
UNIT 8 INTRODUCTION TO WORKING STRESS METHOD AND FLEXURAL MECHANICS OF SINGLY REINFORCED RECTANGULAR SECTION

Structure

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8.1 INTRODUCTION

R.C. structures or their elements may be designed by any of the three methods :

- (i) Limit State Method
- (ii) Working Stress Method, and
- (iii) Methods Based on Experimental Investigations

In this unit, Working Stress Method only shall be discussed. According to basic principles of design, structures or their elements must be safe and serviceable under design loads. Here design loads are actual loads (characteristic loads) on a structure. Safety of a structure is measured against permissible stresses due to design loads. These permissible stresses are 'Characteristic Strengths' divided by 'Factor of Safety' which are different for different materials. General design and detailing requirements including Limit State of Serviceability are the same as those applicable for Limit State Design except Redistribution of Moments in continuous beams and frames. Accordingly moments over supports for any assumed arrangement of loading, including the dead load moments may be increased or decreased by not more than 15%, provided that the modified moments over supports are used for calculation of the corresponding moments in the spans. *Thus, Working Stress Method of design of a structure or its elements is one in which under design loads the stresses developed are within permissible limits, and the detailing of concrete section as well as reinforcements are so as to meet the serviceability requirements.*

Objectives

Through this unit a student will be able to learn the following :

- Basics of Working Stress Method of Design
- Permissible Stresses in Concrete and Reinforcement used in design, and
- Analytical Aspect of Design of a structure and its elements under design loads

SAQ 1

- (i) Explain various methods of designing a R.C structure.
- (ii) Define
 - (a) Working Stress Method
 - (b) Factor of Safety
 - (c) Permissible Stress
 - (d) Design Loads in Working Stress Method

8.2 BASIC ASSUMPTIONS

All structures under design loads are analysed according to linear elastic theory. The design of different sections for providing adequate concrete and reinforcements are based on simplifying assumptions enumerated and discussed as follows :

- (i) A plane section of a structural element remains plane before and after bending
- (ii) The strain-stress ($\epsilon-\sigma$) relationship for concrete and steel under design loads (working or service loads) is linear.
- (iii) The tensile stress resistance of concrete in bending is zero, except specifically permitted, and
- (iv) The modular ratio, $m = \frac{280}{3\sigma_{cbc}}$

Assumptions (i), (ii) and (iii) need no explanation as assumptions (i) and (iii) have been explained in Limit State Method and assumption (ii) is self explanatory. According to assumption (iv), for example, for M 15 concrete the modular ratio,

$$m = \frac{280}{3\sigma_{cbc}} = \frac{280}{3 \times 5} = 18.18 \approx 19$$

As per Strength of Materials, generally, modular ratio, $m = \frac{E_s}{E_c}$. From code, modulus of

elasticity of steel, $E_s = 2 \times 10^5$ MPa and short term static modulus of elasticity of concrete,

$$E_c = 5700\sqrt{f_{ck}} = 5700\sqrt{15} = 227076 \text{ MPa and accordingly, } m = \frac{E_s}{E_c} = \frac{2 \times 10^5}{227076} = 9.06$$

This discrepancy is due to the fact that the value calculated as per assumption takes into account the *long-term* effects such as creep. The creep or any long-term effect goes on continuously deforming the elements during the whole life time of a R.C. structure and, in effect, lowers the modulus of elasticity of concrete. Thus *actual smaller* value of E_c results in higher modular ratio, m .

SAQ 2

- (i) Explain all the assumptions made in Working Stress Method.
- (ii) Why the prescribed value of modular ratio, m , of any grade of concrete is

much greater than those obtained by general formula, $m = \frac{E_s}{E_c}$?

8.3 PERMISSIBLE STRESSES IN CONCRETE AND STEEL

Permissible stresses for various types of forces on structural concrete element is obtained by dividing the corresponding characteristic strength by appropriate 'Factor of Safety'. Accordingly, these stresses are mentioned in the following tables :

Table 8.1 : Permissible Direct Tension σ_t in R.C. Members in N/mm²

| Grade of concrete | M 10 | M 15 | M 20 | M 25 | M 30 | M 35 | M 40 |
|----------------------------------------|------|------|------|------|------|------|------|
| Permissible Tensile Stress, σ_t | 1.2 | 2.0 | 2.8 | 3.2 | 3.6 | 4.0 | 4.4 |

The permissible stresses in direct tension are for members in which the reinforcement

alone takes all tension and the direct tension, f_t , is calculated as $f_t = \frac{F_t}{A_c + mA_{st}}$

Where F_t = total tension on the member minus pretension, if any, before concreting, and

A_c = cross sectional area of concrete excluding any finishing material and reinforcing steel

Table 8.2 Permissible Stress in Concrete in Bending & Direct Compression and in Bond.

