UNIT 2 APPROACHES AND TECHNIQUES OF TEACHING MATHEMATICS

Structure

2.1 Introduction

2.2 Objectives

2.3 Basic Principles of Methods of Teaching Mathematics
   2.3.1 Principles of Child Development and Learning
   2.3.2 Trends in Organizing Content
   2.3.3 Problem-Solving Approach to Teaching
   2.3.4 Language of Mathematics

2.4 Methods of Teaching Mathematics
   2.4.1 Induction and Deduction
   2.4.2 Analytic and Synthetic Methods
   2.4.3 Heuristic or Discovery Method

2.5 Techniques in Teaching Mathematics
   2.5.1 Drill and Practice
   2.5.2 Oral and Written Work
   2.5.3 Play Way Technique
   2.5.4 Assignment and Home Work
   2.5.5 Unit Planning and Lesson Planning
   2.5.6 Materials and Teaching Aids
   2.5.7 Laboratory Approach to Teaching of Mathematics

2.6 Let Us Sum Up

2.7 Unit-end Activities

2.8 Answers to Check Your Progress

2.9 Suggested Readings

2.1 INTRODUCTION

The teaching and learning of mathematics have always been a major concern in education. Various commissions and committees have laid great emphasis on raising the quality of instruction in mathematics. The National Policy of Education (1986) lays down the importance of mathematics as a vehicle for developing creativity. Recent researches in the area of learning have led to a deeper understanding of "how pupils learn". As a result a broad range of new approaches to the teaching of mathematics have been suggested to achieve optimal learning. The highly structured nature of mathematics, its language and methods of proof have also attracted the attention of psychologists and educationists. Consequently, the old methods of mathematics teaching which relied heavily upon rote learning and drill have been replaced by methods which rely upon discovery and problem solving approaches.

The present unit presents the various approaches and techniques of teaching mathematics which will help the teacher to plan instruction in the classroom in the most effective manner.

2.2 OBJECTIVES

At the end of the unit, the teacher will be able to:

- interpret the principles of child development for planning lessons;
- understand the principles of learning;
Appreciate the role of the language of mathematics in learning and teaching;

Help students to apply problem-solving skills in solving mathematical problems;

Select and apply different techniques of teaching;

Make proper choice of materials and teaching aids for each lesson; and

Organize construction activities, projects, etc., to enrich teaching.

2.3 Basic Principles of Methods of Teaching Mathematics

Mathematics has always been the most important subject in the school curriculum. Traditional mathematics teaching has been found to be unsatisfactory. During recent years the demand has grown to make mathematics teaching more imaginative, creative and interesting for pupils. Clearly the demands made on the mathematics teacher are almost unlimited. The teacher must have a specialized understanding of the foundations of mathematical thinking and learning. He/she should also possess skills to put together the whole structure of mathematics in the minds of his/her students. He, like a master technician, should decide what kind of learning is worth what; realize and make use of motivation and individual differences in learning. He/she should be able to translate his/her training into practice. Finally, he/she should plan or design the instruction so that an individualized discovery-oriented (or problem-solving) learning is fostered.

A few of the current trends in the methods and media used in mathematics instruction are mentioned here. These include the basic features of more recent ideas which are the gift of educationists and psychologists. It is expected that teachers would try to fit them into their practical scheme of teaching.

2.3.1 Principles of Child Development and Learning

How does one teach most effectively? Very simple: teach the child in the way he learns best. Therefore, it is necessary that the teacher understands how a child learns, and the factors which affect learning. Thus, the teacher has to understand the way in which growth and development affect learning. Some aspects of learning are now discussed.

1. A child learns best when he is clear about the purpose or goals to be achieved. It is better if he/she is guided by a self-selected goal. His/her purpose determines what he learns and the degree to which he learns.

2. Children grow physically, mentally and socially at different times and with different growth rates. Various growth curves giving data about heights-weights, age, intelligence and interest or aptitude inventories which apply to children of a given age group are available. However, deviations are observed many a time in a given group of children.

The studies of Jean Piaget make it clear to us that a child's mental growth is a continuous process from birth and that his thought processes are by no means those of an adult. The stages of cognitive development which Piaget claims are important for the teaching of mathematics are:

Stage 1: Sensory motor operations: This stage lasts for about the first eighteen months since birth.

Stage 2: Concrete thinking operations: This stage lasts until about eleven or twelve years of age.

Stage 3: Formal thinking operations: This stage comes to form at the age of about fourteen or fifteen.

The actual age at which each stage is attained varies considerably from child to child because of the differing cultural backgrounds and environment. There is no clear borderline between the end of one stage and the beginning of the next. However, what is important is
that Piaget considers that the order in which the stages appear is fixed and this provides a framework against which we can examine the teaching strategy.

