UNIT 10 SHEAR AND TORSION

Structure

10.1 INTRODUCTION

Analytical mechanics and design principles for shear & torsion for a member are *exactly* the same as those mentioned in Limit State Method (Unit 3 & 4). The basic assumptions and methods of evaluations of design loads and permissible stresses for Working Stress Methods (Unit 8) are also applicable here. The permissible stresses in concrete and steel have been given in Section 8.3.

Objective

The objective of this Unit is to acquaint the students of solving different types of problems of shear and torsion by Working Stress Method

10.2 TYPES OF PROBLEMS

Four types of problems in shear and one type in torsion have been exemplified to tackle any problem on these topics and their combinations thereof.

They are :

(i) *Types of problems in Shear*

Type I Design of shear reinforcement as vertical stirrups for a given R.C. section for applied shear force

Type II Design of shear reinforcement as bent up bars along with stirrups for a given R.C. section and applied shear force.

Type III Design of shear reinforcement in the form of inclined stirrups or bars for a given R.C. section for applied shear force, and

Type IV Design of shear reinforcement for beams of varying depth.

(ii) *Types of problems in Torsion*

Type I Design of reinforcements for a R.C. section for applied bending moments, shear force and torsion.

Example 10.1

Design shear reinforcement providing verical stirrups for a beam having a cross-section of $b \times D = 250 \times 500$ reinforced with 4 # 20 and having an effective cover of 40 mm. The shear force at the cross-section is 85 kN. Use M 15 concrete and Fe 415 steel.

Solution

$$\tau_v = \frac{V}{bd} = \frac{85 \times 10^3}{250 \times 460} = 0.739 \frac{N}{mm^2}$$

$$\frac{100A_s}{bd} = \frac{100 \times 4 \times \frac{\pi}{4}(20)^2}{250 \times 460} = 1.09\%$$

From Table 8.3 $\tau_c = 0.37 + \frac{(0.40 - 0.37)}{(1.25 - 1)} \times (1.09 - 1)$

$= 0.38$ N/mm^2

From Table 8.4 $\tau_{c,\max} = 1.6$ N/mm^2

Since $\tau_c < \tau_v < \tau_{c,max}$, shear reinforcements will be provided for

$V_s = V - \tau_c bd = 85 - 0.38 \times 250 \times 460 \times 10^{-3}$

$= 41.21$ kN

For # 8 two legged vertical stirrups

$A_{sv} = 2 \times \rho/4 \times 8^2 = 100$ mm^2

From code $V_s = \frac{\sigma_{sv} A_{sv} d}{s_v}$

or, $s_v = \frac{\sigma_{sv} A_{sv} d}{V_s} = \frac{230 \times 100 \times 460}{41.21 \times 10^3} = 256.7$ c/c

Maximum permissible spacing

$$s_v \le \frac{A_{sv} \times 0.87 f_y}{0.4b}$$

or, $s_v \le \frac{100 \times 0.87 \times 415}{0.4 \times 250} = 362.96$

Further, maximum spacing is the least of

(i) $0.75\,d = 0.75 \times 460 = 345$, and

(ii) 450

Hence, provided # 8 two-legged vertical stirrups at a spacing of 250 c/c.

Exapmle 10.2

Design a R.C. beam at support for shear reinforcement for the following data : $b = 400$; $d = 760$, $D = 800$; S.F. = 100 kN; M 15 concrete & Fe 250 steel. The tensile reinforcement consists of 5 ϕ 20 of which two bars together are bend up at 45^0 near the support to take shear.

Solution

$$\tau_v = \frac{V}{bd} = \frac{100 \times 10^3}{400 \times 760} = 0.329 \frac{N}{mm^2}$$

$$\frac{100A_s}{bd} = \frac{100 \times 3 \times \frac{\pi}{4} \times 20^2}{400 \times 760} = 0.31\%$$

From Table 8.3 τ_c $= 0.22 + \frac{(0.29 - 0.22)}{0.25} \times (0.31 - 0.29)$

$= 0.24 \text{ N/mm}^2$

From Table 8.4 $\tau_{c,\max}$ $= 1.6 \text{ N/mm}^2$

Since $\tau_c < \tau_v < \tau_{c,\max}$, shear reinforcement will be provided in the form of bent up bars and vertical stirrups for shear force,

