UNIT 13 DESIGN OF CIRCULAR SLABS AND SLABS WITH CONCENTRATED LOADS

Structure

13.1 INTRODUCTION

Circular slabs in a structure are provided as per architectural or structural need. They may be floor and / or roof of buildings, water tanks, silos & bunkers etc. Simply supported or fixed edge circular slabs with uniformly distributed loads will only be discussed here.

Rectangular slabs having simple or other types of supports with uniformly distributed loads have been discussed in Unit 12. In this Unit a rectangular slab supported only on two opposite edges or a cantilever slab carrying concentrated loads shall be discussed. Such slabs are provided to support load of storage tanks, machines, shelves, almirahs, vehicles etc. on floors/roofs of buildings or moving loads on bridges and culverts.

Objectives

After going through this Unit, a student will be able to design the followings :

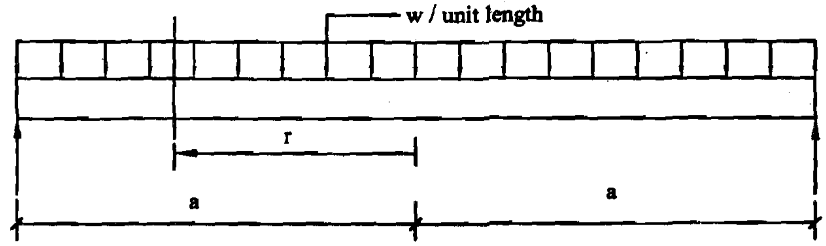
- Simply supported and fixed edge circular slabs carrying uniformly distributed loads, and
- Rectangular solid slabs supported on two opposite edges or cantilever slabs carrying concentrated loads.

13.2 GENERAL DESIGN PRINCIPLES OF CIRCULAR SLABS

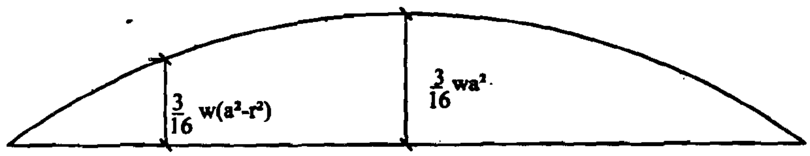
Bending of a circular slab like a saucer (i.e. bending in all directions) is quite different form bending in one or two distinct orthogonal directions for a rectangular slab under uniformly distributed loads. In other words, the stresses caused in circular slabs are radial and circumferential necessiating reinforcements in these directions. Provision of such reinforcements becomes difficult due to congestion in the central portion of the slab where maxmium bending moments in both directions occur. Hence, in practice, reinforcements are provided in two orthogonal directions as those for two way rectangular slabs. Analysis of such slabs are done by Theory of Elasticity assuming Poisson's Ratio as zero. As such analysis are complex and lengthy, formulae for determining bending moments in radial as well as circumferencial directions and for evaluation of shear force are given here for design purposes.

Simply Supported Circular Slabs

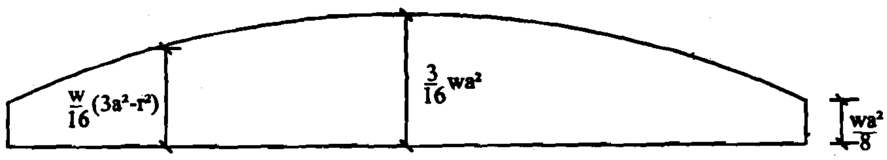
A simply supported circular slab carrying a udl of w per unit area is shown in Figure 13.1(a). The values of moments and shear force per unit width at any radius r from centre are given as follows (Figure 13.1)



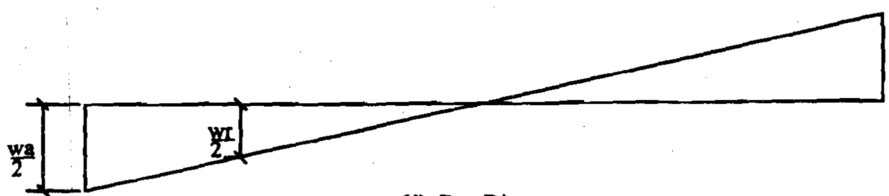
(a) Loaded Circular Slab



(b) M_r - Diagram



(c) M_θ - Diagram



(d) F_r - Diagram

Figure 13.1 : Simply Supported Circular Slab Carrying Udl

Radial B.M. at distance *r* from centre per unit witdh of slab,

$$M_r = \frac{3}{16} w(a^2 - r^2) \qquad \ldots (13.1)$$

Circumferential B.M. at distance *r* from centre per unit witdh of slab,

$$M_q = \frac{w}{16}(3a^2 - r^2) \qquad \ldots (13.2)$$

Radial S.F. at distance *r* from centre per unit witdh of slab, $F_r = \frac{wr}{2}$ ... (13.3)

Example 13.1

Design a circular roof slab of inside dia. 5.625m, supported on brickwall of 375 for the the following data :

Roof slab thickness	= 180
Lime concrete thickness	= 150
Live Load on roof	= 0.75 kN/m²

Use M 15 concrete and Fe 415 steel.