| Grade of Concrete | Permissible Stress in Compression | | Permissible Stress in Bond (Average) for plain Bars in Tension |
|-------------------|-----------------------------------|----------------------|----------------------------------------------------------------|
| | Bending σ_{cbc} | Direct σ_{cc} | |
| M 10 | 3.0 | 2.5 | --- |
| M 15 | 5.0 | 4.0 | 0.6 |
| M 20 | 7.0 | 5.0 | 0.8 |
| M 25 | 8.5 | 6.0 | 0.9 |
| M 30 | 10.0 | 8.0 | 1.0 |
| M 35 | 11.5 | 9.0 | 1.1 |
| M 40 | 13.0 | 10.0 | 1.2 |

* (i) The bond stress shall be increased by 25% for bars in compression, and

(ii) For deformed bars conforming to 15:1786 - 1979, the bond stress shall be increased by 40%

Table 8.3 Permissible Shear Stress in concrete without Shear Reinforcement

| $\frac{100 A_s}{bd}$ | Permissible Shear Stress in Concrete, τ_c N/mm ² | | | | | |
|----------------------|------------------------------------------------------------------|------|------|------|------|------|
| | GRADE OF CONCRETE | | | | | |
| | M 15 | M 20 | M 25 | M 30 | M35 | M 40 |
| 0.25 | 0.22 | 0.22 | 0.23 | 0.23 | 0.23 | 0.23 |
| 0.50 | 0.29 | 0.30 | 0.31 | 0.31 | 0.31 | 0.32 |
| 0.75 | 0.34 | 0.35 | 0.36 | 0.37 | 0.37 | 0.38 |
| 1.00 | 0.37 | 0.39 | 0.40 | 0.41 | 0.42 | 0.42 |
| 1.25 | 0.40 | 0.42 | 0.44 | 0.45 | 0.45 | 0.46 |
| 1.50 | 0.42 | 0.45 | 0.46 | 0.48 | 0.49 | 0.49 |
| 1.75 | 0.44 | 0.47 | 0.49 | 0.50 | 0.52 | 0.52 |
| 2.00 | 0.44 | 0.49 | 0.51 | 0.53 | 0.54 | 0.55 |
| 2.25 | 0.44 | 0.51 | 0.53 | 0.55 | 0.56 | 0.57 |
| 2.50 | 0.44 | 0.51 | 0.55 | 0.57 | 0.58 | 0.60 |
| 2.75 | 0.44 | 0.51 | 0.56 | 0.58 | 0.60 | 0.62 |
| 3.00 and above | 0.44 | 0.51 | 0.57 | 0.60 | 0.62 | 0.63 |

Note : A_s = Area of longitudinal tensile reinforcement which continues at least 'd' beyond the section being considered except at supports where the full area of tension reinforcements may be used provided the detailing conforms to codal provisions.

Table 8.4 : Maximum Shear Stress with Shear Reinforcement, $\tau_{c, \max}$

| Grade of concrete | M 15 | M 20 | M 25 | M 30 | M35 | M 40 |
|---------------------------------------|------|------|------|------|-----|------|
| $\tau_{c, \max}$ (N/mm ²) | 1.6 | 1.8 | 1.9 | 2.2 | 2.3 | 2.5 |

Table 8.5: Permissible Stresses in Steel Reinforcement

| S. No. | Type of Stress in Reinforcement | Permissible Stresses in N/mm ² | | |
|--------|------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------|
| | | Mild Steel Bars Conforming to Grade I of IS: 432 (Part I) 1966 or Deformed Mild Steel Bars Conforming to IS : 1139-1996 | Medium Tensile Steel Conforming to IS 432 (Part I)- 1996 or Deformed Medium Tensile Steel Bars Conforming to IS : 1139-1996 | High Yield Strength Deformed Bars Conforming to IS : 1139-1996 or IS : 1786 - 1979 (Grade Fe 415) |
| (1) | (2) | (3) | (4) | (5) |
| (i) | Tension (σ_{st} or σ_{sv}) | 140 | Half the guaranteed yield stress subject to a maximum of 190 | 230 |
| | (a) Up to and including 20 mm | 130 | | |
| | (b) Over 20 mm | | | |
| (ii) | Compression in column bars (σ_{sc}) | 130 | 130 | 190 |
| (iii) | Compression in bars in a beam or slab when the compressive resistance of the concrete is taken into account | The calculated compressive stress in the surrounding concrete multiplied by 1.5 times the modular ratio or σ_{sc} whichever is lower | | |
| (iv) | Compression in bars in a beam or slab where the compressive resistance of the concrete is not taken into account | | | |
| | (a) Up to and including 20 mm | 140 | Half the guaranteed yield stress subject to a maximum of 190 | |
| | (b) Over 20 mm | 130 | | |

- Note:**
- (i) 0.2% proof stress may be used for yield stress (f_y) for those steel for which there is no clearly defined yield point, and
 - (ii) When mild steel conforming to Grade II of IS 432 : (Part I) - 1966 is used, the permissible stresses shall be 90% of the permissible stresses of Grade of IS : 432 (Part) - 1966, but if the area of reinforcement have already been designed and detailed as per Grade of IS : 432 (Part I) - 1966 steel, the area of reinforcement shall be increased by 10% of that required for Grade I steel.

8.4 ANALYTICAL ASPECT OF DESIGN OF REINFORCED CONCRETE BEAMS

Bending moment causes internal strains and stresses perpendicular to the cross-sections of a beam (Figure 8.1(a)).