3. Learning is a continuous development process. It is change in behaviour brought about by thinking while facing situations that call for making discoveries, recognizing patterns and formulating abstractions or generalizations in mathematics. A child grows through experiences which provide both security and adventure. A learner learns what he does himself. Inefficient rote learning does not cause permanent learning and results in frustration and dislike for the concept/subject. If an experience is motivating only then it stimulates the creative faculty of the child and encourages exploration and ensures the fullest development of the child's mathematical potential. "Learning by doing" or the "discovery approach" through carefully controlled situations or chosen problems has proved to be a sound teaching strategy and a highly motivating activity.

4. A closer examination of the vast literature on "mathematics learning" reveals mainly four levels or steps of learning.


The necessary conditions leading to the acquisition of new responses are (1) Real situations: first-hand experiences with concrete things, (2) intuition, exploration, discovery through investigation, (3) formulation: verbal or symbolic representation based on logical reasoning and (4) assimilation, classification, generalization or concept formation through thinking and reasoning.

New concepts are developed as an extension of previous learning. The process of learning as well as the product should be emphasized. Generalizations in mathematics are formed inductively and applied deductively.

### 2.3.2 Trends in Organizing Content

Owing to the influence of professional mathematicians and due to the recommendations of national groups concerning updating the school mathematics curriculum, new considerations have come to be strongly emphasized during the past 25-30 years. These have a decisive impact on the planning of instructional strategies in mathematics.

1. Recent trends in selection of topics: The advancements and extensive use of technology has replaced manual computations almost completely. Thus, many traditional mathematical topics (e.g., vulgar fractions H.C.F., L.C.M. of large numbers, complicated questions on areas, volumes etc.) and skills (e.g., tedious simplifications with brackets and complicated calculations with very large numbers) have now become obsolete and are not emphasized any more. Arithmetic and algebra are now taught more meaningfully and in an integrated manner. The emphasis has shifted from deductive proofs in geometry to constructions and applications of geometrical properties. A clear distinction is made between the numerical system and the numeration system. The language of sets, relations and mappings is now used in verbal, symbolic and diagrammatic forms.

2. In planning instruction, mathematics does not appear as a static, ready-made, prefabricated body of knowledge any longer. Rather, it is presented as an ever-expanding, growing and lively subject. Pupils are being given more opportunity to experience typical processes of mathematical activity like looking for patterns; making quizzes, puzzles, analogies and proving arguments, etc.

3. The new textual material presents mathematics as a unified discipline of broad key concepts and fundamental structures. The emphasis is on developing conceptual, meaningful mathematics without minimizing the importance of proficiency in computational skills. It is now clarified to pupils "how and "why" different operations take place before expecting them to master computational skills.

4. There are increasing efforts to show mathematics as a useful tool for studying other subjects.
Better coordination between the teaching of mathematics and instruction in other subjects has been recommended.

2.3.3 Problem-Solving Approach to Teaching

It is a fault that the attitude of teachers and pupils towards the learning of mathematics is not clear. Some teachers lack confidence and feel insecure. They prefer to follow rigid and stereotyped curricula and methods, rely heavily on texts and use punishment as a mode of getting assignments done. This is because their own mathematics is often too fragmented to cope with the necessary understanding of extension of a topic and they find it difficult to relate one topic to another. The crisis of attitude among children is very well reflected in their performance, failure and dislike for the subject. It is felt that problem-solving in mathematics presents to both the teacher and the pupils an opportunity to redeem this very sad situation. Problem-solving is an individual or a small group activity, most efficient when done cooperatively with free opportunity for discussion. As a consequence, it permits the incorporation of a wide range of levels and styles of thinking and development. Problem-solving reflects the process of mathematics. It increases a child’s ability to think mathematically. The method of problem-solving is a method of thinking, of analysing, and of learning how to find the answer to a question or problem using known ideas. Learning through problem-solving is a progression from known ideas to unknown ideas, from old ideas to new ideas and from the simple to the complex. Problem-solving essentially results in an increased ability to think and generate ideas of mathematics. Problem-solving does not mean doing the block of exercises at the end of each chapter or unit.

The process of problem-solving involves

a) Sensing, accepting and defining a problem which is intriguing or meaningful to children of the relevant age. The problem need not always be real. The only important factor is acceptance of the problem by children as their own.

b) Considering the relationships which exist among the elements of the situation. Identifying data and information, making knowns and unknowns explicit, presenting data, etc., are a few skills required at this stage.

c) Pursuing the plan of action to a tentative answer. This includes techniques such as trial and error, defining terms and relationships using empirical arguments and control of variables.

d) Testing the result.

e) Accepting the result and acting on it.

The problems have many sources. They may be found in the environment or may be related to some area of living, they may be real (project type) or mental (puzzle or quiz) type.

