UNIT 6 DESIGN AND DETAILING OF BEAMS

Structure

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6.1 INTRODUCTION

This unit is meant to design different types of beams based upon the mechanics of reinforced concrete and principles involved in design & detailing processes- such as safety and serviceability requirements discussed in Units 1 to 5. Through the design of a simply supported beam, a cantilever beam, a continuous beam and a beam with overhang, analysis, design and detailing have been explained in a systematic sequence.

Objectives

After going through this unit a student will be able to design & detail all types of beams. One will learn the following :

- sequence to be followed in the design of beams,
- analysis of beam, and
- structural design & detailing of beams.

6.2 DESIGN OF A SIMPLY SUPPORTED BEAM

Example 6.1

Design a simply supported beam of 6 m clear span. The beam is supported on 375 thick wall and loaded with a super-imposed dead weight of 16 kN/m as well as a live load of 12 kN/m. Use M 15 concrete and Fe 250 steel.

Solution

Depth (D)

(i) *Thumb Rule*

D lying between $\frac{l_{ef}}{10}$ to $\frac{l_{ef}}{20}$

Assuming $l_{ef} \approx 6 + 0.375 = 6.375$m to start with

and taking $D = \frac{l_{ef}}{10} = \frac{6.375 \times 10^3}{10} \approx 638$

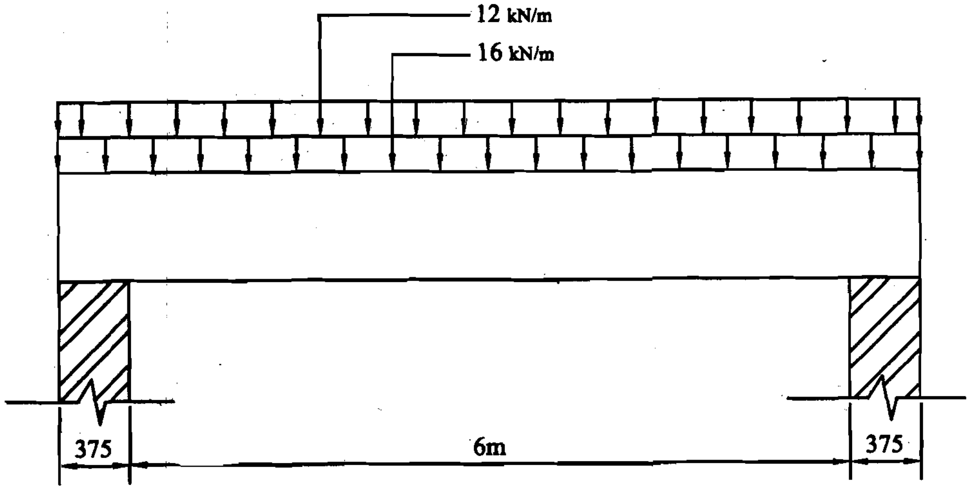


Figure 6.1 : A Simply Supported Beam

(ii) *From Control of Deflection Criteria*

$$d \nless \frac{l_{ef}}{K_B K_1 K_2 K_3}$$

where $K_B = 20$

For M 15 concrete and Fe 250 steel, the amount of balanced section steel = 1.32% and correspondingly $K_1 = 1.3$

$K_2 = K_3 = 1$

Substituting these values in the above equation

$$d \nless \frac{6.375 \times 10^3}{20 \times 1.3 \times 1 \times 1} = 230.77$$

Adopted $D = 800$ and $d = 800 - 40 = 760$

$b = 400$ (i.e. lying between $\frac{1}{3}$rd to $\frac{2}{3}$rd of D)

(iii) *From Moment of Resistance Consideration*

Loads

Self	$= 0.8 \times 0.4 \times 1 \times 25$	= 8.0 kN/m
DL		= 16.0 kN/m
Total *DL*		= 24.0 kN/m
LL		= 12.0 kN/m
Total *DL* + *LL*		= 36.0 kN/m
Design Load, $w_u = 1.5 \times 36.0$		= 54.0 kN/m

$$\text{Maximum B. M., } M_u = \frac{w_u l_{ef}^2}{8} = \frac{54 \times 6.375^2}{8} = 274.32 \text{ kNm}$$

(vide Figure 6.2)

