UNIT 9 FLEXURAL MECHANICS OF DOUBLY REINFORCED BEAMS AND FLANGED BEAMS

Structure

9.1 INTRODUCTION

A cross section of beam which cannot resist an applied bending moment as a singly-reinforced balanced section may either be additionally reinforced in tension only or in tension as well as in compression. In working stress method of design *additional* moment beyond balanced section moment may also be taken care of by over-reinforcing the beam as discussed in Unit 8; but the economic limitation of of over reinforcing a beam may be gauged from the fact that for double the tensile reinforcement of that for balanced section, the increase in moment resisting capacity is only 22%. Therefore, provision of doubly reinforced beam is more efficient and economical propostion than an over-reinforced section. It may also be noted that as per code the calculated compressive stress in steel in compression, $f_{sc} = (1.5\ \mathrm{m})\ *f'_{cbc} = m_c f'_{cbc}$. Increase in Modular Ratio from m to m_c results from more decrease in value of Elastic Modulus of concrete in compression (m_c) than that for tension (m) taking long-term effect into consideration.

General principles for analysis and design of flanged beams are the same as explained in Limit State Design (Unit 2)

Objectives

After going through this unit students will be able to analyse and design doubly reinforced beams and flanged beams for flexure.

9.2 TYPES OF PROBLEMS IN DOUBLY REINFORCED RECTANGULAR SECTION

There are three types of problems :

Type I — To determine Moment of Resistance (M_R) for a given cross section

Type II — To determine maximum stress in concrete and in reinforcing steel for an applied bending moment on a cross section, and

Type III — To determine compressive as well as tensile reinforcements for a given cross section of *concrete* only and applied bending moment.

* f'_{cbc} = stress in surrounding concrete at the level of compressive steel, and
m_c = Modular Ratio at the level of compressive steel.

Type I To determine Moment of Resistance (M_R) for a given cross section

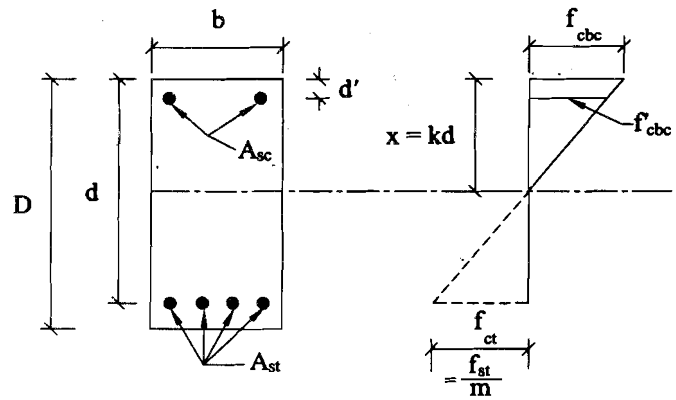


Figure 9.1 : Given Cross-Section with Stress Diagram

For the known cross section, the position of n.a. (kd) is determined by equating moment of areas of concrete in compression and transformed area of steel about n.a., (Figure 9.1) i.e.,

$$kd\frac{kd}{2} + m_c A_{sc}(kd - d') - A_{sc}(kd - d') = mA_{st}(d - kd)$$

or

$$k^2 . \frac{d^2}{2} + (m_c - 1)A_{sc}(kd - d') = mA_{st} d(1 - k) \qquad \ldots (9.1)$$

(a) If $k < k_B$, the section is under-reinforced i.e. the tensile stress will reach permissible stress value (σ_{st}) and the maximum concrete stress in compression will be $f_{cbc} < \sigma_{cbc}$ as shown in Figure 9.2.

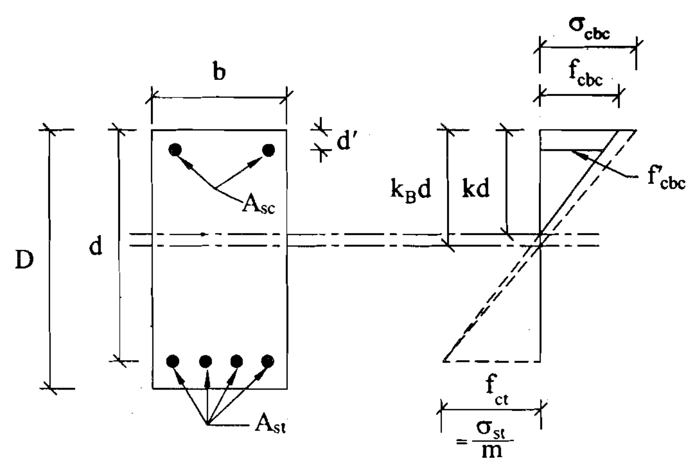


Figure 9.2 : Cross-Section of a Doubly Reinforced Beam having $k < k_B$ (Under-Reinforced Section)

The moment of resistance M_R, is determined by taking moment of compression forces about tensile reinforcement as follows :

$$M_R = \frac{1}{2} f_{cbc} bkd\left(d - \frac{kd}{3}\right) + (m_c - 1)f'_{cbc} A_{st}(d - d') \qquad \ldots (9.2)$$

Where $f_{cbc} = \dfrac{\sigma_{st} . kd}{m(d - kd)}$

and $f'_{cbc} = f_{cbc} \dfrac{(kd - d')}{kd} = \dfrac{\sigma_{st} . kd(kd - d')}{m(d - kd) . kd}$

$$= \frac{\sigma_{st} . k(kd - d')}{m(1 - k) . kd}$$

(b) If $k = k_B$, the permissible tensile stress in tensile steel and the permissible compressive stress in the topmost fibre of concrete will reach simultaneously as shown in Figure 9.3.

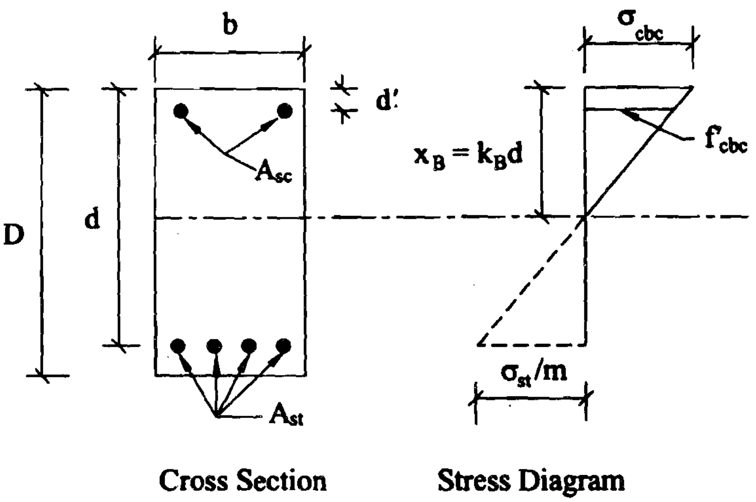


Figure 9.3 : Cross-Section of a Doubly Reinforced Beam having $k = k_B$ (Balanced Section)

In this case, moment of resistance,

$$M_R = \frac{1}{2} \sigma_{cbc} b . k_B d . \left(d - \frac{k_B d}{3} \right) + (m_c - 1) f'_{cbc} A_{sc} (d - d') \qquad \ldots (9.3(a))$$

where $f'_{cbc} = \sigma_{cbc} \dfrac{(kd - d')}{kd}$

or $$M_R = A_{st} \sigma_{st} (d - \bar{x}) \qquad \ldots (9.3(b))$$

where $\bar{x}$ = distance of C.G. of forces in compression in concrete & steel from extreme fibre of concrete in compression.