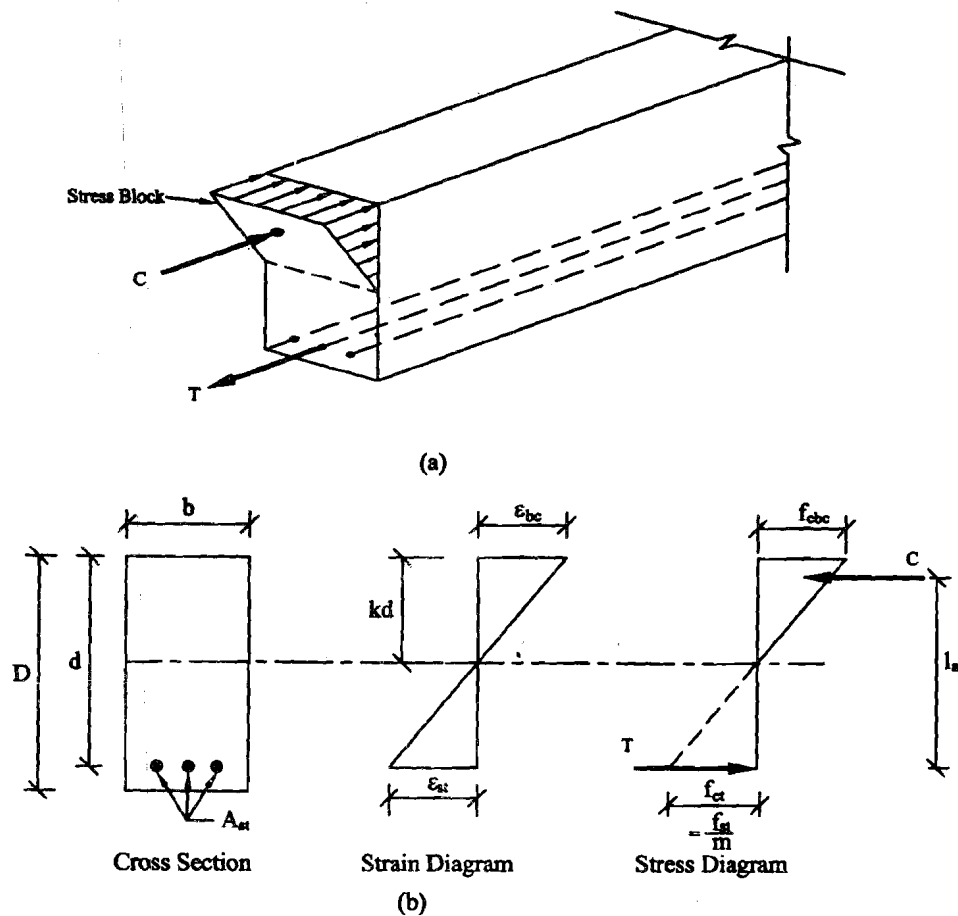


Figure 8.1 : Explaining Bending Mechanics of a Rectangular Beam

From the assumption (Basic Assumption(i)) that plane section remains plane before and after bending, the strain diagram is linear across the section (Figure 8.1(b)). If these strains are multiplied by the modulus of elasticity of concrete, E_c , the resulting stress diagram may be obtained (Basic Assumption (ii)). As tensile resistance of concrete is zero (Basic Assumption (iii)), a fictitious tensile stress in concrete, f_{ct} , (i.e. f_{ct} would have developed if concrete were resisting tensile stress at the level of reinforcing steel) is assumed to evaluate the tensile stress, f_{st} . The moment of resistance may then be calculated as follows :

(i) From *Stress Diagram*

$$\frac{f_{cbc}}{kd} = \frac{f_{ct}}{(d - kd)}$$

$$\text{or } f_{ct} = \frac{f_{cbc}}{kd} (d - kd) = \frac{f_{cbc} (1 - k)}{k}$$

Reinforced concrete being a composite material (i.e. perfect bond develops between concrete and steel), the strains at the level of centroid of steel both in concrete and steel are the same i.e.,

(ii) From *Strain Diagram*

$\epsilon_{st} = \epsilon_{ct}$ where ϵ_{ct} = strain in concrete at the level of centroid of steel area

$$\text{or } \frac{f_{st}}{E_{st}} = \frac{f_{ct}}{E_c}$$

$$\text{or } f_{ct} = \frac{E_c}{E_{st}} f_{st} = \frac{f_{ct}}{m}$$

$$\text{or } f_{st} = m f_{ct} \text{ where } m = \frac{280}{3\sigma_{cbc}} \text{ (Basic Assumption iv)}$$

(iii) From *Force Equilibrium*

$$C - T = 0$$

$$\text{or } C = T$$

$$\text{or } 1/2 f_{cbc} bkd = f_{st} A_{st}$$

(iv) From *Moment Equilibrium*

Applied Bending Moment = Resisting Bending Moment

$$\text{or } M_{\text{applied}} = M_R = C.l_a = T.l_a$$

$$\text{or } M_R = \frac{1}{2} f_{cbc} bkd \left(d - \frac{kd}{3} \right) = f_{st} A_{st} \left(d - \frac{kd}{3} \right)$$

$$= \frac{1}{2} f_{cbc} k \left(1 - \frac{k}{3} \right) b d^2 = f_{st} A_{st} \left(1 - \frac{k}{3} \right) d$$

$$= \frac{1}{2} f_{cbc} k j b d^2 = f_{st} A_{st} j d \quad \text{where } j = \left(1 - \frac{k}{3} \right)$$

$$= R b d^2 = f_{st} A_{st} j d \quad \text{where } R = \frac{1}{2} f_{cbc} k j \quad \dots (8.1)$$

SAQ 3

Derive from the basic principles the Moment of Resistance, M_R , of a singly reinforced concrete section.