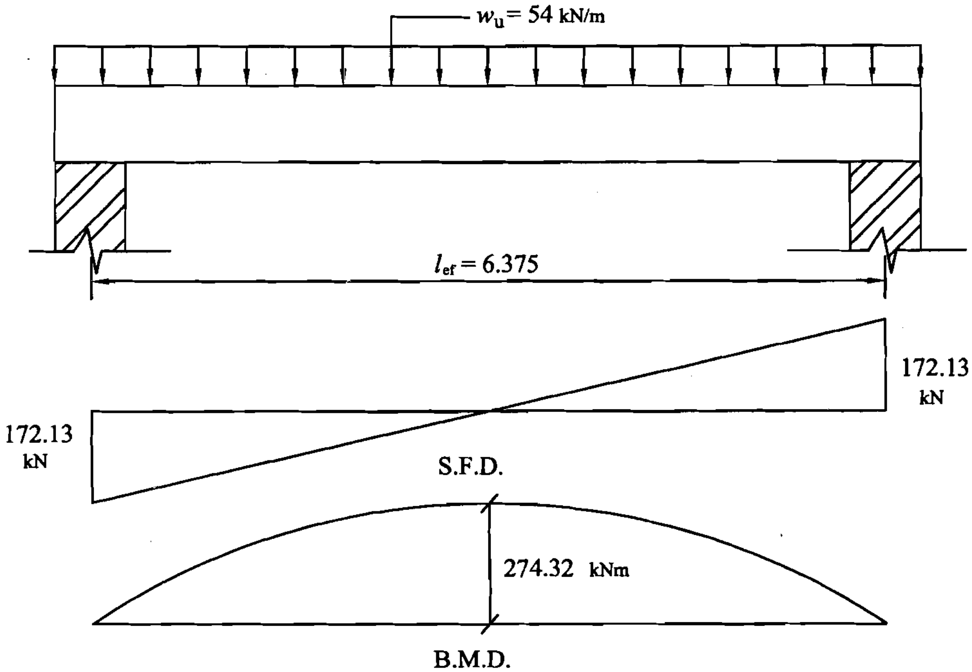


Figure 6.2 : S. F. D. and B. M. D. for the Beam

$M_{u,lim} = 0.148\, f_{ck}\, bd^2$

$$\text{or, } d = \sqrt{\frac{274.32 \times 10^6}{0.148 \times 15 \times 400}} = 555.8 < 760$$

Now l_{ef} = lesser of

(i) 6 + 0.375 = 6.375 m, and

(ii) 6 + 0.76 = 6.76 m

Thus l_{ef} = 6.375 m

Check for b from Lateral Stability Consideration

(i) $60b = 60 \times 400 \times 10^{-3} = 24\text{m} >> l_{ef}\ (6.375\text{m})$, and

(ii) $\frac{250b^2}{d} = \frac{250 \times 400^2}{760} = 52.632 \text{ m} >> l_{ef}\ (6.375 \text{ m})$

A_{st}

$$M_u = 0.87 f_y A_{st} d \left(1 - \frac{A_{st} f_y}{b d f_{ck}}\right)$$

$$\text{or, } 274.32 \times 10^6 = 0.87 \times 250 \times A_{st} \times 760 \left(1 - \frac{A_{st} \times 250}{400 \times 760 \times 15}\right)$$

or, $9.06\, A_{st}^2 - 165300\, A_{st} + 274.32 \times 10^6 = 0$

or, $A_{st} = 1846.38 \text{ mm}^2$

$$A_{st,min} = \frac{0.85}{f_y} bd = \frac{0.85 \times 400 \times 760}{250} = 1033.6\,mm^2 < 1846.38\,mm^2$$

$$A_{st,max} = \frac{1.32\,bd}{100} = \frac{1.32 \times 400 \times 760}{100} = 4012.8\,mm^2 > 1816.38\,mm^2$$

Provided 400×800 section with 6φ22 (A_{st} = 1888.4 mm² > 1846.38mm²) in one layer at d = 760

Curtailment and Detailing of Tension Reinforcement

If three out of six bars are curtailed, then moment of resistance of the section with continuing 3φ20,

$$M_u = 0.87 f_y A_{st} d \left(1 - \frac{A_{st} f_y}{bd f_{ck}}\right)$$

$$= 0.87 \times 250 \times 3 \times \frac{\pi}{4} \times 20^2 \times 760 \left(1 - \frac{3 \times \frac{\pi}{4} \times 20^2 \times 250}{400 \times 760 \times 15}\right) = 147.74\text{ kNm}$$

Let x be the distance of theoretical cut-off point from the L.H. supports, then from

M_u = R.x – w_u x² where R = Reaction at L.H. support

or, 147.74 = 172.13 x – 54 x²/2

or, x' = 1.02 m and 5.355 m from L.H. support

i.e. 2.17 m from either side of mid point of the span.

Actual cut-off point from mid section

= 2.17 + (> of 12 φ or d)

= 21.7 + 0.76 = 2.93 m

$$L_d = \frac{0.87 f_y d}{4\tau_{bd}} = \frac{0.87 \times 250 \times 20}{4 \times 1} = 1.0875 < 2.93\text{m}$$

Length available beyond the actual cut-off point towards supports

$$= \frac{6.375}{2} - 2.93 = 0.2575\text{m}$$

Provision of Shear Reinforcement

(a) Shear reinforcement at the *face* of the support

$$V_u = \frac{w_u l}{2} = \frac{54 \times 6}{2} = 162\text{kN}$$

$$\tau_v = \frac{V_u}{bd} = \frac{162 \times 1000}{400 \times 760} = 0.533 \text{N}/\text{mm}^2$$

$$\frac{100A_s}{bd} = \frac{100 \times 3 \times \frac{\pi}{4} \times 20^2}{400 \times 760} = 0.31$$

Accordingly, $\tau_c = 0.35 + \frac{(0.46 - 0.35)}{(0.5 - 0.25)} \times (0.31 - 0.25) = 0.38 \frac{\text{N}}{\text{mm}^2}$

$t_c < t_n$; hence shear reinforcement shall be provided for

$$V_{us} = V_u - \tau_c bd = 162 - 0.38 \times 400 \times 760 \times 10^{-3} = 46.48 \text{kN}$$

$$V_{us} = \frac{0.87 f_y A_{sv} d}{s_v} = \frac{0.87 \times 250 \times 100.53 \times 760}{s_v}$$

or $s_v = 357.52$ c/c

Check for minimum shear reinforcement

s_v shall be the least of

(i) $0.75d = 0.75 \times 760 = 570$

(ii) 450, and

(iii) $\frac{0.87 f_y A_{sv}}{0.4b} = \frac{0.87 \times 250 \times 100.53}{0.4 \times 400} = 136.65$

Hence £ 8-2 legged vertical stirrups @135 c/c may be provided.

