UNIT 21 RETAINING WALLS

Structure

21.1 INTRODUCTION

Walls, used to retain masses of earth or other loose material in a vertical (or nearly vertical) position at locations where an abrupt change in ground level occurs, are called as 'retaining walls'. The abrupt change in ground level may occur when the width of an excavation, cut or embankment is restricted by conditions of ownership, use of structure, or economy. For example, in railway or highway construction, the width of the *right of way* is fixed and the cut of embankment must be contained within that width. Similarly, the basement walls of a building must be located within the property and must retain soil surrounding the basement.

Retaining walls are generally used for roads in hilly areas, swimming pools, underground water tanks, basement of building, constructing a building on a site where filling is required and at the ends of the bridges in the form of abutments.

Retaining wall prevents the retained earth from assuming its natural angle of repose. This causes the retained earth to exert a lateral pressure on the wall, thereby tending to bend, overturn and slide the retaining wall. The wall, therefore must be suitably designed to be stable under the effects of lateral earth pressure and also to satisfy the usual requirements of strength and serviceability.

Objectives

After studying this unit, you should be able to

- describe various types of retaining wall,
- explain forces acting on retaining walls, and
- design centilever walls and counterfort walls.

21.2 TYPES OF RETAINING WALL

Retaining walls can be broadly classified into two catagories :

(i) Free standing retaining walls, and

(ii) Walls which form part of structures.

Gravity wall, cantilever wall, counterfort wall and buttress wall are the most common examples of free standing retaining walls. Basement wall, wall-type bridge abutments, and side walls of box culvert are the examples of retaining walls which form part of a bigger structure.

Various types of retaining wall are described below.

21.2.1 Gravity Wall

The gravity wall provides stability by virtue of its own weight, and therefore, is rather massive in size. It is usually built in stone masonry and occasionally in plain concrete. The thickness of the wall is governed by the need to eliminate or limit the resulting tensile stress to its permissible limit. For obvious economic reasons, plain concrete gravity walls are not used for heights exceeding about 3 m.

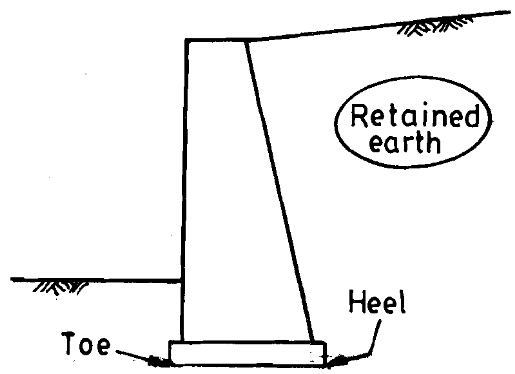
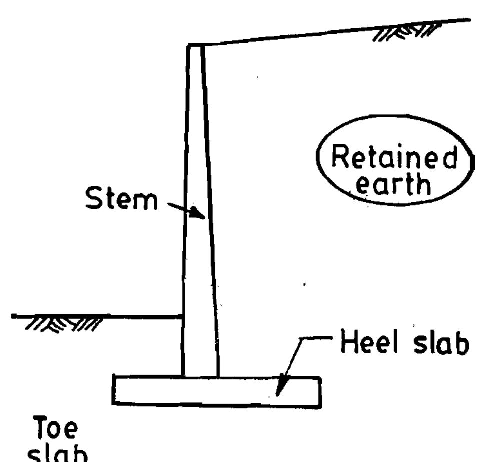


Figure 21.1 : Gravity Wall

21.2.2 Cantilever Wall

It is the most common type of retaining structure and is generally economical for heights up to about 7 m. It consists of a vertical arm (called as stem) which retains the earth and a base slab. Base slab is made up of two distinct regions, viz. a heel slab and a toe slab. These three components behave as one-way cantilever slabs : the, 'stem' acts as a vertical cantilever under the action of lateral earth pressure; the 'heel slab acts as a horizontal cantilever under the action of the weight of the retained earth (minus soil pressure acting upwards from below); and the 'toe slab' also acts as a horizontal cantilever under the action of the resulting soil pressure (acting upward). Reinforcement is provided on the flexural tension faces. The stability of the wall is maintained essentially by the weight of the earth on the heel slab plus the self weight of the structure.



21.2.3 Counterfort Wall

For the large heights, in a cantilever retaining wall, the bending moments developed in the *stem, heel slab* and *toe slab* become very large and require large thickness, thereby making the structure uneconomical. The bending moments (and hence the thickness of stem and slab) can be considerably reduced by introducing transverse supports, called counterforts. These counterforts are spaced at regular intervals of about one-third to one-half of the wall height and interconnect the stem with the heel slab. The counterforts are concealed within the retained earth (on the rear side of the wall). Such a retaining wall structure is called the 'counterfort wall' and is economical for heights above 7 m approximately. The counterforts subdivide the vertical slab (stem) into rectangular panels and support them on two sides (suspender-style), and themselves behave as vertical cantilever beams of T-section and varying depth. The stem and heel slab panels between the counterforts are now effectively 'fixed' on three sides (free at one edge), and for the stem the predominant direction of bending and flexural reinforcement is now horizontal (spanning between counterforts), rather than vertical as in the case of cantilever wall.

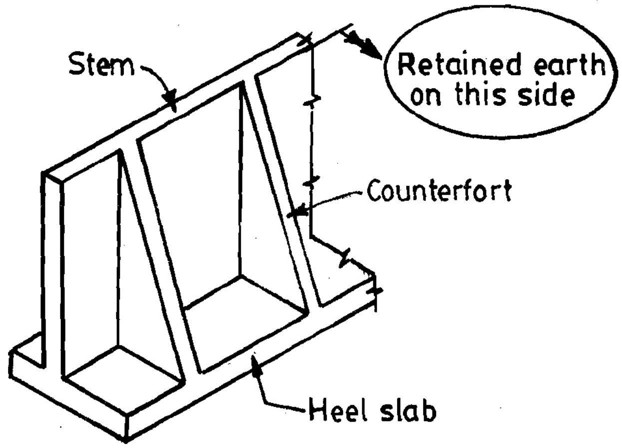


Figure 21.3 : Counterfort Wall

21.2.4 Buttress Wall

When the counterforts are provided on the front of the wall and not on the soil side, it is known as 'buttress wall'. Buttress wall is similar to the counterfort wall, except that the transverse stem supports called *buttresses*, are located in the front side, interconnecting the *stem* with the *toe slab* (not with the heel slab as in case of counterfort wall). Although buttresses are structurally more efficient (and more economical) than counterforts, the counterfort wall is generally preferred to the buttress wall as it provides free usable space (and better aesthetics) in front of the wall.

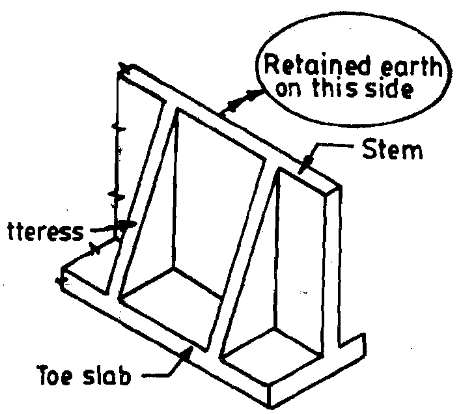


Figure 21.4 : Buttress wall

21.2.5 Other Types of Wall

Retaining walls often form part of a bigger structure, in which case their structural behaviour depends on their interaction with the rest of the structure. For example, the exterior walls in the *basement* of a building (Figure 21.5(a)) and wall type *bridge abutment* (Figure 21.5(b)) act as retaining walls. Slab at the ground floor level (in the case of basement wall) and bridge deck (in the case of bridge abutment) provide an additional horizontal restraint at the top of the vertical stem.

The stem is accordingly designed as a beam, fixed at the base and simply supported or partially restrained at the top. The side walls of *box culverts* also acts as retaining walls. In this case, the box culvert (with single / multiple cells) acts as a closed rigid frame, resisting the combined effects of lateral earth pressure, dead load (due to self weight and earth above), and the live load due to highway traffic.

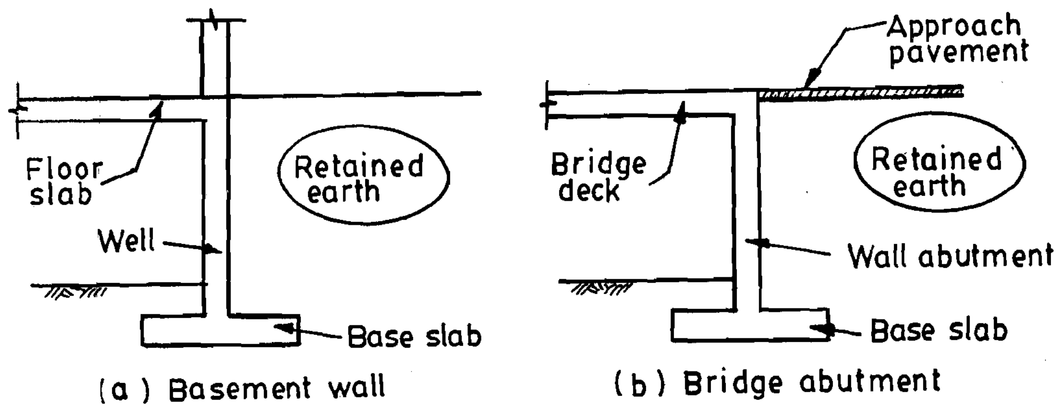


Figure 21.5 : Other Types of Wall

SAQ 1

What is the purpose of a reaining wall? What are the different types of concrete retaining walls?

21.3 FORCES ON RETAINING WALLS

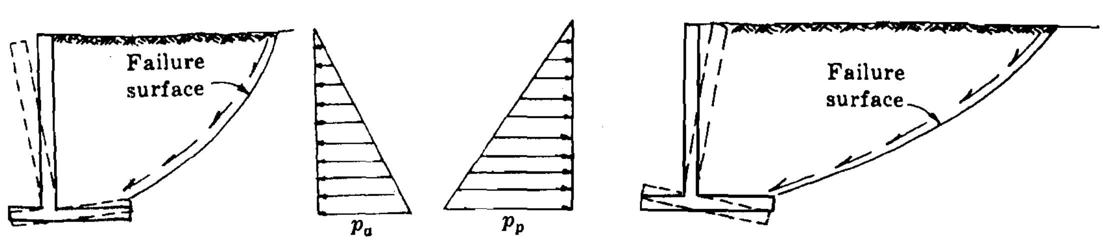
The two forces which act on a retaining wall are :

(a) Gravity loads due to the weight of the materials, and

(b) Lateral force due to earth pressure.

The usual gravity loads due to the weights of the materials do not present great problems with respect to retaining walls. The lateral force due to earth pressure constitutes the main force acting on the retaining wall. The determination of the magnitude and direction of the earth pressure is based on the principles of soil mechanics.

Under confinement, the soil has a tendency to slide and thereby exerts pressure on the wall. If the wall is absolutely rigid, *earth pressure at rest* will develop. If the wall deflects or moves a very small amount away from the earth, active earth pressure will develop (Figure 21.6(a)). If the wall moves towards the earth, *passive earth pressure* will develop (Figure 21.6(b)).



(a) Active Earth Pressure **(b) Passive Earth Pressure**

Figure 21.6 : Active and Passive Earth Pressures

The magnitude of earth pressure at rest lies at some value between active and passive earth pressure.

Under normal conditions earth pressure at rest is so intense that the wall deflects, relieving itself of this type of pressure, and active pressure results. For this reason, most retaining walls are designed for active pressure due to the retained soil.