V_s $= V - \tau_c bd$

$= 100 \times 10^3 - 0.24 \times 400 \times 760 \times 10^{-3}$

$= 27.04$ kN

From code shear resistance of two bent up bars

$= \sigma_{sv} A_{sv} \sin \alpha$

$= 140 \times (2 \times \pi/4 \times 20^2) \times \sin 45^\circ \times 10^{-3}$

$= 62.2 \text{ kN} > \frac{27.04}{2}$ kN

Therefore, the effective shear resistance of the bent up bars will be taken as

$$V_{s1} = 27.04 \times \frac{50}{100} = 13.52 \text{ kN}$$

Taking additional shear reinforcement consisting of ϕ 8 two-legged vertical stirrups

$$A_{sv} = 2 \times \frac{\pi}{4} \times 8^2 = 100 \text{ mm}^2$$

$$s_v = \frac{\sigma_{sv} A_{sv} d}{V_{s1}} = \frac{140 \times 100 \times 760}{13.52 \times 10^3} = 786.9 \text{ c/c}$$

Maximum permissible s_v is given by the following conditions :

(i) $s_v = \frac{0.87 \times f_y A_{sv}}{0.4b} = \frac{0.87 \times 250 \times 100}{0.4 \times 400} = 135.93$ c/c

(ii) $0.75\, d = 0.75 \times 760 = 570$ c/c, and

(iv) 450

Hence provided ϕ 8 two-legged vertical stirrups @135 c/c in addition to two bent up bars Ans

Example 10.3

Design shear reinforcement for the beam shown in Fig 10.1 for the following data : b = 300; d = 550; S.F. = 170 kN. M 15 concrete and Fe 250 steel may be used

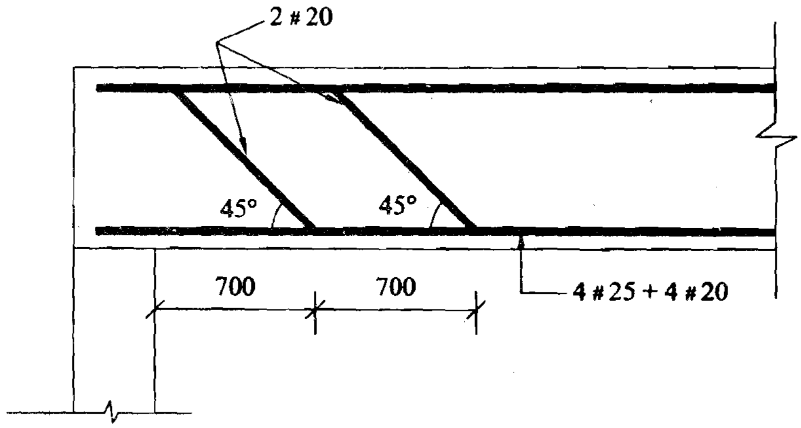

Figure 10.1 : Details of the Beam

Solution

$$\tau_v = \frac{V}{bd} = \frac{170 \times 10^3}{300 \times 550} = 1.03 \frac{N}{mm^2}$$

$$\frac{100 \times A_s}{bd} = \frac{100 \times 4 \times \frac{\pi}{4} \times 25^2}{300 \times 550} \times 100 = 1.19\%$$

$$\tau_c = 0.37 + \frac{(0.4 - 0.37)}{0.25} \times (1.19 - 1)$$

$$= 0.393 \text{ N/mm}^2$$

From table 8.4 $\tau_{c,max}$ = 1.6 N/mm² For M 15,

Since $\tau_c < \tau_v < \tau_{c,max}$

$$V_s = V - \tau_c bd$$

$$= 170 - 0.393 \times 300 \times 550 \times 10^3$$

$$= 105.16 \text{ kN}$$

Shear resistance of bent-up bars at different cross section

$$= \frac{\sigma_{sv} A_{sv} d}{s_v} \times (\sin\alpha + \cos\alpha) \times 10^3$$

$$= \frac{140 \times 2 \times \frac{\pi}{4} \times 20^2 \times 550}{700} \times (\sin 45^0 + \cos 45^0) \times 10^{-3}$$

$$= 97.74 \text{ kN} > \frac{50}{100} \times 105.16 \text{ kN}$$

$$\therefore \; V_{s1} = 0.5 \times 105.16 = 52.58 \text{ kN}$$

Adopting ϕ 8 two-legged vertical stirrups

$$s_v = \frac{\sigma_{sv} A_{sv} d}{V_{s1}} \quad \text{(where } A_{sv} = 2 \times 50 = 100 \text{ mm}^2\text{)}$$

or, $$s_v = \frac{140 \times 100 \times 550}{52.58 \times 10^3} = 146.44 \text{ c/c}$$