(b) Shear reinforcement at the *cut-off* section

$$V_u = \frac{w_u l_{ef}}{2} w_u (3 - 2.93)$$

$$= 162 - 54 \times 0.07 = 158.22 \text{ kN}$$

$$\tau_v = \frac{V_u}{bd} = \frac{158.22 \times 10^3}{400 \times 760} = 0.52 \frac{\text{N}}{\text{mm}^2} > 0.38 \frac{\text{N}}{\text{mm}^2} (\tau_c)$$

$$V_{us} = V_u - \tau_c bd = 158.22 - 0.38 \times 400 \times 760 \times 10^{-3} = 42.7 \text{kN}$$

This shear reinforcement shall be provided over a distance of

$$\frac{3}{4} d = \frac{3}{4} \times 760 = 570$$

from the cut-off section.

$$A_{sv} = \left(\frac{V_{us} s_v}{0.87 f_y d} + \frac{0.4 b s_v}{250} \right)$$

$$\text{or } 100.53 = \left(\frac{42.7 \times 10^3}{0.87 \times 250 \times 760} + \frac{0.4 \times 400}{250} \right) s_v$$

$$\text{or } s_v = 111.91 \text{ c/c}$$

$$= s_{v,\max} = \frac{d}{8\beta_b} = \frac{760}{8 \times 0.5} = 190 \quad \text{c/c (where } \beta_b = \frac{\text{Area of bars cut-off}}{\text{Total area of bars}} = 0.5)$$

Hence provided ϕ 8 - 2 legged vertical reinforcement @ 110 c/c towards both supports. As the distance 570 > (3000 – 2930 = 70), the same reinforcement shall continue upto the face of support. The shear reinforcements may be provided in an usual way (Unit 3) beyond cut-off point towards centre of beam.

Detailing Near the Supports

(i) $L_d = 1088$

Length of bars inside the support for L_d beyond the cut-off point

$= 1088 - (3000 - 2930) = 1018$

The bars shall be bent up with a standard hook of 90° bend.

(ii) Again $L_d = \frac{1.3 M_1}{V} + L_0$

where $M_1 = 147.74$ kNm

$$V_u = \frac{w_u l_{ef}}{2} = \frac{54 \times 6.375}{2} = 172.13\text{kN}$$

$$\text{or, } 1080 = \frac{1.3 \times 147.74 \times 10^6}{172.13 \times 10^3} + L_0$$

or, $L_0 = (-)\ 27.8$

This means that L_0 is not required for the diameter of bars provided.

(iii) Minimum length of bar inside the support

$$= \frac{L_d}{3} = \frac{1080}{3} = 362.7 < 1018$$

Side Face Reinforcement

$$A_s = \frac{0.1}{100} \times 400 \times 760 = 304 \text{mm}^2$$

Provided 2ϕ10 bars (A_s = 156 mm² > 304/2 mm²) on each face

The details have been shown in Figure 6.3.

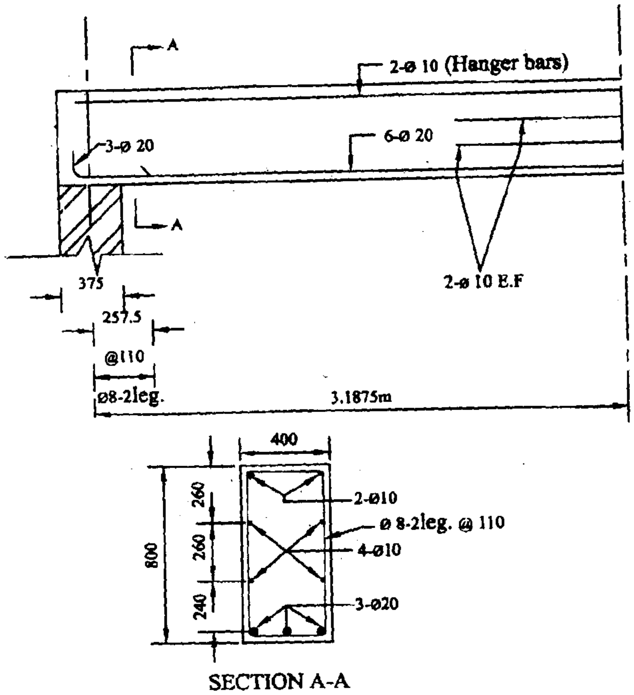


Figure 6.3 : Details of the Beam Designed

SAQ 1

Design and detail a simply supported R.C. beam of 5m clear span supported on walls 380 thick to carry a total dead and live load (excluding self load) of 25 kN/m. Use M 15 concrete and Fe 415 steel.

6.3 DESIGN OF A CANTILEVER BEAM

Example 6.2

Design a cantilever beam for the superimposed dead and live loads shown in Figure 6.4 below. Use M15 concreate and Fe 250 steel.

Solution

Depth (*D*)

(i) *From Control of Deflection Criteria*

$$d \nless \frac{l_{ef}}{K_B K_1 K_2 K_3}$$

where $K_B = 7$

For M15 concrete and Fe 250 steel, the balanced section steel area = 1.32% for which

$K_1 = 1.3$

$K_2 = K_3 = 1$

Substituting above values

$$d \nless \frac{4000}{7 \times 1.3 \times 1 \times 1} = 439.56 \text{ mm}$$

Adopted $D = 850$ at fixed end and $b = 400$ throughout.