In general, the behaviour of lateral earth pressure is analogous to that of a fluid with the magnitude of the pressure p increasing nearly linearly with increasing depth y for moderate depths below surface :

$$p = k \gamma y \qquad \ldots (21.1)$$

where, γ is the unit weight of the earth and k is a coefficient that depends on its physical properties and also on whether the pressure is *active* or *passive*. The coefficient to be used in Eq. (21.1), is the *active pressure coefficient*, K_a, in case of active pressure, and the passive pressure coefficient, K_p, in the case of passive pressure; the latter (K_p) is generally much higher than the former (K_a) for the same type of soil.

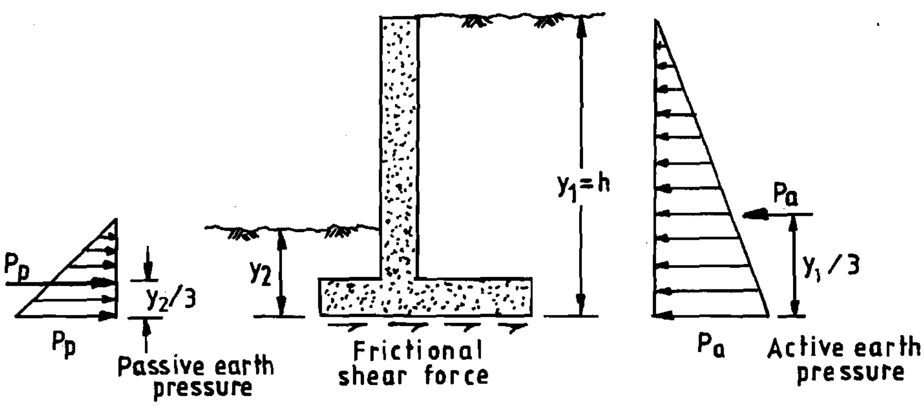


Figure 21.7 : Pressure Diagram

In the absence of more detailed information, the following expressions for K_a and K_p based on Rankine's theory of earth pressure may be used for *cohesionless soils and level backfills* :

$$K_a = \frac{1 - \sin \phi}{1 + \sin \phi} \qquad \ldots (21.2(a))$$

$$K_p = \frac{1 + \sin \phi}{1 - \sin \phi} \qquad \ldots (21.2(b))$$

Here ϕ is the angle of repose (or angle of shearing resistance). For a typical granular soil (such as sand), $\phi \approx 30^0$, corresponding to which, $K_a = 1/3$ and $K_p = 3.0$.

Thus pressure intensity at any depth y may be stated as $p_a = K_a \gamma y$ for active pressure and $p_p = K_p \gamma y$ for passive pressure.

When the backfill is sloped (Figure 21.8), the expression for K_a can be modified as follows :

$$K_a = \left[\frac{\cos \delta - \sqrt{\cos^2 \delta - \cos^2 \phi}}{\cos \delta + \sqrt{\cos^2 \delta - \cos^2 \phi}} \right] \cos \delta \qquad \ldots (21.3)$$

Here δ is the angle of inclination of the backfill i.e., the angle of its surface with respect to the horizontal.

The direction of active pressure, p_a, is parallel to the surface of the backfill. The pressure has a maximum value at the heel, and can be stated as

$$p_a = K_a \gamma h' \qquad \ldots (21.4)$$

Here h' is the height of the backfill, measured vertically above the heel. For the case of a level backfill, $\delta = 0$ and $h' = h$, and the direction of the lateral pressure is horizontal backfill of height h' above the heel, is accordingly obtained from the triangular pressure distribution as :

$$P_a = \frac{1}{2} K_a \gamma (h')^2 \qquad \ldots (21.5)$$

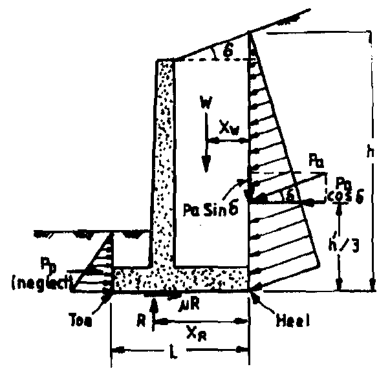


Figure 21.8 : Forces Acting on a Cantilever Retaining Wall

This force has units of kN per meter length of the wall, and acts at a height $h'/3$ above the heel at an angle of inclination δ with the horizontal. The force, P_p, developed by passive pressure on the toe side of the retaining wall is generally small (due to small height of earth) and usually not included in the design calculations, as this is conservative.

Effect of Surcharge on a Level Backfill

Loads are often imposed on the soil surface (level backfill) due to the construction of buildings and the movement of vehicles near the top of the retaining wall. These additional loads can be assumed to be static and uniformly distributed on the top of the backfill. This distributed load W_s (kN / m^2) can be treated as statically equivalent to an additional height, h_s, such that

$$h_s = W_s/\gamma \qquad \ldots (21.6)$$

This additional height of backfill is called surcharge and is expressed either in terms of height, h_s, or in terms of the distributed load W_s (Figure 21.9).

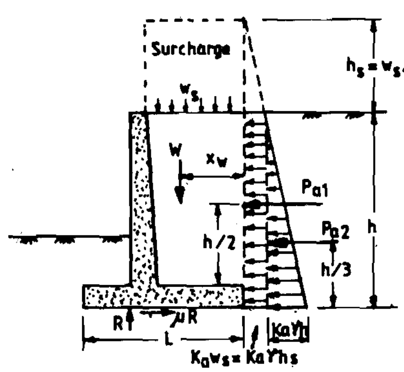


Figure 21.9 : Effect of Surcharge on a Level Backfill

The presence of the surcharge not only adds to the gravity loading acting on the heel slab, but also increases the lateral pressure on the wall by $K_a \gamma h_s = K_a W_s$.

The total force due to active pressure acting on the wall is given by

$$P_a = P_{a1} + P_{a2} \qquad \ldots (21.7)$$

where, $\quad P_{a1} = K_a W_s h = K_a \gamma h_s h \qquad \ldots (21.8)$

$$P_{a2} = \frac{1}{2} K_a \gamma h^2 \qquad \ldots (21.9)$$

Lines of action of P_{a1} and P_{a2} are at $h/2$ and $h/3$ above the heel respectively.

SAQ 2

Distinguish between active and passive pressures of earth, in relation to retaining wall structures?

21.4 STABILITY REQUIREMENTS

The factors of safety (FS) against overturning and sliding should not be less than 1.4. The stabilising forces (due to dead loads) should be factored by a value of 0.9 for calculating the factor of satety.

$$FS = 0.9 \times \frac{\text{(stabilising force or moment)}}{\text{destabilising force or moment}} \geq 1.4 \qquad \ldots (21.10)$$

21.4.1 Overturning

In case of overturning of retaining wall, the toe will act as the centre of rotation. Overturning moment, M_0, and the stabilising (restoring) moment, M_r ,depend on the lateral earth pressure and the geometry of the retaining wall.

For *sloping backfill* (Figure 21.8).

$$M_0 = (P_a \cos \delta)\ (h'/3) = \left[K_a \gamma (h')^3/6\right] \cos \delta \qquad \ldots (21.11)$$

$$M_r = W\ (L - x_w) + (P_a \sin \delta)\ L \qquad \ldots (21.12)$$

where, W = total weight of the reinforced concrete wall structure + retained earth resting on the heel slab, and

x_w = distance of the line of action of W from the heel.

(In the context of overturning, there will be no upward reaction R acting over the base width L. The weight of the earthfill above the toe slab is usually (conservatively) ignored. Similarly, the passive earth pressure P_p is also usually ignored.)

For a *level backfill with surcharge* (Figure 21.9).

$$M_0 = P_{a1}\ (h/2) + P_{a2}\ (h/3) \qquad \ldots (21.13)$$

and

$$M_r = W\ (L - x_w) \qquad \ldots (21.14)$$

Here, P_{a1} and P_{a2} are as given by Eqs. (21.8) and (21.9) respectively.

The factor of safety required against overturning can be obtained as

$$(FS)_{\text{overturning}} = \frac{0.9\, M_r}{M_0} \geq 1.4 \qquad \ldots (21.15)$$

21.4.2 Sliding

The resistance against sliding is essentially provided by the friction between the base slab and the supporting soil, given by

$$F = \mu R \qquad \ldots (21.16)$$

Here, $R = W$ is the resulted soil pressure acting on the footing base. In a sloping backfill, R will also include the vertical component of earth

μ = Coefficient of static friction between soil and concrete.

The value of μ varies between about 0.35 (for silt) to about 0.60 (for rough rock).

The factor of safety against sliding can be obtained as

$$(FS)_{\text{sliding}} = \frac{0.9\,F}{P_a \cos\delta} \geq 1.4 \qquad \ldots (21.17)$$

When active pressure is relatively high (as in case when surcharge is involved), it will be generally difficult to mobilize the required factor of safety against sliding, by considering frictional resistance below the footing alone. In such a situation, a *shear key* projecting below the footing base and extending throughout the length of the wall can be employed (Figure 21.10). The shear key must develop a sufficient passive pressure force to resist the excess lateral force. A simple and conservative estimate of the passive pressure force (P_p) developed, can be obtained by considering the pressure developed over a region ($h_2 - h_1$), below the toe.

$$P_p = K_p \gamma \left(h_2^2 - h_1^2\right)/2 \qquad \ldots (21.18)$$

Here, h_1 and h_2 are as indicated in Figure 21.10. The overburden due to the top 0.3 m earth below ground level is usually ignored for calculations.

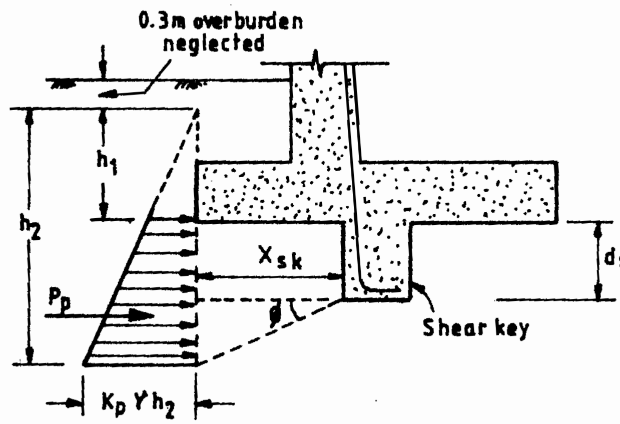
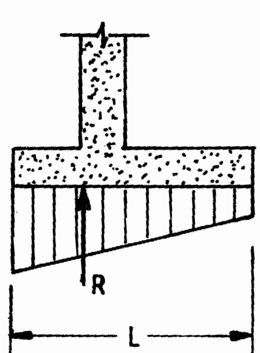


Figure 21.10 : Passive Resistance Due to Shear Key

The shear key is best positioned at a distance x_{sk} from the toe in such a way that the flexural reinforcement from the stem can be extended straight into the shear key near the toe.

21.4.3 Soil Bearing Pressure

The width L of the base slab must be adequate to distribute the vertical reaction R to the foundation soil without causing excessive settlement or rotation. The required foundation depth and the associated allowable pressure q_a can be determined based on the soil study. Tilting of the footing can be avoided by avoiding a highly non-uniform base pressure in week soils. Generally, the pressure diagram will be of trapezoidal shape as shown in Figure 21.11.



SAQ 3

What are the stability requirements of a retaining wall?

SAQ 4

What is the purpose of a shear key? Describe its action.

21.5 PROPORTIONING OF CANTILEVER AND COUNTERFORT WALLS

The first step of the analysis and design of retaining wall structure is to assume preliminary dimensions of various elements of the structure using certain approximations. Subsequently, these dimensions may be suitably revised, if so required by design considerations.