(c) If $k > k_B$, the section will be over-reinforced and, therefore, at this stage when permissible stress will reach in extreme fibre of concrete, the stress in tensile steel will be $f_{st} < \sigma_{st}$ (Figure 9.4).

The moment of resistance

$$M_R = \frac{1}{2} \sigma_{cbc} b . kd \left(d - \frac{kd}{3} \right) + (m_c - 1) f'_{cbc} A_{sc} (d - d') \qquad \ldots (9.4)$$

where $f'_{cbc} = \sigma_{cbc} \dfrac{(kd - d')}{kd}$

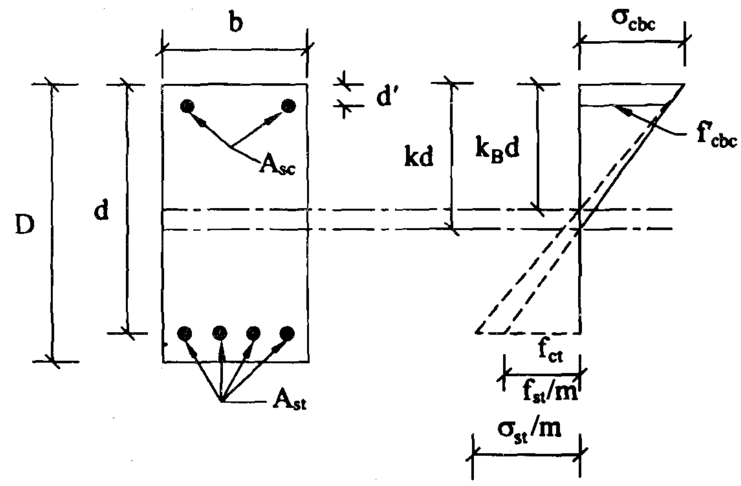


Figure 9.4 : Cross-Section of a Doubly Reinforced Beam having k > k_B (Over-Reinforced Section)

Type II **To determine maximum stress in concrete and reinforcing steel for an applied bending moment on a cross section**

In this case, the maximum compressive stress in concrete is determined first and then the stress in steel is calculated from stress diagram as follows :

(a) The n.a. depth (kd) is determined by taking moment of areas of concrete as well as transformed area of steel in compression and transforced area of tensile steel about n.a.

(b) The applied moment, M, is equated to the moment of compressive forces in concrete & steel about centroid of tensile force in steel (Figure 9.1) i.e.,

$$M = \frac{1}{2} f_{cbc} b . kd \left(d - \frac{kd}{3} \right) + (m_c - 1) f'_{cbc} A_{sc} (d - d') \qquad \ldots (9.2)$$

where $f'_{cbc} = f_{cbc} \dfrac{(kd - d')}{kd}$

(c) Once the maximum stress in concrete f_{cbc} is known, the stresses in compressive as well as tensile steel may be calculated as follows :

Stress in steel in compression,

$$f_{sc} = (m_c - 1) f_{cbc} \frac{(kd - d')}{kd} \qquad \ldots (9.5)$$

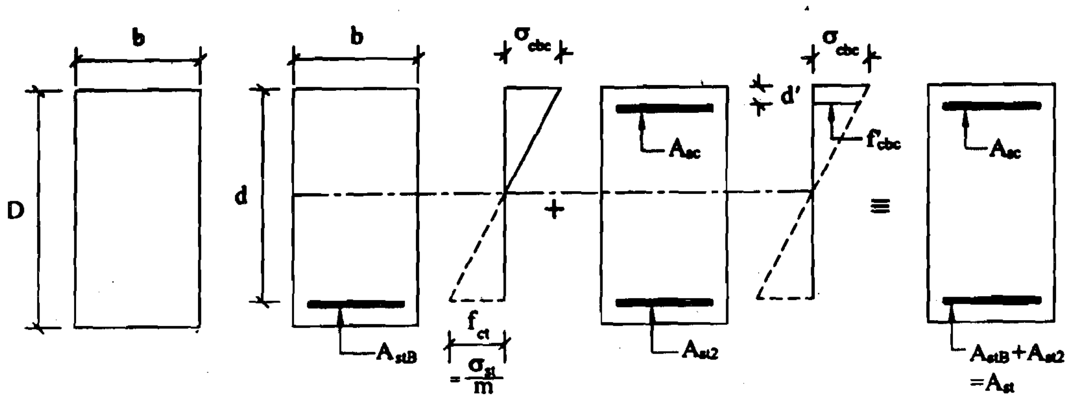
and the stress in tensile steel,

$$f_{st} = m f_{cbc} \frac{(d - kd)}{kd} = m f_{cbc} \frac{(1 - k)}{k} \qquad \ldots (9.6)$$

SAQ 1

Why doubly reinforced section should be preferred to over-reinforced singly reinforced section ?

Type III **To determine compressive as well as tensile reinforcements for a given cross section of concrete only and applied bending moment.**



(a) Concrete Cross Section (b) Concrete Cross Section with A_{stB} for M_B (c) Concrete Cross Section with A_{sc} & A_{st2} for M_2 (d) Concrete Cross Section with A_{sc} & A_{st} for total M

Figure 9.5 : Diagramatic Representation for Provision of Reinforcement for an Applied Moment on a given Concrete Cross-Section

The steel A_{stB} is calculated assuming the section to be balanced one (Figure 9.5(b)) as

$$A_{stB} = \frac{M_B}{\sigma_{st} j_B d}$$

From $M_2 = (M - M_B)$

$$A_{st2} = \frac{M_2}{\sigma_{st}(d-d')} \text{ (Figure 9.5(c))}$$

$$\therefore A_{st} = A_{stB} + A_{st2} \text{ (Figure 9.5(d))}$$

Again $$A_{sc} = \frac{M_2}{f_{sc}(d-d')} \text{ (Figure 9.5(c))} \qquad \ldots (9.7)$$

where $$f_{sc} = m_c \frac{f_{cbc}(kd-d')}{kd} \qquad \ldots (9.8)$$

Example 9.1

Determine Moment of Resistance, M_R, for a $b \times d = 250 \times 415$ reinforced with 2#16 in compression as well as 4#20 in tension and having an effective cover of 35 to compression steel. The grades of concrete and steel used are M 20 and Fe 415 respectively.