Maximum spacing of vertical stirrups is governed by

(i) $$s_v = \frac{f_y A_{sv}}{0.4b} = \frac{250 \times 100}{0.4 \times 300} = 208.33 \text{ c/c}$$

(ii) $0.75\,d = 0.75 \times 550 = 412.5$ c/c, and

(iii) 450 c/c

Hence provided ϕ8 two legged vertical stirrups @ 145 c/c in addition to bent up bars Ans

Example 10.4

Design tensile and shear reinforcement of support for a cantilever beam of span 3 m and of constant width 300 mm. The depth of the beam is linearly varying from 800 mm at support to 350 mm at free end. The beam is loaded with a *udl* of 30 kN/m including its self weight. Use M 20 concrete, Fe 415 steel and #20 bars with 40 mm effective cover.

Solution

For checking whether the beam with given dimension is balanced, over-reinforced, under-reinforced or doubly-reinforced; the constants and depth of the beam for balanced section are calculated as follows :

$$m = \frac{280}{3\sigma_{cbc}} = \frac{280}{3 \times 7} = 13$$

$$k_B = \frac{m\sigma_{cbc}}{m\sigma_{cbc} + \sigma_{st}} = \frac{13 \times 7}{13 \times 7 + 230} = 0.283$$

$$j_B = 1 - \frac{k_B}{3} = 1 - \frac{0.283}{3} = 0.906$$

$$R_B = \frac{1}{2}\sigma_{cbc} k_B j_B = \frac{1}{2} \times 7 \times 0.283 \times 0.906 = 0.897$$

Applied B.M. at support section

$$= \frac{wl^2}{2} = \frac{30 \times 3^2}{2} = 135 \text{ kNm}$$

$$d_B = \sqrt{\frac{M}{R_B b}} = \sqrt{\frac{135 \times 10^6}{0.897 \times 300}} = 708.3 \text{ mm} < 760$$

Hence the beam will be under-reinforcemed.

$$A_{st} \approx \frac{M}{\sigma_{st} jd} = \frac{135 \times 10^6}{230 \times 0.906 \times 760} = 852.4 \text{mm}^2$$

Hence provided 3#20 (A_{st} = 942.5 mm²)

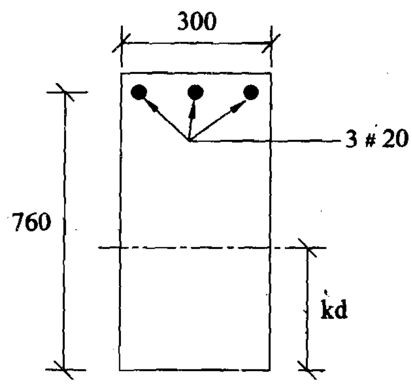

Figure 10.2 : Cross-Section of the Beam at Support

Check

Taking moment of areas about n.a.

$$\frac{b(kd)^2}{2} = mA_{st}(d - kd)$$

or $$\frac{300k^2 \times 760^2}{2} = 13 \times 852.4(760 - k \times 760)$$

or $$150\,k^2 \times 760 = 13 \times 852.4 - 13 \times 852.4\,k$$

or $$k^2 + 0.0972k - 0.0972 = 0$$

or $$k = \frac{-0.972 \pm \sqrt{+0.00945 + 0.38888}}{2}$$

or $$k = \frac{-0.972 \pm 0.631}{2} = 0.26 < 0.283$$

Hence the beam is under-reinforced.

Moment of resistance of the given section,

$$M_R = \sigma_{st} A_{st} \left(d - \frac{kd}{3}\right) = 230 \times 942.5 \left(760 - \frac{0.266 \times 760}{3}\right) \times 10^6$$

$$= 150.141 \text{ kNm} > 135 \text{ kNm}$$

$$\frac{A_{st,min}}{bd} = \frac{0.85}{f_y}$$

or $$A_{st,min} = \frac{0.85 \times 300 \times 760}{415} = 466.9 \text{ mm}^2 < 942.5 \text{ mm}^2$$