21.5.1 Position of Stem for Economical Design

An important consideration in the design of cantilever and counterfort walls is the position of the vertical stem on the base slab. It can be shown that an economical design of the retaining wall can be obtained by proportioning the base slab so as to align the vertical soil reaction R at the base with the front face of the stem. Figure 21.12 shows the case of a level backfill. The location of the resultant soil reaction, R, is dependent on the magnitude and location of the resultant vertical load, W, which in turn depends on the width of the heel slab, X (inclusive of the stem thickness). Dimension X may be expressed as a fraction, α of the full width L of the base slab ($X = \alpha L$). Assuming an average unit weight γ for all material (earth + concrete) behind the front face of the stem (rectangle $a\ b\ c\ d$) and neglecting the weight of concrete in the toe slab and weight of the earth above it.

$$R = W = \gamma\ h\ X = \gamma\ h\ (\alpha L)$$

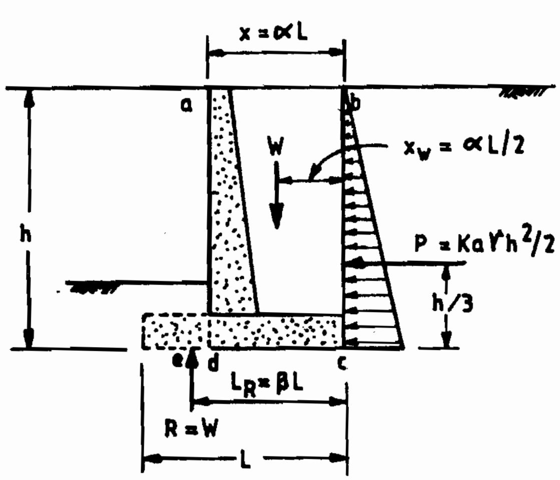


Figure 21.12 : Proportioning of Retaining Wall

For a given location of R corresponding to a chosen value of X, the toe projection of the base slab (and hence its total width, L) can be selected so as to give the desired distribution of base soil pressure.

If we represent the distance (L_R) of R from heal as a fraction β of base width L, the base pressure will be uniform if L is so selected as to make $\beta = 0.5$. Similarly, for $\beta = \frac{2}{3}$, the base pressure distribution will be triangular having zero value at the heel and maximum at the toe. Thus for any selected distribution of base pressure β is a constant and the required base width $L = \frac{L_R}{\beta}$.

Considering static equilibrium and taking moments about reaction point e, and assuming $x_w = \alpha\ L/2$,

$$W\ (\beta L - \alpha\ L/2) = P_a\ (h/3)$$

$$\alpha L h \gamma \left(\beta L - \alpha \frac{L}{2}\right) = K_a \gamma h^3/6$$

$$\gamma h L^2 \left(\alpha \beta - \frac{\alpha^2}{2}\right) = K_a \gamma h^3/6$$

$$\frac{L}{h} = \frac{\sqrt{K_a/3}}{2\alpha\beta - \alpha^2} \qquad \ldots (21.19)$$

For economical proportioning for a given height of wall (h), the width of the base (L) must be minimum i.e., $\left(\frac{L}{h}\right)$ should be minimum. From Eq. (21.19), this implies that $(2\alpha\beta - \alpha^2)$ should be maximum. The location of R, and hence the base width for any selected pressure distribution, is dependent on the variable X, i.e., α. For maximum value of $(2\alpha\beta - \alpha^2)$,

$$\alpha = \beta$$

or,
$$\alpha L = \beta L = X$$

Thus for an economical design the soil pressure resultant should line-up with the front face of the wall.

21.5.2 Base Width

An approximate expression for the minimum width of the base slab of a given height of wall can be obtained from Eq. (21.19).

$$\left(\frac{L}{h}\right)_{\min} \approx \left(\frac{1}{\alpha}\right) \sqrt{K_a/3}$$

$$\Rightarrow L_{\min} \approx \left(\frac{h}{\alpha}\right) \sqrt{K_a/3} \qquad \ldots (21.20)$$

Alternatively, the minimum width of heel slab is given by,

$$X_{\min} = \alpha L_{\min} = \beta L_{\min} = h \sqrt{K_a/3} \qquad \ldots (21.21)$$

The above expression is for a level backfill. The effect of surcharge or sloping backfill may be taken into account, approximately, by replacing h with $h + h_s$ or h', respectively.

Alternatively the heel slab width, X (Figure 21.12) may be obtained by equating moments of W and Pa about the point d. The required L can be worked out based on the desired base pressure distribution.

Note that the total height h of the retaining wall is the difference in elevation between the top of the wall and the bottom of base slab. The depth of the foundation (i.e., bottom of base slab) can be decided based on the geotechnical considerations (availability of firm soil) and is usually not less than 1.0 m below the ground level on the toe side of the wall.

After fixing up the trial width of the heel slab (= X) for a given height of wall and backfill pressure distribution may be assumed, resulting in $L = 1.5X$. Using other approximations (discussed in next section) related to stem thickness and base slab thickness, proper analysis should be done to ascertain that

(i) the factor of safety against overturning is adequate,

(ii) the allowable soil pressure, q_a, is not exceeded, and

(iii) the factor of safety against sliding is adequate.

Condition (i) is generally satisfied, however if it is not, the dimensions L and X may be suitably increased. If condition (ii) is not satisfied, i.e., if $q_{max} > q_a$, the width of the base, L should be increased by suitably extending the width of the toe slab; the dimension X is not changed. If condition (iii) is not satisfied, which is usually the case, a suitable shear key should be designed.

21.6 DESIGN OF CANTILEVER WALL

21.6.1 Initial Thickness of Base Slab and Stem

For preliminary calculations, the thickness of the base slab may be taken as about 8 per cent of the height of the wall plus surcharge (if any); it should not be less than 300 mm. The base thickness of the vertical stem may be taken as slightly more than that of the base slab. For economy, the thickness may be tapered linearly to a minimum value (but not less than 150 mm) at the top of the wall; the front face of the stem is maintained vertical. If the length of the heel slab and / or toe slab is excessive, it will be economical to provide a tapered slab.

With the above preliminary proportions, the stability check and determination of soil pressure (at the base) may be performed, and dimensions L and X of the base slab may be (Figure 21.12) finalised. It may be noted that changes in thickness of base slab and stem, if required at the design stage, will be marginal and will not affect significantly either the stability analysis or the calculated soil pressures below the base slab.

21.6.2 Design of Toe Slab, Heel Slab and Stem

The three elements of the retaining wall, viz., stem, toe slab and heel slab have to be designed as cantilever slabs to resist the factored moments and shear forces. For this a load factor of 1.5 is to be used.

Toe slab

Toe slab is subjected to upward pressure from the soil and the self weight acting downward, thus the net pressure is obtained by deducting the weight of the concrete in the toe slab from the upward acting soil pressure. The net loading acts upward (as in the case of usual footings) and the flexural reinforcement has to be provided at the bottom of the toe slab. The critical section for moment is at the front face of the stem, while the critical section for shear is at a distance '*d*' from the face of the stem. A clear cover of 75 mm may be provided in base slabs.

Heel Slab

In the case of the heel slab, the pressures acting downward, due to the weight of the retained earth (plus surcharge, if any), as well as the concrete in the heel slab, exceed the gross soil pressures acting upward. Hence, the net loading acts downward, and the flexural reinforcement has to be provided at the top of the heel slab. The critical section for moment is at the rear face of the stem base.

The critical section for shear in the heel slab should be taken at the face of the support and not d away from it, because there is no compression introduced by the support reaction, and the probable inclined crack may extend ahead of the rear face of the stem.

Stem

It is subjected to lateral pressures which cause maximum BM and SF at the junction of stem with base slab. In the case of the stem (vertical cantilever), the critical section for shear may be taken d from the face of the support (top of the base slab), while the critical section for moment should be taken at the face of the support. For the main bars in the stem, a clear cover of 50 mm may be provided.

slab). The flexural reinforcement is provided near the rear face of the stem, and may be curtailed in stages for economy.

Temperature and shrinkages reinforcement ($A_{st,\min}$ = 12 per cent of gross area) should be provided transverse to the main reinforcement. Nominal vertical and horizontal reinforcement should also be provided near the front face which is exposed.

SAQ 5

Briefly describe the behaviour of various elements ofa cantilever retaining wall.

21.7 DESIGN OF COUNTERFORT WALL

21.7.1 Spacing of Counterforts

The spacing of counterforts depends upon the height of the wall, cost of steel and concrete, allowable bearing capacity of soil and the cost of formwork. The thickness of vertical slab and heel slab can be reduced by keeping the counterforts closure. But the reduction in cost of concrete and steel due to less thickness of vertical and heel slabs is off-set by the quantity of concrete and steel required for large number of counterforts which are required. The cost of formwork required for large number of counterforts also off-sets the above factor. The best spacing is the one which makes the design economical. The most economical spacing is *one-third* to *one-half* the height of the wall.

The triangular shaped counterforts are provided in the rear side of the wall, interconnecting the stem with the heel slab. Sometimes, small butteresses are provided in the front side below the ground level, interconnecting the toe slab with the lower portion of the stem.

21.7.2 Initial Thickness of Various Elements

The presence of counterforts enables the use of stem and base slab thickness that are much smaller than those normally required for a cantilever wall. For preliminary calculations, the stem thickness and base slab thickness may be taken as about 4 per cent of the height of the wall, but not less than 300 mm. If the front buttress is not provided, the thickness of the toe slab should be taken as in the case of the cantilever wall. The thickness of the counterforts may be taken as about 5 per cent of the height of the wall, but not less than 300 mm.

21.7.3 Design of Stem, Toe Slab and Heel Slab

Each panel of the stem and heel slab, between two adjacent counterforts, may be designed as two-way slabs fixed on three sides, and free on the fourth side (free edge). These boundary conditions are also applicable to the toe slab, if buttresses are provided; otherwise the toe slab behaves as a horizontal cantilever, as in the case of the cantilever wall.

The loads acting on these elements are identical to those acting on the cantilever wall discussed earlier. For the stem, bending in the horizontal direction between counterforts is generally more predominant than bending in the vertical direction. Near the counterforts, the main reinforcement will be located close to the rear face of the stem, whereas midway between counterforts, the reinforcement will be close to the outside face. These two-way slabs, subject to triangular / trapezoidal pressure distributions may be designed by the use of moment and shear coefficients (based on plate theory), available in IS Code for the design of liquid storage structures, viz., IS 3370 (Part 4). Alternatively, the slabs may be designed by the yield line theory.

21.7.4 Design of Counterforts

The main counterforts should be firmly secured (by additional ties) to the heel slab, as them from the counterforts. In addition, the counterforts should be designed to resist the lateral (horizontal) force transmitted by the stem tributary to it. The counterfort is designed as a vertical cantilever, fixed at its base. As the stem acts integrally with the counterfort, the effective section resisting the cantilever moment is a flanged section, with the flange under compression. Hence, the counterforts may be designed as T-beams with the depth of section varying (linearly) from the top (free edge) to the bottom (fixed edge), and with the main reinforcement provided close to the sloping face. Since these bars are inclined (not parallel to the compression face), allowance has to be made for this in computing the area of steel required.

SAQ 6

Briefly describe the behaviour of the various elements of a counterfort retaining wall.

Design Example 21.1

Design a cantilever retaining wall to retain a level earthfill of 4.0 m above ground level. The surcharge on the earthfill is 40 kN / m^2 (due to construction of a building). Assume good soil for foundation at a depth of 1.25 m below the ground level with a safe bearing capacity of 160 kN / m^2. The unit weight and the angle of repose of soil are 16 kN / m^3 and 30° respectively. Assume the coefficient of friction between soil and concrete to be 0.5. Use M20 concrete and Fe 415 steel.

