UNIT 15 ECCENTRICALLY LOADED COLUMNS

Structure

15.1 INTRODUCTION

All columns are deemed to be eccentrically loaded (vide Section 14.3) i.e. there will be direct load and bending in all columns due to positioning of live loads, inaccuracies of construction, accidental loads, monolithic construction, lateral loads, eccentricity of loads etc. Due to above mentioned reasons, even an axially loaded column is designed for a minimum eccentricity (e_{min} defined in Section 14.3) that is for limited amount of bending. Bending of a column may be either *uniaxial* (ie. bending only about one of its principal axis or *biaxial* (i.e. bending about both of its principal axes. Direct load causes uniform stress on the entire cross-section where as bending about a principal axis causes triangular stress distribution about that axis (Figure 15.1).

Objectives

After studying this unit, you should be able to

- study the characteristics of RC Columns subjected, eccentric loading, (or bending moment with axial loading)
- identify the conditions under which the columns crack under direct load and bending, and
- distinguish between the design of cracked and uncracked concrete columns.

The resultant of these two types of stresses of any point may be either compressive or tensile depending on the position of eccentric load. To be more clear, if *the entire cross-section is taken effective both in compression as well as in tension (uncracked section),* application of axial load P and a moment *M_y about its principal y-axis, (*uniaxial bending*) will cause a stress *along a line* at distance x from neutral axis

$$f_x = \frac{P}{A} \pm \frac{M_y}{I_y} x$$

or

$$f_x = \frac{P}{A} \pm \frac{Pe_x}{I_y} x \qquad \text{...(15.1(a))}$$

* A concrete load P and a moment M_y about the principal y-axis is ***equivalent to*** a load P at an eccentricity $e_x = \frac{M_y}{P}$

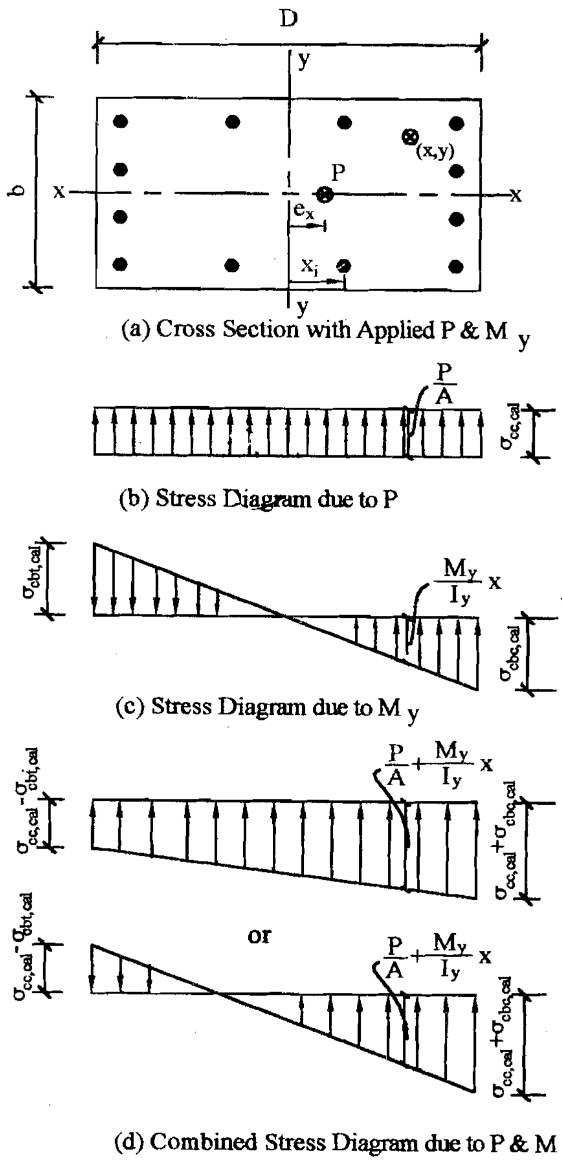


Figure 15.1: Stress Diagram Due to Direct Load & Uniaxial Bending for Uncracked Section

Similarly if a direct load P and a moment, M_x, about its principal x-axis are applied over a cross-section along a line at distance y from neutral axis

$$\left.\begin{aligned} f_y &= \frac{P}{A} \pm \frac{M_x}{I_x} y \\ &= \frac{P}{A} \pm \frac{Pe_y}{I_x} y \end{aligned}\right\} \qquad \ldots(15.1b)$$

where,

A = Equivalent concrete area of the section = $A_c + 1.5\ m\ A_s$

I_x, I_y = Moment of inertia of equivalent concrete area of cross-section about x-axis and y-axis respectively.

Therefore, the maximum and the minimum stress which occur along edges may be given by the equations

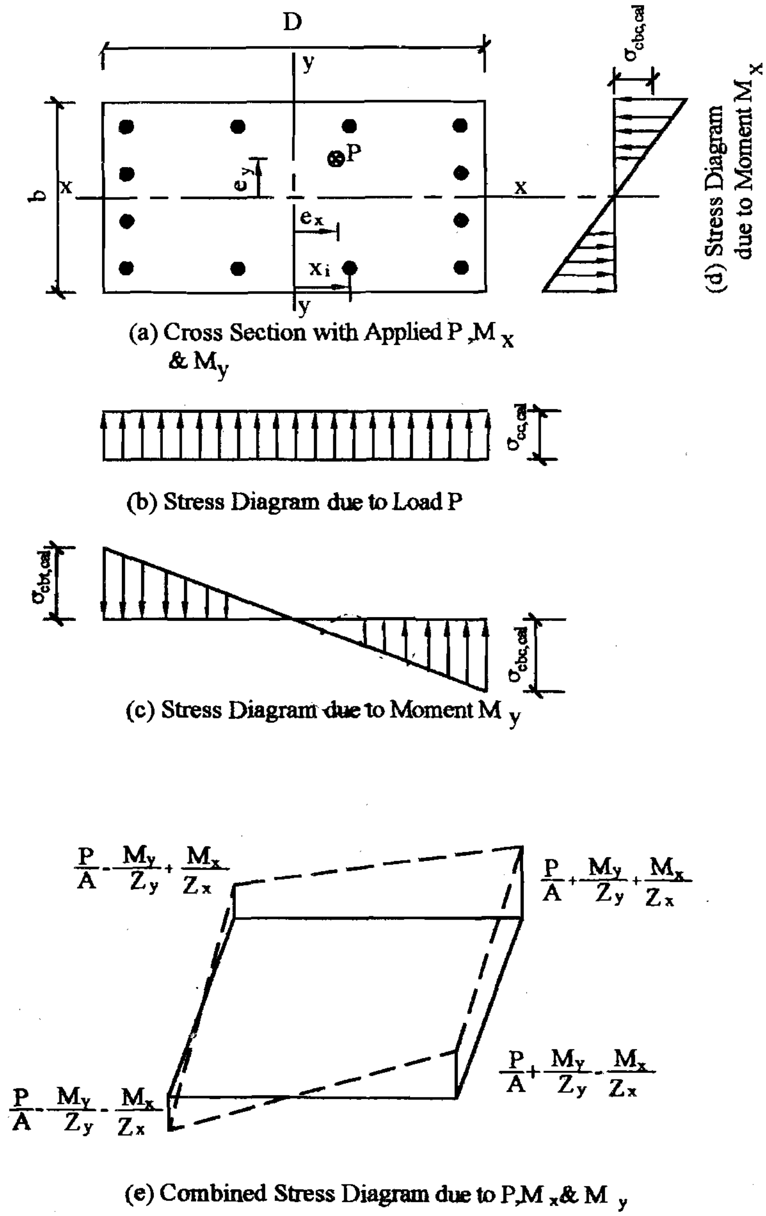


Figure 15.2: Stress Diagram Due to Direct Load & Biaxial Bending for Uncracked Section

$$f_{max,min} = \frac{P}{A} \pm \frac{M}{Z} \qquad ...(15.2)$$

where,

$M = M_x$ or M_y

and Z = Modulus of Section about x-axis or y-axis

The (+)tive or (–)tive sign of the stress due to bending or due to eccentricity of load depend upon (+)tive or (–)tive sign of distance x or y from the neutral axis.