Solution

(i) *Design constants*

For Fe 415 bars, σ_{st} = 230 N/mm², and for M 15 concrete σ_{cbc} = 5 N/mm²

$k_B = 0.292, j_B = 0.904, R_B = 0.658$

(ii) *Load*

Self wt. = 0.18 × 1 × 25	= 4.5 kN/m²
LC = 0.15 × 1 × 19	= 2.85kN/m²
LL	= 0.750 kN/m²
Total load	= 8.10 kN/m²

(iii) *Design of Section*

l_{ef} »5.625 + 0.375 = 6m

Radial and circumferential B.M. at centre per meter width of slab:

$$(M_r)_c = (M_\theta)_c = \frac{3}{16} wa^2 = \frac{3}{16} \times 8.1 \times 3^2 = 13.67 \text{ kNm}$$ vide Eqs. (13.1) and (13.2)

Circumferential B.M. at support per meter width of slab

$$(M_\theta)_e = \frac{2}{16} wa^2 = \frac{2}{16} \times 8.1 \times 3^2 = 9.113 \text{ kNm}$$

Radial B.M. at support per meter width of slab, $(M_r)_e = 0$

Design Bending Moment = 13.67 kNm

$$\therefore d = \sqrt{\frac{13.67 \times 10^6}{0.658 \times 1000}} = 144$$

Keeping total depth D = 180

and using #10 bars and 15 clear cover

d = 180– 15– 5 = 160 mm for bottom layer, and

d = 160– 10 = 150 for upper layer

(iv) A_{st}

Radial and circumferencial reinforcement required at the centre

$$A_{st} = \frac{13.67 \times 10^6}{230 \times 0.904 \times 150} = 438 \text{ mm}^2$$

Using #10 bars (A_s = 78.5mm²)

$$\text{Spacing } s = \frac{1000 \times 78.5}{438} = 179 \text{ mm}^2$$

Hence provided a mesh of #10 @175

To avoid slipping at the edges, provide extra circumferential reinforcement in the form of rings.

Available depth = 150 − 10 = 140 mm

$$(A_{st})_\theta = \frac{9.113\times10^6}{230\times0.904\times140} = 313\,\text{mm}^2$$

$$\text{Spacing} = s = \frac{1000\times78.5}{313} = 250.75$$

$(M_\theta)_e$ is $\frac{2}{3}$rd of maximum moment at the centre. The circumferential steel is to be provided for a length of $\frac{2}{3}(68\phi) = \frac{2}{3}\times68\times10 = 453.33$ mm

(i.e. a length required to develop a tensile stress of $\frac{2}{3}\times230$ = 153.3 N/mm² by bond). Hence provided rings of #10 @250 c/c. Total no. of rings = $\frac{453}{250}+1\approx3$

i.e., spacing of rings = $\frac{453}{2} = 226.5 \approx 225$ c/c

Provided 3 rings of # 10 @ 225 c/c at the edge

(v) *Check for shear*

Maximum S.F. at the edge is given by

$$V_r = \frac{1}{2}wa = \frac{1}{2}\times8.100\times\frac{5.625}{2} = 11.4\ \text{kN/m width}$$

$$\tau_v = \frac{V_r}{bd} = \frac{11.4\times10^3}{1000\times150} = 0.076\ \text{N/mm}^2\text{(very low)}$$

Check for Development Length

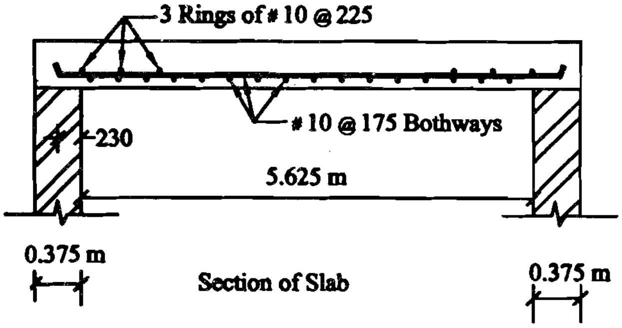
$$\frac{1.3M_1}{V}+l_0 \qquad \text{Assuming } l_0 = 0$$