8.5 TYPES OF SINGLY REINFORCED RECTANGULAR SECTIONS

Based on fundamentals of analysis in the above section, a singly reinforced rectangular section may be put in any of the three categories :

- (i) a Balanced Section
- (ii) an Under-Reinforced Section, and
- (iii) an Over-Reinforced Section

(i) *Balanced Section*

A section is balanced when the extreme fibre of concrete in compression and the tensile steel reach their respective permissible stresses simultaneously under applied bending moment

Under applied bending moment, M_{applied} , the maximum stress in concrete in compression and tensile stress in steel are their respective permissible values, σ_{cbc} and σ_{st} (Figure 8.2).

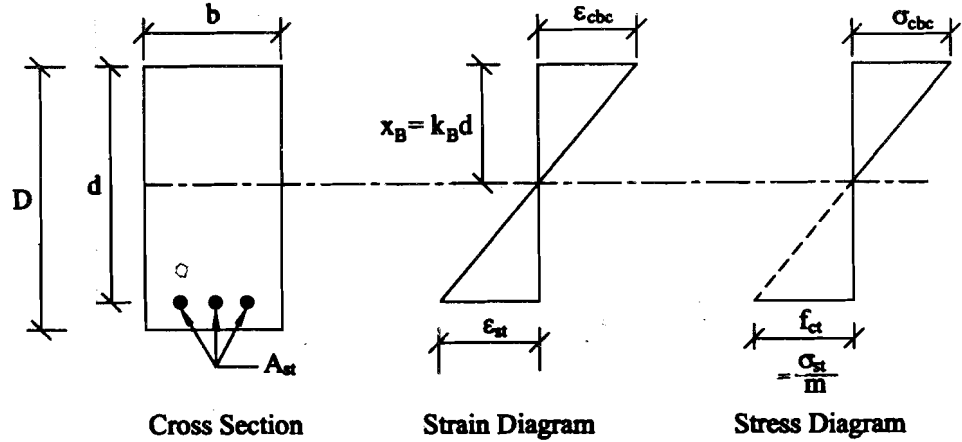


Figure 8.2 : A Balanced Section

Neutral-axis depth in this case is denoted by $k_B d$ or x_B where k_B = coefficient for n.a. depth for balanced section.

Therefore, Moment of Resistance

$$M_R = \frac{1}{2} \sigma_{cbc} k_B j_B b d^2 = \sigma_{st} A_{st} j d$$

or $M_R = R_B b d^2 \dots (8.2)$

(ii) *Under-Reinforced Section*

As the name suggests an under-reinforced section is one in which the area of tensile reinforcement provided is less than that required for balanced section.

Let a cross section of reinforced concrete may be taken in which the area of tensile steel is less than that required to make it a balanced section (Figure 8.3). If applied moment is increased gradually on such section, the permissible tensile stress in steel will reach first since $A_{st} < A_{stB}$, and at this stage the stress in concrete will be $f_{cbc} < \sigma_{cbc}$ (Figure 8.3(d)).

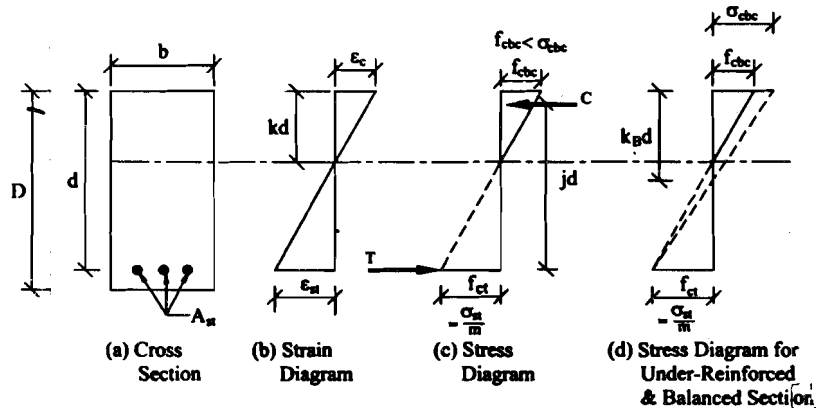


Figure 8.3 : An Under Reinforced Section

The n.a. depth may be determined by equating moment of area of concrete in compression and equivalent area of tensile steel in terms of concrete about n.a.

$$b.k.d.\frac{kd}{2} = mA_{st}(d - kd)$$

The resulting value of $kd < k_B d$ (Fig : 8.3 d) and hence

$$M_R = A_{st}\sigma_{st}\left(d - \frac{kd}{3}\right) = A_{st}\sigma_{st}jd \text{ (Fig : 8.3 c)}$$

(iii) *Over Reinforced Section*

An over Reinforced Section is one in which the tensile reinforcing steel area is more that required for a balanced section.