If a section is subjected to direct load and bending about both of its principal axis (Figure 15.2) the resultant stress at any *point*, (x, y),

$$f_{x,y} = \frac{P}{A} \pm \frac{M_y}{I_y} x \pm \frac{M_x}{I_x} y$$

or

$$f_{x,y} = \frac{P}{A} \pm \frac{Pe_x}{I_y} x \pm \frac{Pe_y}{I_y} y \qquad ...(15.3)$$

Therefore, the maximum and the minimum stress which occur at any two corners may be given by equations

$$f_{max,min} = \frac{P}{A} \pm \frac{M_y}{Z_y} \pm \frac{M_x}{Z_x} \qquad ...(15.4)$$

If the applied moment is small, that is the eccentricity of load is small there will be compressive stress in the entire cross-sectional area. Even if there is small tensile stress anywhere within the section which is below the permissible limits prescribed by the I.S. Code for cracking, the section may be taken as *uncracked.* (i.e. whole cross-section may be considered to be effective for analysis and design purposes). But if the applied moment is large *'*no stress line*' will fall within the section and the tensile stress in concrete is greater than the permissible limit for uncracked section, the section may be considered a cracked one and the *cracked sectional area* is neglected in analysis and design.

15.2 DESIGN BASED ON UNCRACKED SECTION

15.2.1 Design Based on Uncracked Section for Uniaxial Bending

A section is considered to be *uncracked* if the maximum tensile stress in concrete is not greater than 25% of the resultant compression nor does it exceed 3/4th of the 7 days modulus of rupture of concrete.

The adequacy or safety of an uncracked section under applied load under direct load *P* and *uniaxial bending* M_x or M_y is checked by Equations 15.5 derived below:

For safety (vide Figure 15.1)

$$\sigma_{cc,cal} + \sigma_{cbc,cal} \le \sigma$$

or $$\frac{\sigma_{cc,cal}}{\sigma} + \frac{\sigma_{cbc,cal}}{\sigma} \le 1 \qquad ...(15.5)$$

where $\sigma = \sigma_{cc}$ for pure axial compression and $\sigma = \sigma_{cbc}$ for pure bending

Hence, the above equation is written as

$$\frac{\sigma_{cc,cal}}{\sigma_{cc}} + \frac{\sigma_{cbc,cal}}{\sigma_{cbc}} \le 1 \qquad ...(15.6)$$

where,

$$\sigma_{cc,cal} = \frac{P}{A_c + 1.5m\,A_{sc}}$$

and $$\sigma_{cbc,cal} = \frac{M}{Z}$$

If $$M = M_x ; Z = Z_x = \frac{I_{xx}}{b/2}$$

where, $$I_{xx} = \frac{Db^3}{12} + (1.5m - 1)\,\Sigma A_{si} {y_{si}}^2$$

* 'No stress line' is determined from the equations of equilibrium vide Section 15.3.1. It shall not be confused with neutal axis because the latter is the geometrical property of a section i.e. a line passing through C.G. of the area.

or where, $M = M_y$; $Z = Z_y = \frac{I_{yy}}{D/2}$

Where, $I_{yy} = \frac{bD^3}{12} + (1.5m - 1)\,\Sigma\, A_{si}\, x_{si}^2$

15.2.2 Design Based on Uncracked Section for Biaxial Bending

A section is considered to be *uncracked* if the maximum tensile stress is not greater than 35% of the resultant compression nor does it exceed 3/4th of the 7 days modulus of rupture of concrete.

The adequacy or safety of an uncracked section under applied load and *biaxial bending* M_x and M_y is checked by Equation 15.6,

where, $\sigma_{cbc,cal} = \sigma_{cbcx,cal} + \sigma_{cbcy,\,cal}$

Example 15.1

Check the adequacy of a rectangular column section $b \times D = 300 \times 500$ mm reinforced on two opposite faces parallel to the major axis with 10#20 ϕ applied with an axial load, $P = 500$ kN and a bending moment about major axis, $M_y = 60$ kNm. Use M 20 and Fe 415 steel. An effective cover to reinforcement on all sides is taken to be 50 mm.

Solution

a) **Check for status of cracking** (Figure 15.3)

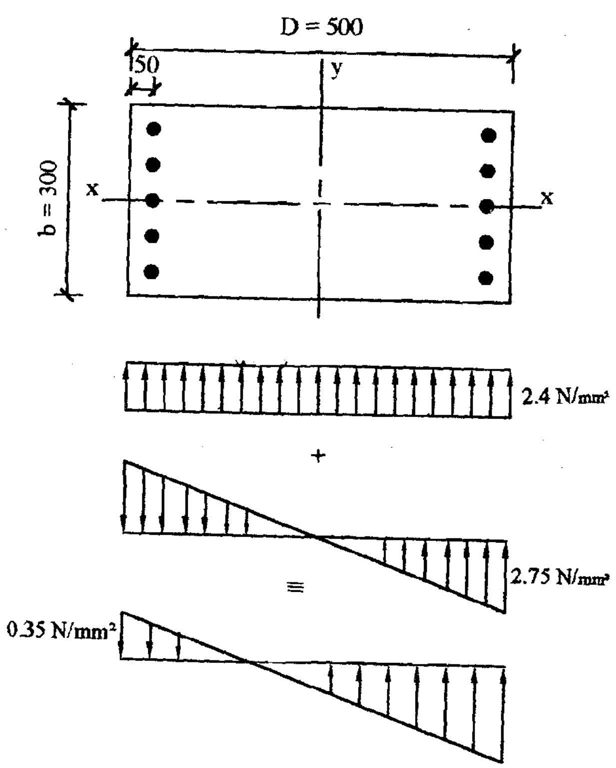


Figure 15.3 : Combined Stress Diagram for P & M_y

$$m = \frac{280}{3\sigma_{cbc}} = \frac{280}{3 \times 7} = 13.33 \approx 13$$

$A = A_c + 1.5\, m\, A_{sc}$

$= (300 \times 500 - 10 \times \frac{\pi}{4} \times 20^2) + 1.5 \times 13 \times 10 \times \frac{\pi}{4} \times 20^2$

$= 146858.41 + 61261.06$

$= 208119.47 \text{ mm}^2$

$$Z_y = \frac{I_y}{D/2} = \frac{\frac{1}{12} bD^3 + (1.5m - 1)A_{sc}\left(\frac{D}{2} - d'\right)^2}{D/2}$$

$$= \frac{\frac{1}{12} \times 300 \times 500^3 + (1.5 \times 13 - 1) \times 10 \times \frac{\pi}{4} \times 20^2 (250 - 50)^2}{250}$$

$$= \frac{5449.78 \times 10^6}{250} \text{mm}^2 = 21.80 \times 10^6 \text{ mm}^2$$

$$\sigma_{cc,cal} = \frac{P}{A} = \frac{500 \times 10^3}{208119.47} = 2.40 \ N/\text{mm}^2$$

$$\sigma_{cc,cal} = \frac{M_y}{Z_y} = \frac{60 \times 10^6}{21.80 \times 10^6} = 2.75 \ N/\text{mm}^2$$

∴ Resultant compression

$= \sigma_{cc,cal} + \sigma_{cbc,cal}$

$= 2.40 + 2.75 = 5.15 \text{ N/mm}^2$

Resultant tension

$= \sigma_{cc,cal} - \sigma_{cbc,cal}$

$= 2.40 - 2.75 = -0.35 \text{ N/mm}^2$

$\leq 0.25 \times 5.15$ (=1.29 N/mm²)

$\leq \frac{3}{4} \times 2.4$ (=1.8 N/mm²) (7 days modulus of rupture of concrete for M–20 = 2.4 N/mm²)