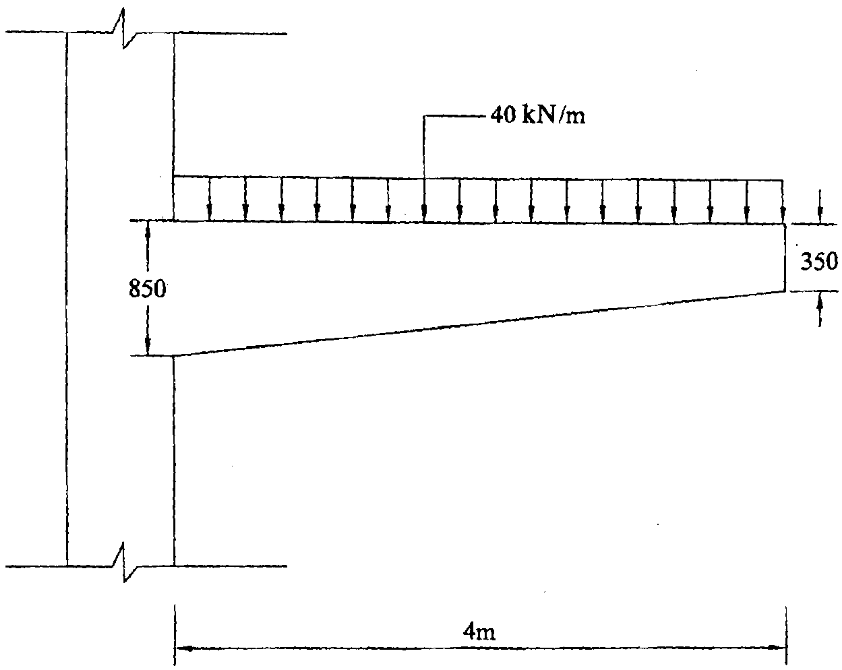


Figure 6.4 : A Cantilever Beam

(ii) *From Moment of Resistance Consideration*

Loads

Self $= \left(\frac{0.85 + 0.35}{2} \right) \times 0.4 \times 1 \times 25$ $= 6\text{kN/m}$

Superimposed Load $= 40$ kN/m

Total load $= 46$ kN/m

Design loads, $w_u = 1.5 \times 46$ $= 69$ kN/m

Design Moment, $M_u = \frac{w_u l_{ef}^2}{2} = \frac{69 \times 4^2}{2}$ $= 55.2$ kN/m

Effective depth (d) for resisting applied moment is obtained by

$M_{u,lim} = 0.148 f_{ck} \, bd^2$

$552 \times 10^6 = 0.148 \times 15 \times 400 \times d^2$

or, $d = 788.43$

Assuming effective cover 50,

$d = 850 - 50 = 800 > 788.43$

Check for b from lateral stability consideration

$25\,b = 25 \times 400 \times 10^{-3} = 10\text{m} >> l_{ef}\,(4\text{m})$

$$\frac{100b^2}{d} = \frac{100 \times 400^2}{800} \times 10^{-3} = 20 >> l_{ef}\,(4\text{m})$$

A_{st}

$$M_u = 0.87 f_y A_{st} d \left(1 - \frac{A_{st} f_y}{bdf_{ck}}\right)$$

or, $$55.2 \times 10^6 = 0.87 \times 250 \times A_{st} \times 800 \left(1 - \frac{A_{st} \times 250}{400 \times 800 \times 15}\right)$$

or, $9.06\,A_{st}^2 - 174000\,A_{st} + 552 \times 10^6 = 0$

or, $A_{st} = 4009.46\ \text{mm}^2$

$$A_{st,min} = \frac{0.85bd}{f_y} = \frac{0.85 \times 400 \times 800}{250} = 1088\,\text{mm}^2 < 4009.46\,\text{mm}^2$$

Hence provided 400 × 850 section at fixed end with 5φ32 (A_{st} = 4021.24 mm²)

Curtailment and Detailing of Tension Reinforcement

Let 2φ32 be curtailed at x from the free end, then moment of resistance of the section of continuing 3φ32 with 50 cover throughout the length

$$M_R = 0.87 f_y A_{st} \left\{\left(d_e + 0.125x\right) - \frac{A_{st} f_y}{bf_{ck}}\right\}$$

where d_e = effective depth at free end

$$= 0.87 \times 250 \times 3 \times \frac{\pi}{4} \times 32^2 \left\{\left(300 + 0.125x\right) - \frac{3 \times \frac{\pi}{4} \times 32^2 \times 250}{400 \times 15}\right\}$$

$= 1.0467 \times 10^8 + 65596.456\,x$

Applied B.M. at x from free end

$$M_u = \frac{69x^2}{2} = 34.5x^2$$

Equating M_R with M_u

$$34.5\,x^2 = (1.046 \times 10^8 + 65596.456x) \times 10^{-6}$$

or x = 1.74 m

$\therefore d_x = 300 + 0.125\,x = 300 + 0.125 \times 1740 = 517.5$

Actual cut-off point from *fixed* end

$= 4 - 1.74 + (> \text{of } 12\phi \text{ or } d_x) = 2.26 + 517.5 \times 10^{-3}$

$= 2.78$ m

$$L_d = \frac{\phi\sigma_s}{4\tau_{bd}} = \frac{32 \times 0.87 \times 250 \times 10^{-3}}{4 \times 1} = 1.74\,\text{m}$$

The tensile reinforcement will extend both sides of the face of the support, a length $\geq L_d$, and the continuing 3ϕ32 towards free end from cut-off section shall also extend $\geq L_d$ (Figure 6.5).

Provision of Shear Reinforcement

(i) At support

$V_u = 69 \times 4 = 276$ kN

$M_u = 552$ kNm

$$\tau_v = \frac{V_u - \frac{M_u}{d}\tan\beta}{bd}$$

$$\text{where } \tan\beta = \frac{850 - 350}{4000} = 0.125$$

$$\text{or } \tau_v = \frac{276 \times 10^3 - \frac{552 \times 10^6}{800} \times 0.125}{400 \times 800} = 0.593 \frac{\text{N}}{\text{m}^2}$$

$$\frac{100A_s}{bd} = \frac{100 \times 5 \times \frac{\pi}{4} \times 32^2}{400 \times 800} = 1.26\%$$

$$\therefore \tau_c = 0.64 + \frac{(0.68 - 0.64)}{(1.5 - 1.25)} \times (1.26 - 1.25)$$

$$= 0.642 \frac{\text{N}}{\text{mm}^2} > \tau_v \left(= 0.593 \frac{\text{N}}{\text{mm}^2}\right)$$