Problem-solving situations may be used by the teacher for three purposes: (a) for helping children develop mathematical ideas, (b) for the application of known mathematical ideas in new situations, (c) for the analysis of the method of problem-solving.

The basic techniques which help are the same for all the three categories. These are drawing a diagram, restating the problem in one’s own words, dramatizing the situation or preparing a model, replacing the numbers (quantitative aspects) by variables and rearranging data, estimating an answer, arguing backwards logically, i.e. from “to prove” to “what is given” and discover the relationships between the known and the unknown.

2.3.4 Language of Mathematics

It is known that language can either help or hinder learning. If language is used correctly and with clarity, it helps in thinking but if it reveals imperfect meanings it creates a misunderstanding. Since mathematics deals in abstractions and itself is a way of thinking, it creates a dependent relationship between the notions and the language used to describe them. Mathematical language facilitates thinking by complementing ordinary language. Consider how a child gets the notion of a “circle”. A child handles, manipulates and observes shapes of objects like a wheel, bangles, the ring, etc. He may experiment with a model or may stand in a circle while playing. In all
these actions he must abstract the properties or features which make a mental picture called a circle. Having understood that to be a circle, a shape has to consist of a set of points in the same plane such that they all are at the same distance from a given point. He then generalizes his understanding to all possible circles and their relations and components. He leaves out the concrete modes and subconsciously enters the symbolic level. To use or communicate that abstract idea, one requires language. So mathematical language walks hand in hand with the growth of mathematical understanding, permeating the general linguistic development of children. Also, mathematics itself is a language; it has its own symbols and rules for correct usage. In spoken language, usage indicates what words mean, in mathematics careful defining sharpens word meanings. Mathematical language is clear, concise, consistent and cogent. Pupils who get the idea and describe it in correct language are less confused than pupils who memorize terms representing ideas which remain as strange as the terms themselves.

Mode-Building: Translating verbal language to the language of mathematics, that is solving a word problem, involves three stages: (i) encoding, (ii) operations, (iii) decoding.

Encoding is the process of building a mathematical model from a given verbal statement. Suppose we say that "a father's age is 5 years more than twice his son's age". If we assume the two ages to be $x$ and $y$ years respectively, then the corresponding mathematical model is

$$x = 2y + 5$$

After a model has been set up, we operate on it according to given conditions, obtain a solution and then translate it back into verbal language.

The skill of model-building requires a clear understanding of the mathematical equivalent of words which have mathematical meanings. Words such as more, less, times, difference, is equal to, square, etc., have to be identified and used in the model for the verbal statement.

Check Your Progress

Notes:  a) Write your answers in the space given below.
       b) Compare your answers with those given at the end of the unit.

1. Give three important benefits of using problem-solving approach to teaching of mathematics.

......................................................................................................................................
......................................................................................................................................
......................................................................................................................................
......................................................................................................................................
......................................................................................................................................
.......................................................................................................................................

2.4 METHODS OF TEACHING MATHEMATICS

We have seen that it is difficult to weave patterns with ideas which are essentially abstract. The elements of mathematics and the roles of intuition, logic, generalization and structure are all difficult areas to delimit or explore. What then, are proof structures or formats for organizing teaching-learning situations which a teacher can use? A few important lesson patterns are mentioned below.

2.4.1 Induction and Deduction

Mathematics in the making is experimental and inductive. Induction is that form of reasoning in which a general law is derived from a study of particular objects or specific processes. The child can use measurement, manipulatory or constructive activities, patterns, etc., to discover a relationship which he shall later formulate in symbolic form as a law or rule. The law, the rule or definition formulated by the child is the summation of all the particular or individual instances.
In all induction the generalization that is evolved is regarded as a tentative conclusion.

Example 1: Ask pupils to draw a number of triangles. Ask them to measure the angles of each triangle and find their sum.

Conclusion: The sum of 3 angles of a triangle is 180° (approximately).

You can also ask children to cut the three corners of the triangles and put them at a point so that they form a straight line.

Fig. 2.1

Example 2:

3 + 5 = 8 sum of 2 odd numbers is an
5 + 7 = 12 even number.
9 + 11 = 20

In deduction the law is accepted and then applied to a number of specific examples. The child does not discover laws but develops skills in applying them. We proceed from the general to the particular or from the abstract to the concrete. In actual practice the combination of induction and deduction is practised. The laws discovered by pupils inductively are further verified deductively through applications to new situations.