Hence the section is uncracked

b) **Check for safety**

$$\frac{\sigma_{cc,cal}}{\sigma_{cc}} + \frac{\sigma_{cbc,cal}}{\sigma_{cbc}}$$

$$= \frac{2.4}{5} + \frac{2.75}{7}$$

$= 0.87 < 1$

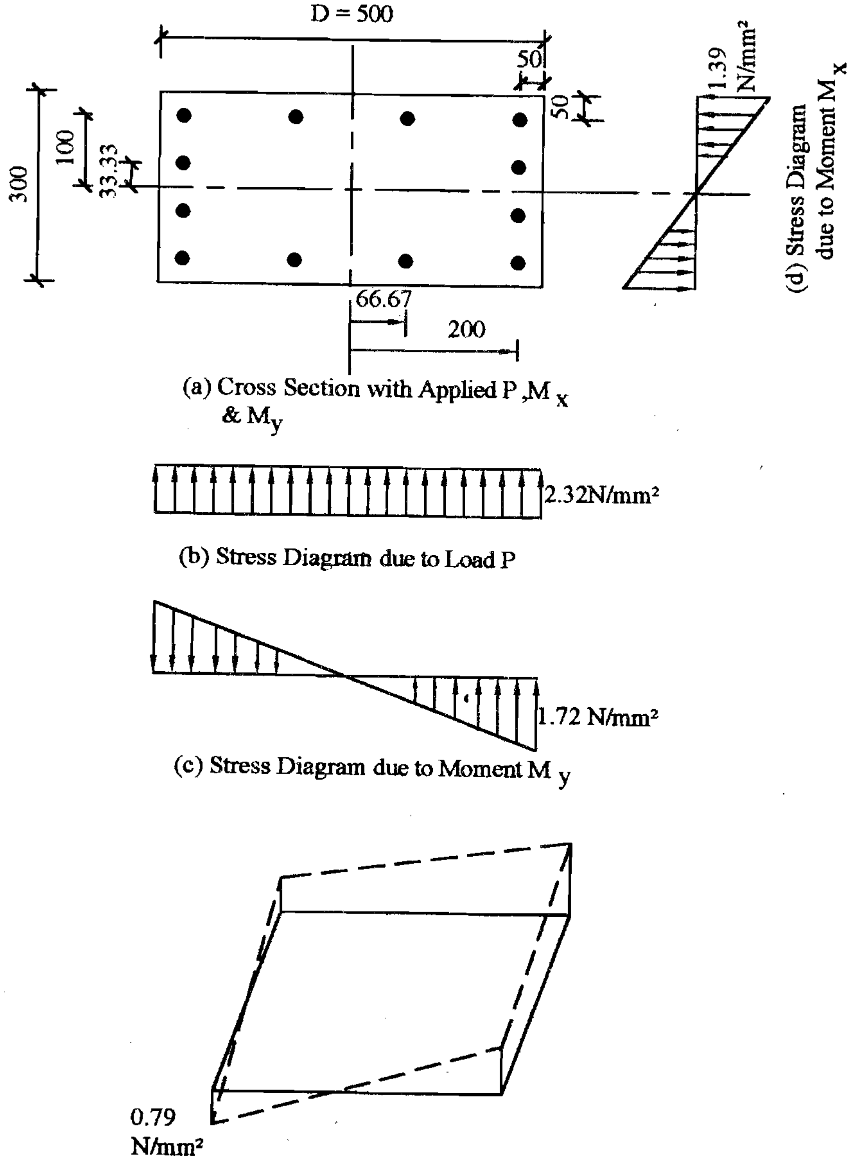
Hence the section is safe. Ans

Example 15.2

Check the adequacy of a rectangular column section $b \times D = 300 \times 500$ mm reinforced with 12#20 ϕ distributed equally on all sides applied with an axial load, $P = 500$ kN and a bending moment about minor axis, $M_x = 15$ kNm and bending about major, $M_y = 35$ kNm. Use M 20 and Fe 415 steel. Cover to reinforcement on all sides is taken equal 50 mm.

Solution

a) **Check for status of cracking** (Figure 15.4)



(a) Cross Section with Applied P, M_x & M_y

(b) Stress Diagram due to Load P

(c) Stress Diagram due to Moment M_y

(e) Combined Stress Diagram due to P, M_x & M_y

Figure 15.4 : Stress Diagram Due to Direct Load & Biaxial Bending for Uncracked Section

$$m = \frac{280}{3\sigma_{cbc}} = \frac{280}{3 \times 7} = 13.33 \approx 13$$

$$A = A_c + 1.5\, m\, A_{sc}$$

$$= (300 \times 500 - 12 \times \frac{\pi}{4} \times 20^2) + 1.5 \times 13 \times 12 \times \frac{\pi}{4} \times 20^2$$

$= 146230.08 + 18.5 \times 3769.91$

$= 215973.42\ \text{mm}^2$

$$I_{xx} = \frac{1}{12} Db^3 + (1.5m - 1) \Sigma A_{si} y_{si}^2$$

$$= \frac{1}{12} \times 500 \times 300^3 + 18.5 \times (8 \times \frac{\pi}{4} \times 20^2 \times 100^2 + 4 \times \frac{\pi}{4} \times 20^2 \times 33.3^2)$$

$= 1125 \times 10^6 + 490.73 \times 10^6$

$= 1615.78 \times 10^6\ \text{mm}^4$

$$\therefore\ Z_x = \frac{I_{xx}}{b/2} = \frac{1615.73 \times 10^6}{150} = 10.77 \times 10^6\ \text{mm}^3$$

Similarly $I_{yy} = \frac{1}{12}\ bD^3 + (1.5\ \text{m} +) \Sigma A_{si} x_{si}^2$

$$= \frac{1}{12} \times 300 \times 500^3 + 18.5\ (8 \times 20 \times 200^2 \times 4 \times \frac{\pi}{4} \times 20^2 \times 66.67^2)$$

$= 5088.16 \times 10^6\ \text{mm}^4$

$$\therefore\ Z_y = \frac{I_{yy}}{D/2} = \frac{5088.16 \times 10^6}{250} = 20.35 \times 10^6\ \text{mm}^3$$

Now $\sigma_{cc,cal} = \frac{P}{A} = \frac{500 \times 10^3}{215973.42} = 2.32\ \text{N/mm}^2$

$$\sigma_{cbc,x,cal} = \frac{M_x}{Z_x} = \frac{15 \times 10^6}{1077.10^6} = 1.39\ \text{N/mm}^2$$

$$\sigma_{cbc,y,cal} = \frac{M_y}{Z_y} = \frac{35 \times 10^6}{20.35 \times 10^6} = 1.72\ \text{N/mm}^2$$

Resultant compression

$$= \frac{P}{A} + \frac{M_x}{Z_x} + \frac{M_y}{Z_y}$$

$= 2.32 + 1.39 + 1.72 = 5.43\ \text{N/mm}^2$

Resultant tension

$= \sigma_{cc,cal} - \sigma_{cbc,x,cal} - \sigma_{cbc,y,cal}$

$= 2.32 - 1.39 - 1.72 = -0.79\ \text{N/mm}^2$

$\leq 0.35 \times 5.43\ (=1.9\ \text{N/mm}^2)$

$\leq \frac{3}{4} \times 2.4\ (=1.8\ \text{N/mm}^2)$

Hence the section is uncracked

b) **Check for safety**

$$\frac{\sigma_{cc,cal}}{\sigma_{cc}} + \frac{\sigma_{cbc,x,cal}}{\sigma_{cbc}} + \frac{\sigma_{cbc,y,cal}}{\sigma_{cbc}}$$

$= \frac{2.32}{5} + \frac{1.39}{7} + \frac{1.72}{7}$

$= 0.908 < 1$ **Hence the section is safe. Ans**

SAQ 1

i) Discuss the conditions under which an eccentrically loaded column will be treated as uncracked and safe.

ii) Derive the equation of safety for uncracked section of an eccentrically loaded column.

iii) Check the safety of rectangular R.C. column section $b \times D = 250 \times 350$ mm reinforced with 4 # 16 ϕ on each of its short faces subjected to an axial load of 450 kN and a bending moment of 15 kNm acting about its major axis. Use M 20 concrete and effective cover of 50 mm.

iv) Check the safety of a rectangular column section $b \times D = 300 \times 600$ mm reinforced equally on four sides with 16 # 16 ϕ and having an effective cover of 50 mm. Use M 20 concrete and Fe 250 steel (Figure 15.5).