Solution

To determine x_B

$$x_B = K_B\, d = \frac{1}{1+\frac{\sigma_{st}}{m\sigma_{cbc}}}\, d = \frac{1}{1+\frac{230}{13\times 7}}\, d = 0.283 \times 415 = 117.65$$

where $m = \frac{280}{3\sigma_{cbc}} = \frac{280}{3\times 7} \approx 13$

To determine depth of n.a (x)

Equating moment of areas of concrete and transformed area of steel in compression to that of transformed area of steel in tension about n.a.,

$$\frac{bx^2}{2} + (m_c - 1)\, A_{sc}\, (x - d') = m\, A_{st}\, (d - x)$$

or $$\frac{250x^2}{2} + (1.5 \times 13 - 1) \times 402.1\, (x - 35) = 13 \times 1256.6\, (415 - x)$$

or $x = 160.56 > 117.65$ indicating that the section is over-reinforced. Therefore, due to gradually applied bending moment the permissible stress in concrete will reach first and hence,

$$M_R = \frac{1}{2}\sigma_{cbc} b.x.\left(d - \frac{x}{3}\right) + (1.5m - 1) f'_{cbc}\, A_{sc}(d - d') \text{ (vide Figure 9.4)}$$

where $$f'_{cbc} = \sigma_{cbc}\frac{(x - d')}{x} = 7 \times \frac{(160.56 - 35)}{160.56} = 5.474\ \text{N/mm}^2$$

$$\therefore M_R = \left\{\frac{1}{2} \times 7 \times 250 \times 160.56 \times \left(415 - \frac{160.56}{3}\right) + (1.5 \times 13 - 1) \times 402.1 \times 5.474(415 - 35)\right\} \times 10^{-6}$$

$= 66.26$ kNm Ans

Example 9.2

Find maximum compressive stress in concrete and compressive as well as tensile stresses in steel in a doubly reinforced section for the following data :

$M = 110$ kNm $\quad b = 300 \quad d = 450$

$d' = 45 \quad A_{st} = 5\#25 \quad A_{sc} = 2\#25 \quad m = 13$

Solution

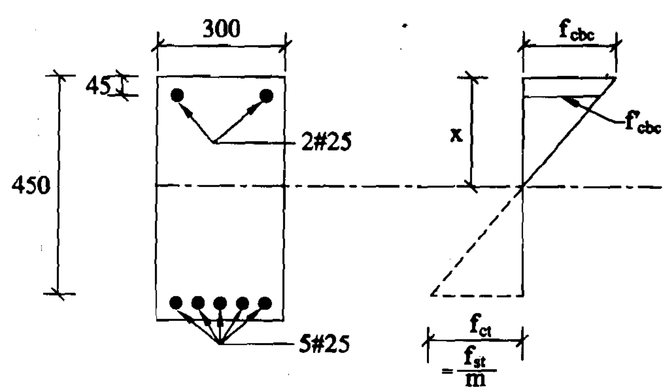


Figure 9.6 : Showing Cross-Section and Stress Diagram

To Determine x

Equating moment of concrete in compression and transformed area of compressive steel to that of transformed area of tensile steel (Figure 9.6)

$$\frac{bx^2}{2} + (m_c - 1)\, A_{sc}\, (x - d') = m\, A_{st}\, (d - x)$$

$$\frac{300x^2}{2} + (1.5 \times 13 - 1) \times 981.75\,(x - 45) = 13 \times 2454.37\,(450 - x)$$

or $\quad x = 192.3$ mm

Now

$$M = \frac{1}{2} f_{cbc}\, b.x\,(d - x/3) + (m_c - 1) f'_{cbc} A_{sc}\,(d - d')$$

where $f'_{cbc} = f_{cbc} \dfrac{(x - d')}{x} = \dfrac{(192.3 - 45)}{192.3} \times f_{cbc} = 0.766 f_{cbc}$

$$\therefore \quad 110 \times 10^6 = \frac{1}{2} f_{cbc} \times 300 \times 192.3 \left(450 - \frac{192.3}{3}\right) + (1.5 \times 13 - 1) \times 0.766 f_{cbc}$$

$$\times 981.75\,(450 - 45)$$

or $110 \times 10^6 = 11131286 f_{cbc} + 5634513.6 f_{cbc}$

f_{cbc} **= 6.56 N/mm² Ans**

From stress diagram

$$f_{sc} = m_c f_{cbc} \left(\frac{x - d'}{x}\right)$$

$$= (1.5 \times 13) \times 6.56 \left(\frac{192.3 - 45}{192.3}\right)$$

= 97.99 N/mm² Ans

and $\quad f_{st} = m f_{cbc} \left(\dfrac{d - x}{x}\right) = 13 \times 6.56 \left(\dfrac{450 - 192.3}{192.3}\right)$

= 114.28 N/mm² Ans

Example 9.3

Determine reinforcements to be provided for a R.C. beam of 6m effective span and concrete cross sectional area $b \times D = 300 \times 600$ and loaded with a super-imposed load of 25 kN/m. Concrete and reinforcements to be provided are of specifications M 15 and Fe 415 respectively.

Solution

Design Bending Moment (M)

Self weight	$= 0.3 \times 0.6 \times 25$	= 4.5 kN/m
Super-imposed load		= 25.0 kN/m
Total load, w		= 29.5 kN/m

Therefore $M = \dfrac{w l_{ef}^2}{8} = \dfrac{29.5 \times 6^2}{8} \quad = 132.75$ kN/m

The above value M is first compared with bending moment resistance capacity, M_B, to know whether a singly or doubly reinforced section is required.

$$M_B = \frac{1}{2}\sigma_{cbc}.b.k_B d\left(d - \frac{k_B d}{3}\right)$$

where
$$k_B = \frac{1}{1 + \dfrac{\sigma_{st}}{m\sigma_{cbc}}}$$

$$\left(m = \frac{280}{3\sigma_{cbc}} = \frac{280}{3 \times 5} \approx 19\right)$$

or
$$k_B = \frac{1}{1 + \dfrac{230}{19 \times 5}} = 0.292$$

$$\therefore M_B = \frac{1}{2} \times 5 \times 0.292 \times 560 \times 300\left(560 - \frac{0.292 \times 560}{3}\right) = 61.98 \text{ kNm}$$

Hence a doubly reinforced section is to be provided. Additional moment, $M_2 = (132.75 - 61.98) = 70.77$ kN/m. for which additional reinforcements both in tension, A_{st2}, and in compression, A_{sc} are to be provided.