The difference of inductive and deductive method observed in teaching of mathematics is as follows:

<table>
<thead>
<tr>
<th>Inductive Method</th>
<th>Deductive Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Proceeds from the particular to the general; from the concrete to the abstract.</td>
<td>1. Proceeds from the general to the particular; from the abstract to the concrete.</td>
</tr>
<tr>
<td>2. It takes care of the needs and interests of children. It is a developmental process.</td>
<td>2. The child is provide with information of facts, principles &amp; theories.</td>
</tr>
<tr>
<td>3. It encourages “discovery” and stimulates thinking.</td>
<td>3. It establishes linkage with real life observations &amp; knowledge already gained.</td>
</tr>
<tr>
<td>4. The generalization or rule is formulated by the child therefore he remembers it with ease.</td>
<td>4. The rule is first learn &amp; then derived by the child. So, he/she is likely to forget it.</td>
</tr>
<tr>
<td>5. The “how” and “why” of the rule/ generalization are made clear through reasoning.</td>
<td>5. The process is accepted by the child without much reasoning.</td>
</tr>
<tr>
<td>6. It starts from observation and direct experiences and ends in developing a rule in the abstract form.</td>
<td>6. It starts with a rule and provides for practice and applications.</td>
</tr>
<tr>
<td>7. It encourages child participation and group work.</td>
<td>7. It demands individual learning and treats a child as a passive recepient.</td>
</tr>
</tbody>
</table>
2.4.2 Analytic and Synthetic Methods

We have seen that in its early stages, most mathematics originates in ideas and concepts associated with physical form and shape. It is then presented as a systematic abstract structure in logico-deductive form. The ability to understand and work out a rigorous deductive structure using logic or reasoning is of great importance.

Analysis and synthesis are methods which use reasoning and arguments to discover relationships. Synthetic Euclidean geometry is a good model of a deductive structure and is favourable to the learning of reasoning and to the development of precision of thought. In any proposition we have (i) a hypothesis, which may be the information given in the proposition or a set of axioms, definitions, principles or relationships which have been proved earlier, and (ii) a conclusion, that is, the result to be proved or arrived at. Study the example given below.

Example: Prove that the sum of the three angles of a triangle is two right angles.

Here, the hypothesis is “a triangle (or the three angles of a triangle)” and the knowledge of results related to angles such as alternate angles, corresponding angles and the angle pairs which add up to two right angles (linear pair), etc., which are relations/definitions already proved prior to proving the given proposition.

The conclusion is “the sum of the three angles is two right angles”.

In analysis we start from the conclusion and break it up into simpler arguments establishing connections with the relationships assumed in the hypothesis. In so doing, we find the missing logical connections and formulate a pattern for the proof. This pattern, when retraced from hypothesis to conclusion, gives the synthetic proof.

Split to simpler steps and establish logical connections

(Analysis)

Express the pattern in deductive form

(Synthesis)

Step 1: Let ABC be the triangle with angles 1, 2, and 3. To prove: \[ \angle 1 + \angle 2 + \angle 3 = 2 \text{ right angles}. \]

Step 2: We know that a linear pair measures 2 right angles. Can we get a linear pair in fig. 2?
Step 3: Can we prove $\angle 1 + \angle 2 = \angle 4$?

We can at least cut an angle equal to $\angle 2$ from $\angle 4$ if we draw a line $CD$ parallel to $BA$ (corresponding angles are equal).

$\therefore \angle 2 = \angle OCE$

Construction: Draw $CD \parallel BA$

Now can we show $\angle 1$ to be equal to $\angle ACD$?
Yes, This is true because $\angle 1$ and $\angle ACD$ are alternate angles. Hence the result. We can see how through arguments in step 2 and step 3 we arrive at constructions which help in developing the logical connections between the conclusion and the hypothesis.

The same sequence when written from the hypothesis to the conclusion gives the synthetic proof.

Given: $ABC$ is a triangle.

To prove: $\angle A + \angle B + \angle C = 2$ right angles.

Construction: Draw $CD \parallel BA$ and extend $BC$ to $E$ (Fig. 2.4)

\[ \angle ABC = \angle DCE \text{ (corresponding angles)} \]

\[ \angle 1 + \angle 2 + \angle 3 = \angle 3 + \angle 4 \]

\[ \angle 2 = \angle OCE \]

Fig. 2.4

Add $\angle ACB$ to both the sides

$\angle ABC + \angle BAC + \angle BCA = \angle DCE + \angle ACD + \angle ACB = 2$ right angles.

Q.E.D.

Analytic Method

1. It proceeds from the conclusion to the hypothesis.
2. It involves breaking up the conclusion into simpler steps and setting up relationships with what is given or known. It applies intuition and inductive reasoning.
3. It is a method of discovery. The solution or proof is arrived at through systematic reasoning.

Synthetic Method

1. It proceeds from the hypothesis to the conclusion.
2. It involves writing out the steps in the proof in proper sequence using accepted deductive reasoning.
3. It is a method of presenting facts already discovered in a logical format.
4. It takes care of psychological considerations, self-learning, active participation of students, organized thinking and reasoning power. It builds up a scientific attitude, originality and creativity among the students.