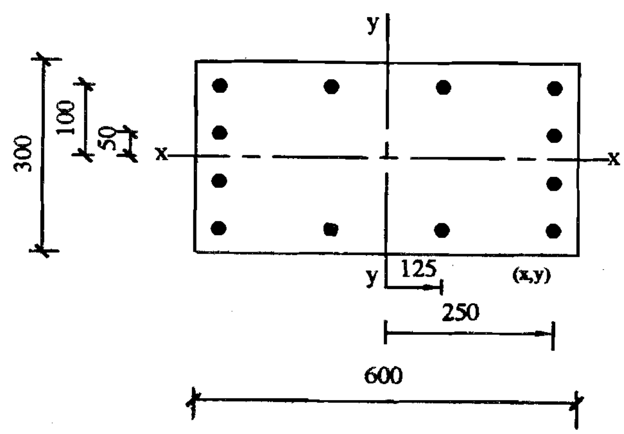


Figure 15.5: Cross-section of the Column

15.3 DESIGN BASED ON CRACKED SECTION

15.3.1 Design Based on Cracked Section for Uniaxial Bending

For a given column section applied with direct load and bending, the section is *first* checked for the two conditions of safety as uncracked section (vide Section 15.2.1). If it does not comply with those above-mentioned conditions, it is assumed to have *cracked.* If a section has cracked, the cracked portion of area of concrete can not resist tension. It is demarked by a 'no stress line' within the cross-section (Figure 15.6). Next, the cross-section may be considered safe if the maximum stress in concrete and the maximum tensile stress in steel do not exceed their respective permissible limits. It may be noticed here that as per *linear* elastic theory, the strains and the corresponding stresses are related and, therefore, if only one, say, the maximum compressive stress in concrete, f_c, is taken as unknown, the stresses in concrete as well as steel elsewhere on the section may be defined in terms of f_c. It may, therefore, be concluded that *only two* unknowns – that is (i) position of 'no stress line' and (ii) maximum compressive stress in concrete f_c – are to be determined by analysis for design. This may be done by the two equations of equilibrium mentioned below :

i) *The external load is equal to the algebraic sum of forces in concrete (ignoring the tensile force in concrete) and longitudinal steel.*

Mathematically, from equilibrium of forces (Figure 15.6)

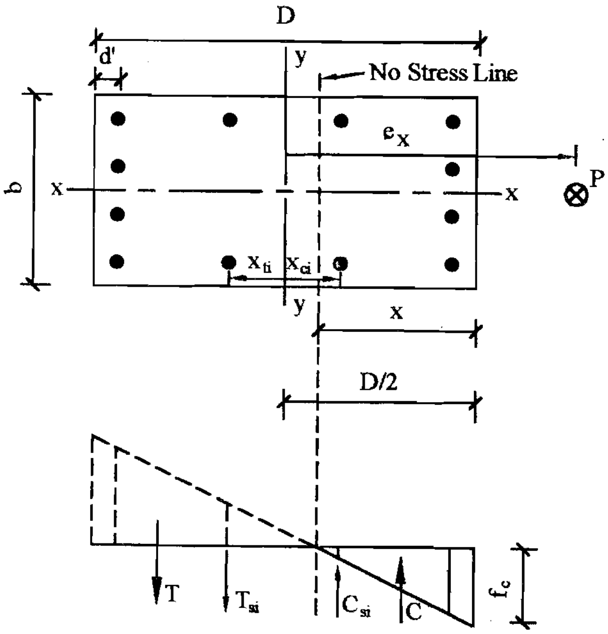


Figure 15.6: Stress Diagram Due to Direct Load & Uniaxial Bending for Cracked Section

$$\Sigma F = 0$$

or $$-P + C - T = 0$$

or $$P = C - T$$

$$= C_c + C_s - T$$

where,

P = Externally applied load at an eccentricity e ($e = \frac{M}{P}$) from the centroidal axis

$$C_c = \frac{1}{2} f_c x b - \frac{\Sigma f_{sci} A_{sci}}{1.5\, m}$$

$$\left\{ f_{sci} = 1.5\, m \frac{f_c}{x}\left(x_{ci} - \frac{D}{2} + x \right) \right\}$$

$$C_s = \Sigma f_{sci} A_{sci}$$

and $T = \Sigma f_{sti} A_{sti}$

$$\left\{ f_{sti} = m \frac{f_c}{x}\left(x_{sti} + \frac{D}{2} - x \right) \right\}$$

Substituting the above values

$$P = \frac{1}{2} f_c x b + (1.5\, m - 1) \frac{f_c}{x} \Sigma \left(x_{ci} - \frac{D}{2} + x\right) A_{sci} - m \frac{f_c}{x} \Sigma \left(x_{sti} + \frac{D}{2} - x\right) A_{sti} \qquad \ldots(15.6)$$

ii) *The moment of external force about any reference line is equal to the moment of forces in concrete (ignoring the tensile force in concrete) and steel about the same line.*

Mathematically, from equilibrium of moment of forces about a reference line, say, about the line parallel to y-axis and passing through the point of application of P

$$\Sigma T_{si} (e + x_{sti}) = C_c \left(e - \frac{D}{2} + \frac{x}{3}\right) + \Sigma C_{si} (e - x_{sci})$$

where, T_{si} = Tensile force in ith row of reinforcement at x_{sti} from y-axis

and C_{si} = Compressive force in ith row of reinforcement at x_{sci} from y-axis

or $$\Sigma \frac{mf_c}{x} \left(x_{sti} + \frac{D}{2} - x \right) A_{sti} (e + x_{sti})$$

$$= \frac{1}{2} f_c x b \left(e - \frac{D}{2} + \frac{x}{3} \right) + \Sigma \frac{(1.5\,m - 1) f_c}{x} \left(x_{sci} - \frac{D}{2} + x \right) A_{sci} (e - x_{sci})$$

or $$m \left(x_{sti} + \frac{D}{2} - x \right) A_{sti} (e + x_{sti})$$

$$= \frac{1}{2} b x^3 \left(e - \frac{D}{2} + \frac{x}{3} \right) + \Sigma (1.5\,m - 1) \left(x_{sci} - \frac{D}{2} + x \right) A_{sci} (e - x_{sci}) \qquad \ldots(15.7)$$

Example 15.3

Check status of cracking and safety of a column of rectangular cross-section $b \times D$ = 300 × 500, reinforced on two faces parallel to major axis with 10#20 applied with an axial load, P = 240 kN and a bending moment about major axis, M_y = 70 kNm. Use M 20 concrete and Fe 415 steel. Effective concrete cover of 50 mm may be provided on all sides of reinforcement.