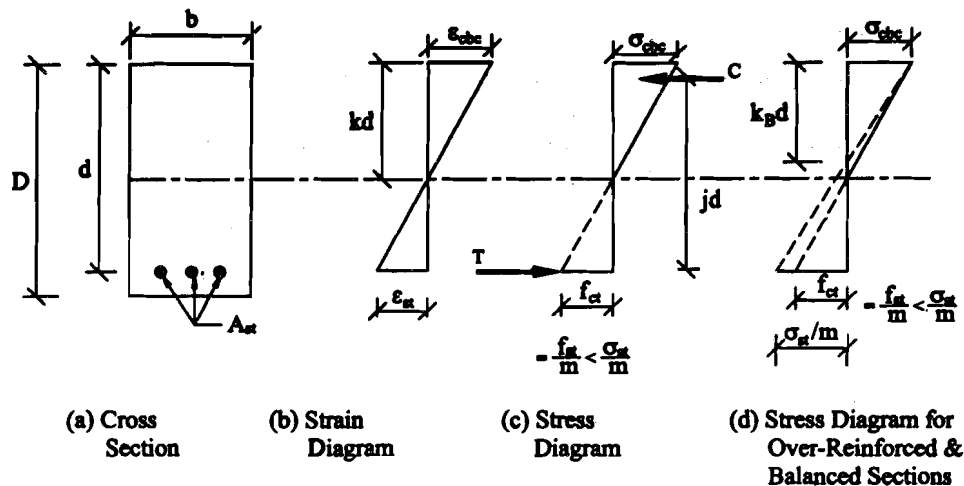


Figure 8.4 : An Over-Reinforced Section

As $A_{st} > A_{stB}$, the permissible stress in concrete (σ_{cbc}) will reach first due to gradual application of bending moment resulting in $kd > k_B d$ (Figure 8.4 d), the value of kd may be obtained by equating moment of area of concrete in compression and equivalent area of tensile steel in terms of concrete about n.a.

$$b.k.d.\frac{kd}{2} = mA_{st}(d - kd)$$

Hence moment of resistance

$$\begin{aligned} M_R &= \frac{1}{2}\sigma_{cbc}b.k.d.\left(d - \frac{kd}{3}\right) \\ &= \frac{1}{2}\sigma_{cbc}kj.bd^2 \end{aligned}$$

SAQ 4

- (i) How many types of singly reinforced sections can be had based on percentage of reinforcement provided?
- (ii) Define and explain Balanced Section
- (iii) Explain the meaning of a Under-Reinforced Section,
- (iv) Is Moment of Resistance of an over-reinforced section is more than that for a Balanced Section, if so, explain why ?

8.6 TYPES OF PROBLEMS IN SINGLY REINFORCED RECTANGULAR SECTIONS

Four types of problems may be encountered :

- Type I : To determine M_R and w for a given cross section
- Type II : To determine maximum compressive stress in concrete (f_{cbc}) and tensile stress in steel (f_{st}) for a given cross section and $M_{applied}$.
- Type III : To determine M_{RB} and A_{stB} when *concrete* cross section is only given, and
- Type IV : To design a singly reinforced section for a given bending moment

The above mentioned types of problems have been given and illustrated with examples given below.

Example 8.1

Determine moment of resistance and uniformly distributed super-imposed load carried by a simply supported singly reinforced R.C. beam having effective span 5m and a cross section of 300×555 ($b \times d$) reinforced with $5 \phi 20$. Use M 15 concrete and Fe 250 steel.

Solution

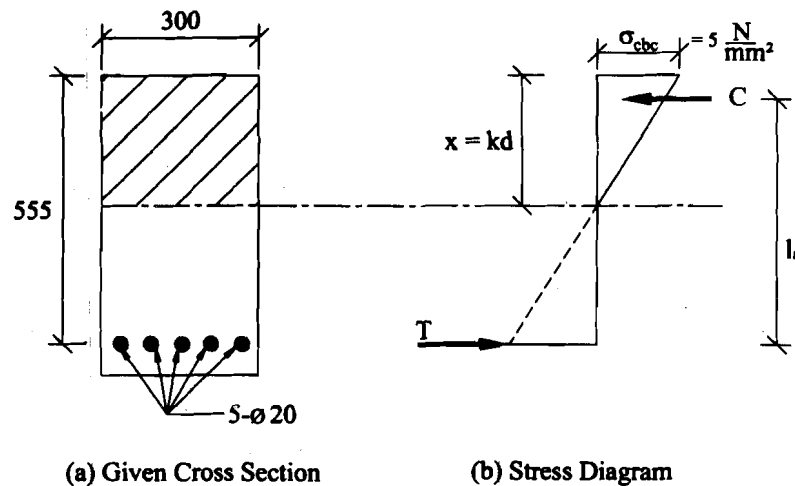


Figure 8.5 : Showing Cross-Section and Stress Diagram of the Beam

Equating moment of area of concrete in compression and equivalent area of tensile steel in terms of concrete about n.a. (Figure 8.5(a)).