To determine A_{st} & A_{sc}

Tensile Reinforcement for M_B

$$A_{stB} = \frac{M_B}{\sigma_{st} j_B d} = \frac{61.98 \times 10^6}{230 \times 0.903 \times 560} = 532.9 \text{ mm}^2 \text{ (vide Fig. 9.5)}$$

where $j_B = \left(1 - \dfrac{k_B}{3}\right) = \left(1 - \dfrac{0.292}{3}\right) = 0.903$

Additional Tensile Reinforcement for M_2

$$A_{st2} = \frac{M_2}{\sigma_{st}(d - d')} = \frac{70.77 \times 10^6}{230(560 - 40)} = 591.71 \text{ mm}^2 \text{ (taking } d' = 40)$$

Therefore, total tensile reinforcedment $A_{st} = A_{stB} + A_{st2} = 532.9 + 591.71$ **= 1124 mm² Ans**

Compression reinforcement, A_{sc} is determined by considering equibilibrium of forces due to M_2 i.e., $C_2 = T_2$

or
$$(m_c - 1)\frac{\sigma_{cbc}}{k_B d}(k_B d - d')A_{sc} = \sigma_{st} A_{st2}$$

or
$$A_{st} = \frac{230 \times 591.71}{(1.5 \times 19 - 1)\dfrac{5}{0.292 \times 560} \times (0.292 \times 560 - 40)} = \mathbf{131.3 mm^2 \ Ans}$$

SAQ 2

(i) How many types of problems may arise in the analysis and design of doubly reinforced section ?

(ii) Explain the methods of solution of all types of problems in doubly reinforced section.

(iii) Find Moment of Resistance of a R.C. beam for the following data :

$b = 300; d = 450; d' = 45$
$A_{st} = 5\#25$; $A_{sc} = 2\#25$
$f_y = 415$ *M*Pa and $f_{ck} = 20$ *M*Pa

(iv) For an applied bending moment 65 kNm, determine the maximum stress in concrete and stresses in compressive as well as tensile steel. Take m = 13. (Figure 9.7).

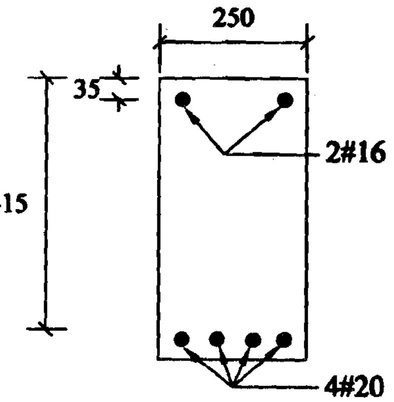


Figure 9.7 : Cross-Section of the Beam

(v) Determine reinforcement for a R.C beam of a rectangular cross section having breadth 250 and effective depth of tensile and compressive reinforcements from the farthest fibre of concrete in compression are 40 and 500 respectively. The cross section has to carry a bending moments of 70 kNm. Take M 20 concrete and Fe 450 steel.

9.3 TYPES OF PROBLEMS IN FLANGED BEAMS

Three types of problems are encountered in the analysis and design of a flanged beam in flexure :

Type I To determine Moment of Resistance (M_R) for a Specified Cross Section

Type II To determine Maximum Stresses in Concrete and Steel for Applied Bending Moment, M, and

Type III To design a section for a specified Bending Moment, M.

Type I To determine M_R for a Specified Cross Section

(i) *N.A. in the flange*

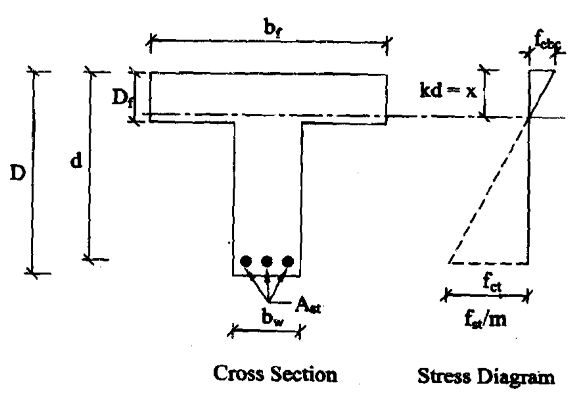


Figure 9.8 : Showing Cross-Section and Stress Diagram of a T-Beam when $x < D_f$

Assuming that n.a. falls in the flange Figure 9.8, the moments of areas of concrete in compression and transformed area of tensile steel about n.a. are equated to determine the position of n.a., i.e.,

$$kd.\frac{kd}{2} = mA_{st}(d - kd) \qquad \ldots (9.9)$$

From the above Equation, if $kd \leq D_f$, the section will be analysed and M_R will be found out just as that of a rectangular beam of width b_f and depth d (Unit 8), since concrete in tension, whatever its shape may be, does not play any role in the analysis.

(ii) *N.A. in the web*

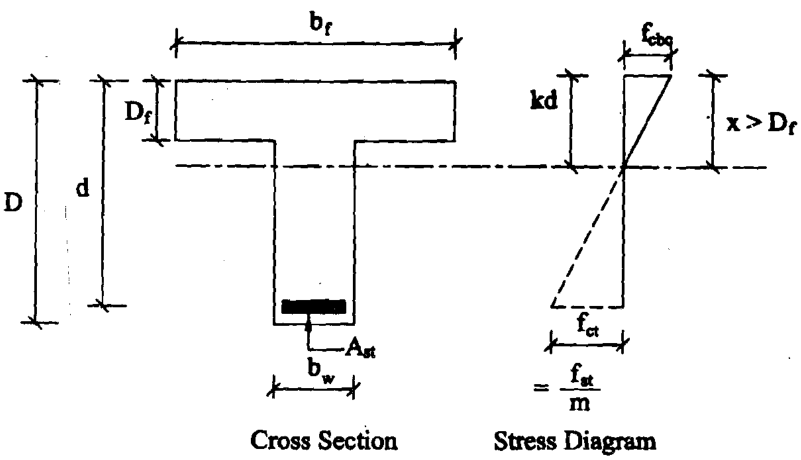


Figure 9.9 : Showing Cross-Section and Stress Diagram of a T-Beam when $x > D_f$

If from Equation 9.9 $kd > D_f$ (i.e. n.a. falls in the web Figure 9.9), kd is determined by equating moment of area of concrete in compression and transformed area of tensile steel about n.a. as

$$b_f D_f\left(kd - \frac{D_f}{2}\right) + b_w(kd - D_f)\frac{(kd - D_f)}{2} = mA_{st}(d - kd) \qquad \ldots (9.10)$$

whether n.a. is falling in the flanged or in the web, the coefficient for n.a. depth for the balanced section is determined by the same formula as for singly reinforced beam * i.e. $k_B = \dfrac{\sigma_{cbc}}{m\sigma_{cbc} + \sigma_{st}}$

(a) If $k < k_B$, the section is under-reinforced and the stress in steel and the topmost fibre of concrete will be σ_{st} and $f_{cbc} < \sigma_{cbc}$ respectively (Figure 9.10).