Solution

(i) *Data Given*

Height of backfill		= 4.0 m
Angle of repose of soil,	ϕ	= 30°
Unit wt. of soil,	γ	= 16 kN / m^3
Safe bearing capacity of soil,	q_a	= 160 mN
Surcharge pressure,	w_s	= 40 kN / m^2
Coefficient of friction,	μ	= 0.5

(ii) *Preliminary Proportions*

$$\text{Earth pressure coefficient, } K_a = \frac{1 - \sin \phi}{1 + \sin \phi} = \frac{1 - \sin 30^\circ}{1 + \sin 30^\circ} = \frac{1}{3}$$

$$K_p = \frac{1}{K_a} = 3.0$$

Depth of foundation below ground level

$$= \frac{q_a}{\gamma}\left(\frac{1 - \sin \phi}{1 + \sin \phi}\right)^2 = \frac{160}{16}\left(\frac{1 - \sin 30^\circ}{1 + \sin 30^\circ}\right)^2$$

$$= 1.11 \text{ m}$$

It is given that good soil for foundation is at a depth of 1.25 m below ground level, so take depth of foundation below ground level as 1.25 m.

$\therefore$ Total height of wall above foundation level, $h = 4.0 + 1.25 = 5.25$ m

$$\text{Now, Equivalent height of earth as surcharge, } h_s = \frac{w_s}{\gamma} = \frac{40}{16} = 2.5 \text{ m}$$

Thickness of footing base slab $\approx 0.08\,(h + h_s)$

$$= 0.08\,(5.25 + 2.5) = 0.62 \text{ m}$$

Assume a thickness of 620 mm.

Assume a stem thickness of 650 mm at the base of the stem and tapering to a value of 200 mm at the top of the wall.

For an economical proportioning of the length, L, of the base slab, it can be assumed that the vertical reaction, R, at the footing base is in line with the front face of the stem. For such a condition, the length of the heel slab (inclusive of stem thickness).

$$X \approx (h + h_s)\ \sqrt{K_a/3} = 7.75\ \sqrt{\frac{(1/3)}{3}}$$

$$= 2.58 \text{ m}$$

Take $X = 2.60$ m

Assuming a triangular soil pressure distribution below the base,
$L = 1.5\,X = 1.5\,(2.60) = 3.90$ m

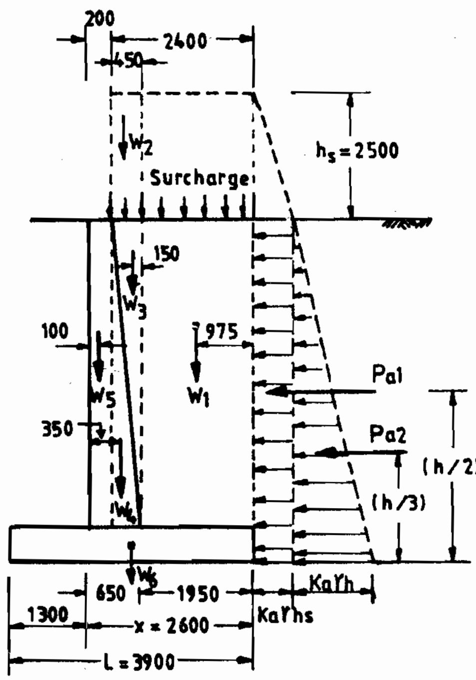


Figure 21.13 : Forces on Wall

(iii) *Stability against Overturning*

(a) Forces due to active earth pressure (per meter length of wall) :

$$P_{a1} = (K_a \gamma\, h_s)\ h = \frac{1}{3} \times 16 \times 2.5 \times 5.25 = 70.0 \text{ kN}$$

$$P_{a2} = \frac{1}{2} K_a \gamma\, h^2 = \frac{1}{2} \times \frac{1}{3} \times 16 \times (5.25)^2 = 73.5 \text{ kN}$$

$\therefore$ Total active pressure, $P_a = P_{a1} + P_{a2} = 70.0 + 73.5$

$$= 143.5 \text{ kN}$$

(b) Overturning moment, M_o :

$$M_o = P_{a1}\ (h/2) + P_{a2}\ (h/3)$$

$$= (70.0)\left(\frac{5.25}{2}\right) + (73.5)\left(\frac{5.25}{3}\right)$$

$$= 183.75 + 128.625$$

$$= 312.375 \text{ kN} - \text{m (per meter length of wall)}$$

(c) Resultant vertical force and its line of action :

Consider 1 m length of the wall and by applying statics we can get the following :

Force (kN)	Distance from Heel (m)	Moment About Heel (kN – m)
$W_1 = 16 \times 1.95 \times (7.75 - 0.62) = 222.456$	0.975	216.895
$W_2 = 16 \times 0.45 \times (2.5) = 18.00$	$1.95 + 0.225 = 2.175$	39.15
$W_3 = (1/2) \times 16 \times 0.45 \times (5.25 - 0.62) = 16.668$	$1.95 + 0.15 = 2.10$	35.003
$W_4 = (1/2) \times 25 \times 0.45 \times (5.25 - 0.62) = 26.044$	$2.60 - 0.35 = 2.25$	58.600
$W_5 = 25 \times 0.20 \times (5.25 - 0.62) = 23.15$	$2.60 - 0.10 = 2.50$	57.875
$W_6 = 25 \times 0.62 \times 3.9 = 60.45$	1.95	117.878
$W \approx 366.80$		$M_W = 525.4$

Thus resultant vertical force, $W = 366.80$ kN. Distance of resultant vertical force from heel, $x_w = \dfrac{M_W}{W} = \dfrac{525.4}{366.8} = 1.432$ m

Stabilising moment (about toe) :

$$M_r = W\,(L - x_w)$$

$$= 366.8\ (3.9 - 1.432)$$

$$= 905.26 \text{ kN – m (per meter length of wall)}$$

Factor of safety against overturning

$$(FS)_{\text{overturning}} = \frac{0.9\,M_r}{M_o} = \frac{0.9 \times 905.26}{312.375}$$

$$= 2.61 > 1.4 \text{ (OK)}$$

(iv) *Soil Pressure at the Base of Footing*

Resultant vertical reaction, $R = W = 366.8$ kN

Distance of R from heel, $L_R = \dfrac{M_W + M_o}{R}$

$$= \frac{525.4 + 312.375}{366.8}$$

$$= 2.28 \text{ m}$$

Eccentricity, $e = L_R - (L/2) = 2.28 - (3.9/2) = 0.33 \text{ m} < \dfrac{L}{6}\ (= 0.5)$

Hence the resultant lies within the middle third of the base, which is desirable. (In case the resultant, R, lies outside the middle third of the base, then base length has to be accordingly revised to bring the resultant well within the middle third of the base.)

Now $$q_{\max} = \frac{R}{L}\left(1 + \frac{6\,e}{L}\right) = \frac{366.8}{3.9}\left(1 + \frac{6 \times 0.33}{3.9}\right)$$

$$= 141.8 \text{ kN/m}^2 < q_a = 160 \text{ kN/m}^2 \text{ OK}$$

and $$q_{\min} = \frac{R}{L}\left(1 - \frac{6\,e}{L}\right) = \frac{366.8}{3.9}\left(1 - \frac{6 \times 0.33}{3.9}\right)$$

Soil pressure distribution is shown in Figure 21.14.

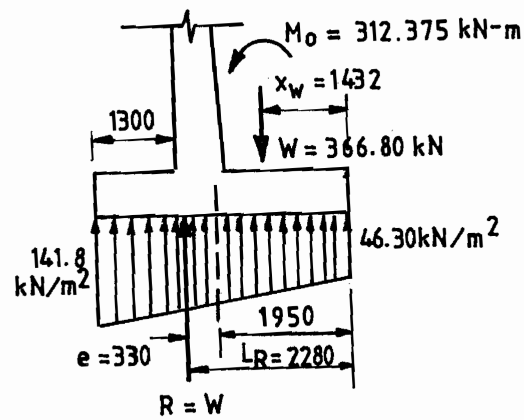


Figure 21.14 : Soil Pressure Distribution

(v) *Stability against Sliding*

Sliding force, = P_a = 143.5 kN (per meter length of wall)

Resisting force (ignoring passive pressure)

$$F = \mu R$$

$$= 0.5 \times 366.8$$

$$= 183.4 \text{ kN}$$

Factor of safety against sliding

$$(FS)_{\text{sliding}} = \frac{0.9\,F}{P_a} = \frac{0.9 \times 183.4}{1433.5}$$

$$= 1.15 < 1.40$$

Hence a shear key needs to be provided to mobilise the balance force through passive resistance.

Providing a shear key 300 mm × 300 mm at 2 m from toe (refer Figure 21.15)

$$h_2 = 0.95 + 0.3 + 2.0 \tan 30^\circ = 2.40 \text{ m}$$

$$P_p = K_a \gamma \left(h_2^2 - h_1^2\right)/2$$

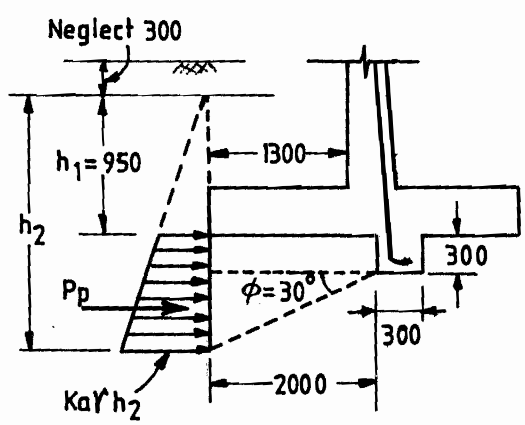


Figure 21.15 : Shear Key

$$= 3 \times 16 \left(2.4^2 - 0.95^2\right)/2$$

$$(FS)_{\text{sliding}} = \frac{0.9\ (183.4 + 116.6)}{143.5}$$

$$= 1.88 > 1.4 \quad \text{(OK)}$$

(vi) *Design of Toe Slab*

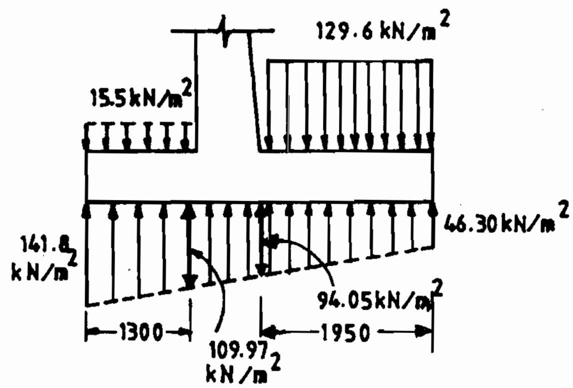
The loads considered for the design of the toe slab are shown in Figure 21.16(a). The net pressures acting upward are obtained by reducing the uniformly distributed self weight of the toe slab from the gross pressure at the base.

Loading due to self weight of slab = $25 \times 0.62 = 15.5\ \text{kN/m}^2$

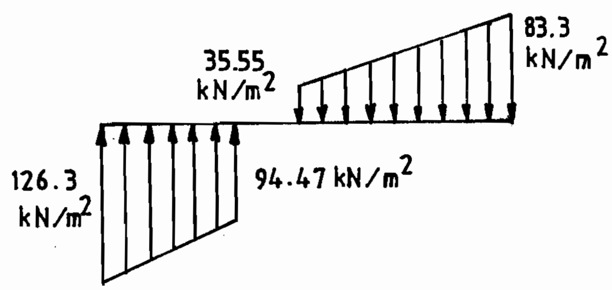
Thus net upward pressure varies from 126.3 kN / m^2 (= 141.8 – 15.5) to 94.47 kN / m^2 (= 109.97 – 15.5) as shown in Figure 21.16(b).