5. The teacher acts as a guide and plans situations for discovery learning by students.

2.4.3 Heuristic or Discovery Method

The modern philosophy of mathematics instruction lays emphasis upon meaning, understanding and application. This is in contrast to the "traditional" or "drill" theory. Children should see the meaning and feel the importance of what they are learning. Under the "drill" theory they are told the facts which they memorize through repeated practice. In "meaningful" learning, the child is a participant in finding the answer. He reasons it out. He discovers the interrelationships by doing, experimenting and actively participating in situations. The drill or practice is given only when an understanding has been achieved. Various phrases such as "learning by doing", "activity approach" or "child-centred approach", etc., have been used to put greater emphasis on "child as a learner" as against the content matter supposed to be learnt.

The word "discovery" is not new. It has been in use under the title "the heuristic method". The word heuristic is derived from the Greek word 'heurisko' which means "I find". Schultze, in his book "The Teaching of Mathematics in Secondary Schools", has listed the advantages and disadvantages of the discovery method. Students think for themselves, acquire a real understanding of the subject, take more interest and are willing to learn, and a perception of relationships is made possible. The method does not work well with all teachers, it does not follow a textbook, it is slow and requires better class management techniques, etc. These are some of the disadvantages. Recently Jerome Bruner, in his book "Process of Education" has defined discovery as "a matter of rearranging or transforming evidence in such a way that one is enabled to go beyond the evidence as assembled to new insights".

According to Piaget, new experiences are met by the child by either assimilation or accommodation. If he has available a scheme with which the new experience ties in, he can assimilate it. If, on the other hand, the element of novelty is too great for assimilation to take place, he will have to accommodate the new situation by modifying existing and relevant schemata. It is through such alteration of existing structures that new ones emerge and the child's conceptual equipment grows.

Thus all discovery methods are invariably closely linked to practical work or problem situations which bring the child face to face with the inadequacy of existing schemata. Secondly, the teachers using the discovery method should guide and offer help to the child to develop a positive attitude and interpret controlled situations for the verification of their hypotheses to deduce new relations. The teacher's role is one of support and encouragement.

Some of the new experiments conducted in the past 20 years show that discovery or inquiry-oriented teaching techniques are more successful if the learner is allowed to speculate, hypothesize, commit errors without embarrassment, learn from contradictions or inconsistencies, make "mistakes" and produce and have a first hand experience of the growth of mathematical ideas.

Some of the common methods labelled as "project method", "problem method", "activity method", "induction method" fall under the discovery approach.

Check Your Progress

Notes: a) Write your answers in the space given below.
     b) Compare your answers with those given at the end of the unit.
Give two reasons to clarify why discovery method is a better method of teaching mathematics as compared to other methods.

........................................................................................................................................
........................................................................................................................................
........................................................................................................................................
........................................................................................................................................

2.5 TECHNIQUES IN TEACHING MATHEMATICS

Whatever method a mathematics teacher may adopt for teaching, the daily mathematics lesson tends to follow a fairly standard pattern. Each lesson builds upon the lessons previously taught. Hence, reinforcement by adequate practice or drill of previously learnt skills becomes an important task. Similarly, for gaining mastery of the new skills learnt, proper assignments/home-work need to be planned. As a rule, oral recitation and written work both form vital components of any lesson. Some of the techniques in teaching mathematics are discussed here.

2.5.1 Drill and Practice

Drill is one of the most essential ways (or methods) of learning in mathematics. The controlling purpose of all teaching activity is to reduce necessary learning to habit. Gaining mastery requires acquisition of habits, hence drill/practice plays an important role in acquiring mastery. By and large, practice lessons are of three types. The first category of lessons for mastery is of basic subject matter, e.g., multiplication tables, addition combinations, fractional equivalents of decimals and percentages, factorization, construction in geometry, etc. These include subject matter which must be thoroughly mastered so that speed and accuracy is ensured on which future learning can be based.

The second category includes lessons for the mastery of procedures. In mathematics one has to adhere to a systematic arrangement of steps, follow correct algorithms to scrutinize and check the correctness of each step, label appropriately parts in a diagram, sort out data, translate problems into symbolic form, practice short cuts, etc.

The third category consists of lessons which strive to develop the power of thinking and reasoning, and increase the concentration and interest of the learner. Such lessons include quizzes, puzzles and historical material which does not form part of a regular lesson.

Although, a certain amount of formal drill is inevitable, preference should be given to functional or meaningful drill. Meaningful drill implies prior understanding of content and its appropriate application. This drill is purposeful and is determined by need as well as by use. An effective drill lesson should be organized keeping in view the following considerations:

1. Drill should follow learning and understanding of basics. It should not encourage rote memorization without understanding the subject matter.