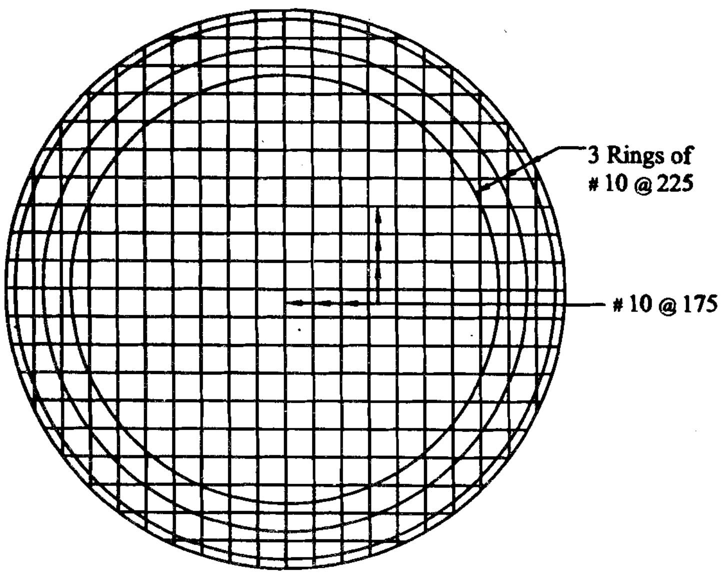
$$= \frac{1.3\times9.113\times10^6}{11.4\times10^3}+0 = 1039.2$$

and, $L_d = 68\times10 = 680 > 1039.2$ O.K.

Also $\frac{L_d}{3} = \frac{680}{3} = 226.67 \approx 230 < (375-25) = 350$

The detailing of the reinforcement has been shown in Figure 13.2.





Plan

Figure 13.2 : Details of Reinforcement of the Designed Slab

Fixed Edge Circular Slabs

A fixed edge circular slab carrying udl of w per unit width is shown in Figure 13.3(a). The values of moments and shear force per unit width of any radius r from centre are given as follows (Figure 13.3) :

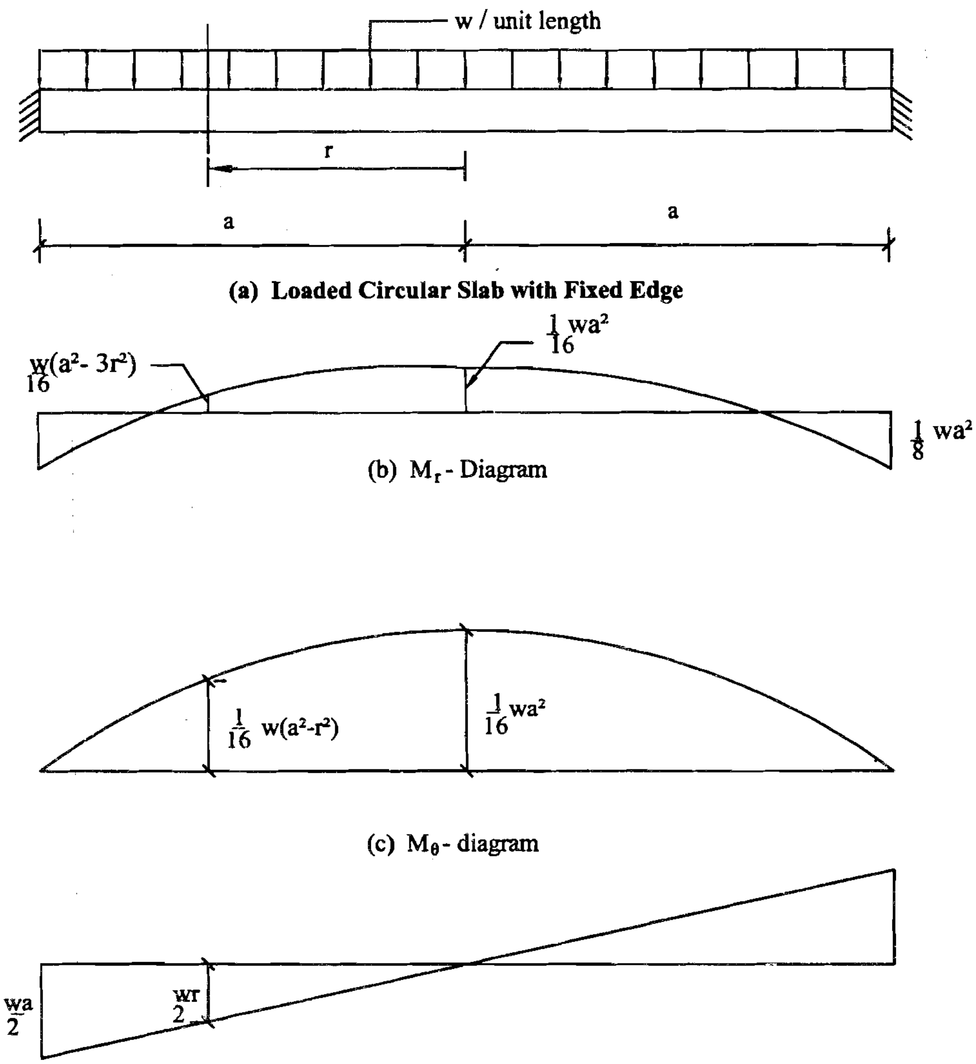
Radial B.M. at distance r from centre per unit witdh of slab,

$$M_r = \frac{w}{16}(a^2 - 3r^2) \qquad \ldots (13.4)$$

Circumferential B.M. at distance r from centre per unit witdh of slab,

$$M_\theta = \frac{w}{16}(a^2 - r^2) \qquad \ldots (13.5)$$

Radial S.F. at distance r from centre per unit witdh of slab, $F_r = \frac{wr}{2}$ . . . (13.6)