Hence,

$$M_R = \frac{1}{2} f_{cbc} b_f D_f \left(d - \frac{D_f}{3}\right) + \frac{1}{2} f^{f}{}_{cbc} b_f D_f \left(d - \frac{2D_f}{3}\right)$$

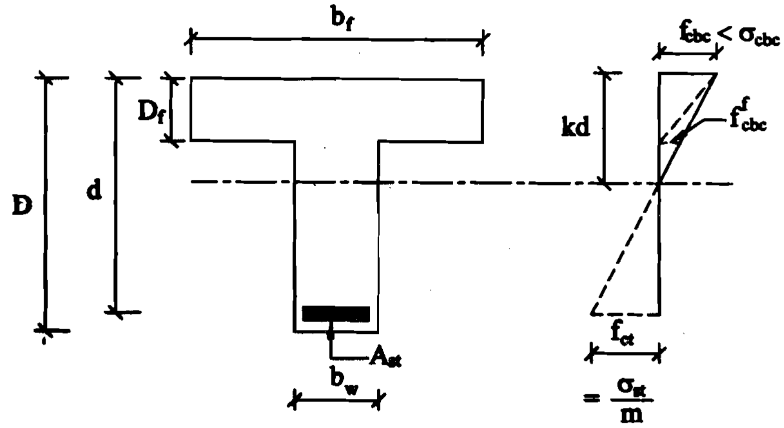
$$+ \frac{1}{2} f^{f}{}_{cbc} b_w (kd - D_f)\left\{d - D_f - \frac{kd - D_f}{3}\right\} \qquad \ldots (9.11)$$

where $f_{cbc} = \dfrac{\sigma_{st}}{m} \cdot \dfrac{kd}{d - kd}$, and

---

* If may noted that the formula is independent of dimension of a cross section and therefore it is applicable for any shape of a cross section.

$$f^{f}_{cbc} = f_{cbc} \frac{kd - D_f}{kd}$$



(a) Cross Section (b) Stress Diagram

Figure 9.10 : Cross-Section and Stress Diagram when $x > D_f$ and $k < k_B$

(b) If $k = k_B$, the section is a balanced one in which case permissible stresses in concrete and steel will reach simultaneously due to applied bending moment. Hence substituting σ_{cbc} in place of f_{cbc} in above Equation 9.11

$$M_R = \frac{1}{2}\sigma_{cbc} b_f D_f \left(d - \frac{D_f}{3}\right) + \frac{1}{2} f^{f}_{cbc} b_f D_f \left(d - \frac{2D_f}{3}\right)$$

$$+ \frac{1}{2} f^{f}_{cbc} b_w (k_B d - D_f) \left\{ d - D_f - \frac{k_B d - D_f}{3} \right\} \quad \ldots (9.12)$$

where $f_{cbc} = \sigma_{cbc} \frac{k_B d - D_f}{k_B d}$

(c) If $k > k_B$, the section is over-reinforced and the permissible stress in concrete will reach first and the stress in tensile steel at that stage will be $f_{st} < \sigma_{st}$. The moment of resistance, M_R, for such section may be written similar to Eq. (9.12) changing k_B to k as given below

$$M_R = \frac{1}{2}\sigma_{cbc} b_f D_f \left(d - \frac{D_f}{3}\right) + \frac{1}{2} f^{f}_{cbc} b_f D_f \left(d - \frac{2D_f}{3}\right)$$

$$+ \frac{1}{2} f^{f}_{cbc} b_w (kd - D_f) \left\{ d - D_f - \frac{kd - D_f}{3} \right\} \quad \ldots (9.13)$$

where $\sigma_{cbc} = \sigma_{cbc} \frac{kd - D_f}{kd}$

II To determine Stresses in Concrete and Steel for a specified Cross Section for Applied Bending Moment, *M*.

(i) Assuming that n.a., falls in the flange the value of kd is determined by equating the areas of concrete in compresson and transformed area of tensile steel about n.a. e.g.

$$b_f \, kd \, kd/2 = m A_{st} \, (d - kd)$$

If $kd \le D_f$, the section may be analysed as a rectangular beam of width b_f and effective depth d,

(ii) If from above analysis $kd > D_f$, moment of areas of concrete in compression and transformed area of tensile steel about n.a. e.g.

$$b_f . D_f . \left(kd - \frac{D_f}{2} \right) + b_w \left(kd - D_f \right) \left(\frac{kd - D_f}{2} \right) = mA_{st} \left(d - kd \right)$$

Knowing the value of kd, the applied resistance may be equated to moment due to internal forces to get f_{cbc} from equation mentioned below

$$M_R = \frac{1}{2} f_{cbc} b_f D_f \left(d - \frac{D_f}{3} \right) + \frac{1}{2} f^f{}_{cbc} b_f D_f \left(d - \frac{2D_f}{3} \right)$$

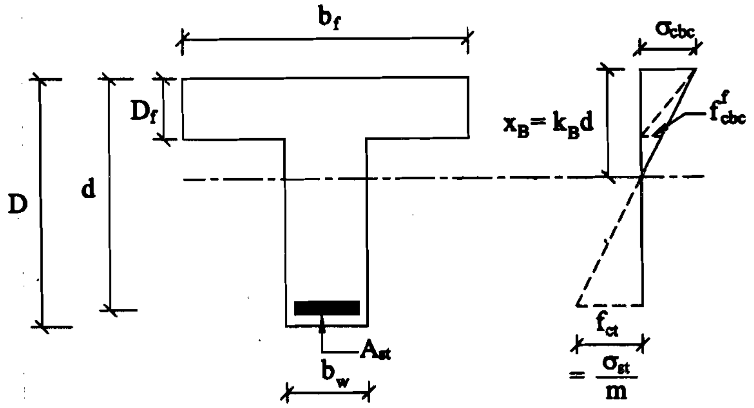
$$+ \frac{1}{2} f^f{}_{cbc} b_w \left(kd - D_f \right) \left\{ d - D_f - \frac{kd - D_f}{3} \right\} \qquad \ldots (9.14)$$

where $f^f{}_{cbc} = f_{cbc} \dfrac{kd - D_f}{kd}$

The stress in steel f_{st} may be evaluated from similarity of D^s of stress diagram

$$\frac{f_{st}}{m(d - kd)} = \frac{f_{cbc}}{kd} \qquad \ldots (9.15)$$

III To determine M_R and % of Tensile Reinforcement for Specified *Concrete* Section



(a) Cross Section **(b) Stress Diagram for Balanced Section**

Figure 9.11 : Cross-Section and Stress Diagram for Balanced Section when $x > D_f$ and $k = k_B$

Let the *concrete* cross section is a shown in Figure 9.11. For determination of M_R for balanced section in this case, the coefficient for n.a. depth will be determined by the formula

$$k_B = \frac{m\sigma_{cbc}}{m\sigma_{cbc} + \sigma_{st}}$$

If $k_B d$ falls in the flange, the M_{RB} will be determined by just as for rectangular section of width b_f i.e.

$$M_{RB} = \frac{1}{2} \sigma_{cbc} b_f d . \left(d - \frac{k_B d}{3} \right) \qquad \ldots (9.16)$$

If $k_B d > D_f$

$$M_{RB} = \frac{1}{2}\sigma_{cbc} b_f D_f \left(d - \frac{D_f}{3}\right) + \frac{1}{2} f^f{}_{cbc} b_f D_f \left(d - \frac{2D_f}{3}\right)$$

$$+\frac{1}{2} f^f{}_{cbc} b_w (d - D_f)\left\{d - D_f - \frac{k_B d - D_f}{3}\right\} \quad \ldots (9.17)$$