2. Drill should be varied. Some routine procedures make learning monotonous and uninteresting.

3. Drill should be individualized and rewarding to each pupil. Each child should see its purpose and utility.

4. Drill periods should be short and the learner’s achievement should be frequently tested.

5. Drill should not be planned merely to keep pupils “busy”. It should be based upon thought-provoking situations to avoid the repetition of any process mechanically.

6. Drill may also provide diagnostic information about pupils.
2.5.2 Oral and Written Work

Typical mathematical situations which people meet outside the school demand oral computation or involve a visual impression not completely described in words. A good deal of socio-economic information requires a quick response to a quantitative data. It is, therefore, desirable to develop a high rate of performance consistent with accuracy. Oral work helps each child work at the optimum rate which insures maximum accuracy for him. In any lesson, both oral and written (especially when modern practice or work books are used), work should mingle freely. Oral work provides a rapid drill designed to habituate a fundamental process, a mode of thinking, or a set of facts. It helps in completing more work in any given period. Generally, material for oral work should not be read from a textbook. Spontaneity in grasping the data, and organization of thought in a limited time, are important aspects to test the pupil's responses.

However, when a teacher finds it necessary to inspect/check work done by each child or to give children practice in independent work, written work becomes a necessity. Throughout written work accuracy in computation, legibility of figures and symbols, speed consistent with accuracy, proper algorithm, i.e., logical and sequential arrangement of steps in the solution, neatness of work and correctness of results should be kept in mind. Written work can also be kept as a collective record which may help in assessing the pupil's progress over a period.

2.5.3 Play Way Technique

The most recent technique in the teaching of mathematics is the Play Way technique or teaching through games. A game is a planned activity which the students undertake with the guidance of the teacher. Although only some concepts can be taught through games, the most important use of games like the quiz, the puzzle, the guessing game, etc., is the drill or oral practice of various mathematical concepts.

For example, here we demonstrate a quiz for class IX. Divide the class into two groups and ask questions one by one from each team on, say, factorisation. Then, ask a question on the factor theorem (which has not been taught). If the students are unable to answer, they would be motivated to learn the new concept and continue with the game. Thus, Play Way techniques can be used for concept formation and motivation too.

2.5.4 Assignment and Home Work

Every mathematics teacher assigns home work. The usual argument in favour of home work is that it provides additional time for practice and develops the habit of self-study and self-reliance. It is assumed that classroom time is only for teaching and not for practice. However, home work presents a number of problems — the study is unsupervised, it encourages the use of cheap notes and guides, etc. Often the unsupervised home work develops undesirable study habits. Home work, which is an extension of or supplementary to class work and which does not take away much of the free time of the child, is considered legitimate. Any home work assigned to pupils should be corrected and kept as a cumulative record in notebooks rather than on loose sheets of paper.

In recent years the concept of home work is being replaced by differentiated assignments which are adjusted to the individual progress of each child and which encourage each one to do his/her own learning. Planning the assignment represents one of the most important phases of teaching. It is that part of instructional activity which is devoted to (i) organizing a task to be done, and (ii) fitting to the task an appropriate procedure for accomplishment. It assumes that the most effective learning is the product of self-imposed pupil activity. A few principles governing a good assignment are listed here:

1. The assignment should be clear and definite. It should be brief but fairly explanatory to enable each child to understand the task assigned.

2. It should anticipate difficulties in the work to be done, and suggest ways to overcome them.

3. It should connect the new lesson to past experiences and correlate the topic with all related subject matter.
4. It should be interesting, motivating and thought provoking.

5. The activities suggested for the assignment should be varied and adapted to the needs and interests of the students.

Dr. Lorene Fox, in his article on “Home Work is What We Make It” has suggested the following criteria for good home work:

1. Is the home work challenging to the students?

2. Does it grow out of or is it related to the everyday life and interests of the students?

3. Does it encourage individual choice and creativity?

4. Does it foster the habits of working together, planning and execution through democratic ways?

5. Does it encourage discovery and the use of a variety of sources and ways of learning?

6. Is it well adjusted to the available time frame of the child and inculcates good study habits?

2.5.5 Unit Planning and Lesson Planning

An important aspect of teaching relates to planning and conducting daily lessons. While most teachers base their daily teaching on the subject matter presented in the prescribed textbook, there are many who extend the subject matter to include vital experiences which have their source in the need or interest of the learner. The subject matter is organized into units to provide for as many types of functional activities as possible.

A unit is a long-range plan to direct the instructional plan. A unit takes care of the logical unity of the subject matter and the psychological considerations of the learner — his needs, interests and ability to learn. There are many advantages in the unit organization of content:

a) The teacher directs the instruction programme and pupils carry it out with the cooperation of teachers and other pupils.

b) Units cut across subject matter lines, thereby providing for a better correlation between different branches in mathematics and with other subject areas. Thus, learning is more integrated rather than fragmented.

c) A variety of activities/experiences provided for meeting individual differences in learning. Learning is not forced upon the child.

d) Drill becomes functional and problem-solving skills more effective if developed in meaningful situations. Critical thinking is, thus, stimulated.