(a) Loaded Circular Slab with Fixed Edge

(b) M_r - Diagram

(c) M_θ - diagram

(d) F_r - Diagram

Figure 13.3 : Fixed Edge Circular Slab Carrying Udl

Example 13.2

Design a fixed edge roof of 6m inside diameter for the following specifications :

thickness of slab = 150; wt. of roof covering and *LL* = 3.9 kN/m^2; concrete of grade M 15 and steel of grade Fe 415

Solution

(i) *Design Constants*

$k_B = 0.292, j_B = 0.904, R_B = 0.658$

(ii) *Load*

DL of slab = 0.15 × 25.00	= 3.75 kN/m
wt. of roof covering and *LL*	= 3.9 kN/m
Total w	= 7.65 kN/m

(iii) *Design of Secion*

$$(M_\theta)_c = (M_r)_c = \frac{1}{16}\times wa^2 \qquad = \frac{1}{16}\times 7.65\times 3^2 \quad = 4.303 \text{ kNm}$$

$$(M_r)_e = \frac{-2}{16}\times wa^2 \qquad = \frac{-2}{16}\times 7.65\times 3^2 \quad = -8.606 \text{ kNm}$$

$(M_\theta)_c = 0$

The radial moment changes sign, the point of inflection being at a radius $\frac{a}{\sqrt{3}} = \frac{3}{\sqrt{3}} = 1.732$m or at a distance of 3 − 1.732 = 1.268 m from the edge of the support.

Maximum B.M. = 8.606 kNm

$$\therefore d = \sqrt{\frac{8.606\times 10^6}{0.658\times 1000}} = 114.4$$

Taking total thickness = 150 mm and using #10 bars and a clear cover of 15 mm, d = 130 mm for the bottom layer and 130 − 10 = 120 mm for the top layer.

(iv) A_{st}

At centre, in mutually perpendicular dirction,

$$A_{st} = \frac{4.303\times 10^6}{230\times 0.904\times 120} = 172.5 \text{ mm}^2$$

$$A_{stmin} = \frac{0.12}{100}\times 1000\times 150 = 1180 \text{ mm}^2$$

Using # 10 bars (A_s = 78.5 mm²)

$$\text{Spacing} = \frac{1000\times 78.5}{1180} = 436$$

maximum spacing = $3d$ = 3 × 120 = 360

or 450

Provided mesh reinforcement consisting of #10 @360 c/c in two mutually perpendicular directions at as bottom reinforcement.

Circumferential reinforcement at the edge -

At the edge there is no requirement of circumferential reinforcement since there is no circumferential moment.

Radial negative reinforcement at the edge -

Radial distance from the face of the support upto which reinforcement will be provided will be greater of the followings :

(i) Distance of pt. of inflexion from edge + d = 1268 + d
= 1268 + 130 = 1398

(ii) Distance of pt. of inflextion from edge + 12ϕ

$= 121268 + 12\phi = 1268 + 12 \times 10 = 1388$

(iii) Distance of pt. of inflextion from edge + $\frac{l_{ef}}{16}$

$= 1268 + \frac{1}{16}(6000) = 1268 + 375 = 1643$

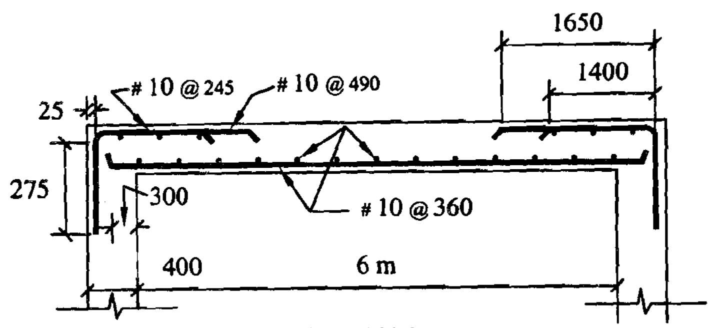
(iv) $L_d^{'} = 68\phi = 68 \times 10 = 680$