Assuming a clear cover of 75 mm and 20 ϕ bars, effective depth of the slab, $d = 620 - 75 = 535$ mm

Applying a load factor of 1.5, the design shear force (at d = 535 mm from the front face of the stem)



(a)



(b)

Figure 12.16 : Net Soil Pressure Acting on Base Slab

$$V_u = 1.5 \times \left(\frac{126.3 + 94.47}{2}\right) \times (1.3 - 0.535) = 126.67\ \text{kN}$$

and the design moment at the face of the stem

$$M_u = 1.5 \times \left[(94.47 \times 1.3^2/2) + (126.3 - 94.47) \times 0.5 \times 1.3^2 \times \frac{2}{3}\right]$$

$$= 146.64\ \text{kN} - \text{m}$$

$$\text{Nominal shear stress, } \tau_v = \frac{V_u}{bd} = \frac{126.67 \times 10^3}{1 \times 10^3 \times 535}$$

$$= 0.237\ \text{MPa}$$

From Table 13 of IS : 456-1978, for a $\tau_c = 0.24$ MPa, the required

$$p_t \left(= \frac{100\,A_{st}}{bd}\right) = 0.25 \text{ with M20 concrete.}$$

$$\frac{M_u}{bd^2} = 0.87\,f_y \left(\frac{p_t}{100}\right)\left[1 - \frac{f_y}{f_{ck}}\left(\frac{p_t}{100}\right)\right]$$

or

$$p_t = \frac{100 \times (A_{st})_{\text{required}}}{bd} = \frac{f_{ck}}{2\,f_y}\left[1 - \sqrt{1 - \frac{4.598}{f_{ck}} \times \frac{M_u}{bd^2}}\right] \times 100$$

$$= \frac{20}{2 \times 415}\left[1 - \sqrt{1 - \frac{4.598}{20} \times \frac{146.64 \times 10^6}{1 \times 10^3 \times 535^2}}\right]$$

$= 0.146 < 0.25$ which is required for shear

So adopt $p_t = 0.25$

$\therefore\ (A_{st})_{\text{required}} = (0.25 / 100) \times 10^3 \times 535$

$= 1338\ \text{mm}^2$ (per m length of wall)

Using 20 ϕ bars, spacing required $= \left(\frac{\pi}{4} \times 20^2\right) \times 10^3 / 1338 \approx 235$ mm

Provide 20 ϕ bars, @ 230 c/c at the bottom of the toe slab.

For Fe 415 steel and M20 concrete, the development length,

$$L_d = \frac{0.87\,\sigma_y\,\phi}{4\,\tau_{bd}}$$

$$= \frac{0.87 \times 415}{4 \times 1.2\,(1 + 0.6)}\,\phi = 47\,\phi$$

(refer section 25.2.1.1 of IS : 456-1978).

$\therefore$ The bars should extend by atleast a distance $L_d = 47\,\phi = 47 \times 20 = 940$ mm beyond the front face of the stem, on both sides. As the toe slab is 1.3 m long, no curtailment of bars is required.

(vii) *Design of Heel Slab*

The loads considered for the design of the heel slab are shown in Figure 21.16(a).

The distributed load acting downward on the heel slab in caused by

(a) Overburden + surcharge : $16 \times (7.75 - 0.62) = 114.08\ \text{kN/m}^2$

(b) heel slab : $25 \times 0.62 = 15.5\ \text{kN/m}^2$

Thus total distributed load acting downward on the heel slab

$= 114.08 + 15.5$

$= 129.6\ \text{kN/m}^2$

$\therefore$ The net pressure acting downward varies from $35.55\ \text{kN/m}^2$ ($= 129.6 - 94.05$) to $83.3\ \text{kN/m}^2$ ($= 129.6 - 46.30$) as shown in Figure 21.16(b).

Using a load factor of 1.5, the design shear force at the rear face of stem,

$$V_u = 1.5\left(\frac{35.55 \times 83.3}{2}\right) \times 1.95 = 173.82\ \text{kN}$$

and bending moment,

$$M_u = 1.5 \times \left[\left(\frac{35.55 \times 1.95^2}{2}\right) + 83.3 - 35.55) \times 0.5 \times 1.95^2 \times \frac{2}{3}\right]$$

$$= 1.5\ (67.589 + 60.523)$$

$$= 192.17 \text{ kN} - \text{m}$$

Assuming a clear cover of 75 mm and 20 ϕ bars, $d = 620 - 75 = 535$ mm

Nominal shear stress, $\tau_v = \frac{V_u}{bd} = \frac{173.82 \times 10^3}{1 \times 10^3 \times 535} = 0.325$ MPa

From Table 13 of IS : 456-1978, for $\tau_c = 0.325$, with M 20 concrete, $(p_t)_{\text{required}} = 0.25$

$$(p_t)_{\text{required}} = \frac{100 \times (A_{st})_{\text{required}}}{bd} = \frac{f_{ck}}{2 f_y}\left[1 - \sqrt{1 - \frac{4.598}{f_{ck}} \times \frac{M_u}{bd^2}}\right] \times 100$$

$$= \frac{20}{2 \times 415}\left[1 - \sqrt{1 - \frac{4.598}{20} \times \frac{192.17 \times 10^6}{1 \times 10^3 \times 535^2}}\right] \times 100$$

$= 0.193 < 0.25$ which is required for shear.

$$(A_{st})_{\text{required}} = \left(\frac{0.25}{100}\right) \times 10^3 \times 535$$

$= 1338 \text{ mm}^2$ per meter length of wall

Using 20 ϕ bars, spacing required $= \left(\frac{\pi}{4} \times 20^2\right) \times 10^3 / 1338 \approx 235$mm

Provide 20 ϕ bars @ 230 c/c near upper face of the heel slab. The bars should extend by at least a distance $L_d = 47\,\phi = 47 \times 20 = 940$ mm beyond the rear face of the stem, on both sides. The bars may be curtailed part way to the heel; however since the length is relatively short, it is not done in this example.

(viii) *Design of Vertical Stem*

Height of cantilever above base, $y = 5.25 - 0.62 = 4.63$ m

Assuming a clear cover of 50 mm and 20 ϕ bars,
d (at the base) = 650 – 50 – 10 = 590 mm

Assuming a load factor of 1.5, maximum design moment,

$$M_u = 1.5\left[K_a\, w_s\left(y^2/2\right) + K_a\,\gamma\left(y^3/6\right)\right]$$

$$= 1.5 \times \left(\frac{1}{3}\right)\left[40 \times \left(\frac{4.63^2}{2}\right) + 16 \times \left(\frac{4.63^3}{6}\right)\right]$$

$$= 346.7 \text{ kN} - \text{m}$$

$$\frac{(P_t)_{\text{required}}}{100} = \frac{(A_{st})_{\text{required}}}{bd} = \frac{20}{2 \times 415}\left[1 - \sqrt{1 - \frac{4.598}{20} \times \frac{346.7 \times 10^6}{1 \times 10^3 \times 590^2}}\right]$$

$$= 2.94 \times 10^{-3}$$

$$\therefore\ (A_{st})_{\text{required}} = (2.94 \times 10^{-3}) \times 1 \times 10^3 \times 590$$

$\approx 1735 \text{ mm}^2$ (per meter length of wall)

$$\text{Use 16 } \phi \text{ bars, spacing required} = \frac{\frac{\pi}{4}(16^2) \times 10^3}{1735}$$

$$= 116 \text{ mm}$$

Provide 16 ϕ @ 110 c/c bars extending into the shear key. It will provide the anchorage more than the minimum required, i.e., $L_d = 47 \times 16 = 752$ mm.

Check for shear at the base : Critical section at $d = 0.59$ m above base i.e., at $y_s = y - 0.59 = 4.63 - 0.59 = 4.04$ m below the top edge. Shear force at the critical section,

$$V_u = 1.5 \left[K_a \, w_s \, y_s + K_a \, \gamma \, y_s^2/2\right]$$

$$= 1.5 \times \left(\frac{1}{3}\right) \left[40 \times 4.04 + 16 \times \frac{4.04^2}{2}\right]$$

$$= 146.09 \text{ kN}$$

$$\tau_v = \frac{146.09 \times 10^3}{1 \times 10^3 \times 590} = 0.248 \text{ MPa}$$

From Table 13, IS : 456-1978, for M20 and $p_t = 0.294$

$$\tau_v = 0.36 + \frac{0.48 - 0.36}{0.50 - 0.25} (0.294 - 0.25) = 0.38$$

Thus $\tau_v < \tau_c \Rightarrow$ it is safe in shear.

Curtailment of bars : The curtailment of reinforcement bars may be done in two stages i.e., at one-third and two-third height of the stem above the base as shown in Figure 21.17. It can be verified that the curtailment satisfies the codal requirements.

Temperature and shrinkage reinforcement in stem : In the lower most one-third height of the stem above the base, nominal steel equal to 0.12% of gross cross-sectional area is provided.

$$A_{st} = \frac{0.12}{100} \times 1 \times 10^3 \times 650 = 780 \text{ mm}^2 \text{ (for one meter length of wall)}$$

Provide two-third of the reinforcement i.e., 520 mm^2 $\left(= \frac{2}{3} \times 780\right)$ near the front face (which is exposed to weather) and the remaining one-third i.e., 260 mm^2 near the rear face.

$$\text{Using 8 } \phi \text{ bars, spacing required} = \frac{\frac{\pi}{4}(8^2) \times 1 \times 10^3}{520}$$

$$\approx 97 \text{ mm} \approx 100 \text{ mm}$$

Provide 8 ϕ @ 100 c/c near front face and 8 ϕ @ 200 c/c near rear face in the lower most one-third height of the wall; 8 ϕ @ 200 c/c near front face and 8 ϕ @ 400 c/c in the middle one-third height; and 8 ϕ @ 300 c/c near front face and 8 ϕ @ 600 c/c near the rear face in the top and third height of the wall.

Also provide nominal bars (vertically) 10 ϕ bars @ 300 c/c near the front face of the wall.

Heel and Toe Slab : Provide nominal reinforcement of 0.12% of gross cross-sectional area

$$A_{st} = \frac{0.12}{100} \times 1 \times 10^3 \times 620 = 744 \text{ mm}^2$$

$$\text{using 10 } \phi \text{ bars, spacing required} = \frac{\frac{\pi}{4}(10^2) \times 1 \times 10^3}{744}$$

$$\approx 105 \text{ mm}$$

Provide 10 ϕ bars @ 100 c/c across the direction of tensile reinforcement.

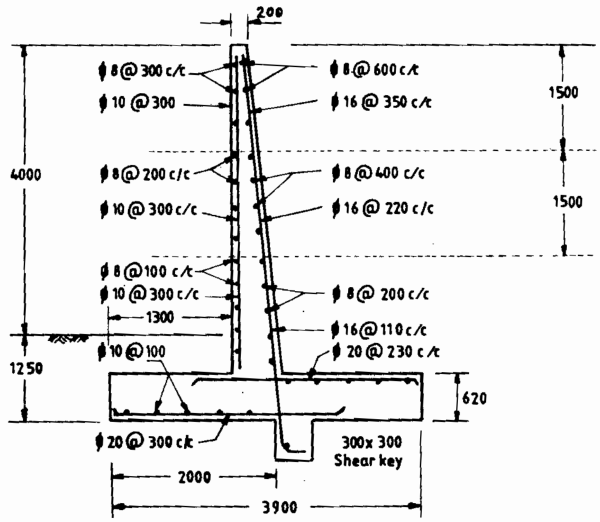


Figure 21.17 : Reinforcement Details of Cantilever Wall

Design Example 21.2

Design a RC counterfort retaining wall to support the difference in ground elevation of 7.5 m. The depth of foundation may be taken as 1.5 m below ground level, with a allowable bearing capacity of 150 kN / m^2. The top of the earth retained is horizontal with a unit weight of 16 kN / m^3 and an angle of shearing resistance of 30^o. The coefficient of friction between soil and concrete may be taken as 0.5.