Design of Shear Reinforcement

$V = wl = 30 \times 3 = 90$ kN

Nominal shear stress, $$\tau_v = \frac{V - \frac{M}{d}\tan\beta}{bd}$$

$$= \frac{90 \times 10^3 - \frac{135 \times 10^6}{760} \times \frac{450}{3000}}{300 \times 760} = 0.278 \frac{N}{mm^2}$$

% of tensile reinforcement at support

$$= \frac{100 A_{st}}{bd} = \frac{100 \times 942.5}{300 \times 760} = 0.41\%$$

$$\therefore \tau_c = 0.22 + \frac{(0.30 - 0.22)}{0.25} \times (0.41 - 0.25)$$

$$= 0.271 \frac{N}{mm^2} < \tau_v$$

$\tau_{c,\,max} = 1.8$ . . . (Table 8.4)

Since $\tau_c < t_v < \tau_{c,max}$

$V_s = V - \tau_c bd$

$= 90 - 0.271 \times 300 \times 760 \times 10^3$

$= 28.21$ kN

Providing #8 two-legged vertical stirrups

$$A_{st} = 2 \times \frac{\pi}{4} 8^2 = 100 mm^2$$

$$s_v' = \frac{\sigma_{sv} A_{sv} d}{V_s} = \frac{230 \times 100 \times 760}{28.21 \times 10^3} = 619.6 \text{ c/c}$$

Maximum spacing is governed by the following :

(i) $s_v = \frac{0.87 A_{sv} f_y}{0.4 b} = \frac{0.87 \times 100 \times 415}{0.4 \times 300} = 302$ c/c

(ii) 0.75 d = 0.75 × 760 = 570, and

(iii) 450

Hence provided #8 two-legged vertical stirrups@ 300 Ans

Example 10.5

Design a section of a R.C. beam of cross section 300 mm wide and 840 mm deep with effective cover of 40 mm. The section is applied with a bending moments of 130 kNm, twisting moment of 30 kN and a shear of 65 kN. Use M 15 concrete and Fe 250 steel.

Solution

$$V_e = V + 1.6 \frac{T}{b} = 65 + 1.6 \frac{30}{0.3} = 225 \text{ kN}$$

Equivalent nominal shear

$$\tau_{ve} = \frac{V_e}{bd} = \frac{225 \times 10^3}{300 \times 800} = 0.94 \text{MPa}$$

As percentage of tensile reinforcement is not known at the out set, the value of τ_c cannot be calculated to compare if withτ_{vc}, it is assumed that $\tau_{vc} > \tau_c$ to calculate M_{e1}, for longitidinal reinforcement.

$$M_{e1} = M + M_t = M + M_t = 130 + \frac{T\left(1+\frac{D}{b}\right)}{1.7}$$

$$= 130 + \frac{130\left(1+\frac{840}{300}\right)}{1.7} = 130 + 67.06 = 197.06 \text{ kNm}$$

M_{RB} $= R_B\, bd^2 = 0.87 \times 300 \times 800^2$

$= 167.04 \text{ kNm} < M_{e1}$

Hence a doubly reinforced section is required

$$A_{stB} = \frac{M_B}{\sigma_{st} j_B d} = \frac{167.04 \times 10^6}{140 \times 0.865 \times 800} = 1724.2 \text{ mm}^2$$

$$A_{st2} = \frac{M_{e1} - M_B}{\sigma_{st}(d-d')} = \frac{(197.06 - 167.04)\times 10^6}{140 \times (800-40)} = 281.9 \text{ mm}^2$$

Total area of tension reinforcement,

$A_{st} = A_{stB} + A_{st2} = 1724.2 + 281.9 = 2006.1 \text{ mm}^2$

*Provided 3ϕ25 + 2ϕ20 (A_{st} = 2100 mm^2) as tension reinforcement

Taking moment of area of compression reinforcement (A_{sc}) and additional tension reinfrocemnet (A_{st2}) about n.a.