Solution

a) **Check for Sections of Cracking** (Figure 15.7)

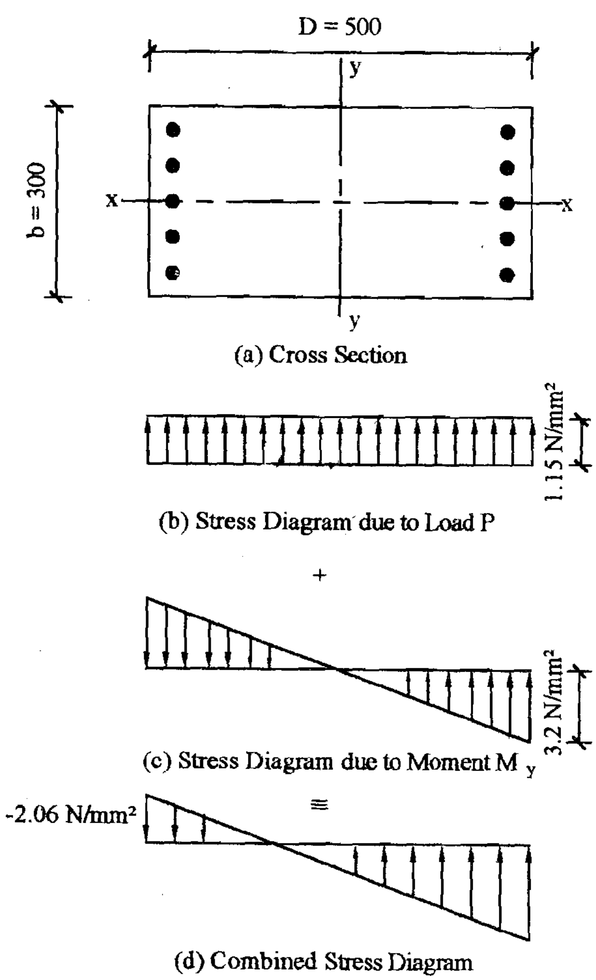


Figure 15.7 : Check for Status of Cracking

$$m = \frac{280}{3\sigma_{cbc}} = \frac{280}{3 \times 7} = 13.33 \approx 13$$

$$A = A_c + 1.5\, m\, A_s$$

$$= (300 \times 500 - 10 \times \frac{\pi}{4} \times 20^2) + 1.5 \times 1.3 \times 10 \times \frac{\pi}{4} \times 20^2$$

$$= 146858.41 + 61261.06$$

$$= 208119.47 \text{ mm}^2$$

$$Z_y = \frac{I_y}{D/2} = \frac{\frac{1}{12} bD^3 + (1.5m - 1)A_{sc}\left(\frac{D}{2} - d')\right)}{D/2}$$

$$= \frac{\frac{1}{12} 300 \times 500^3 + (1.5 \times 13 - 1)10 \times \frac{\pi}{4} \times 20^2 \times (250 - 50)^2}{250}$$

$$= 21.80 \times 106 \text{ mm}^3$$

$$\sigma_{cc,cal} = \frac{P}{A} = \frac{240 \times 10^3}{208119.47} = 1.15 \frac{N}{mm^2}$$

$$\sigma_{cbc,cal} = \frac{M_y}{I_y} = \frac{70 \times 10^3}{21.80 \times 10^6} = 3.21 \frac{N}{mm^2}$$

Result compression,

$$f_{cc,max} = \frac{P}{A} + \frac{M_y}{I_y} = 1.15 + 3.21 = 4.36 \frac{N}{mm^2}$$

Resultant tension

$$f_{ct,max} = \frac{P}{A} - \frac{M_y}{I_y} = 1.15 - 3.21 = -2.06 \frac{N}{mm^2}$$

$$> 0.25 \times 4.36\ (=1.09 \text{ N/mm}^2)$$

$$> 0.75 \times 2.4\ (=1.80 \text{ N/mm}^2)$$

Hence the section is cracked

b) **Check for safety** (Figure 15.8)

$$P = \frac{1}{2} f_c x b + (1.5\, m - 1)A_{sc} . \frac{f_c}{x}(x - d') - mA_{st} . \frac{f_c}{x}(d - x)$$

$$= \frac{1}{2} f_c x \times 300 + (1.5 \times 13 - 1) \times 5 \times \frac{\pi}{4} \times 20^2 . \frac{f_c}{x}(x - 50) - 13 \times 5 \times \frac{\pi}{4} \times 20^2 . \frac{f_c}{x}(450 - x)$$

$$= 150 f_c x + 18.5 \times 1570.80 \times \frac{f_c}{x}(x - 50) - 13 \times 1570.8 \times \frac{f_c}{x}(450 - x)$$

$$= \frac{f_c}{x}(150x^2 + 49480.13x - 10642166.5) \quad \text{...(i)}$$

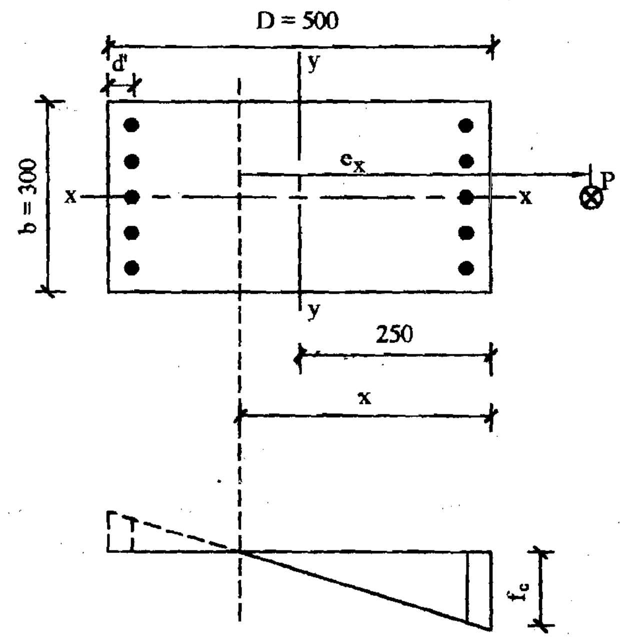


Figure 15.8: Stress Diagram Due to *P* & M_y for Cracked Section

$$e_x = \frac{M}{P} = \frac{70 \times 10^6}{240 \times 10^3} = 291.67$$

Taking moment of all forces about tensile steel

$$P\left(\frac{D}{2} - d' + e_x\right) = f_c x b\left(d - \frac{x}{3}\right) + (1.5\,m - 1)\,A_{sc}\,\frac{f_c}{x}(x - d')(d - d')$$

or $$P(250 - 50 + 291.67) = \frac{1}{2} f_c x \times 300\left(450 - \frac{x}{3}\right) + (1.5 \times 13 - 1) \times 5 \times \frac{\pi}{4} \times 20^2$$

$$\frac{f_c}{x}(x - 50)(450 - 50)$$

or $$P = \frac{f_c}{x}(-0.1017x^3 + 137.29x^2 + 23641.65x - 1182082.70) \qquad \ldots\text{(ii)}$$

Equating I & II

$$x^3 + 124.9754x^2 + 254065.68x - 93019506.4 = 0$$

or $x = 261.292$

From I

$$f_c = \frac{P.x}{150x^2 + 49480.13x - 10642166.5}$$

Substituting x from above

$f_{cc} = 5 \leq 5$ N/mm²

$$f_{sc} = (1.5m - 1) f_c \frac{(x - d')}{x}$$

$$= (1.5 \times 13 - 1) \times 5 \times \frac{(261.292 - 50)}{261.292} = 74.8 \frac{N}{mm^2} < 190 \frac{N}{mm^2}$$

$$f_{st} = m f_c \frac{(d - x)}{x} = 13 \times 5 \times \frac{(450 - 261.292)}{261.292} = 46.94 \frac{N}{mm^2} < 230 \frac{N}{mm^2}$$

Hence the Section is safe. Ans

15.3.2 Design Based on Cracked Section for Biaxial Bending

The analysis and design of a section applied with direct force and biaxial bending becomes complicated. More so the code prescribes that a member subjected to combined direct load and bending and designed by the methods based on elastic theory should be further checked for strength under ultimate load conditions; the design procedure here will, therefore, be based on Limit State Method as follows:

According to Code the design direct load, P_u, and moments M_{ux} and M_{uy} should satisfy the following equation

$$\left[\frac{M_{ux}}{M_{ux1}}\right]^{\alpha_n} + \left[\frac{M_{uy}}{M_{uy1}}\right]^{\alpha_n} \leq 1 \qquad \text{...(15.8)}$$

where, M_{ux}, M_{uy} = bending moments about x and y-axis

M_{ux1}, M_{uy1} = maximum uniaxial bending moment capacity for an axial load of P_u, bending about x and y axis respectively

α_n is related to $\frac{P_u}{P_{uz}}$

where, $P_{uz} = 0.45 f_{ck} \cdot A_c + 0.75 f_y A_{sc}$

For values of $\frac{P_u}{P_{uz}}$ = 0.2 to 0.8, the values of α_n vary linearly form 1 to 2. For values less than 0.2, α_n is 1 and for values of greater than 0.8, α_x is 2

Here again the evaluation of M_{ux1} and M_{uy1} and P_{uz} involves lengthy and complicated calculations.