A unit generally consists of three parts: (1) the purposes or objectives, (2) learning experiences to carry out these purposes, and (3) evaluation tests to find out how well the purposes have been achieved.

1 Purposes are stated in terms of the understanding or the ability of the learners. The desired attitudes and appreciations which the teacher wants to see developed are also listed as outcomes.

2 The learning experiences or activities are such that they contribute to the growth of the child and help him/her move towards the stated purposes. Since learning is individual, it is desirable to suggest a wide variety of activities suitable for both group and individual work. These may be:

   a) Preparatory, which orient and motivate pupils for purposeful activity or test preliminary abilities of pupils.

   b) Developmental, which enable pupils to gain desired skills, abilities, attitudes, and
understandings. These activities involve discussion, problem-solving experiences, construction and other forms of creative experiences and field trips.

c) Culminating, which will be a sort of non-conventional assignment, organizing exhibits, preparing reports, individual record-books, short plays or a review test.

3. Evaluation includes plans to determine whether growth has taken place, how far it has gone, and in what direction.

Lesson planning relates to the organization of a forty-minute period for teaching. Since mathematics is a sequential subject, each day's lesson is a necessary foundation for understanding a subsequent lesson. A well-planned lesson gives a teacher a sense of security, keeps him/her on the right track, checks waste of time and ensures a smooth transition from one part of the subject matter to the next. It is advisable that the teacher draw up a schedule of lessons he/she plans to teach in advance, preferably in the beginning of each week. The lesson plan should contain the following six parts:

(1) Aims or objectives, (2) background material or previous knowledge, (3) introductory or motivational activities, (4) developmental activities, (5) summary, and (6) application.

1. The aim should be stated in behavioural terms. The statements should be simple, clear and in direct present tense.

2. The teacher must take note of previous learning required as a foundation for the lesson to be taught. This should clarify possible sources of errors or confusion and make sure pupils understand all that is necessary.

3. Motivation should be ensured through short, simple application problems.

4. During the development of the lesson, the teacher should take the role of guide and avoid dominating the class. He/she should ensure pupil participation, make use of proper questioning techniques, elicit from pupils full and accurate statements, employ correct vocabulary, present a sufficient number and variety of situations, provoke students to formulate generalizations.

When pupils submit their work, the teacher should see its correctness, point out errors, if any, and suggest remedial work, if necessary.

2.5.6 Materials and Teaching Aids

For many teachers, the only means of communicating with their pupils is through textbooks and blackboard work. The textbook has been the major source for providing explanatory material, directions for processes and procedures, a set of solved model examples, diagrammatic representations of quantitative relationships, practice exercises and model test papers. A good textbook saves the time and labour of the teachers. It also makes unnecessary the writing of exercises and problems dictated by the teacher. The usefulness of a textbook is increased if it includes suitable illustrations and diagrams. Too often difficulty arises because of the limitations of language or of readability in a textbook. A good textbook also provides exercises that call for oral rather than written responses. The mechanical requisites of paper, print size, format, and binding must measure up to approved standards in a good textbook. These days textbooks are supplemented by work-books and practice books which provide well distributed practice exercises arranged according to their difficulty level.

The most common aid of the teacher is the chalkboard. Almost every classroom in every school contains one or more chalkboards. Class furniture is arranged so that the students usually sit facing the board. Pupils' work can also be placed on the board and their errors can be easily corrected. Materials can be prepared at home on large sheets of paper which can be taped to the board. Colour is easily applied to the chalkboard to emphasize key ideas.

These days a wide variety of learning aids are included in teaching to clarify concepts, their applications and uses. These include:
a) Concrete and semi-concrete materials including measuring instruments. Some of these materials can be easily collected from community sources, and some may be purchased.

b) Excursions and exhibits.

Contact with real life situations through tours and trips enables pupils to grasp the role of mathematics in life.

Children find collections of pictures, clippings, posters and charts enjoyable and educationally profitable.

c) Construction activities.

Construction activities afford an excellent opportunity for learning by doing.

d) Multisensory-aids, that is, motion pictures, film strips and slides.

Audio-visual aids help in communication in a manner that is stimulating, expedient, enjoyable and profitable to both the teacher and the learner.

While selecting any aid the teacher should raise the following questions:

1. Is the aid appropriate? That is, can it represent the idea or concept in a more satisfactory manner? Is it adapted to the age level?