$(1.5m - 1)\, A_{sc}\,(x_B - d') = mA_{st2}\,(d - x_B)$

or $(1.5 \times 19 - 1)\, A_{sc}\,(323.2 - 40) = 19 \times 281.9\,(800 - 323.2)$

or $A_{sc} = 327.91 \text{ mm}^2$

Hence provided 3ϕ12 (A_{sc} = 339 mm^2) as compression reinforcement

It may be noted that additional longitudinal reinforcement on flexural compression force is not needed as $M_t < M$ (i.e. 67.06 kNm < 130 kNm)

Provision of Shear Reinforcement

$$\frac{100A_{st}}{bd} = \frac{100 \times 2100}{300 \times 800} = 0.875\%$$

* (2006.1 – 628) mm^2 = 1378.1 mm^2 area have been provided of ϕ 25 bars for which σ_{st} = 130 N/mm^2. Therefore, area to be provided by ϕ 25 bars = 1378.1 × $\frac{140}{130}$ = 1484.11 mm^2 - resulting in A_{st} = (1484.11 + 628) = 2112.11 mm^2. Though 12 mm^2 difference $\left(\frac{12}{2006.1} \times 100 = 0.6\%\right)$ is too small % of total area to be taken cognigance of, even then it is also compensated by increased depth for 2ϕ20 bars (Figure 10.3)

$$\therefore \tau_c = 0.34 + \frac{(0.37-0.34)}{0.25}(0.875-0.75) = 0.355 \text{ MPa}$$

= 0.355 MPa < 0.94 MPa Hence the assumption that $\tau_{vc} > \tau_c$ is correct.

It is proposed to provided ϕ10 two legged vertical stirrups, A_{sv} = 157 mm²

$d_1 = 800 - 40 = 760$ and

$b_1 = 300 - (25 + 12.5) - (25 + 12.5) = 225$ vide Figure 10.3

$$\therefore A_{sv} = \frac{T s_v}{b_1 d_1 \sigma_{sv}} + \frac{V s_v}{25 d \sigma_{sv}}$$

or,

$$s_v = \frac{\sigma_{sv} A_{sv}}{\frac{T}{b_1 d_1} + \frac{V}{2.5 d_1}} + \frac{140 \times 157}{\frac{30 \times 10^6}{225 \times 760} + \frac{65 \times 10^6}{25 \times 760}} = 104.8$$

Also, $A_{sv} > \frac{(\tau_{sv} - \tau_c) b s_v}{\sigma_{sv}}$

or

$$s_v = \frac{\sigma_{sv} A_{sv}}{(\tau_{ve} - \tau_c) b} = \frac{140 \times 157}{(0.94 - 0.355) \times 300} = 125$$

Again

$x_1 = 300 - (25 - 5) - (25 - 5) = 260$

$y_1 = 760 + (12.5 + 5) + (6 + 5) = 788$

Also the spacing of transverse reinforcement shall not exceed the following

(i) $x_1 = 260$

(ii) $\frac{x_1 + y_1}{4} = \frac{260 \times 788.5}{4} = 262.13$

(iii) 300

(iv) $0.75\, d = 0.75 \times 800 = 600$

Hence provided ϕ10 two-legged vertical stirrups @100 c/c

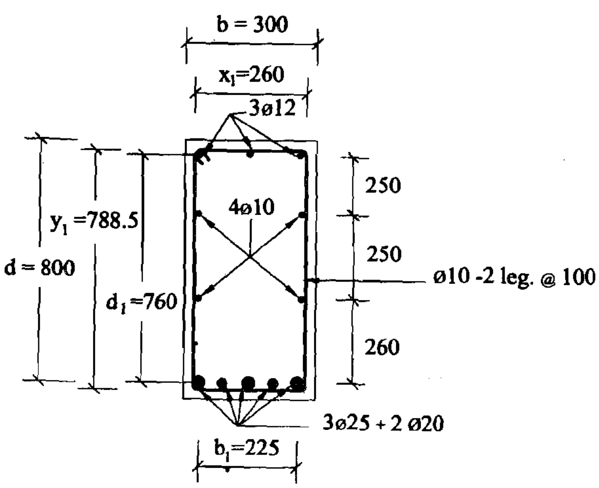

Figure 10.3 : Reinforcement Details of the Designed Beam

As $d > 450$, additional side reinforcement of 0.1% of web area equally distributed on both faces shall be provided (i.e. $\frac{0.1}{100} \times 300 \times 840 = 252$ mm^2).