Solution

(1) *Data Given*

Height of wall, $h = 7.5 + 1.5 = 9.0$ m

Unit weight of earth, $\gamma = 16$ kN / m^3

Angle of internal friction of earth, $\phi = 30^o$

Allowable bearing capacity of soil, $q_a = 150$ kN / m^2

Coefficient of active earth pressure, $K_a = \dfrac{1 - \sin \phi}{1 + \sin \phi} = 1/3$

Coefficient of passive earth pressure, $K_p = \dfrac{1}{K_a} = 3.0$

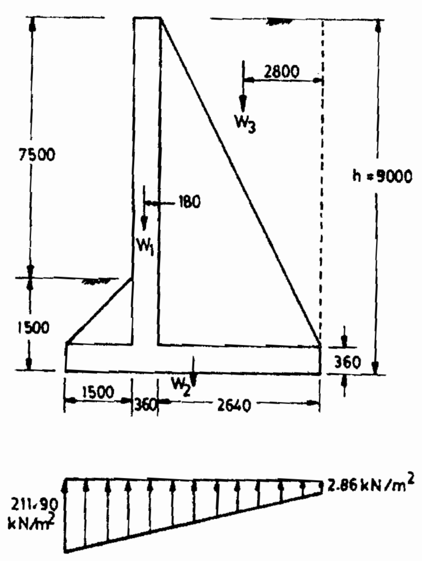


Figure 21.18 : Preliminary Dimensions of Counterfort Retaining Wall and Soil Pressure

(2) *Preliminary Proportioning*

Thickness of stem and heel slab = 0.04 h = 360 mm

Providing front buttresses, the thickness of toe slab can also be kept as 360 mm.

Thickness of counterforts = 0.05 h = 450 mm

For an economical proportioning of base slab, vertical reaction at the base of the footing shall be in line with the front face of the stem. For such a condition, the length of heel slab (including thickness of stem) :

$$X = h\sqrt{\frac{K_a}{3}} = 9\sqrt{\frac{1}{9}} = 3.0 \text{ m}$$

Assuming triangular base pressure distribution, total width of base slab,

$$L = 1.5\,X = 1.5 \times 3.0 = 4.5 \text{ m}$$

c/c spacing of counterforts $h/3$ to $h/2$ = 3.0 to 4.5 m = 4.0 m (say)

The preliminary proportions are shown in Figure 21.18.

(3) *Stability against Overturning*

Considering 1 m length of wall

Force due to active earth pressure,

$$P_a = \frac{1}{2}\,K_a\,\gamma\,h^2 = \frac{1}{2} \times \frac{1}{3} \times 16 \times 9^2 = 216 \text{ kN}$$

$$\text{Overturning moment, } M_o = P_a\,\frac{h}{3} = 216 \times \frac{9}{3} = 648 \text{ kNm}$$

The weight of various components and their moment has been calculated in Table given below.

Sl. No.	Component	Force (kN)	Distance from heel (m)	Moment about heel (kN–m)
1.	Weight of stem, W_1	$0.36 \times 8.64 \times 25 = 77.76$	$2.64 + \frac{0.36}{2} = 2.82$	219.28
2.	Weight of base slab, W_2	$0.36 \times 4.5 \times 25 = 40.50$	$\frac{4.5}{2} = 2.25$	91.13
3.	Weight of earth on heel, W_3	$2.64 \times 8.64 \times 16 = 364.95$	$\frac{2.64}{2} = 1.32$	481.73
	Total	$W = 483.21$		$M_W = 792.14$

$$\text{Distance of resultant vertical force from heel, } x_w = \frac{M_W}{W} = \frac{792.14}{483.21} = 1.64 \text{ m}$$

$$\text{Stabilizing moment (about toe), } M_r = W(L - x_w) = 483.21 \times (4.5 - 1.64)$$

$$= 1381.98 \text{ kNm}$$

Factor of safety against overturning

$$= \frac{0.9\,M_r}{M_o} = \frac{0.9 \times 1381.98}{648} = 1.92 \text{ m} > 1.4 \text{ (OK)}$$

(4) *Soil Reaction under the Base Slab*

Resultant vertical reaction, $R = W = 483.21$ kN

$$\text{Distance of } R \text{ from heel, } L_R = (M_W + M_o)/R = \frac{(792.14 + 648)}{483.21} = 2.98 \text{ m}$$

$$\text{Eccentricity, } e = L_R - L/2 = 2.98 - 4.5/2 = 0.73 \text{ m} < L/6 \ (= 0.75 \text{ m})$$

Hence, the resultant lies within the middle third of the base.

Soil pressure under the toe and heel, $q = \frac{R}{L}\left(1 \pm \frac{6\,e}{L}\right) = \frac{483.21}{4.5}\left(1 \pm \frac{6 \times 0.73}{4.5}\right)$

$$q_{max} = 211.90 \text{ kN / m}^2 \quad \text{(at toe)}$$

$$q_{min} = 2.86 \text{ kN / m}^2 \quad \text{(at heel)}$$

Soil pressure diagram is shown in Figure 21.18.

(5) *Stability against Sliding*

Sliding force, = $P_a = 216$ kN

Resisting force, $F = \mu R = 0.5 \times 483.21 = 241.60$ kN

Factor of safety against sliding = $\frac{0.9\,F}{P_a} = \frac{0.9 \times 241.60}{216} = 1.01 < 1.4$

Hence, providing a shear key 300 × 300 mm at a distance of 1.7 m from toe (Figure 21.19) to resist the unbalanced force by passive resistance.

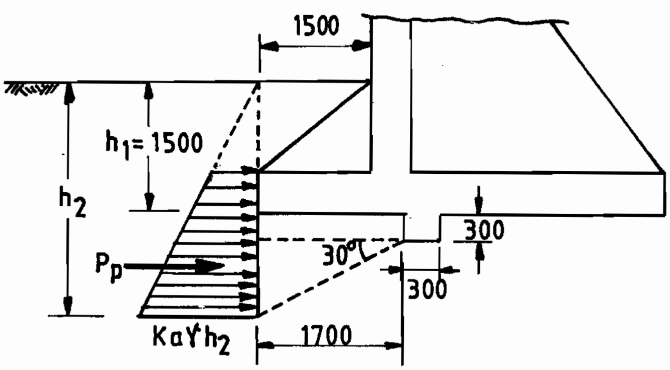


Figure 21.19 : Shear Key and Passive Pressure

Distance $h_2 = 1.5 + 0.3 + 1.7 \tan 30^\circ = 2.78$ m

Passive pressure,

$$P_p = \frac{1}{2} K_p \gamma \left(h_2^2 - h_1^2\right) = \frac{1}{2} \times 3 \times 16 \times (2.78^2 - 1.5^2) = 131.48 \text{ kN}$$

Factor of safety against sliding = $\frac{0.9\,(241.60 + 131.48)}{216} = 1.55 > 1.4$ (OK)

(6) *Design of Heel Slab*

Considering 1 m wide strip of heel slab near the outer edge.

Downward load due to earth and concrete

$$= 8.64 \times 16 + 0.36 \times 25 = 147.24 \text{ kN / m}^2$$

Upward pressure at the end = 2.86 kN / m^2

Therefore, net downward load, $p = 147.24 - 2.86 = 144.38$ kN / m^2

Effective span of slab, l = clear span = $4.0 - 0.45 = 3.55$ m

Factored BM at counterfort,

$$M_u = \frac{1.5\,p\,l^2}{12} = \frac{1.5 \times 144.38 \times 3.55^2}{12} = 227.45 \text{ kNm}$$

Effective depth required,

$$d = \sqrt{\frac{M_u}{0.138\,b\,f_{ck}}} = \sqrt{\frac{227.45 \times 10^6}{0.138 \times 1000 \times 20}} = 287 \text{ mm} < 335 \text{ mm (OK)}$$

Top steel at support :

$$M_u = 0.87\,f_y\,A_{st}\left(d - \frac{f_y\,A_{st}}{b\,f_{ck}}\right)$$

or, $A_{st} = 2174 \text{ mm}^2$. Using 16 ϕ bars, spacing required

$$= \frac{\pi}{4} \times 16^2 \times 10^3 / 2174 \approx 92 \text{ mm}$$

Providing $\phi 16$ @ 90 mm c/c $\left(\frac{100 A_{st}}{bd} = 0.677\% \right)$

Factored BM at mid span,

$$M_u = \frac{1.5\, p\, l^2}{16} = \frac{1.5 \times 144.38 \times 3.55^2}{16} = 170.85 \text{ kNm}$$

Bottom steel :

$$170.58 \times 10^6 = 0.87 f_y A_{st} \left(d - \frac{f_y A_{st}}{b f_{ck}} \right)$$

$$= 0.87 \times 415 A_{st} \left(355 - \frac{415 A_{st}}{20 \times 1000} \right)$$

or, $A_{st} = 1562 \text{ mm}^2$

Using 16 ϕ bars, spacing required $= \frac{\pi}{4} \times 16^2 \times 10^3 / 1562 = 128.7$ mm

Providing ϕ 16 @ 120 c/c.

Check in shear

Factored SF, $V_u = \frac{1.5\, p\, l}{2} = \frac{1.5 \times 144.38 \times 3.55}{2} = 384.41$ kN

Nominal shear stress, $\tau_v = \frac{V_u}{bd} = \frac{384.41 \times 10^3}{1000 \times 335} = 1.15$ MPa

Design shear strength of concrete, $\tau_c = 0.535 \text{ MPa} < \tau_v$

Shear stirrups are required in the area shown shaded in Figure 21.20 where $\tau_v > \tau_c$.

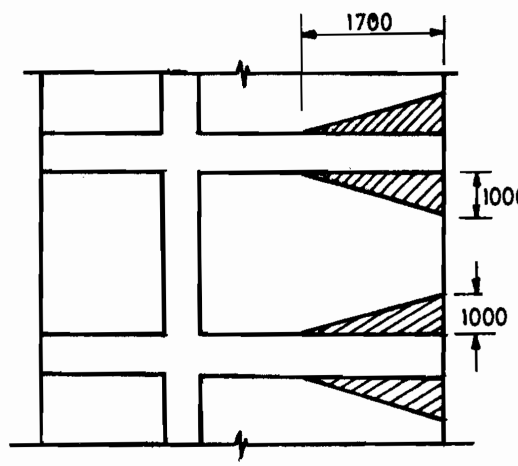
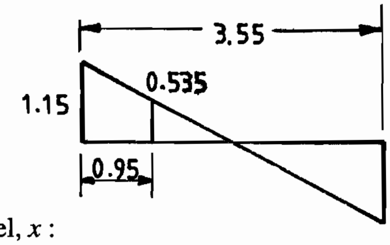


Figure 21.20 : Plan Showing Shear Stirrups Zone in Heal Slab

Length of the area from end towards centre of span



$$= \frac{3.55}{2} - \frac{0.535}{1.15} \times \frac{3.55}{2}$$

$$= 0.95 \text{ m} \approx 1.0 \text{ m (say)}$$

Length of the area from end of the heel, x :

$$1.5\ (144.38 - 46.45\, x) \times \frac{3.55}{2} \times \frac{10^3}{1000 \times 335} = 0.535$$

or, $\quad x = 1.66 \text{ m} \approx 1.7 \text{ m (say)}$

Spacing of 8 mm dia vertical legs (66.7 legs in 1 m i.e., at a spacing of 150 mm c/c)

$$S_v = \frac{0.87 f_y A_{sv}}{(\tau_v - \tau_c)\, b} = \frac{0.87 \times 415 \times 6.67 \times 50}{(1.15 - 0.535) \times 1000} = 195 \text{ mm}$$

Therefore providing 8 mm dia vertical stirrups at a spacing of 150 mm c/c in both directions.