$$b \frac{x^2}{2} = m A_{st} (d - x)$$

Here $m = \frac{280}{3\sigma_{cbc}} = \frac{280}{3 \times 5} \approx 19$ and

$$A_{st} = 5 \times \frac{\pi}{4} \times 20^2 = 1570.8 \text{ mm}^2$$

or $\frac{300x^2}{2} = 19 \times 1570.8(555 - x)$

$$\text{or } x^2 + 198.97x - 110427.24$$

$$\text{or } x = 247.4$$

The value of x_B is next determined to compare it with x and to know the type of the section.

$$x_B = k_B d = \frac{1}{1 + \frac{\sigma_{st}}{m\sigma_{cbc}}} d = \frac{1}{1 + \frac{140}{19 \times 5}} = 0.4d$$

$x_B = 0.4 \times 555 = 222 < 247.4$ Hence the section is over-reinforced (Figure 8.4(d)) and the stress distribution across the section is as shown is Figure 8.5(b)

$$\begin{aligned} \text{Thus } M_R &= \frac{1}{2} \sigma_{cbc} b \cdot x \cdot \left(d - \frac{x}{3} \right) \\ &= \frac{1}{2} \times 5 \times 300 \times 247.4 \times \left(555 - \frac{247.4}{3} \right) \times 10^{-6} \\ &= 87.68 \text{ kNm Ans} \end{aligned}$$

As the beam is simply-supported with a u.d.L. (w) over the whole span,

$$M_R = \frac{wl_{ef}^2}{8}$$

$$\text{or } 87.68 \times 10^6 = \frac{w5^2}{8}$$

$$\text{or } w = 28.05 \text{ kN/m (including self weight)}$$

$$\text{Self weight} = 0.3 \times 0.6 \times 1 \times 25 = 4.5 \text{ kN/m (taking 45 as effective cover)}$$

$$\begin{aligned} \therefore \text{ Super-imposed load } w_s &= 28.05 - 4.5 \\ &= 23.55 \text{ kN/m Ans} \end{aligned}$$

Example 8.2

Calculate the maximum compressive stress in concrete and tensile stress in reinforcing steel for a R.C. beam of 3.6m effective span having a cross section of 300×600 ($b \times D$) with $4\phi 20$ and clear concrete cover of 25. The beam is loaded with a super-imposed u.d.l. of 80 kN. Use $m = 19$

Solution

$$\begin{aligned} \text{Total u.d.l.} &= \text{Super-imposed load} + \text{self weight} \\ &= 80 + 0.3 \times 0.6 \times 3.6 \times 25 = 96.2 \text{ kN} \end{aligned}$$

$$M_{\max} = \frac{WL}{8} = \frac{96.2 \times 3.6}{8} = 43.29 \text{ kNm}$$

Equating moment of areas of concrete in compression and transformed area of tensile steel about n.a.

$$\frac{bx^2}{2} = mA_{st}(d-x)$$

$$\text{or } \frac{300 \times x^2}{2} = 19 \times 1256.6(565 - x)$$

$$\text{or } x^2 + 159.1x - 89930.67 = 0$$

$$\text{or } x = 230.68$$

The maximum stress in concrete and steel may now be evaluated as under :

$$M = \frac{1}{2} f_{cbc} b x \left(d - \frac{x}{3} \right)$$

$$\text{or } 43.29 \times 10^6 = \frac{1}{2} f_{cbc} \times 300 \times 230.68 \left(565 - \frac{230.68}{3} \right)$$

$$\text{or } f_{cbc} = 2.56 \text{ N/mm}^2 \text{ Ans}$$

Again

$$M = f_{st} A_{st} \left(d - \frac{x}{3} \right)$$

$$\text{or } 43.29 \times 10^6 = f_{st} \times 1256.6 \left(565 - \frac{230.68}{3} \right)$$

$$\text{or } f_{st} = 70.58 \text{ N/mm}^2 \text{ Ans}$$

Example 8.3

Determine moment of resistance and the area of tensile steel required for a section of R.C. beam of 300×550 ($b \times d$). Use M 15 concrete and Fe 415 steel.

Solution

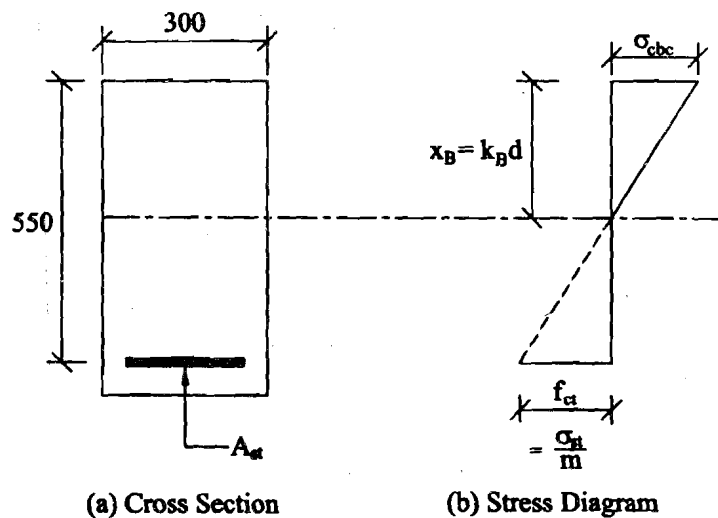


Figure 8.6 : Showing Cross-Section and the Stress Diagram for Balanced Section

As M_R and A_{st} are to be determined for **balanced section**, the value of k_B is first determined Figure 8.6.