From equilibrium of forces

$$C_B = T_B$$

$$\frac{1}{2}\sigma_{cbc} b_f D_f + \frac{1}{2} f^f{}_{cbc} b D_f + \frac{1}{2} f^f{}_{cbc} b_w (d - D_f) = A_{st}\sigma_{st} \quad \ldots (9.18)$$

Once A_{st} is found out from the above mentioned equation 9.18

$$p\% = \frac{A_{st}}{b_w d} \times 100\%$$

Example 9.4

Determine the moment of resistance, M_R for T-beam of the following configuration : $d = 500$; $b_f = 1000$; $D_f = 130$; $b_w = 250$ and $A_{st} = 4\#28$. Use M 15 concrete and Fe 415

Solution

$$m = \frac{280}{3\sigma_{cb}} = \frac{280}{3 \times 5} \approx 19$$

Assuming that the n.a. lies in the flange then

$$b_f kd \frac{kd}{2} = mA_{st}(d - kd)$$

or $\quad k^2 \times \dfrac{1000 \times 500}{2} = 19 \times 4 \times \pi/2 \times 28^2 (1 - k)$

or $\quad 250000\, k^2 = 46979.16\,(1 - k)$

or $\quad k^2 + 0.187K - 0.187 = 0$

or $\quad k = 0.35$

$\therefore\ kd = 0.35 \times 500 = 175 > 130$

Hence n.a. will fall in the web (Figure 9.12)

Now equating moment of area of concrete in compression to that of transformed area of tensile steel about n.a.

$$b_f D_f\left(x - \frac{D_f}{2}\right) + \frac{b_w}{2}\left(x - \frac{D_f}{2}\right)^2 = mA_{st}(d - x)$$

or $\quad 1000 \times 130\,(x - 130/2) + 250/2\,(x - 130)^2 = 19 \times 4 \times \pi/4 \times 28^2 (500 - x)$

or $\quad 13000\,x - 8450000 + 125\,x^2 - 32500x + 2112500 = 23398582 - 46797.16x$

or $\quad x^2 + 1154.38x - 237889 = 0$

or $$x = \frac{-1154.38 \pm \sqrt{1154.38^2 + 4 \times 237889}}{2} = 178.48$$

Now $$x_B = \frac{1}{1 + \frac{\sigma_{st}}{m\sigma_{cbc}}} \times d = \frac{1}{1 + \frac{230}{19 \times 5}} \times 500 = 146 < 178.48$$

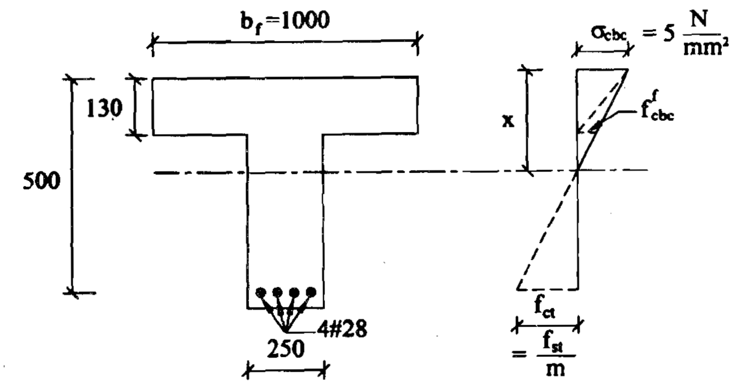


Figure 9.12 : Showing Cross-Section and Stress Diagram

Hence the section is over-reinforced. The permissible stress in concrete in compression will reach first and, therefore, moment of resistance, M_R may be calculated by the Eq. (9.13).

$$M_R = \frac{1}{2}\sigma_{cbc} b_f D_f \left(d - \frac{D_f}{3}\right) + \frac{1}{2} f^f{}_{cbc} b_f D_f \left(d - \frac{2D_f}{3}\right)$$

$$+ \frac{1}{2} f^f{}_{cbc} b_w (x - D_f) \left\{d - D_f - \frac{x - D_f}{3}\right\}$$

Where $f^f{}_{cbc} = \sigma_{cbc} \frac{x - D_f}{x} = 5 \times \frac{178.48 - 130}{178.48} = 1.358 \text{ N/mm}^2$

$$= 1/2 \times 5 \times 1000 \times 130\,(500 - 130/3) + 1/2 \times 1.358 \times 1000 \times \left(500 - \frac{2 \times 130}{3}\right)$$

$$+ 1/2 \times 1.358 \times 250\,(178.48 - 130) \left(500 - 130 - \frac{178.48 - 130}{3}\right)$$

$= 1.48917 \times 10^8 + 29119192.2$

= 187.82 kNm Ans

Example 9.5

Determine the maximum stress in compression in concrete and tensile stress in steel for an applied bending moment of 180 kNm on a T-beam section of the configuration shown in Figure 9.13. Take m = 19.

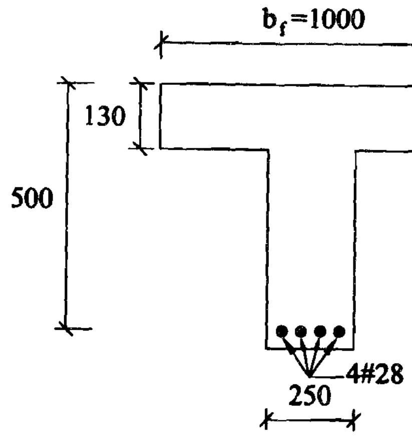


Figure 9.13 : Showing Cross-Section of the Beam

Solution

Assuming that the n.a. falls in the flange and equating moment of area of concrete in compression to that of transformed area of tensile steel about n.a. i.e.,

$$b_f \frac{x^2}{2} = mA_{st}(d - x)$$

or $$1000 \frac{x^2}{2} = 19 \times 4 \times \pi/4 \times 28^2 (500 - x)$$

or $$500 \frac{x^2}{2} = 23398582 - 46797.16\,x$$

or $$x^2 + 93.59\,x - 46797.16 = 0$$

or $$x = \frac{-93.59 \pm \sqrt{93.59^2 + 4 \times 46797.16}}{2} = 174.53 > 130$$

Hence n.a. will be in web

Again equating moment of area of concrete in compression to that of transformed tensile reinforcement area about n.a., i.e.