Hence provided 2ϕ10 on each face (A_s = 314 mm^2) at spacing less than 300

SAQ 1

(i) Design shear reinforcement in the form of two legged vertical stirrups of # 8 for abeam for the following data :

$b = 250$ $D = 540$ $d = 500$ $A_{st} = 5$ # 16

S.F. = 62 kN $f_y = 415$ N/mm^2 $f_{ck} = 15$ N/mm^2

(ii) Design shear reinforcement for section x-x of the beam shown in Fig. 10.4 for the data given as follows :

$M_x = 100$ $V_x = 90$ kN $b_x = 300$

$d_x = 800$ $f_y = 250$ MPa $f_{ck} = 15$ MPa

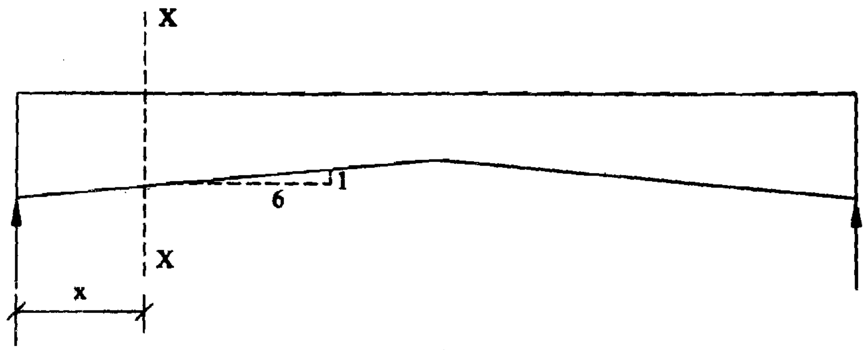

Figure 10.4 : Longitudinal View of the Beam

(iii) Design a rectangular section of a R.C. beam having a cross section of 300 mm × 790 mm. Due the loading on the beam, the section is applied with a bending moment of 60 kNm, a torque of 30 kNm and a shear of 60 kNm. Use M 20 concrete and Fe 415 steel and an effective cover of 40 mm.

10.3 SUMMARY

Basic mechanics of analysis and general principles of design for shear and torsion have been decribed in Unit 3 & 4. Here given examples have been explained with the methods of solution of various types of problems faced in practice.

10.5 ANSWERS TO SAQs

SAQ 1

(i) Provided ϕ8 two-legged vertical stirrups @ 375 c/c

(ii) For checking whether the beam with given dimensions will be balanced over-reinforced, under-reinforced or doubly-reinforced; the constants and the depth of the beam for balanced section are calculated as follows :

$$m = \frac{280}{3\sigma_{cbc}} = \frac{280}{3 \times 5} = 19$$

$$k_B = \frac{m\sigma_{cbc}}{m\sigma_{cbc} + \sigma_{st}} = \frac{19 \times 5}{19 \times 5 + 1.40} = 0.404$$

$$j_B = \left(1 - \frac{k_B}{3}\right) = \left(1 - \frac{0.404}{3}\right) = 0.865$$

$$R_B = \frac{1}{2}\sigma_{cbc} k_B j_B = \frac{1}{2} \times 5 \times 0.404 \times 0.865 = 0.87$$

$$d_B = \sqrt{\frac{M}{R_B b}} = \sqrt{\frac{100 \times 10^6}{0.865 \times 300}} = 620.77 < 800$$

Hence the section is under-reinforced.

$$A_{st} \approx \frac{M}{\sigma_{st} j_B d . \cos\beta} = \frac{100 \times 10^6}{140 \times 0.865 \times 800 \times 0.986} = 1046.8 \text{ mm}^2.$$

Provided 4ϕ20 (A_{st} = 1256 mm²)

$$\frac{A_{st,max}}{bd} = \frac{0.85}{250}$$

or $$A_{st,min} = \frac{0.85 \times 300 \times 800}{250} = 816 \text{ mm}^2 < 1256 \text{ mm}^2$$

The moment of resistance, M_R, of the section with 4ϕ20 is calculated as follows.

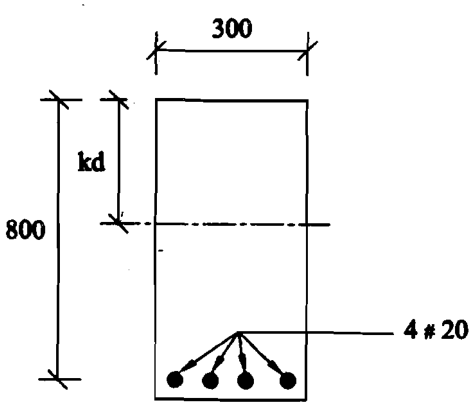

Figure 10.5 : Cross-Section of the Beam at X - X

Taking moment of areas about n.a.