Taking ϕ8 - 2 legged vetical stirrups as shear reinforcement

$$s_{v,max} = \frac{0.87 f_y A_{sv}}{0.4b} = \frac{0.87 \times 250 \times 100.53}{0.4 \times 400} = 136.66$$

$$s_v < 0.75\,d\ (= 0.75 \times 800 = 600)$$

$$s_v < 450$$

(ii) At cut-off section

Actual cut-off section from *free* end

$= 4 - 2.78 = 1.22$ m

$\therefore$ Shear force and B.M. at cut-off section,

$V_u = w_u x' = 69 \times 1.22 = 84.18$ kN

$$M_u = \frac{w_u x'^2}{2} = \frac{69 \times 1.22^2}{2} = 51.35 \text{kN}$$

$d_x = 300 + 0.125\, x' = 300 + 0.125 \times 1220 = 452.5$

$$\tau_v = \frac{V_u - \frac{M_u}{d_x}\tan\beta}{bd} = \frac{84.18 \times 10^3 - \frac{51.35 \times 10^6 \times 0.125}{452.5}}{400 \times 452.5} = 0.387 \frac{\text{N}}{\text{mm}^2}$$

$$\frac{100 A_s}{bd} = \frac{100 \times 3 \times \frac{\pi}{4} \times 32^2}{400 \times 452.5} = 1.333\%$$

$$\therefore \tau_c = 0.64 + \frac{(0.68 - 0.64)}{(1.5 - 1.25)} \times (1.33 - 1.25)$$

$$= 0.65 MPa > 1.5\tau_v \left(= 1.5 \times 0.387 = 0.58 \frac{\text{N}}{\text{mm}^2} \right)$$

$s_{v,max} = 0.75\, d = 0.75 \times 452.5 = 339.375$ or 450

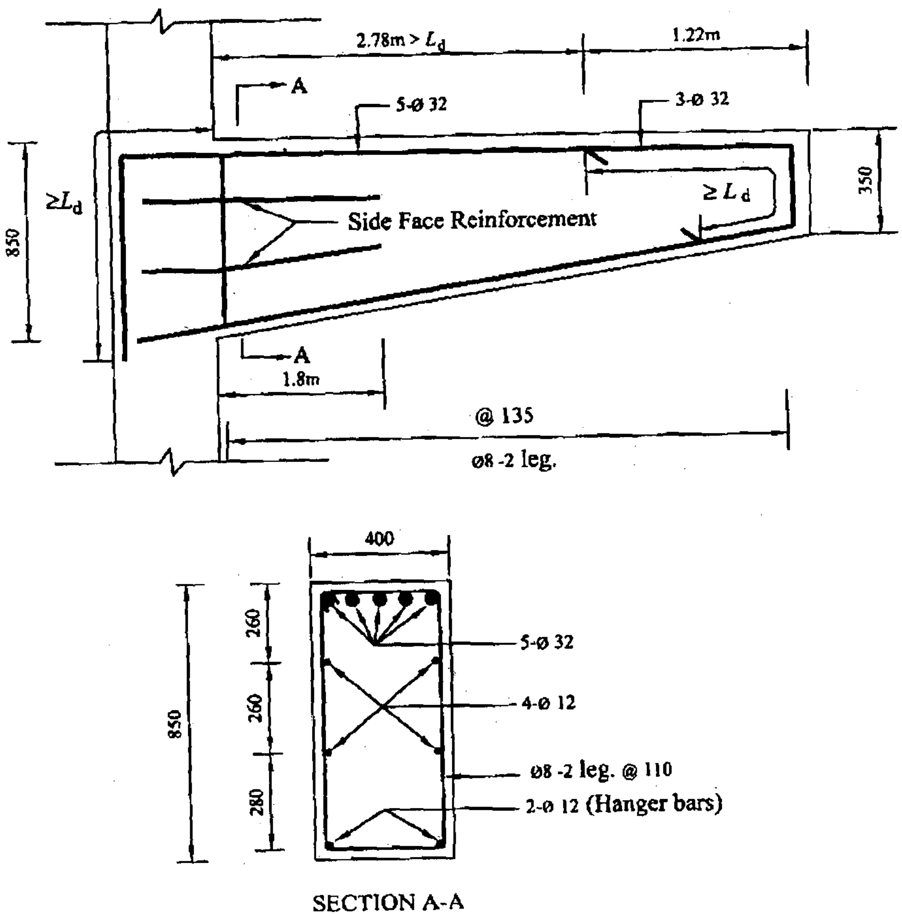


Figure 6.5 : Details of the Designed Beam

Provided $\phi 8$ - 2 legged stirrups @ 135 throughout

Side Face Reinforcement

Since $D>750$, side face reinforcement will be required for a distance x" from the free end where

$750 = 350 + 0.125$ x" or x" = 3.2 m

$A_s = 0.1/100 \times 800 \times 400 = 320\ mm^2$

Provided 2ϕ12 on each side face ($A_s = 452\ mm^2 > 320\ mm^2$)

The details of the designed beam have been shown in Figure 6.5.

SAQ 2

Design and draw a cantilever beam of uniform cross section throughout of a canopy of 3.5 m span for a dead and live loads of 15 kN/m and 8 kN/m respectively. The dead load does not include self load. Use M 20 concrete and Fe 415 steel.