2. How expensive is the aid?

3. Is the aid attractive?

4. Is the aid easy to handle?

2.5.7 Laboratory Approach to Teaching of Mathematics

We are familiar with a laboratory as used in teaching science subjects. Recently, this idea has been extended to the teaching of mathematics. One reason why learning mathematics is considered difficult is the verbalistic quality of teaching. Verbalism is the use of words without emphasis on meaning and practice. It is not uncommon to find pupils who have learnt various concepts, formulae and theorems without a proper understanding of their meaning and use. In a laboratory pupils learn by doing. They participate in experiments, manipulate materials and models, use different instruments and are thus able to give meaning to verbalism. Much of the mathematics comes to life for the child in a laboratory situation. The laboratory provides an atmosphere in which problems are worked out in simulated life situations. Wall charts, models, mathematical instruments, film slides and video tapes and a lot of manipulative material should be provided in the laboratory. Various materials can be assembled from cheap and easily available things by the teachers and pupils.

In a laboratory situation

a) Learning is child-centred, not teacher-dominated. The pupils carry out the activity. The teacher acts as a guide or helper.

b) The work is related to life situations and has significance for the learner.

c) Among pupils more interest is created since they work in concrete situations rather than in the abstract.

d) Many community resources are utilized as the subject matter is organized into functional activities.
Nature, Objectives and Approaches to Teaching of Mathematics

Check Your Progress

Notes: a) Write your answers in the space given below.
       b) Compare your answers with those given at the end of the unit.

3. Give two advantages each of
   a) drill and practice and b) homework

2.6 LET US SUM UP

This unit discusses the principles of child development and learning as the understanding of these principles helps us in planning lessons effectively. The importance of mathematical language and its proper use has also been explained. Recent trends in organising content and importance of problem-solving approach have also been elaborated.

Methods of teaching mathematics are very important as they help the teacher to transact the contents of Mathematics effectively to the learners. The usage of these methods enables the teacher to make mathematics teaching more imaginative, creative and interesting for learners. To give a comprehensive view and understanding of methods to be used for teaching of mathematics, the various methods of teaching mathematics have been discussed in this unit. Considering the practical aspect of the utilisation of methods in real classroom situations, the important methods of teaching mathematics like Inductive and Deductive Analytic and Synthetic, Heuristic and Discovery have been dealt with in detail.

The emphasis has also been laid on the use of different techniques in teaching of mathematics. The various techniques discussed were Drill and Practice, Oral and Written Work, Play Way, Assignment and Home Work. The essential aspect of unit and lesson planning has been elaborated so that unit and lesson planning can be done effectively. The importance and use of material and teaching aids for teaching mathematics has also been highlighted. Finally the laboratory approach to teaching of mathematics has been explained so that it can be utilised for teaching of mathematics effectively.

2.7 UNIT-END ACTIVITIES

1. List the mathematical activities that you must perform in the course of a typical day or week. Compare this list with the course content of secondary mathematics. What difference do you note?

2. Analyse the course of study for secondary classes to determine how it is organised. Select a unit for Class IX and split it into lessons.

   Take a lesson of your choice and list the objectives. Give the development activities for the lesson indicating the principles you utilize.

3. Observe a lesson being taught by a teacher. Analyse the materials and teaching aids being used by him. Comment upon their use and limitations.

4. Study the illustrations given in a book. Comment upon their variety, adequacy, appropriateness and usefulness.

5. Choose any topic out of the topics at secondary level and illustrate how you would use:
   (a) Inductive method (b) Discovery method (c) Play Way techniques to teach the same.

6. Prepare a lesson plan to teach the proof of the Pythagoras Theorem.
2.8 ANSWERS TO CHECK YOUR PROGRESS

1. The three important benefits of the problem-solving approach are:
   a) The real life situation considered in the problem helps in motivating the learner.
   b) It helps in increasing the ability to think and generate new ideas.
   c) It results in a better understanding of concepts.

2. The two reasons which go in favour of the discovery method as compared to other methods are:
   a) It is more child-centered. It provides more opportunities to the child to think and reach conclusions.
   b) The feeling of discovery enhances the motivation for further learning and it also helps in fixing the concepts learnt and retaining them for a longer period.

3. a) The two advantages of drill and practice are:
    i) It helps in memorizing and retaining the concept for a longer period.
    ii) It provides diagnostic information about pupils.
   b) The two advantages of good homework are:
    i) It encourages individual thinking and application.
    ii) It can be adjusted according to the needs of the individual.

2.9 SUGGESTED READINGS


Stephen Krulik and Irwin Kaufman, Multi-Sensory Techniques in Mathematics Teaching Teachers; Practical Press Inc.


Buxton, Lawrie, Mathematics for Every Man, J.M. Dent & Sons Ltd., London.


The National Council of Teachers of Mathematics, Hints for Problem-Solving. Booklet No. 17, Washington D.C., USA.