Therefore, column with biaxial bending shall be designed with the help of Interaction Curves vide Section 15.4.2 only.

15.4 CONSTRUCTION AND USE OF INTERACTION CURVES

15.4.1 Construction and Use of Interaction Curves for Uniaxial Bending

As mentioned above a column with axial load and bending designed by Working Stress Method has to be checked for strength by Limit State Method; therefore, method of design of rectangular column section by Limit State Method *through* an interaction curve $\frac{M_{ux}}{f_{ck} b D^2} - \frac{P_u}{f_{ck} b D}$ for a particular values of $\frac{p}{f_{ck}}$ = 0.16 for Fe 415 steel reinforcement distributed equally on all four sides and having $\frac{d}{D} = 0.1$ has been discussed here (Figure 15.9 & Figure 15.10).

$$P_u = 0.36 f_{ck} x_u b + \sum_{i=1}^{n} A_{si} f_{si} - \sum_{i=1}^{n} A_{si} f_{ci}$$

where, A_{si} = area of reinforcement in the ith row

f_{si} = stress in the ith row of reinforcement, compression being (+)tive and tension being (–)tive.

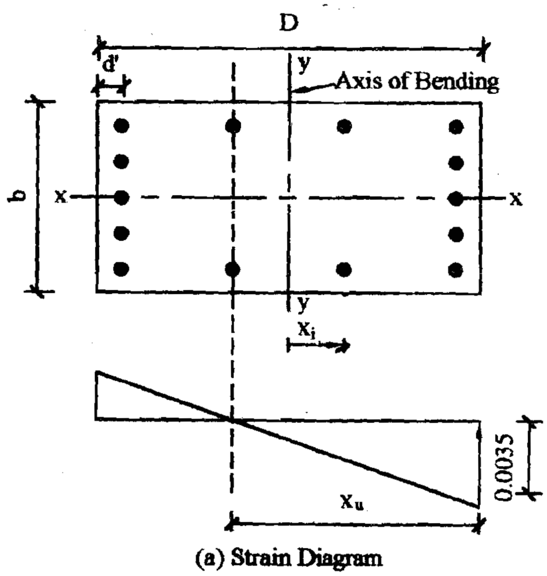


Figure 15.9: Strain Diagram When Neutral Axis is within the Section

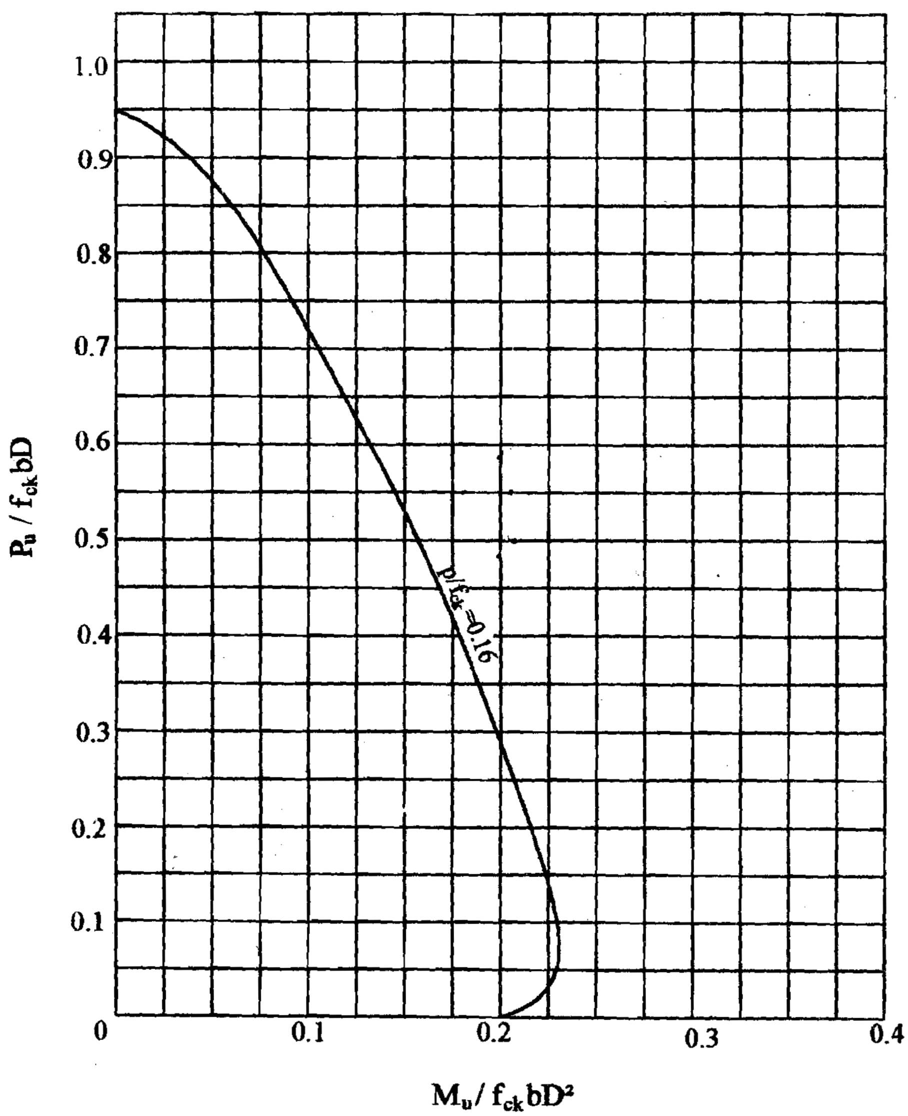


Figure 15.10: Compression with Bending

f_{ci} = stress in concrete at the level of ith row of reinforcement and

n = number of rows of reinforcement.

Dividing each side by $f_{ck}bD$

$$\frac{P_u}{f_{ck}bD} = 0.36\frac{x_u}{D} + \sum_{i=1}^{n} \frac{A_{si}}{f_{ck}bD} f_{si} - \sum \frac{A_{si}}{f_{ck}bD} f_{ci}$$

$$= 0.36\, k + \sum_{i=1}^{n} \frac{p_{ti}}{100 f_{ck}} (f_{si} - f_{ci})$$

where, $k = \frac{x_u}{D}$ and $p_{ti} = \frac{100 A_{si}}{bD}$

Similarly equating moment of forces about centroidal axis of the section

$$M_u = 0.36 f_{ck} x_u b\left(\frac{D}{2} - 0.416 x_u\right) + \left(\sum_{i=1}^{n} A_{si} f_{si} - \sum_{i=1}^{n} A_{si} f_{ci}\right) x_i$$

Dividing each side by $f_{ck}\ bD^2$ and putting $0.416\ \frac{x_u}{D} = C_2$

$$\frac{M_u}{f_{ck}bD^2} = 0.36\, k\,(0.50 - 0.416\, k) + \sum_{i=1}^{n} \frac{p_{ti}}{100 f_{ck}} (f_{si} - f_{ci}) \frac{x_i}{D}$$

Taking $\frac{P_u}{f_{ck}bD}$ on x-axis and $\frac{M_u}{f_{ck}bD^2}$ on y-axis, curves are plotted for a particular value of $\frac{p}{f_{ck}} = 0.16$ and steel grade Fe 415 and constant $\frac{d'}{D} = 0.1$

15.4.2 Construction and Use of Interaction Curves for Biaxial Bending

As mentioned in Section 15.3.2, a column a with direct load and biaxial bending shall satisfy the Equation 15.8,

$$\left[\frac{M_{ux}}{M_{ux1}}\right]^{\alpha_n} + \left[\frac{M_{uy}}{M_{uy1}}\right]^{\alpha_n} \leq 1$$

Here P_{uz} is calculated from Figure 15.11 and M_{ux} and M_{uy} for a particular load P_u are calculated from relevant

$$\frac{P_u}{f_{ck}bD} \sim \frac{M_{ux}}{f_{ck}bD^2}$$

curves for unaxial bending.