$$k_B = \frac{1}{1 + \frac{\sigma_{st}}{m \sigma_{cbc}}} = \frac{1}{1 + \frac{230}{19 \times 5}} = 0.292$$

$$\text{where } m = \frac{280}{3 \times \sigma_{cbc}} = \frac{280}{3 \times 5} \approx 19$$

$$j_B = 1 - \frac{k_B}{3} = 1 - \frac{0.292}{3} = 0.903$$

$$R_B = \frac{1}{2} \sigma_{cbc} j_B = \frac{1}{2} \times 5 \times 0.292 \times 0.903$$

$$= 0.659$$

$$\therefore M_R = M_B = R_B b d^2 = 0.659 \times 300 \times 555^2 \times 10^6$$

$$= 59.86 \text{ kNm Ans}$$

$$A_{st} = \frac{M_b}{\sigma_{st} j_B d} = \frac{59.86 \times 10^6}{230 \times 0.903 \times 550} = 524 \text{ mm}^2 \text{ Ans}$$

Example 8.4

Design for flexure only a R.C. beam of 5m clear span supported on two walls of 300 thickness and carrying a super-imposed load of 20 kN/m. Use M 15 concrete and Fe 415 steel.

Solution

Design coefficients

$$m = \frac{280}{3 \sigma_{cbc}} = \frac{280}{3 \times 5} \approx 19$$

For balanced section

$$k_B = \frac{1}{1 + \frac{\sigma_{st}}{m \sigma_{cbc}}} = \frac{1}{1 + \frac{230}{19 \times 5}} = 0.292$$

$$j_B = 1 - \frac{k_B}{3} = 1 - \frac{0.292}{3} = 0.903$$

$$R_B = \frac{1}{2} \sigma_{cbc} k_B j_B = \frac{1}{2} \times 5 \times 0.292 \times 0.903 = 0.66$$

Depth (D)

(i) *Thumb Rule*

$$D \text{ lying between } \frac{l_{ef}}{10} \text{ to } \frac{l_{ef}}{20}. \text{ Therefore taking } D = \frac{l_{ef}}{12} = \frac{5.3 \times 10^3}{12} = 441.67$$

(where l_{ef} = c/c distance between supports = 5.3m)

(ii) *From Deflection criteria*

$$d \leq \frac{l_{ef}}{K_B K_1 K_2 K_3}$$

$K_B = 20$ for simply supported beam

$$p_{tB} \% = \frac{\sigma_{cbc} k_B}{2\sigma_{st}} \times 100 = \frac{5 \times 0.292}{2 \times 230} \times 100 = 0.32\%$$

and correspondingly $K_1 = 1.4$

$$K_2 = K_3 = 1$$

Substituting these values in the above equation

$$d \leq \frac{5.3 \times 10^3}{20 \times 1.4 \times 1 \times 1} (= 189.29)$$

Taking $D = 450$ and $d = 415$

and taking b (between $D/3$ to $2D/3$) = 225

l_{ef} is lesser of

(i) c/c distance between supports
 $= 5 \text{ m} + 0.3 \text{ m} = 5.3 \text{ m}$, and

(ii) clear span + d
 $= 5 \text{ m} + 0.415 \text{ m} = 5.415 \text{ m}$

Thus $l_{ef} = 5.3 \text{ m}$

(iii) From moment of resistance consideration

Loads

Self weight = $0.225 \times 0.45 \times 1 \times 25 = 2.53 \text{ kN/m}$

Super imposed load = 20.00 kN/m

Total load $w = 22.53 \text{ kN/m}$

$$M_{\max} = \frac{wl_{ef}^2}{8} = \frac{22.53 \times 5.3^2}{8} = 79.1 \text{ kNm}$$

$$\therefore d = \sqrt{\frac{M}{R_B b}} = \sqrt{\frac{79.1 \times 10^6}{0.66 \times 225}} = 729.84 \gg 415$$

Taking $D = 775$, $d = 730$ and $b = 350$

Loads

Self weight = $0.775 \times 0.35 \times 1 \times 25 = 6.78 \text{ kN/m}$

Super imposed load = 20.00 kN/m

Total load $w = 26.78 \text{ kN/m}$

$$M_{\max} = \frac{wl_{ef}^2}{8} = \frac{26.78 \times 5.3^2}{8} = 94.035 \text{ kNm}$$

$$\therefore d = \sqrt{\frac{M}{R_B b}} = \sqrt{\frac{94.035 \times 10^6}{0.66 \times 350}} = 638 < 730 \text{ (to be provided)}$$