$$\frac{b(kd)^2}{2} = mA_{st}(d - kd)$$

or $$\frac{300k^2 \times 800^2}{2} = 19 \times 1256\ (800 - k \times 800)$$

or $$150 \times 800\, k^2 = 19 \times 1256\ (1 - k)$$

or $$k^2 + 0.1988\, k - 0.1988 = 0$$

or $$k = \frac{0.1988 \pm \sqrt{0.03955 + 0.7952}}{2}$$

$$= \frac{0.1988 \pm 0.91365}{2} = 0.357 < 0.409$$

Hence the section is under-reinforced,

$$\therefore \quad M_R = \sigma_{st} A_{st}\left(d - \frac{kd}{3}\right) = 140 \times 1256 \times 800\left(1 - \frac{0.357}{3}\right) \times 10^{-6}$$

$$= 123.9 \text{ kNm} > 100 \text{ kNm Hence O.K.}$$

Design for Shear Reinforcement

$$\tau_v = \frac{V + \frac{M}{d}\tan\beta}{bd} = \frac{900 \times 10^3 + \frac{100 \times 10^6}{800} \times \frac{1}{6}}{300 \times 800} = 0.462 \frac{\text{N}}{\text{mm}^2}$$

% of tensile steel of the section

$$= \frac{100 A_{st}}{bd} = \frac{100 \times 1256}{300 \times 800} = 0.523\%$$

$$\tau_c = 0.29 + \frac{(0.34 - 0.29)}{(0.75 - 0.5)} \times (0.523 - 0.50) = 0.295 \text{ MPa}$$

$\tau_{c,\,max}$= 1.6 MPa for M 15 concrete ....vide Table 8.4

Since $\tau_c < \tau_v < \tau_{c,\,max}$

$$V_s = V - \tau_c bd$$

$$= 90 - 0.295 \times 300 \times 800 \times 10^{-3} = 19.2 \times 10^3 \text{ kN}$$

Providin £8 two-legged vertical stirrups

$A_{sv} = 100 \text{ mm}^2$

$$s_v = \frac{\sigma_{sv} A_{sv} d}{V_s}$$

$$= \frac{140 \times 100 \times 800}{19.2 \times 10^2} = 583.33 \text{ c/c}$$

Maximum spacing

(i) $$s_{v,max} = \frac{f_y A_{sv}}{0.4b} = \frac{250 \times 100}{0.4 \times 300} = 208.33$$

(ii) 0.75 d = 0.75 × 800 = 600

(iii) 450

Hence provided legged ϕ8 vertical stirrups @ 200 Ans

(iii) $V_e = V + 1.6\dfrac{T}{b} = 60 + 1.6 \times \dfrac{30}{0.3} = 220$ kN

$$\tau_{vc} = \frac{V_c}{bd} = \frac{200 \times 10^3}{300 \times 750} = 0.978 \text{ N/mm}^2 < \tau_{c,\,max}(1.8 \text{ MPa})$$

Assuming $\tau_{vc} > \tau_c$ as area of tensile steel is not known at the outset, the equivalent bending moment is evaluated.

$M_{e1} = M + M_t$

$$\text{where } M_t = \frac{T\left(1 + \frac{D}{b}\right)}{1.7} = \frac{30\left(1 + \frac{790}{300}\right)}{1.7} = 64.11 \text{ kNm}$$

$\therefore\ M_{e1} = 60 + 64.11 = 124.11$

To asertain whether the section will be under-reinforced, over-reinforced, doubly-reinforced or balanced section, the moment for balanced section is calculated

$$M_B = R_B\, bd^2$$

$$= 0.897 \times 300 \times 750^2 \times 10^{-3} = 151.36 \text{ kNm} > 124.11 \text{ kNm}$$

Hence the section under applied M_{e1}, will be singly reinforced and under-reinforced.

$$A_{st} \approx \frac{M_{e1}}{\sigma_{st}\, j_B d} = \frac{124.11 \times 10^6}{230 \times 0.906 \times 750} = 794.13 \text{ mm}^2$$

Proved 4#16 ($A_{st} = 804$ mm^2)

Check

Actual n.a. depth is calculated taking moment of areas about n.a.