Available $d = 130$

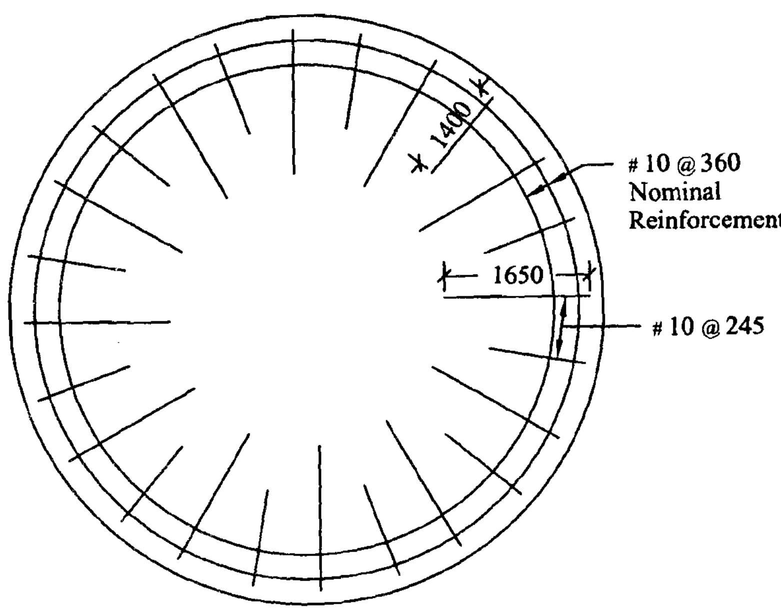
$$A_{st} = \frac{8.606 \times 10^6}{0.904 \times 130 \times 230} = 318.4 \text{ mm}^2$$

$$\text{spacing of \#10 bars} = \frac{1000 \times 78.5}{318.4} = 246.6$$

Provide *d* all bars #10@245 upto 1400 and half the reinforcement upto 1650 from the edge



Section of Slab



Plan of Top Reinforcement

Figure 13.4 : Details of Top Reinforcement of the Designed Slab

(v) *Check for shear*

$$F_r = \frac{1}{2} wa = \frac{1}{2} \times 7.65 \times 3 = 11.48 \text{ kN}$$

$$\therefore \tau_v = \frac{11.48 \times 10^3}{1000 \times 130} = 0.09 \text{ N/mm}^{2n} \text{ (too low)}$$

Distance upto which the negative reinforcement to be extented beyond the curved portion (x) is given by the equation,

$(400 - 25 - 5f) + 8\phi + x = 680$

or $x = 275$

The detailing of reinforcement is shown in Figure 13.4

SAQ 1

(i) How bending of circular slab is different from that of a rectangular two way slab.

(ii) Why reinforcement in a circular slab is provided in *orthogonal* directions rather than in radial and circumferential direction.

(iii) Design and draw the detailing of reinforcement for a simply supported circular slab of effective span 5m for a total laod (including self weight) of 10 kN/m^2. Use M 15 concrete and Fe 250 steel.

(iv) Design and draw the detailing of reinforcement for a fixed edge circular slab of effective span 5m for a total load (including self weight) of 12 kN/m^2. Use M 20 concrete and Fe 415 steel.

13.3 GENERAL DESIGN PRINCIPLES OF SLABS CARRYING CONCENTRATED LOADS

13.3.1 Determination of Bending Moment

A simply supported or a continuous solid slab supported on two opposite edges only as shown in Figure 5 to 8 and loaded with concentrated loads are analysed by Elastic Theory or Johansen's Yield Line Theory or by Strip Method. For design purposes code has proposed simple rules and according to which the load is considered to be spread over an effective width measured parallel to the supported edges as mentioned in the following subsections.

Slab Carrying Single Concentrated Load

If a single concentrated load is, W, placed at x from the nearer supporting edge (Figure 13. 5), the *effective width,* b_{ef}, measured parallel to the supporting edge shall be given by the formula

$$b_{ef} = \left\{ kx\left(1 - \frac{x}{l_{ef}}\right) + a \right\} \leq l' \quad \ldots (13.7)$$

where k = constant having values given in Table 13.1 depending upon ratio of l' to l_{ef}. and

a = width of contact area of the concentrated load measured parallel to the supported edge.

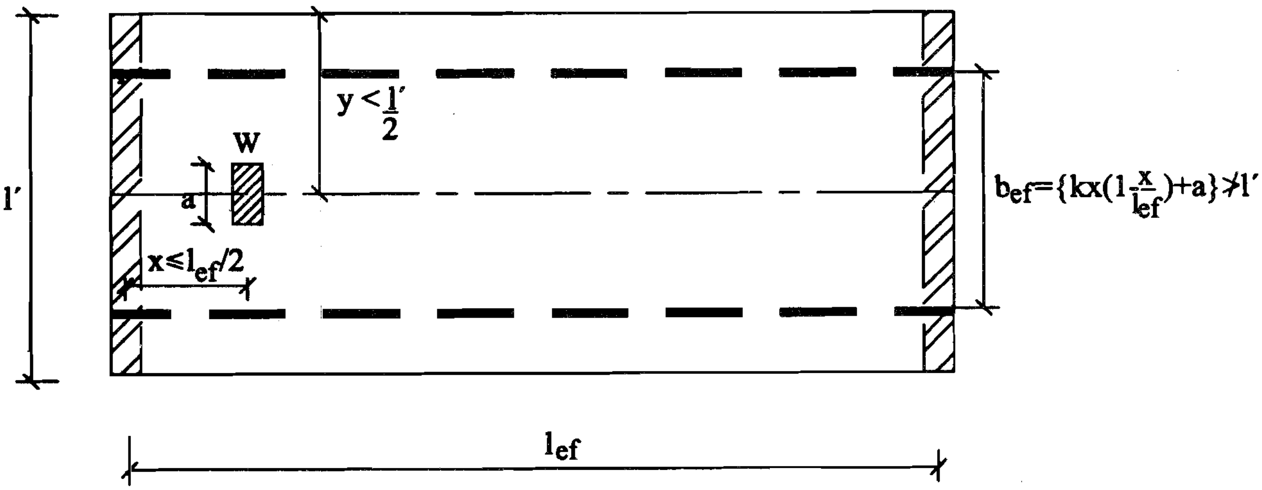


Figure 13.5 : Showing Effective Width for a Concentrated Load *W* on a Slab Supported on Two Opposite Edges