6.4 DESIGN OF A CONTINUOUS BEAM

Example 6.3

Design a continuous beam (Figure 6.6) for superimposed dead and live loads of 10 kN/m and 12 kN/m respectively. Use M15 concrete and Fe 250 steel

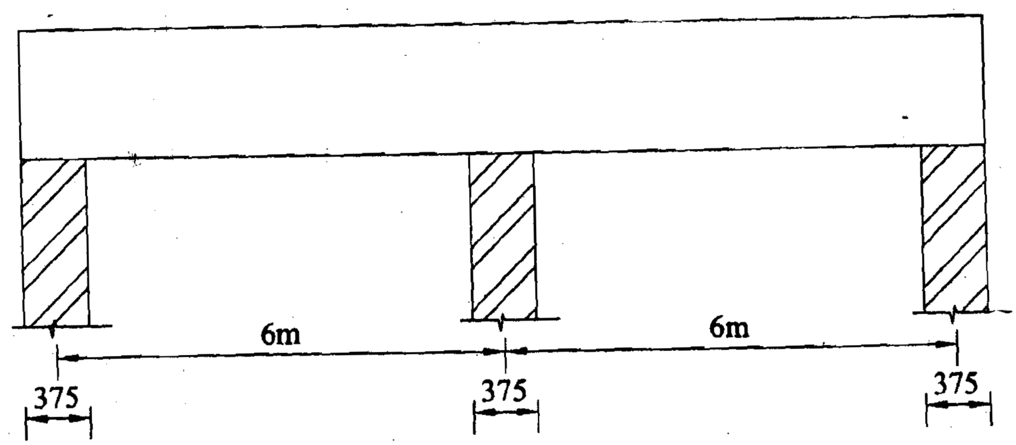


Figure 6.6 : A Continuous Beam

Solution

Depth (*D*)

(i) *Thumb Rule*

$$D \text{ lying between } \frac{l_{ef}}{10} \text{ to } \frac{l_{ef}}{20}$$

To start with taking l_{ef} = 6m

$$D = \frac{l_{ef}}{15} = \frac{6 \times 10^3}{15} = 400$$

(ii) *From Control of Deflection Criteria*

$$d \nless \frac{l_{ef}}{K_B K_1 K_2 K_3}$$

$K_B = \frac{20+26}{2} = 23$, since for each span, one of the supports is simple support and the other is continuous.

For M15 and Fe 250, $p_B\,\% = 1.32\,\%$ and

correspondingly, $K_1 = 1.3$

$K_2 = K_3 = 1$

$$\therefore d \nless \frac{6000}{23 \times 1.3 \times 1 \times 1} = 200.67$$

Assuming $D = 600$; $d = 600 - 55 = 545$ and taking b (between $\frac{1}{3}$rd to $\frac{2}{3}$rd D) = 300

Check b from Lateal Stability Consideration

Let the beam be laterally supported at 6 m interval i.e. at supports,

$l_{ef} < 60b$; or $l_{ef}\,(6\text{m}) < 60 \times 300 (= 18\text{m})$

or $l_{ef} < \frac{250b^2}{d}$; or $b > \sqrt{\frac{545 \times 6000}{250}} = 114.3$

Loads

Self	$= 0.6 \times 0.3 \times 1 \times 25$	= 4.5 kN/m
DL		= 10.0 kN/m
Total *DL*		= 14.5 kN/m
LL		= 12.0 kN/m
Total (*DL* + *LL*)		= 26.5 kN/m
Design *DL*	$= 1.5 \times 14.5$	= 21.75 kN/m
Design *LL*	$= 1.5 \times 12.0$	= 18.0 kN/m
Design (*DL* + *LL*)	$= 1.5 \times 26.5$	= 39.75 kN/m

*** Evaluation of Design B.Ms & S. Fs. at Critical Sections**

$$M_s = K_s\, w_u\, l_{ef}^2 \text{ and } M_n = K_n w_u\, l_{ef}^2$$

where M_s and M_n are span and support B.Ms and K_s and K_n are their corresponding moment coefficients respectively. Similarly V_u = S.F. at the centre of support (Figure 6.7 (a) to (f)).

* Refer Appendix A.