$$b_f D_f \left(x - \frac{D_f}{2} \right) + b_w (x - D_f) \left(\frac{x - D_f}{2} \right) = mA_{st}(d - x)$$

or $$1000 \times 130 \left(x - \frac{130}{2} \right) + \frac{250}{2} (x - 130)^2$$

$$= 13 \times 4 \times \pi/4 \times 28^2 (500 - x)$$

or $x = 178.48$

Now applied moment

$$M = \frac{1}{2} f_{cbc} b_f D_f \left(d - \frac{D_f}{3} \right) + \frac{1}{2} f^f{}_{cbc} b_f D_f \left(d - \frac{2D_f}{3} \right) + \frac{1}{2} f^f{}_{cbc} b_w (x - D_f) \left(d - D_f - \frac{x - D_f}{3} \right)$$

where $$f^f{}_{cbc} = f_{cbc} \frac{x - D_f}{x} = f_{cbc} \times \frac{178.48 - 130}{178.48} = 0.272 f_{cbc}$$

$$180 \times 10^6 = 1/2 f_{cbc} \times 1000 \times 130\left(500 - \frac{2\times130}{3}\right) + 1/2 \times 0.272 f_{cbc} \times 250 \times$$

$$(178.48 - 130) \times \left(500 - 130 - \frac{178.48\times130}{3}\right)$$

$$180 \times 10^6 = (29.683 \times 10^6 + 7.31 \times 10^6 + 0.5832 \times 10^6) f_{cbc}$$

or $\quad$ f_{cbc} **= 4.79 N/mm² Ans**

From similarity of Δ^s in stress diagram

$$\frac{f_{cbc}}{x} = \frac{f_{st}}{m(d-x)}$$

or $$f_{st} = \frac{4.79\times19(500-178.48)}{178.48}$$

= 163.95 N/mm² Ans

Example 9.6

Determine percentage of tensile steel required for developing moment of resistance for balanced section, M_{RB}, for a *concrete* cross section shown in Figure 9.14 below. Use M 15 concrete and Fe 415 steel.

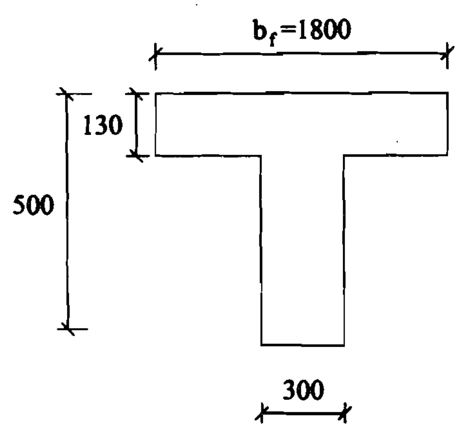


Figure 9.14 : Concrete Cross-Section of the Beam

Solution

$$k_B = \frac{\sigma_{cbc}}{m\sigma_{cbc} + \sigma_{st}} = \frac{1}{1 + \frac{\sigma_{st}}{m\sigma_{cbc}}}$$

where $$m = \frac{280}{3\sigma_{cbc}} = \frac{280}{3\times5} = 18.67 \approx 19$$

or $$k_B = \frac{1}{1 + \frac{230}{19\times5}} = 0.292$$

$$\therefore \; x_B = 0.292 \times 500 = 146 > D_f$$

from Eq. (9.17) and Figure 9.12

$$M_{RB} = \frac{1}{2}\sigma_{cbc}b_f D_f\left(d-\frac{D_f}{3}\right)+\frac{1}{2}f^f{}_{cbc}b_f D_f\left(d-\frac{2D_f}{3}\right)+\frac{1}{2}f^f{}_{cbc}b_w\left(k_B d-D_f\right)\left\{(d-D_f)-\frac{k_B d-D_f}{3}\right\}$$

where $$f^f{}_{cbc} = f_{cbc}\frac{k_B d - D_f}{k_B d}$$

$$= 5\times\frac{0.292\times 500-130}{0.292\times 500} = 0.548$$

$$\therefore \quad M_{RB} = 1/2\times 5\times 1800\times 130\left(500-\frac{130}{3}\right) + 1/2\times 0.548\times 1800\times 130$$

$$\times\left(500-\frac{2\times 130}{3}\right) + 1/2\times 0.548\times 300\,(0.292\times 500-130)$$

$$\times\left(500-130-\frac{0.292\times 500-130}{3}\right)$$

$= 2.6715\times 10^8 + 0.2650\times 10^8 + 0.0047961\times 10^8$

$= 2.9413\times 10^8$ Nmm

= 294.13 kNm Ans

From Equilibrium of Forces

$C_B = T_B$

or $\quad \frac{1}{2}\sigma_{cbc}b_f D_f + \frac{1}{2}f^f{}_{cbc}b_f D_f + \frac{1}{2}f^f{}_{cbc}b_w(x_B - D_f)$

or $\quad \frac{1}{2}\times 5\times 1800\times 130 + \frac{1}{2}\times 0.548\times 1800\times 130 + \frac{1}{2}\times 0.548\times 300$

$(0.292\times 500-130) = A_{st}\times 230$

or $\quad A_{st} = 2828.00\ \text{mm}^2$

$$\therefore \quad p\% = \frac{A_{st}}{b_w\times d}\times 100\%$$

$$= \frac{2828}{300\times 500}\times 100\%$$

= 1.88% Ans

Example 9.7

Determine moment of resistance of a *doubly* reinforced T-section for the following data : $D_f = 150$; $b_w = 250$; $d = 500$; $b_f = 1000$; $d' = 40$; $A_{sc} = 3$ # 20 and $A_{st} = 6$ # 28 Use M 15 concrete and Fe 415 steel.

Solution

Determination of x_B

$$x_B = k_B d = \frac{1}{1+\frac{\sigma_{st}}{m\sigma_{cbc}}}\times 500 = \frac{1}{1+\frac{230}{19\times 5}}\times 500 = 146.15$$

where $m = \frac{280}{3\sigma_{cbc}} = \frac{280}{3 \times 5} \approx 19$

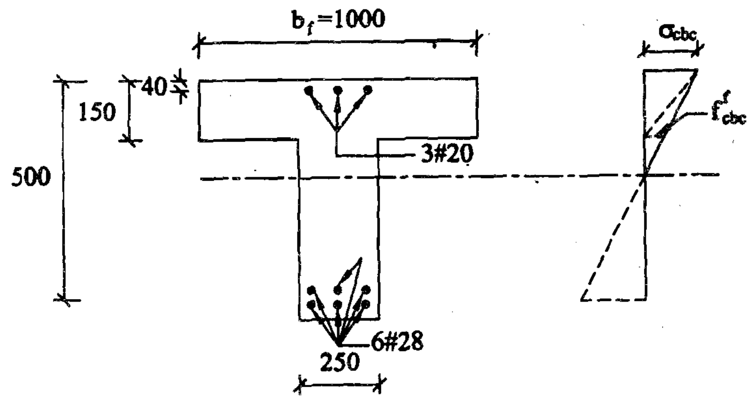


Figure 9.15 : Cross-Section of the Beam

Determination of x

Let n.a. fall in the flange, then equating moment of areas of concrete and transformed area of steel in compression to that of transformed area of tensile reinforcement about n.a.