Table 13.1 : Values of *k* for Simply Supported and Continuous Slab

l'/lef	k for simply supported slabs	k for continuous slabs
0.1	0.4	0.4
0.2	0.8	0.8
0.3	1.16	1.16
0.4	1.48	1.44
0.5	1.72	1.68
0.6	1.96	1.84
0.7	2.12	1.96
0.8	2.24	2.08
0.9	2.36	2.16
1.0 and above	2.48	2.24

In case, the load is near an unsupported edge, b_{ef} shall not exceed the value given the value given in Eq. (13.7) nor $\frac{b_{ef}}{2}$ as calculated plus the distance of the load from the nearer unsupported edge. (i.e. $\frac{b_{ef}}{2} + y$)

Example 13.3

Calculate maximum bending moment per meter width in a simply supported slab (supported on two opposite faces only) for the following data :

l' = 2m; l_{ef} = 5m; x = 2m; y = 0.5m; a = 0.3 m; W = 20 kN

Solution

$$b_{ef} = kx\left(1-\frac{x}{l_{ef}}\right)$$

For $\frac{l'}{l_{ef}} = \frac{2}{5} = 0.4$; k = 1.48 vide Table 13.1

$$\therefore b_{ef} = 1.48\times 2\left(1-\frac{2}{5}\right) = 1.776\text{m}$$

As the load is nearer to the unsupported edge

$$b_{ef} = \frac{b_{ef}\text{ calculated above}}{2} + y = \frac{1.776}{2} + 0.5 = 1.388\text{m}$$

The load per meter width $w' = \frac{20}{1.388} = 14.41$ kN/m width

$$\therefore M_{max} = \frac{w'x(l_{ef}-x)}{l_{ef}} = \frac{14.41\times 2(5-2)}{5} = 17.292 \text{ kNm/m width}$$ **Ans**

Slab Carrying Two or More Concentrated Loads in a Line

If two or more loads are placed *in one line* (Figure 13.6), the effective widths and thereafter loads per m width for each laod are calculated separately and the bending moment at any section for all loads is determined by adding them together (Figure 13.6).

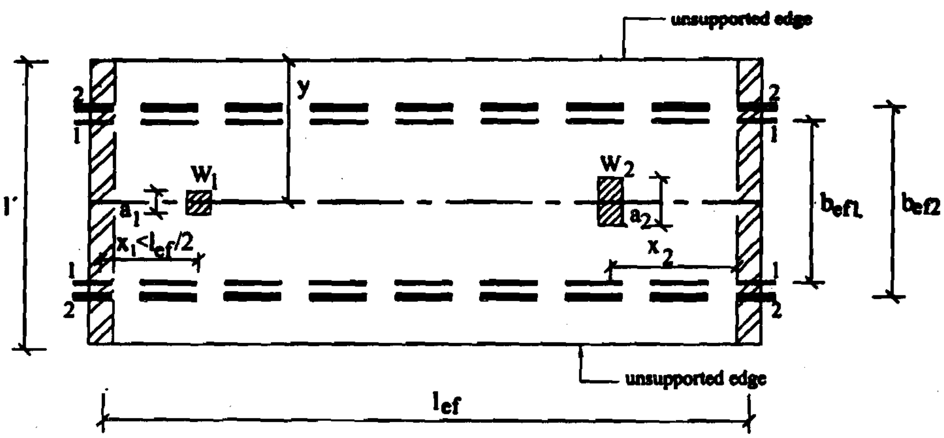


Figure 13.6 : Showing Effective Widths for Concentrated Loads *in One Line* on a Slab Supported on Two Opposite Edges

Example 13.4

Find the bending moment per meter width at the centre of the span for the following data :

l_{ef} = 7.5m; l' = 3m ;	y = 1.5m
x_1 = 1.5 m	x_2 = 2m
a_1 = 0.3m	a_2 = 0.4m
W_1 = 20 kN	W_2 = 30 kN

Solution

For $\frac{l'}{l} = \frac{3}{7.5} = 0.4$, k = 1.48

$$b_{ef1} = kx_1\left(1 - \frac{x_1}{l_{ef}}\right) + a_1 = 1.48 \times 1.5\left(1 - \frac{1.5}{7.5}\right) + 0.3 = 2.076 < 3\text{m}(= l')$$