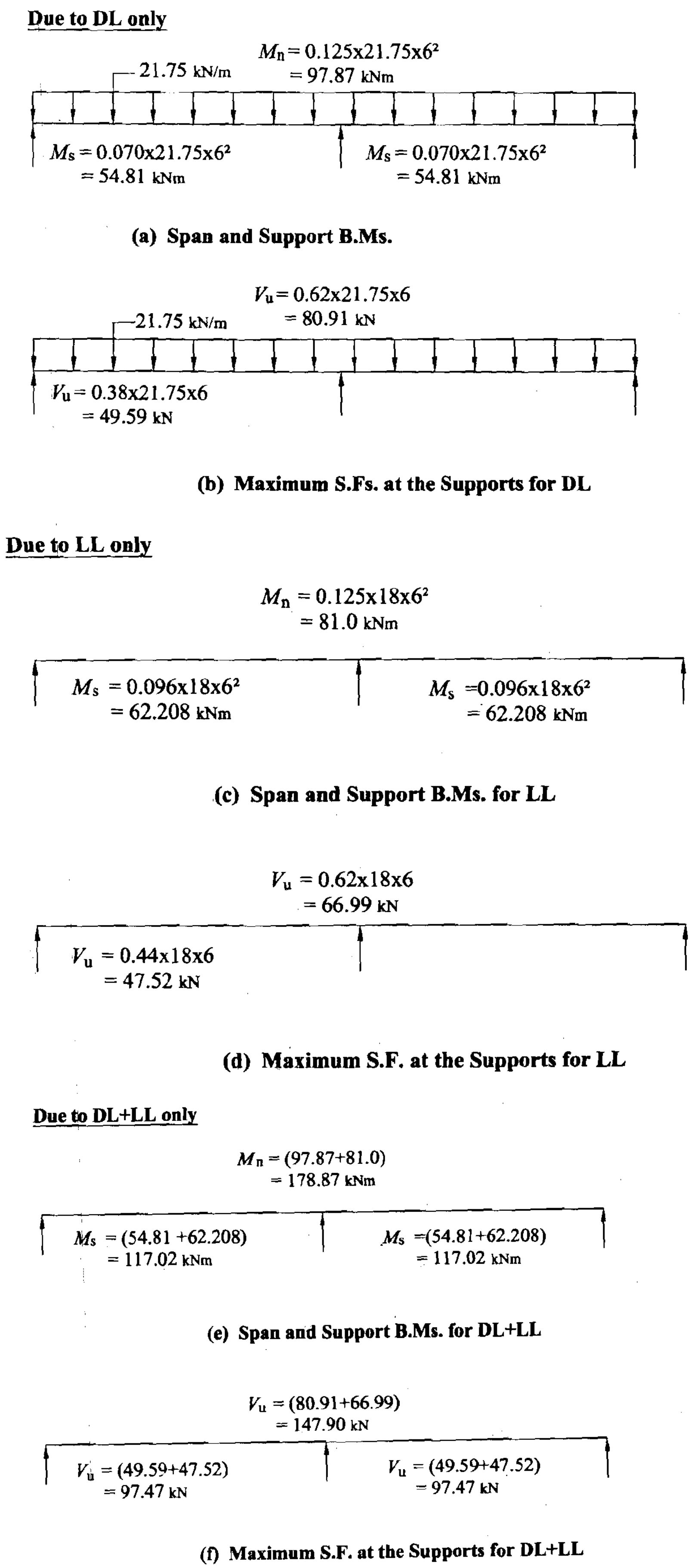


Figure 6.7 : Evaluation of B. Ms and S. Fs at Critical Sections

(iii) *From Moment of Resistance Consideration*

$$M_u = M_{u,\lim} = 0.36 \frac{x_{u,\max}}{d}\left(1 - 0.42 \frac{x_{u,\max}}{d}\right) bd^2 f_{ck}$$

$$178.87 \times 10^6 = 0.36 \times 0.53(1 - 0.42 \times 0.53) \times 300 \times d^2 \times 15$$

or, $d = 518.24 < 542.5$

(where adoted $d = 600 - 25 - 20 - 12.5 = 542.5$ for main reinforcement of $\phi 20$ in two layers and a clear spacing of 25 between two layers of bars)

Moment of resistance for balanced secion

$$M_{u,\lim} = 0.36 \frac{x_{u,\max}}{d}\left(1 - 0.42 \frac{x_{u,\max}}{d}\right) bd^2 f_{ck}$$

$$= 0.36 \times 0.53(1 - 0.42 \times 0.53) \times 300 \times 542.5^2 \times 15 \times 10^{-6}$$

$= 196.01$ kNm > 178.87 kNm

Hence the section is under-reinforced.

Provision of Main Reinforcement

In span

$$M_u = 0.87 f_y A_{st} d\left(1 - \frac{A_{st} f_y}{bd.f_{ck}}\right)$$

$$\text{or } 117.02 \times 10^6 = 0.87 \times 250 \times A_{st} \times 542.5\left(1 - \frac{A_{st} \times 250}{300 \times 542.5}\right)$$

or $12.08\,A_{st}^2 - 117993.75\,A_{st} + 117.02 \times 10^6 = 0$ or $A_{st} = 1120.20\ \text{mm}^2$

$$A_{st,min} = \frac{0.85}{f_y} bd = \frac{0.85}{250} \times 300 \times 542.5 = 553.35\,\text{mm}^2 < 1120.22\,\text{mm}^2$$

Hence provided 6 ϕ 16 (A_{st} = 1206.37 mm²) in two layers

At Intermediate support

$$M_u = 0.87 f_y A_{st} d\left(1 - \frac{A_{st} f_y}{bd.f_{ck}}\right)$$

$$178.87 \times 10^6 = 0.87 \times 250 \times A_{st} \times 542.5\left(1 - \frac{A_{st} \times 250}{300 \times 542.5 \times 15}\right)$$

$$\text{or } 12.08 A_{st}^2 - 117993.75 A_{st} + 178.87 \times 10^6 = 0$$

$$\text{or } A_{st} = 1876.38\,\text{mm}^2 > A_{st,min}\left(553.35\,\text{mm}^2\right)$$

Provided 10 ϕ 16 (A_{st} = 2010.62 mm²) in two layers

Detaling of Main Reinforcement

At Simple Support

Let 4 ϕ16 be curtailed and the remaining 2ϕ16 be continued in the simple support. Moment of resistance of the section with 2ϕ16,

$$M_1 = 0.87 f_y A_{st} d\left(1 - \frac{A_{st} f_y}{bd.f_{ck}}\right)$$

$$= 0.87 \times 250 \times 2\frac{\pi}{4} \times 16^2 \times 542.5\left(1 - \frac{2 \times \frac{\pi}{4} \times 16^2 \times 250}{300 \times 542.5 \times 15}\right) \times 10^{-6}$$

$$= 45.49 \text{kNm}$$

Let x_m be the distance of curtailment of 4ϕ16 from the outer support, then

$$M_1 = 45.49 = 97.47x - \frac{39.75x^2}{2}$$

or $19.875\ x^2 - 97.47\ x + 45.49 = 0$

or x = 0.52 m and 4.38 m

From centre line of end support, the actual cut-off point

$= 0.52 - (> \text{of } 12\phi \text{or } d)$

$= 0.52 - 0.5425 = -0.0225$ m

$$L_d = \frac{0.87 f_y \phi}{4\tau_{bd}} = \frac{0.87 \times 250 \times 16}{4 \times 1} = 870$$

The continuing bars must extend into the support by a distance

$$> \frac{L_d}{3}\left(= \frac{870}{3} = 290\right)$$

V at the centre line of support

= 97.47 kN

$$L_d < \frac{1.3 M_1}{V} + L_0$$

$$\text{or } 870 < \frac{1.3 \times 45.49 \times 10^{-6}}{97.47 \times 10^3} + L_0$$

or $L_0 > 263.28$

Maximum straight length available from centre line of support to the outer face of the bent bars

$$= \frac{375}{2} - 25 = 162.5 \text{ (Figure 6.8)}$$

Now 263.28 = 162.5 + 8 ϕ + x

or x = − 27.22

Hence only standard hook of 90° bend shall be sufficient (Figure 6.8).