Check for Development Length at Point of Inflextion

The point of inflexion lies approximately at 0.2 times the span from supports. At this section, bottom steel is to be checked for development.

Development length, $L_d = 47\,\phi$

$= 47 \times 16 = 752$ mm

Anchorage length, $L_o = d$ or $12\,\phi$ whichever is greater

$= 335$ mm

Factored SF at the point of inflexion, $V_u = \dfrac{(0.5 - 0.2)}{0.5} \times 384.41$

$= 230.65$ kN

curtailing alternate bars, factored M_R of ϕ 16 @ 240 c/c

$$\frac{M_u}{V_u} + L_o = \frac{96 \times 10^6}{230.65 \times 10^3} + 335 = 752 \geq L_d$$

$$\left(A_{st} = \frac{\pi}{4} \times 16^2 \times \frac{1000}{240} = 837.5 \text{ mm}^2\right)$$

$$M_u = 0.87 \times 415 \times 837.5 \left(335 - \frac{415 \times 837.5}{20 \times 1000}\right) \times 10^{-6} \text{ kNm} = 96 \text{ kNm}$$

Therefore alternate bars can be curtailed at (= 365 m $0.2 \times 3550 - 335$) from support the top reinforcement shall continue upto $(0.2\,l + d)$ or L_d whichever is more i.e.,
$0.2 \times 3550 + 335 = 1045$ mm or 752 mm = 1045 mm ≈ 1100 mm (say)

Therefore, provide top reinforcement upto 1100 mm from the face of counterfort.

Now, considering 1 m wide strip of heel slab near the stem slab.

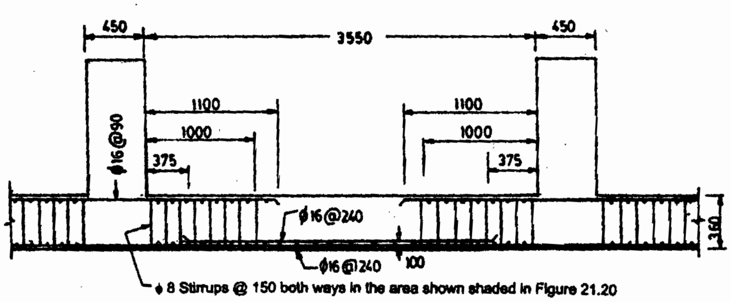
$$\text{Net downward load, } p = 147.24 - \left[\left(\frac{211.9 - 2.86}{4.5}\right) \times 2.64 + 2.86\right]$$

$$= 147.24 - 125.50 = 21.74 \text{ kN/m}^2$$

This is about 15% of the load near the edge of the heel.

The spacing of both top and bottom steels can be gradually increased from ϕ 16 @ 90 (top) and ϕ 16 @ 120 (bottom) at the edge of the heel to ϕ 16 @ 500 (both top and bottom) at its junction with the stem.

Figure 21.21 gives a section of heel slab near its end showing the detail of reinforcement.



(7) *Design of Stem Slab*

Considering 1 m wide strip at the bottom.

Maximum lateral earth pressure, $p = \frac{1}{3} \times 16 \times 8.640 = 46.08 \text{ kN/m}^2$

Factored BM near counterforts,

$$M_u = \frac{1.5\, p\, l^2}{12} = \frac{1.5 \times 46.08 \times 3.55^2}{12} = 72.59 \text{ kNm}$$

Effective depth required,

$$d = \sqrt{\frac{72.59 \times 10^6}{0.138 \times 20 \times 1000}} = 162 \text{ mm} < 335 \text{ mm (OK)}$$

Area of Steel :

$$72.59 \times 10^6 = 0.87 \times 415\, A_{st} \left(335 - \frac{415\, A_{st}}{20 \times 1000}\right)$$

or, $A_{st} = 625 \text{ mm}^2$

Providing ϕ 12 @ 180 mm $\left(\frac{100\, A_{st}}{bd} = 0.19\%\right)$

Maximum factored shear force,

$$V_u = \frac{1.5\, p\, l}{2} = 1.5 \times 46.08 \times \frac{3.55}{2} = 122.69 \text{ kN}$$

Nominal shear stress, $\tau_v = \frac{122.69 \times 10^3}{1000 \times 335} = 0.36 \text{ MPa}$

Design shear strength of concrete, $\tau_c = 0.326$ MPa

For making the slab safe in shear, increase its thickness to 450 mm.

Provide ϕ 12 @ 220 mm throughout the stem slab. Though the BM at mid of span is $p\, l^2 / 16$ but no calculation is required because even the steel near counterfort is only nominal.

Figure 21.22 shows horizontal section of stem slab. For economy either the stem can be tapered or by keeping it 360 mm thick stirrups can be provided.

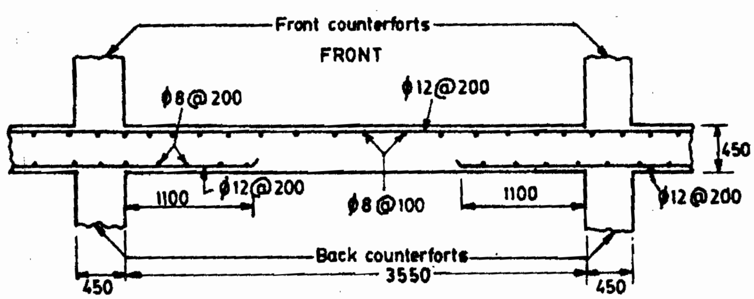


Figure 21.22 : Horizontal Section through Stem Slab Showing Reinforcement

(8) *Design of Toe Slab*

Assuming 400 mm thick slab considering 1 m wide strip near the end of toe slab.

Net upward reaction, $p = 211.90 - 0.4 \times 25 = 201.9 \text{ kN/m}^2$

Factored BM near counterfort, $M_u = \frac{1.5\, p\, l^2}{12} = 1.5 \times 201.9 \times \frac{3.55^2}{12}$

Effective depth required,

$$d = \sqrt{\frac{318.06 \times 10^6}{0.138 \times 20 \times 1000}} = 340 \text{ mm} < 375 \text{ mm (OK)}$$

Bottom steel near counterfort :

$$318.06 \times 10^6 = 0.87 \times 415 A_{st} \left(375 - \frac{415 A_{st}}{20 \times 1000}\right)$$

or, $A_{st} = 2776 \text{ mm}^2$

Providing ϕ 20 @ 100 mm $\left(\frac{100 A_{st}}{bd} = \frac{100 \times 314 \times 10}{1000 \times 375} = 0.837\%\right)$

Top steel at mid span :

$$\frac{3}{4} \times 318.06 \times 10^6 = 0.87 \times 415 A_{st} \left(375 - \frac{415 A_{st}}{20 \times 1000}\right)$$

or, $A_{st} = 1979 \text{ mm}^2$

Providing ϕ 20 @ 150 mm.

Maximum factored SF, $V_u = \frac{1.5\,p\,l}{2} = 1.5 \times 201.9 \times \frac{3.55}{2} = 537.56 \text{ kN}$

Nominal shear stress, $\tau_v = \frac{537.56 \times 10^3}{1000 \times 375} = 1.433 \text{ MPa}$

Design shear strength of concrete, $\tau_c = 0.583 \text{ MPa} > \tau_c$

Therefore shear stirrups are required. The design for shear, bond and curtailment can be done on the same line as that of the heal slab. This is left to the student as an exercise.

The reinforcement detail in toe slab is shown in a vertical section as in Figure 21.23.

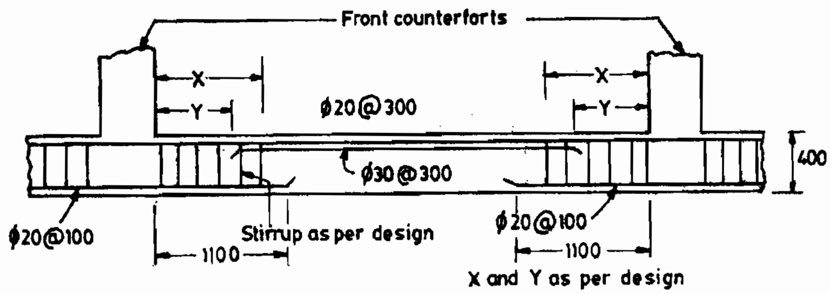


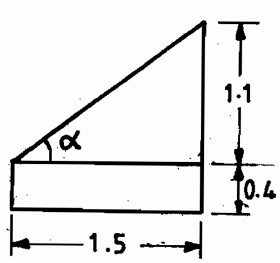
Figure 21.23 : Vertical Section of Toe Slab showing Reinforcement

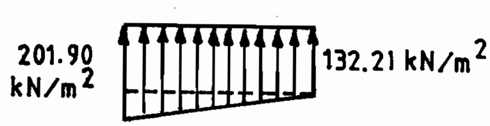
(9) *Design of Front Counterfort*

Each counterfort will resist earth pressure from 4.0 mm length of wall.

It bends as a horizontal cantilever under the action of vertical upward loads.

Inclination of top face of counterfort, $\alpha = \tan^{-1}\left(\frac{1.1}{1.5}\right) = 36.254°$





Net upward reaction at the end of toe

$$= 211.90 - 0.4 \times 25 = 201.90 \text{ kN / m}^2$$

Net upward reaction at the end of face of stem

$$= (2.86 + 46.45 \times 3) - 0.4 \times 25 = 132.21 \text{ kN / m}^2$$

Maximum factored BM,

$$M_u = 1.5 \times \left[132.21 \times 1.5 \times \left(\frac{1.5}{2} \right) \times 4.0 + \frac{1}{2} (201.9 - 132.21) \times 1.5 \times \frac{2}{3} \times 1.5 \times 4.0 \right]$$

$$= 1206.02 \text{ kNm}$$

Effective depth required,

$$d = \sqrt{\frac{1206.02 \times 10^{-6}}{0.138 \times 20 \times 1000 \times \cos^2 36.254}} = 820 \text{ mm} < 1450 \text{ mm (OK)}$$

Area of Steel

$$1206.02 \times 10^6 = 0.87 \times 415 A_{st} \left(1450 - \frac{415 A_{st}}{20 \times 450} \right)$$

or, $A_{st} = 2503 \text{ mm}^2$

Providing six ϕ 25 bars at the bottom of counterforts $\left(\frac{100 A_{st}}{bd} = 0.45\% \right)$.

These bars should be extended straight into the back counterfort by 47 ϕ i.e., 1175 mm ≈ 1200 mm.

Max-factored SF, $V_u = 1.5 \times 4.0 \times (132.21 + 201.90) \times \frac{1.5}{2} = 1503.50 \text{ kN}$

Nominal shear stress, $\tau_v = \frac{1503.50 \times 10^3}{450 \times 1450} = 2.304 \text{ MPa} < 2.77$ OK

Design shear strength of concrete, $\tau_c = 0.458 \text{ MPa} < \tau_v$

Spacing of ϕ 20-2 legged vertical stirrups,

$$S_v = \frac{0.87 \times 415 \times 2 \times 314}{(2.304 - 0.458) \times 450} = 272 \text{ mm}$$

Provide $2L - \phi$ 20 vertical stirrups @ 250 mm c/c.