$\therefore$ Load per m width $w_1' = \frac{20}{2.076} = 9.634$ kN/m width

$\therefore$ B.M. at centre of span due to w_1'

$$M_1' = R_{R1} \times \frac{l_{ef}}{2} = \frac{9.634 \times 1.5}{7.5} \times \frac{7.5}{2} = 7.226 \text{ kNm/m width}$$

Similarly

$$b_{ef2} = kx_2\left(1 - \frac{x_2}{l_{ef}}\right) + a_2$$

$$= 1.48 \times 2\left(1 - \frac{2}{7.5}\right) + 0.4 = 2.571\text{m} < 3\text{m}(= l')$$

$\therefore$ Load per m width $w_2' = \frac{30}{2.571} = 11.669$ kN

$\therefore$ B.M. at centre of span due to w_2'

$$M_2' = R_{R2} \times \frac{l_{ef}}{2} = \frac{11.669 \times 2}{7.5} \times \frac{7.5}{2} = 11.669 \text{ kN/m}$$

Hence total B.M. at the centre due to w_1' & w_2'

$M_c = M_1' + M_2' = 7.226 + 11.669 =$ **18.895 kN/m width Ans**

Slab Carrying Two or More Concentrated Loads *not* in a Line in the Direction of the Span

If two or more loads on a slab are *not* in one line (Figure 13. 7) and the respective effective widths are *not* overlapping, then slab is designed in flexure for each effective width and provided for. But if the effective width of slab for one load overlaps the effective width of slab for an adjacent laod, the overlapping portion of the slab shall be designed for the combined effect of the two loads.

Example 13.5

If in Example 13.4 y_1 i.e. distance of W_1 from unsupported nearer edge is 1.2 m. Calculate the resultant B.M. at the mid span for overlapping portion.

Solution

Resultant B.M per meter width at the mid span for overlapping portion will be same as that for Example 13. 4 i.e. $M_c = 18.895$ **Ans**

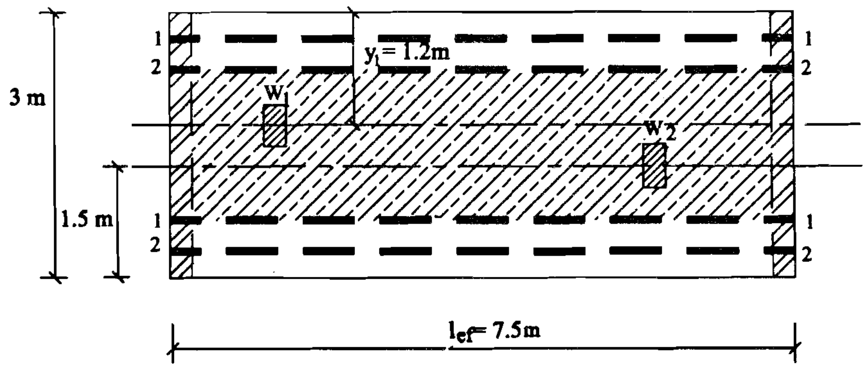


Figure 13.7 : Showing Overlapping of Effective Widths

Cantilever slab carrying concentrated load

If a single concentrated load is placed at a_1 form the fixed edge (Figure 13. 8) effective width,

$$b_{ef} = (1.2\,a_1 + a) \le \frac{l'}{3} \qquad \ldots (13.8)$$

where a = width of contact area of concentrated load measured parallel to the supporting edge.

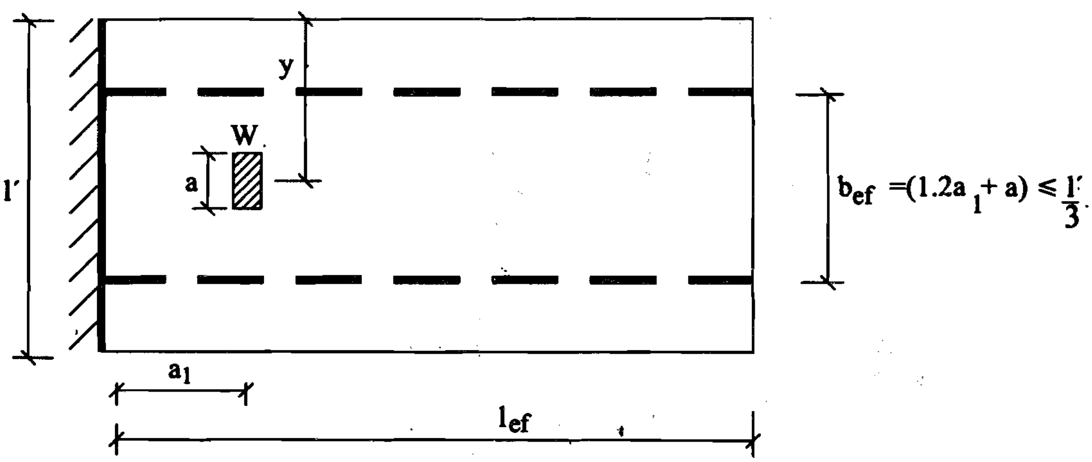


Figure 13.8 : Showing Effective Width for Concentrated Load on a Cantilever Slab

In case, the load is near an unsupported edge, b_{ef} shall not exceed the value given in Eq. (13.8) nor shall it exceed half the above value plus the distance of the concentrated load from the extreme end (y) measured in the direction parallel to the fixed edge.

