078
7	5.203
8	5.307
9	5.394
10	5.469

Activity G

Verify the control chart for the example given in Figure IV. The data regarding the samples is as in Table 4.

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Table 4
Weights of Twenty 1-kg. Boxes

Box	Sample 1	Sample 2	Sample 3	Sample
1	1.00218	1.00266	1.00266	1.00306
2	1.00298	1.00242	1.00290	1.00266
3	1.00210	1.00258	1:00223	1.00234
4	1.00226	1.00250	1.00250	1.00322
5	1.00258	1.00234	1.00301	1.00322
Average	1.00242	1.00250	1.00266	1.00290
Range	0.00088	0.00032	0.00078	0.00088

Determining the **sample size** is an important decision. It is common to use n = 4 or n = 5 in order to obtain low cost feedback. Large samples such as n =15 or 20 are necessary if the process standard deviation is large. To a large extent this will depend on the rate of production, convenience and other considerations as well. Similarly, it is important to decide the frequency of sampling. It should be in general proportional to the average frequency of out-of-control conditions. It is a fairly complex decision to make if one were to look for some optimality. Generally, it is decided by convenience.

Other Control Charts

There are several other control charts, including the p-chart which is used to control the process when the measurement is by attributes. In other words the decision is only to decide whether or not the sample item is acceptable. No measurement is taken. For example in using the GO/NO GO gauges one gets only such a measurement.

The p-chart is based on the fraction defective, p, in a sample of n items. If po represents the normal process defective (i.e. when the process is in control) then the 3 sigma control limits are,

This is based on the fact that the number of defectives has the binomial probability. The control chart is used just like an *X* chart, except that the fraction defective p is calculated rather than *X* for each sample of n items and a lower control limit is often omitted.

$$\text{Po}\pm 3\sqrt{\frac{\text{po(1-po)}}{\text{n}}}$$

This is based on the fact that the number of defectives has the binomial probability. The control chart is used just like an X chart, except that the fraction defective p is calculated rather than X for each sample of n items and a lower control limit is often omitted.

Sample sizes are typically larger for p-charts then for X-charts. Since the information content of a yes/no measurement is much smaller than the actual variable measurement, it can only be expected. In fact, the required sample size can be computed approximately from the following formula.

$$\text{n}=\left[\frac{1.645\sqrt{p_1(1-p_1)}+3p_0(1-p_0)}{p_1-p_0}\right]^2$$

In this formula p_o is the normal process fraction defective, p_t is the specified fraction defective that is unacceptable. (p_o like AQL and pr is like LTPD in acceptance sampling).

For example, consider ABC Company that makes ready-made shirts: It has been found that 4 per cent of the shirts are defective when the process is under control. ABC Company wants to be able to detect a shirt to 12 per cent defective on the basis of one sample of n items. The formula for n suggests a sample size of n = 197. The upper control limits will be

$$p_0+3\sqrt{p_0(1-p_0)/n}=8.19 \text{ per cent}$$

15.8 USE OF COMPUTERS IN QUALITY CONTROL

Now-a-days with most of the computers including personal computers well written quality control packages are available. In some special cases these computers can be linked directly to take the sample measurements and control the process on-line. These packages are likely to take much of the chores associated with the lengthy calculations and make the application of quality control techniques far more easy and vet effective.

15.9 SUMMARY

Quality is fitness for use. Using this definition leads to many opportunities to improve both quality and productivity simultaneously. Many quality problems have causes that cross departmental boundaries. So a good quality management system must make the entire organisation responsible for quality. Statistical methods are important tools for quality control. They separate random variations from real assignable causes of deviations from normal. Acceptance sampling helps in deciding the quality of a large batch (lot) from an inspection of small sample: The operating characteristic curve precisely gives the risks associated with any sampling plan. The design of a sampling plan can be based on Operating Characteristic Curve as well as Average Outgoing Quality. Control charts display the results of inspecting a continuous process. This provides convenient and rapid feedback suggesting when feedback, overhaul or adjustment, may be needed. The design of control chart is based on sound statistical principle regarding the behaviour of sample mean.

The statistical methods described in this unit are used widely in manufacturing and service industry. They are also the basis for many of the commonly used, yet more complex, schemes described in the references.

15.10 KEY WORDS

Consumer's Risk: Probability of accepting a bad lot.

Producer's Risk: Probability of rejecting a good lot: OCC: Operating Characteristic **Curve. AQL:** Acceptable Quality Level.

LTPD: Lot Tolerance Per cent Defective.

AOQ: Average Outgoing Quality.

AOQL: Average Outgoing Quality Limit.

Control Limits: Limits if exceeded imply that the process is out of control.

15.11 SELF-ASSESSMENT QUESTIONS/EXERCISES

1 What is the fundamental difference between the use of acceptance sampling plans and process control charts?

2 Why are averages of samples used in control charts rather than individual readings?

3 Comment on the following:

a) It is important to inspect the inspector.

b) As a quality improvement programme is established, cost of quality increases.

4 In the example in the text, the sampling plan A with n = 35 and c=1 has a = 0.16 and 13 = 0.14, both too large to be acceptable.

a) What would happen to α and β if c were increased but n remained at 35?

b).Why do we need to increase both n and c to lower both a and 13?

c) If a batch contains 8% defective items, what is the probability that it would be rejected by the plan n = 40 and c =1.

d) Find a sampling plan for AQL = 0.008, LTPD = 0.01, a = 5%, 13 =10 per cent.

5 A manufacturing company produces a small product in lots of 10,000. They want to be 90 per cent sure of accepting the lot with fraction defective of 0.01 and 95 per cent sure of rejecting a lot with a fraction defective of 0.08. They do not know anything about sampling plan design. They intuitively decide that they will take a sample size of 100 and accept if not more than 4 defective items are found. The reasoning is that it amounts to 4/100 = 0.04 fraction defective which is roughly the mid point of their acceptable and rejectable quality.

a) Does their plan achieve their goals?

b) Suggest a better plan.

c) What are the AOQL values for these plans?

6 One of the important functions of a hospital laboratory is the perform blood samples. The quality of this process is tested periodically be selecting five blood specimens and dividing each specimen into two equal parts. Approximately 30 minutes after the first batch of five has been processed, its twin are submitted and the results are compared. The following data are taken at four different times in an 8 hours shift.

Batch 1	Batch 2	Batch 3	Batch 4
1.2	0.6	0.6	2.1
1.8	0.3	1.5	0.6
1.5	0.3	1.0	0.6
0.9	0.0	1.0	2.7
0.3	0.6	0.9	2.7

a) Calculate the 3 sigma control limits for the process. Assume normal process average to be 0.9 and the process standard deviation is 0.5.

b) What control limit will be used for range chart?

c) Is the process in control?

7 Using the data of Problem 6 calculate the mean and standard deviation of each of the batch data. Use the average of the means and the average of the standard deviation as the estimate of the process average and process standard deviation (instead of 0.9 and 0.5, respectively).

Rework problems 6(b) and 6(c).

15.12 FURTHER READINGS

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