Reinforcement has been shown in Figure 21.25.

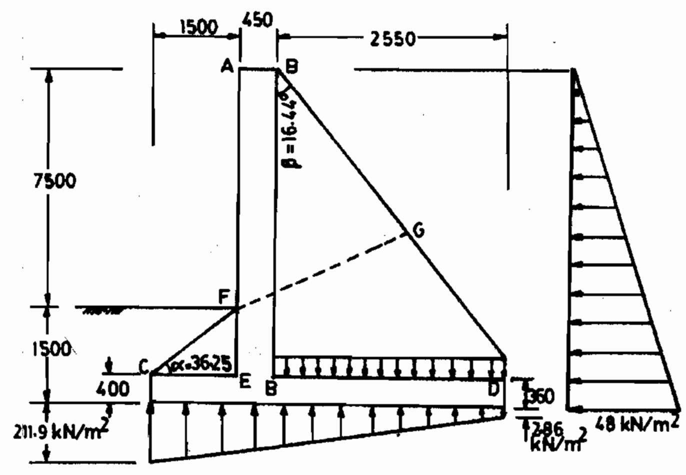


Figure 21.24 : Forces on the Counterfort Wall

(10) *Design of Back Counterfort*

About the top of the counterfort (i.e., point *F*, Figure 21.24), the counterfort is subjected only to earth pressure from its back and bends outward as a cantilever tapered *T*-beam. Below this point the back counterfort gets the support of front counterfort. Therefore, point *F* is critical for its design. But

At a depth h m below top, the earth pressure acting on the counterfort

$$= 4\,(K_a\ \gamma\ h) = 4\left(\frac{1}{3} \times 16\,h\right) = \frac{64}{3}\,h\ \text{kN/m}$$

$$\text{Factored BM at } F,\ M_u = \frac{1.5}{6} \times \left(\frac{64}{3} \times 7.5\right) \times 7.5^2 = 2250\ \text{kNm}$$

$$\text{Factored SF at } F,\ V_u = \frac{1.5}{2}\left(\frac{64}{3} \times 7.5\right) \times 7.5 = 900\ \text{kN}$$

Width of flange,

$$b_f = \frac{0.7\,l}{6} + b_\omega + 6\,D_f = \frac{0.7 \times 3.55}{6} + 0.45 + 6 \times 0.45$$

$$= 3.56\ \text{m} < 4.0\ \text{m}\ \ \text{OK}$$

Assuming neutral axis (NA) to lie in the flange of T-beam (i.e., stem slab), effective depth required,

$$d = \sqrt{\frac{M_u}{0.138\,f_{ck}\,b_f}} = \sqrt{\frac{2250 \times 10^6}{0.138 \times 20 \times 3560}} = 479\ \text{mm} \quad \text{very low}$$

Assuming neutral axis to lie in the flange of T-beam (i.e., stem slab),

Area of Steel

$$M_u = 0.87\,f_y\,A_{st}\left(d - \frac{f_y\,A_{st}}{f_{ck}\,b_f}\right)$$

D, the depth of beam at F normal to the face of reinforcement,

$$D = \left[\left(\frac{3.0 - 0.45}{9 - 0.36}\right) \times 7.5 + 0.45\right] \cos\beta\ (16.44) \qquad \beta = \tan^{-1}\left(\frac{3 - 0.45}{9 - 0.36}\right) = 16.44^\circ$$

$$= 2.664\ \cos\ 16.44$$

$$= 2.555\ \text{m}\ ,\ d \approx 2500\ \text{mm}$$

$$2250 \times 10^6 = 0.87 \times 415\,A_{st}\left(2500 - \frac{415\,A_{st}}{20 \times 3560}\right)$$

or, $\quad A_{st} = 2508\ \text{mm}^2$

Providing six 25 bars at the slant back face of the counterfort

$$\left(100\,\frac{A_{st}}{bd} = \frac{100 \times (6 \times 490)}{450 \times 2500} = 0.26\%\right)$$

Depth of NA,

$$x_u = \frac{0.87\,f_y\,A_{st}}{0.36\,f_{ck}\,b_f} = \frac{0.87 \times 415 \times (6 \times 490)}{0.36 \times 20 \times 3560} = 41.42\ \text{mm} < 450\ \text{mm}$$

Therefore, NA lies in the flange.

Effective factored shear force,

$$V_{ue} = V_u - \frac{M_u \tan\beta}{d'} = 900 - \frac{2250 \tan 16.44}{2.664} = 650.78\ \text{kN}$$

$$\text{Nominal shear stress, } \tau_v = \frac{V_{ue}}{bd'} = \frac{650.78 \times 10^3}{450 \times 2664} = 0.543\ \text{MPa}$$

Design shear strength of concrete, $\tau_c = 0.363\ \text{MPa} < \tau_v$

Therefore, horizontal stirrups are required. These are also required to keep the stem attached with the counterfort and this requirement will give more stirrups which have been calculated latter.

If we consider a section FG (Figure 21.24) of the counterfort passing through F and normal to the reinforcement, it will be seen that the effective lever arm through F. So, same reinforcement is to be provided at G as on the horizontal section through F.

As the BM in the counterfort decreases cubically (because of triangular pressure diagram) towards the top, reinforcement can be curtailed. At least two bars shall continue upto the top and all the six bars should be carried upto the bottom of counterfort. The curtailment has been left to the student as an exercise.

The stem should be tied to the counterforts properly so that it does not separate out. The separating tendency will be there in the top 7.5 m only.

The Factored force causing separation at any depth h below top

$$= \frac{1.5}{3} \times 16\,h \times 3.55 = 28.4\,h$$

At 7.5 m depth, this force = $28.4 \times 7.5 = 213$ kN / m

Therefore, steel required = $\dfrac{213 \times 10^3}{0.87 \times 415} = 590$ mm^2 / m

Providing 2 L – ϕ 10 @ 250 mm c/c. The spacing can be increased gradually to the top to say, 450 mm c/c.

Vertical ties : As the steel receives a net vertical downward load and has a tendency to separate from the counterforts like the vertical slab. Vertical ties are, therefore, required to connect the two together. The end 300 mm strip of the heel may be assumed to be tied to the counterfort through the main reinforcement.

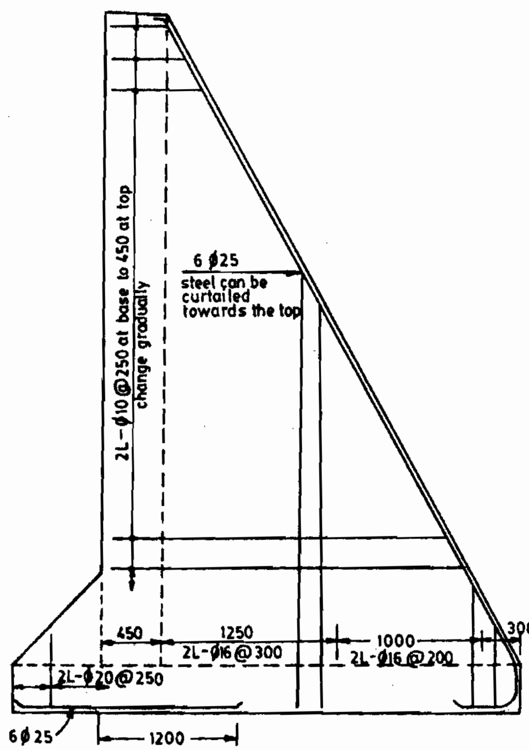


Figure 21.25 : Reinforcement in Counterforts (*Slab Reinforcement not shown*)

Net downward load on heel slab at 0.3 m form the end of heel

$$= 3.55 \times [147.24 - (46.45 \times 0.3 + 2.86)] = 463.08 \text{ kN}$$

Area of steel required, $A_{st} = \dfrac{1.5 \times 463.08 \times 10^3}{0.87 \times 415} = 1924$ mm^2 / m

Providing 2L – ϕ 16 @ 200 mm c/c in 1 m strip of heel.

Net downward load on heel slab at 1.3 m from the end of heel

$$\text{Area of steel required, } A_{st} = \frac{1.5 \times 298.18 \times 10^3}{0.87 \times 415} = 1239 \text{ mm}^2$$

Providing 2 L – ϕ 16 @ 300 mm c/c in the remaining 1.25 m length of heel.

The ties are required to be extended into the counterfort for embedment to act effectively as ties.

Reinforcement has been shown in Figure 21.25.

(11) *Fixing Effect in Stem, Toe and Heel*

Near the junction of the stem and heel, the slabs are rigidly fixed and will be subjected to a BM in a direction at right angles to their normal direction of bending. The problem of determining of BM at the junction is indeterminate and complex. It is considered that a reinforcement equal to 0.24% of the sectional area of concrete may be provided vertically on the back of the stem slab and 0.12% at the top of the heel slab as well as the bottom of toe slab at right angles to their main reinforcement, near their junction.

This steel may be extended into the adjoining slab for anchorage. Provision of this steel is shown in Figure 21.26.

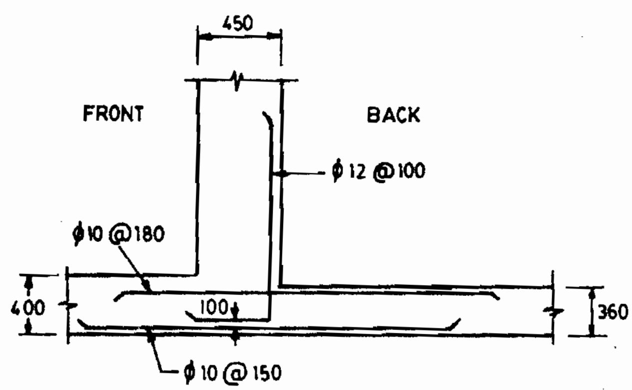


Figure 21.26 : Steel for Providing Fixidity Effect

(12) *Temperature and Shrinkage Steel*

Nominal steel equal to the 0.12% of the gross cross-sectional area is provided.

Stem slab :

$$A_{st} = \frac{0.12}{100} \times 450 \times 1000 = 540 \text{ mm}^2$$

Providing ϕ 8 @ 200 mm c/c on the inside face and ϕ 8 @ 100 mm c/c on the outside face of the stem slab.

Heal slab :

$$A_{st} = \frac{0.12}{100} \times 360 \times 1000 = 432 \text{ mm}^2$$

Providing ϕ 10 @ 180 mm c/c

Toe slab :

$$A_{st} = \frac{0.12}{100} \times 400 \times 1000 = 480 \text{ mm}^2$$

Providing ϕ 12 @ 230 mm c/c.

21.8 SUMMARY

The wall used to retain some material on one or both sides of it can be termed as retaining wall. Generally, it is used to retain soil, at two different levels on either side of the wall. Moreover, the materials to be retained on either side may be different, e.g. the wall of a swimming pool retains soil on one side and water on the other side.

Gravity wall, cantilever wall, counterfort wall and Buttress wall are some of the common types of retaining wall. In case of Gravity Wall the self weight of the structure provides stability against the use of retained earth. Cantilever wall consists of the vertical arm which retains the earth and is held in position by the base slab. In this case, the weight of fill on top of the heel, in addition to the weight of the wall, contributes to the stability of the structure. To reduce the bending moments in vertical wall of greater heights, counterforts are provided spaced at the distance from each other equal to or slightly larger than one-half of the height. Such walls are called as Counterfort Walls.

The process of design of a retaining wall commences with preliminary proportioning of the wall, then the design in checked against the stability requirements of wall and is revised if required. Design of various elements of a wall depend on the end conditions of the elements.

21.9 ANSWERS TO SAQs

Refer the relevant preceding text in the unit or other useful books on the topic listed in the section "Further Reading" to get the answers of the SAQs.