Example 13.6

Calculate bending moment per m width of a cantilever slab for the following data :

Conc. Load	W	=	20 kN
	a_1	=	1.5m
	a	=	0.3m and
	l'	=	2m
	y	=	0.6m

Solution

$$b_{ef} = 1.2\, a_1 + a$$

$$= 1.2 \times 1.5 + 0.3 = 2.1\text{m} > \frac{l'}{3}\left(= \frac{2}{3} = 0.67\text{m}\right)$$

The effective width shall be the least of the followings:

(i) $b_{ef} = 2.1\text{m}$

(ii) $\frac{l'}{3} = \frac{2}{3} = 0.67\text{m}$, and

(iii) $\frac{b_{ef}}{2} + 0.6 = \frac{2.1}{2} + 0.6 = 1.65\text{m}$

Hence b_{ef} = 0.67m

$\therefore$ load/m width, $w' = \frac{20}{0.67} = 30$ kN/m width

Hence $M_{max} = 30 \times 1.5 =$ **45 kNm/m width Ans**

13.3.2 Determination of critical section for cheking shear

The critical section for checking shear strength and for design is taken at effective depth of slab from the face of the concentrated load as shown in Figure 13 .9

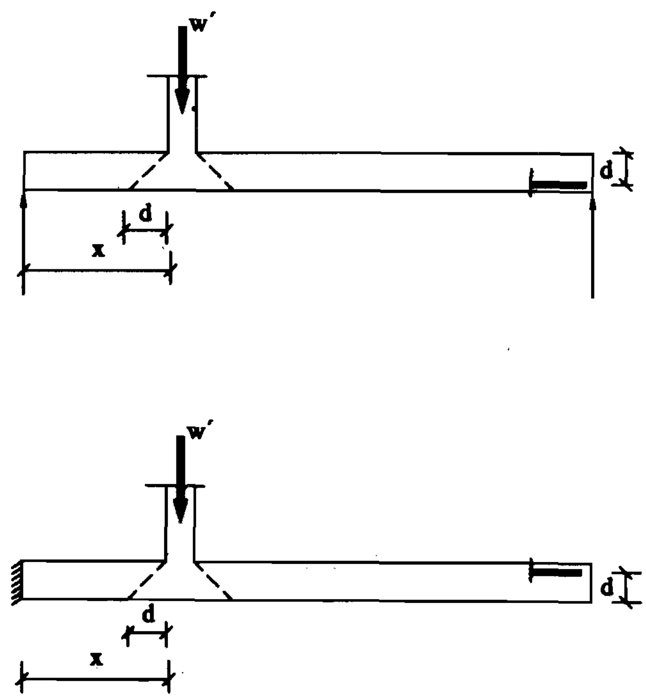


Figure 13.9 : Critical sections for shear

SAQ 2

(i) How bending moments, for concentrated load in a simply supported **slab** (on two opposite faces only) and in a cantilever slab, are evaulated. Illustrate with drawings.

(ii) Illustrate with sketches as to how shear force at any section is evaluated.

(iii) Find the bending moment per meter with at a section 3m from left support of a simply supported slab (supported on two opposite edges only) for the following data:

l_{ef} = 8m; l' = 4m

x_1 = 2m	x_2 = 2.5m
a_1 = 0.4m	a_2 = 0.35m
W_1 = 15 kN	W_2 = 30 kN
y_1 = 0.8m	y_2 = 1.5m

(iv) Evaluate bending moment per meter width of a cantilever slab for the following data: W = 25 kN, a_1 = 2m ; a = 0.4 and l' = 3m and y = 1.5m

13.4 SUMMARY

A circular slab bends in all directions hence radial and circumferential bending moments are considered for design. But the provision of reinforcement is *orthogonal* (similar to that of a two way slab) as there would have been congestion at the centre of the slab if they were provided in radial and circumferential directions. Provision of shear force is governed by the same rules as those for two way slabs.

A solid slab, supported on two opposite faces only, or a cantilever slab loaded with concentrated loads is designed by *Equivalent Effetive Width Method.* In other words, the concentrated load is assumed to have been spread only on 'effective width' (b_{ef}), determined by formulae given by Eqn. 13.7 & 8 . The critical section for checking shear force is supposed to be located at effective depth of the slab (d) from the face of the concentrated load.

13.6 ANSWERS TO SAQs

SAQ 1

(i) Refer text 13.1

(ii) Refer text 13.1

(iii) Refer Example 13.1

(iv) Refer Example 13.2

SAQ 2

(i) Refer Section 13.3.1

(ii) Refer Section 13.3.2

(iii) 23.055 kNm/m width in the overlapping portion.

(iv) 50 kNm/m width