$b.x.x/2 = mA_{st}(d-x)$

or $300x^2 = 13 \times 804\,(750 - x)$

or $x^2 + 69.68x - 52260 = 0$

or $$x = \frac{-69.68 \pm \sqrt{69.68^2 + 4 \times 52260}}{2} = 196.4$$

$$x_B = \frac{m\sigma_{cbc} d}{m\sigma_{cbc}\sigma_{st}} = \frac{13 \times 7 \times 750}{13 \times 7 \times 230} = 212.62 > 196.4$$

$$f_{cbc} = \frac{\sigma_{st} x}{m(d-x)} = \frac{230 \times 196.4}{13(750 - 196.4)} = 6.27 \text{ MPa} < 7 \text{ MPa Hence O.K.}$$

Evaluation of τ_c

$$\frac{100A_s}{bd} = \frac{100 \times 804}{300 \times 750} = 0.357\%$$

$$\therefore \tau_c = 0.22 \times \frac{(0.30 - 0.22)}{(0.5 - 0.25)} \times (0.357 - 0.25) = 0.254 \text{ MPa} < 0.978 \text{ MPa}$$

Hence the assumption that $\tau_c < \tau_{vc}$ is correct.

Further, since $M_t > M$, longitudinal reinforcement on compression face has to be provided for an equivalent bending moment of

$M_e = M_t - M = 64.11 - 60 = 4.11$ kNm

$$A_{st2} = \frac{M_{e2}}{\sigma_{st}(d - d')} = \frac{4.11 \times 10^6}{230 \times (750 - 40)} = 25.2 \text{ mm}^2$$

Provided 2#10 ($A_{s2} = 157$ mm²)

Design of Transerved Reinforcement

$$A_{sv} = \frac{Ts_v}{b_1 d_1 \sigma_{sv}} + \frac{Vs_v}{2.5 d_1 \sigma_{sv}}$$

or
$$s_v = \frac{\sigma_{sv} A_{sv}}{\left(\frac{T}{b_1 d_1} + \frac{V}{2.5 d_1}\right)}$$

Providing #8 two legged vertical stirrups,

$A_{sv} = 100$ mm²

$b_1 = 300 - (25 + 8) - (25 + 8) = 234$

$d_1 = 750 - 40 = 710$

$$\therefore s_v = \frac{230 \times 100}{\frac{30 \times 10^6}{234 \times 710} + \frac{60 \times 10^3}{2.5 \times 710}} = 107.3$$

Also
$$A_{sv} < \frac{(\tau_{ve} - \tau_c) b s_v}{\sigma_{sv}}$$

or
$$s_v = \frac{\sigma_{ve} A_{sv}}{(\tau_{ve} - \tau_c) b} = \frac{230 \times 100}{(0.978 - 0.254) \times 300} = 105.9 \text{ c/c}$$

The spacing should not be more than the least of the following :

(i) $x_1 = 234 + (8 + 4) + (8 + 4) = 258$

(ii) $\frac{x_1 + y_1}{4} = \frac{258 + 731}{4} = 247.25$

(where $y_1 = 710 + (8 + 4) + (5 + 4) = 731$)

(iii) 300

(iv) $0.75\, d = 0.75\ 5\ 750 = 562.5$ c/c

Hence provided #8 two legged vertical stirrups @ 105 c/c

Side face Reinforcement

As $d > 450$, side force reinforcement of 0.1% of web area equally distributed over both faces have to be provided.

$$A_s' = \frac{0.1}{100} \times bD = \frac{0.1}{100} \times 300 \times 790 = 237 \text{ mm}^2$$

Hence provided #10 bars on each face as shown in Figure 10.6.

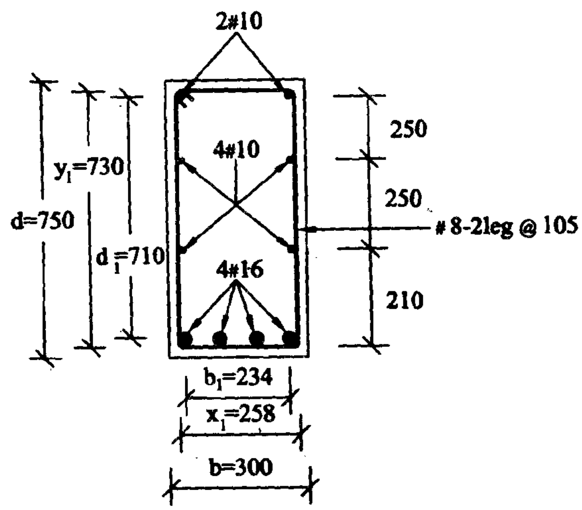

Figure 10.6 : Reinforcement Details of the Designed Section