$$b_f \frac{x^2}{2} + (m_c - 1)A_{sc}(x - d') = mA_{st}(d - x)$$

or $$\frac{1000}{2} x^2 + (1.5 \times 19 - 1) \times 942.48\, A_{sc}(x - 40) = 19 \times 3694.51(500 - x)$$

or $$500\, x^2 + 25918.2x - 1036728 = 35097845 - 70195.69\, x$$

or $$x^2 + 192.22 - 72269.15 = 0$$

or $$x = \frac{-192.22 \pm \sqrt{192.22^2 + 4 \times 72269.15}}{2} = 189.38 > 150$$

Hence n.a. will fall in the web

Equating moment of area of concrete in compression and transformed area of steel in compression to that of transformed area of tensile steel about n.a.

$$b_f D_f \left(x - \frac{D_f}{2} \right) + (m_c - 1)A_{sc}(x - d') + b_w \frac{(x - D_f)^2}{2} = mA_{st}(d - x)$$

or $$1000 \times 150 \left(x - \frac{150}{2} \right) + (1.5 \times 19 - 1) \times 942.48\,(x - 40) + \frac{250}{2}(x - 150)^2$$

$$= 19 \times 3694.51\,(500 - x)$$

or $$150000\,x - 11250000 + 25918.2\,x - 1036728 + 125\,x^2 - 37500\,x + 2812500$$

$$= 35097845 - 70195.69\,x$$

or $$125x^2 + 208613.89\,x - 44572073 = 0$$

or $$x^2 + 1668.91\,x - 356576.58 = 0$$

or $$x = \frac{-1668.91 \pm \sqrt{1668.91^2 + 4 \times 356576.58}}{2}$$

$$= \frac{-1668.91 \pm 2052.21}{2} = 191.65 > 146$$

Hence the section is over-reinforeced. The extreme fibre of concrete in compression will have permissible stress first under gradually applied bending moment.

$$M_R = \frac{1}{2}\sigma_{cbc} b_f D_f \left(d - \frac{D_f}{3}\right) + \frac{1}{2} f^f{}_{cbc} b_f D_f \left(d - \frac{2D_f}{3}\right) + \frac{1}{2} f^f{}_{cbc} b_w (x - D_f) \times$$

$$\left\{d - D_f - \frac{x - D_f}{3}\right\} + (m_c - 1) A_{sc} f'_{cbc} (d - d')$$

where $$f^f{}_{cbc} = f_{cbc} \frac{x - D_f}{x} = 5 \times \frac{191.65 - 150}{191.65} = 1.087 \text{ N/mm}^2$$

and $$f_{cbc} = \sigma_{cbc} \frac{x - 40}{x} = 5 \times \frac{191.65 - 40}{191.65} = 3.956 \text{ N/mm}^2$$

or $$M_R = \frac{1}{2} \times 5 \times 1000 \times 150 \left(500 - \frac{150}{3}\right) + \frac{1}{2} \times 1.087 \times 1000 \times 150 \times$$

$$\left(500 - \frac{2 \times 150}{3}\right) + \frac{1}{2} \times 1.087 \times 250 \times (191.65 - 150)$$

$$\left(500 - 150 - \frac{191.65 - 150}{3}\right) + (1.5 \times 19 - 1) \times 942.48 \times 3.956 \times (500 - 40)$$

$$= 250.427 \text{ kNm}$$

SAQ 3

(i) Determine moment of resistance M_R for the cross section of a T-beam shown in Figure 9.16 using M 20 concrete and Fe 415 steel.

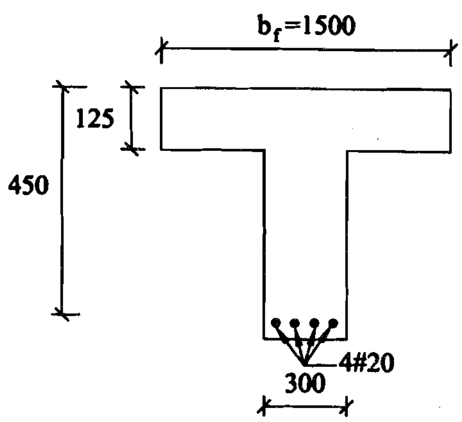


Figure 9.16 : Cross-Section of the Beam

(ii) The cross-section of a T-beam is as shown in Figure 9.17.Determine maximum compressive stress in concrete and tensile stress in reinforcement when applied with a bending moment of 280 kNm.

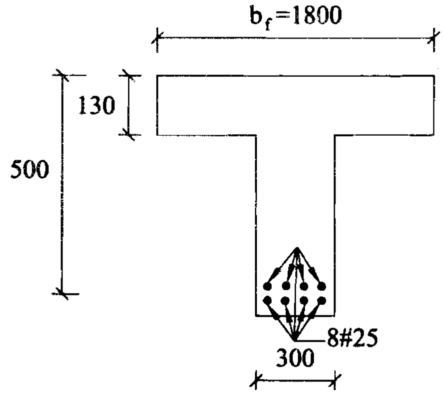


Figure 9.17 : Cross-Section of the Beam

(iii) Determine Moment of Resistance M_B, and percentage of tensile reinforcement, p_{tB}, for the concrete section shown in Figure 9.18 and designed as a balanced section. Use M 15 concrete and Fe 415 steel.

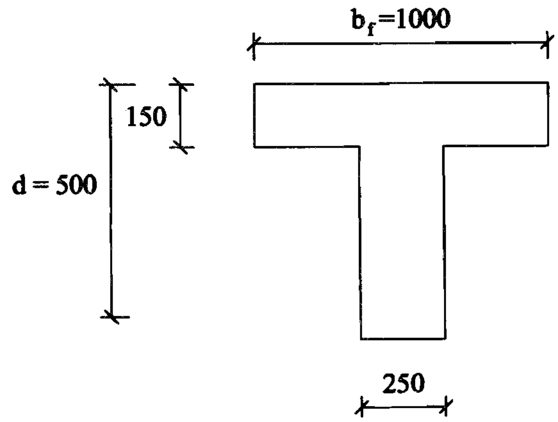


Figure 9.18 : Concrete Cross-Section

9.4 SUMMARY

General principles for the analysis and design of doubly reinforced reactangular beams and singly as well as doubly reinforced flanged beams have been explained in Unit 2. The variation in analysis, if any, have been discussed in section 9.1. Types of problems have been enunciated, explained and illustrated with examples.

9.5 ANSWERS TO SAQs

SAQ 1

Refer text 9.1

SAQ 2

(i) Refer text 9.2

(ii) Refer text 9.2

(iii) $M_R = 117.36$ kNm

(iv) $f_{cbc} = 6.86$ N/mm^2 ; $f_{sc} = 104.61$ N/mm^2 ; $f_{st} = 141.32$ N/mm^2

(v) $A_{sc} = 659.93$ mm^2 and $A_{st} = 853.65$ kNm

SAQ 3

(i) $M_R = 121.52$ kNm

(ii) $f_{cbc} = 4.295$ MPa ; $f_{st} = 159.23$ MPa

(iii) $M_B = 164.74$ kNm; $p_{tB}\% = 1.27$