Check for Lateral Stability

Assuming the lateral restraints have been provided at the centres of supports

(l_{ef}) = 5.3 m ($60 b = 60 \times 0.35 = 21 \text{ m}$) and

$$(l_{ef} = 5.3\text{m}) \left\{ \frac{250b^2}{d} = \frac{250 \times 0.35^2}{0.73} = 41.95\text{m} \right\}$$

Hence provided $D = 775$; $d = 730$ and $b = 350$

A_{st}

$$A_{st} = \frac{M}{\sigma_{st} j_B d} = \frac{9.4035 \times 10^6}{230 \times 0.903 \times 730} = 619.98 \text{ mm}^2$$

$$\frac{A_{st, \min}}{bd} = \frac{0.85}{f_y}$$

$$\text{or } A_{st, \min} = \frac{0.85 \times 350 \times 730}{415} = 523.31 \text{ mm}^2 < 619.98 \text{ mm}^2$$

Hence provided 4#16 ($A_{st} = 804 \text{ mm}^2 > 619.98 \text{ mm}^2$)

Side Face Reinforcement

$$0.1\% \text{ of total Cross Sectional Area of concrete} = \frac{0.1 \times 350 \times 775}{100} = 271.25 \text{ mm}^2$$

Provided 4#10 ($A_s = 314.16 \text{ mm}^2 > 271.25 \text{ mm}^2$)

The designed cross section is shown in Figure 8.7.

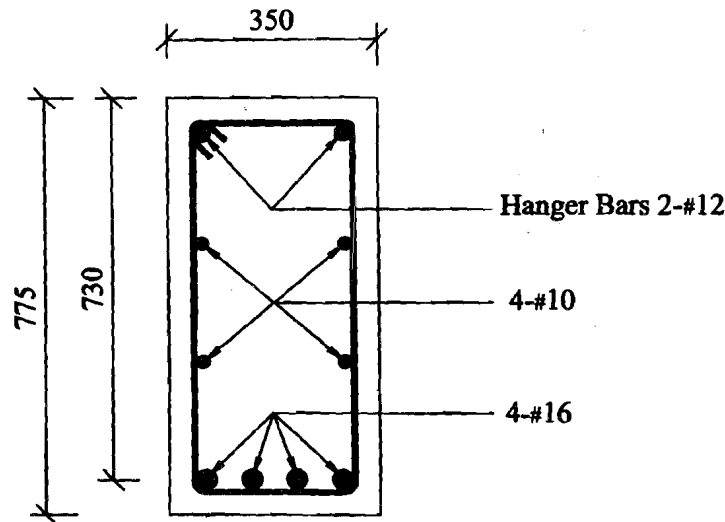


Figure 8.7 : Cross-Section of the Designed Beam

SAQ 5

- (i) Calculate the Moment of Resistance M_R , of a singly reinforced R.C. Section $b \times d = 250 \times 550$ reinforced with 3 # 20. Use M 20 concrete and Fe 415 steel.
- (ii) A singly reinforced cross section, $b \times d = 250 \times 400$ is reinforced with 3 # 20. Determine maximum compressive stress in concrete and tensile stress in steel if a bending moment 30 kNm is applied on it. Use M 15 concrete and Fe 415 steel.
- (iii) Determine Moment of Resistance, M_R and percentage of tensile steel $p_t\%$, for a concrete section $b \times d = 300 \times 500$ taking M 15 concrete and Fe 250 steel.

- (iv) Design for flexure only a R.C. simply supported beam for the following data :

| | |
|------------------------------------|-----------|
| Clear Span | = 5.6m |
| Super imposed Dead Load | = 10 kN/m |
| Live Load | = 15 kN/m |
| Width of Supports | = 375 |
| Use M 15 concrete and Fe 415 steel | |

8.7 SUMMARY

Basic principles involved in the analysis and design for flexure by Working Stress Method have been explained. Assumptions made to simplify the analysis & design have been enunciated and discussed. Flexure Mechanics of R.C. Sections have been explained through derivation of basic equations and parameters of simple singly reinforced sections from the given data. Examples have been solved to illustrate the above mentioned facts.

8.8 ANSWERS TO SAQs

SAQ 1

- (i) Refer text 8.1
- (ii) Refer text 8.1

SAQ 2

- (i) Refer text 8.2
- (ii) Refer text 8.2

SAQ 3

Refer text 8.4

SAQ 4

- (i) Refer text 8.5
- (ii) Refer text 8.5
- (iii) Refer text 8.5
- (iv) Refer text 8.5

SAQ 5

- (i) 90.27 kNm
- (ii) $f_{cbc} = 3.95 \text{ N/mm}^2$ and $f_{st} = 93.45 \text{ N/mm}^2$
- (iii) $M_r = 65.54 \text{ kNm}$, $p_t\% = 0.71$
- (iv) Provide $b \times d = 400 \times 760$ reinforced with 4 # 16 ($A_{st} = 804 \text{ mm}^2 > \text{reqd. } 622.63 \text{ mm}^2$) and side face reinforcement of 4 # 10.

Introduction to Working
Stress Method and
Flexural Mechanics of
Singly Reinforced
Rectangular Section