An Interaction Curve $\frac{M_{ux}}{M_{ux1}} \sim \frac{M_{uy}}{M_{uy1}}$ for a particular value of $\frac{P_u}{P_{uz}} \leq 0.2$ and $= 0.6$ have been plotted in Figure 15.12.

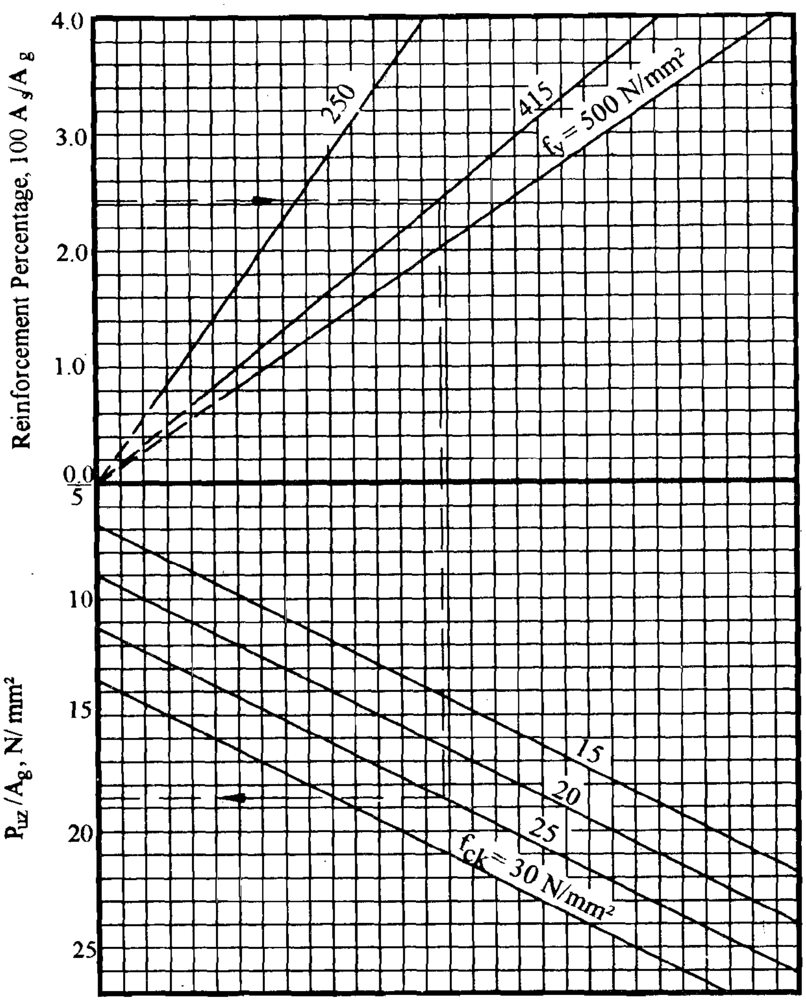


Figure 15.11: Values of P_{uz} for Compression Members

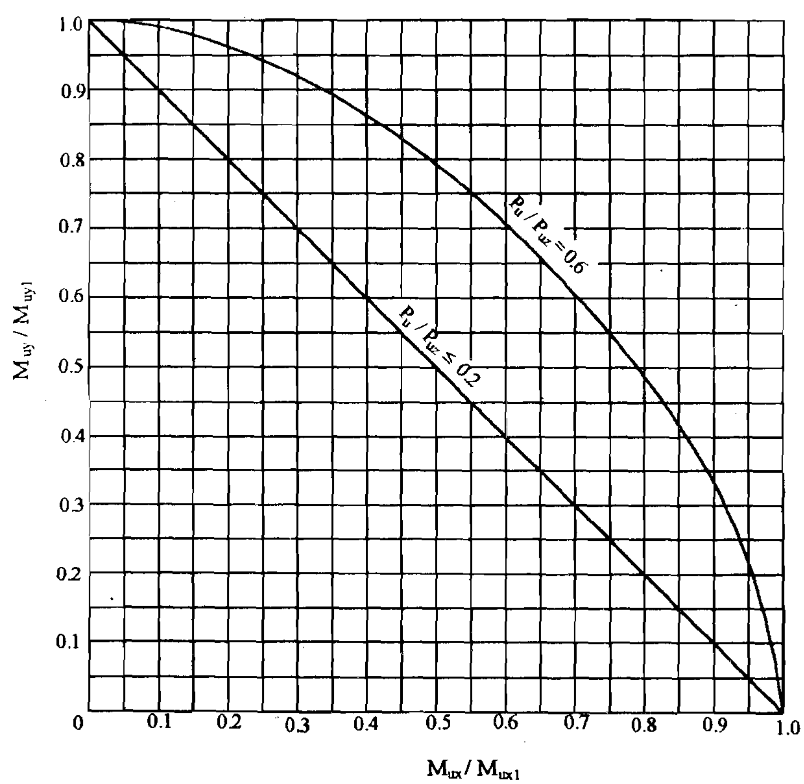


Figure 15.12: Biaxial Bending in Compression Member

Example 15.4

Check the adequacy of the section shown in Figure 15.13 for $P = 500$ kN ; $M_y = 60$ kNm ; $f_{ck} = 20$ MPa and $f_y = 415$. Reinforcement consists of 10#16ϕ.

Solution

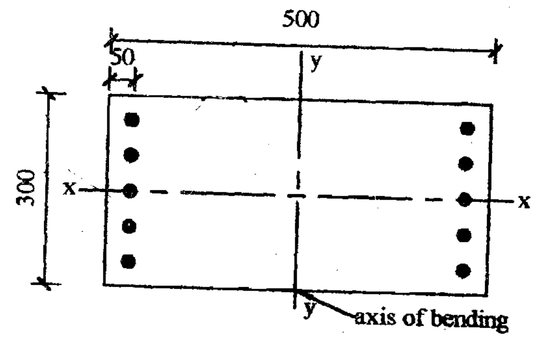


Figure 15.1[illegible] Cross-section of Column

$$\frac{d'}{D} = \frac{50}{500} = 0.1$$

$$P_u = 1.5 \times 500 = 750 \text{ kN}$$

$$M_u = 1.5 \times 60 = 90 \text{ kN}$$

$$\frac{p}{f_{ck}} = \frac{100 \times A_s}{bDf_{ck}} = \frac{100 \times 10 \times \frac{p}{4} \times 16^2}{300 \times 500 \times 20} = 0.067$$

$$\frac{P_u}{f_{ck}bD} = \frac{750 \times 10^3}{20 \times 300 \times 500} = 0.25$$

$$\frac{M_u}{f_{ck}bD^2} = \frac{90 \times 10^6}{20 \times 300 \times 500^2} = 0.06$$

From Chart 32 of SP 16, for $\frac{P_u}{f_{ck}bD} = 0.25$

and $\frac{M_u}{f_{ck}bD^2} = 0.06$; the value of $\frac{p}{f_{ck}} = 0.005 << 0.067$ (provided)

Hence the Section is safe. Ans

Example 15.5

Using *Interaction Diagram* check the adequacy of the column cross-section of Example 15.2 for the same loading as given in that Example.

Solution

$$P_u = 1.5P = 1.5 \times 500 = 750 \text{ kN}$$

$$M_{ux} = 1.5\, M_x = 1.5 \times 15 = 22.5 \text{ kN}$$

$$M_{uy} = 1.5\, M_y = 1.5 \times 35 = 52.5 \text{ kN}$$

$$\frac{p}{f_{ck}} = \frac{100 \times A_s}{bDf_{ck}} = \frac{100 \times 12 \times \frac{p}{4} \times 20^2}{300 \times 500 \times 20} = 0.13$$

$$P_{uz} = 0.45 f_{ck} \cdot A_c + 0.75 f_y A_{sc}$$

$$= 0.45 \times 20 \times 146230.09 + 0.75 \times 415 \times 12 \times \frac{\pi}{4} \times 20^2$$

$$= 2489.46$$

Alternatively P_{uz} may be evaluated from Chart 63 of SP 16 as follows:

$$\frac{A_s}{A_g}\% = \frac{12 \times \frac{\pi}{4} \times 20^2}{300 \times 500} \times 100 = 2.5\%$$

For the above % of reinforcement of grade Fe 415 and concrete of grade M 20 (Figure 15.9)

$$\frac{P_{uz}}{A_g} = 16.60$$

or $P_{uz} = 16.60 \times 300 \times 500 = 2490$

Values of M_{ux1} and M_{uy1} have been evaluated from Chart 44 of SP 16 as follows :

$$\frac{P}{f_{ck}} = 0.13 \text{ as already calculated}$$

$$\frac{P_u}{f_{ck}bD} = \frac{750 \times 10^3}{20 \times 300 \times 500} - 0.25$$

$$\frac{M_{ux1}}{f_{ck}Db^2} = 0.18$$

or $M_{ux1} = 0.18 \times 20 \times 500 \times 3$ 62 kNm

Similarl

$$\frac{M_{uy1}}{f_{ck}bD^2} = 0.18$$

or $M_{uy1} = 0.18 \times 20 \times 300 \times 500^2 = 270$ kNm

$$\frac{P_u}{P_{uz}} = \frac{750}{2490} = 0.30$$

$$\frac{M_{ux}}{M_{ux1}} = \frac{22.5}{162} = 0.1388$$

$$\frac{M_{uy}}{M_{uy1}} = \frac{52.5}{270} = 0.1944$$

From Chart 64 or SP 16 for provided value of $\frac{M_{uy}}{M_{uy1}} = 0.1944$

and $\frac{P_u}{P_{uz}} = 0.3$, the value of $\frac{M_{ux}}{M_{ux1}} = 0.91 > 0.1388$ (provided)

Hence the section is safe . Ans

Aternatively by calculation

$$\text{for } \frac{P_u}{P_{uz}} = 0.3;\ \alpha_n = 1 + \frac{(2-1))}{0.6} \times (0.3 - 0.2) = 1.167$$

$$\therefore \left(\frac{M_{ux}}{M_{ux1}}\right)^{\alpha_n} + \left(\frac{M_{uy}}{M_{uy1}}\right)^{\alpha_n} \leq 1$$

or $(0.1388)^{1.167} + (0.1944)^{1.167} = 0.2476 < 1$

Hence the section is safe. Ans

SAQ 2

i) Solve Example 15.3 Using Interaction Diagrams of SP 16.

ii) Using Interaction Diagram of SP16 check whether the section provided (Figure 15.14) is safe for P_u = 1500 kN, M_{ux} = 100 kN and M_{uy} = 200 kNm ; f_{ck} = 20 N/mm^2 and f_y = 415 N/mm^2.

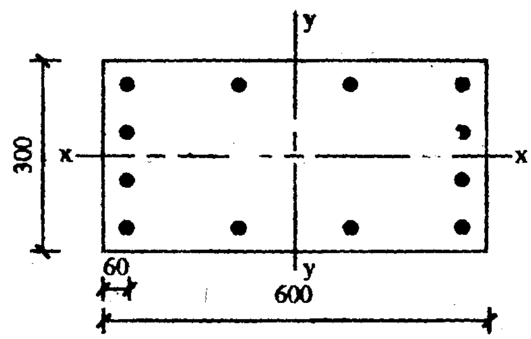


Figure 15.14: Cross-section of the Column

15.5 SUMMARY

An eccentrically loaded column section may either crack or remains uncracked depending upon whether the tensile stress in the cross-section is within the permissible stress or not. If it is uncracked, the whole area of concrete and that reinforcement are taken effective and the cross-section is considered safe if it satisfies the Equation,

$$\frac{\sigma_{cc,cal}}{\sigma_{cc}} + \frac{\sigma_{cbc,cal}}{\sigma_{cbc}} \leq 1$$

irrespective of whether the bending is uniaxial or biaxial. But if a section cracks, the concrete area in tension beyond *no stress line* become ineffective. For uniaxial bending, the position of '*no stress line*' and the stresses in concrete and steel are determined from the two equation of equilibrium for direct load P and bending moment M_x or M_y. For biaxial bending for cracked section, the analysis becomes more involved hence not tried analytically.

The method of drawing and use of interaction curves for the design of a column section have also been dealt with.

15.6 ANSWERS TO SAQs

SAQ 1

i) Refer text Section 15.2

ii) Refer text Section 15.2

iii) Resultant tension,

$$\sigma_{cc,cal} - \sigma_{cbc,cal}$$

$$= (2.56 - 3.22)$$

$$= -0.86$$

$$\leq 0.25\ (2.56 + 3.22)\ \text{N/mm}^2$$

$$\leq 0.75 \times 2.4\ (=1.8\ \text{N/mm}^2)$$

Hence the section is uncracked

Check for safety

$$\frac{\sigma_{cc,cal}}{\sigma_{cc}} + \frac{\sigma_{cbc,cal}}{\sigma_{cbc}}$$

$$= \frac{2.56}{5} + \frac{3.22}{7}$$

$$= 0.972 < 1$$

Hence the section is safe

iv) Resultant tension,

$$\sigma_{cc,cal} - \sigma_{cbcx,cal} - \sigma_{cbcy,cal}$$

$$= 2.09 - 1.71 - 1.87$$

$$= -1.49$$

$\leq 0.35 \times 5.67\ (=1.98)$

$< 0.75 \times 2.4\ (=1.8\ N/mm^2)$

Hence the section is uncracked

Check for safety

$$\frac{\sigma_{cc,cal}}{\sigma_{cc}} + \frac{\sigma_{cbcx,cal}}{\sigma_{cbc}} + \frac{\sigma_{cbcy,cal}}{\sigma_{cbc}}$$

$$= \frac{2.09}{5} + \frac{1.71}{7} + \frac{1.87}{7}$$

$= 0.93 < 1$

Hence the section is safe.

SAQ 2

i) The Section is safe

ii) $$\frac{p}{f_{ck}} = \frac{100 \times A_s}{bD \times f_{ck}} = \frac{100 \times 12 \times \frac{\pi}{4} \times 20^2}{300 \times 600 \times 20} = 0.1047$$

From Chart 63 of SP16 the value of P_{uz} may be calculated as follows :

$$\frac{A_s}{A_g} = \frac{12 \times \frac{\pi}{4} \times 20^2}{300 \times 600} \times 100 = 2.09\%$$

For the above % of reinforcement of grade Fe 415 and concrete of grade M 20

$$\frac{P_{uz}}{A_g} = 15.5$$

or $P_{uz} = 15.5 \times 300 \times 600 = 232$ kN

Value of M_{ux1} and M_{uy1} have been evaluated from Chart 44 of SP16 as follows:

$$\frac{p}{f_{ck}} = 0.1047 \text{ as calculated earlier}$$

$$\frac{P_u}{f_{ck}bD} = \frac{1500 \times 10^3}{20 \times 300 \times 600} = 0.4166$$

$$\frac{M_{ux1}}{f_{ck}Db^2} = 0.125$$

or $M_{ux1} = 0.125 \times 20 \times 600 \times 300^2 = 135$ kNm

Similarly

$$\frac{M_{uy1}}{f_{ck}bD^2} = 0.125$$

or $M_{uy1} = 0.125 \times 20 \times 300 \times 600^2 = 270$ kNm

$$\frac{P_u}{P_{uz}} = \frac{1500}{2325} = 0.645$$

$$\frac{M_{ux}}{M_{ux1}} = \frac{100}{135} = 0.741$$

$$\frac{M_{uy}}{M_{uy1}} = \frac{200}{270} = 0.741$$

From Chart 64 or SP 16 for $\frac{M_{ux}}{M_{ux1}} = 0.71$

and $\frac{P_u}{P_{uz}} = 0.645$ the value of $\frac{M_{uy}}{M_{uy1}} = 0.625 < 0.741$ (provided)

Hence the section is *unsafe*. Ans