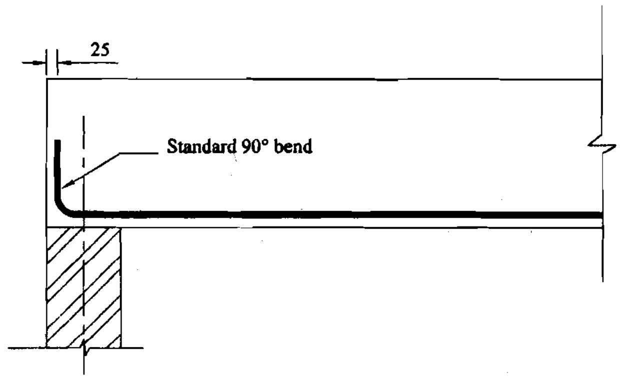


Figure 6.8 : Detailing of Positive Reinforcement at End Support

***Positive* Bending Moment Reinforcement at Intermediate support**

Actual cut-off section from centre line of end support

= 4.38 + (>12 ϕ or *d*) = 4.38 + 0.5425

≐ 4.923 m

i.e. 1.077 m from intermediate support. The continuing bars shall extend into the support by at least $\frac{L_d}{3} = \frac{870}{3} = 290$

Let point of inflexion be at x from end support, then

$$0 = 97.47x - 39.75\frac{x^2}{2} \text{ ; or } x = 4.9 \text{ m}$$

S.F. at point of inflexion, V = 97.4 − 39.75 × 4.9

= 97.305 kN

$$L_d > \frac{M_1}{V} + L_0$$

$$\text{or } 870 > \frac{45.49 \times 10^6}{97.305 \times 10^3} + L_0$$

or L_0 > 0.402 m < 1.18 m (Figure 6.9)

Negative B.M. Reinforcement at Intermediate Support

Let half of the (-) tive B.M. reinforcement be curtailed, then moment of resistance of the section with remaining 5 ϕ16

$$M_u = 0.87 f_y A_{st} d\left(1 - \frac{A_{st} f_y}{bd.f_{ck}}\right)$$

$$= 0.87 \times 250 \times 5 \times \frac{\pi}{4} \times 16^2 \times 542.5 \left(1 - \frac{5 \times \frac{\pi}{4} \times 16^2 \times 250}{300 \times 5 \times 542.5 \times 15} \right) \times 10^{-6} = 106.41 \text{kNm}$$

Let this B.M. occur at x from centre line of end support then

$$-106.41 = 97.47x - \frac{39.75x^2}{2} \text{ ; or, } x = 5.824 \text{ m}$$

i.e. theoretical cut-off point from centre line of intermediate support

$= 6 - 5.824 = 0.176$ m

Therefore, actual cut-off point from centre line of intermediate support

$= 0.176 + (> \text{of } 12\,\phi \text{ or } d)$

$= 0.176 + 0.5425 = 719 \text{ m} < 0.87 \text{ m } (L_d)$

Hence, all (-)tive B.M. reinforcement shall extend upto 0.87 m from intermediate support. The remaining 5 $\phi 16$ must extend beyond the inflexion point for a distance greater of $d = 542.5$

or $12\,\phi = 12 \times 16 = 192$

or $\frac{L_c}{16} = \frac{1}{16}\,(6000 - 375) = 351.56$

Therefore remaining 5 $\phi 16$ shall extend upto $(1.077 + 0.5425)$ m = 1.62 m (Figure 6.9)

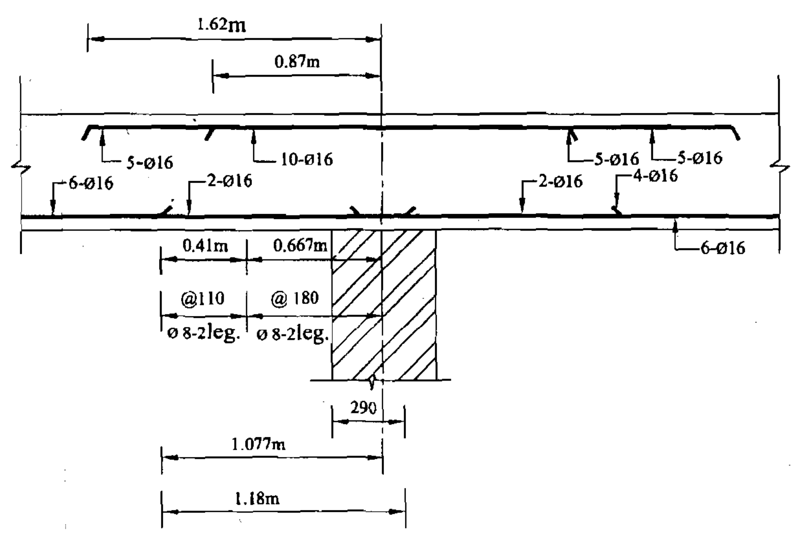


Figure 6.9 : Showing Details of Reinforcement Near Intermediate Support

Detailing of Transverse Reinforcement at Intermediate Support

S.F. at the *face* of support,

$$V_u = 147.9 - 39.75 \times \frac{0.375}{2} = 140.45 \text{ kN}$$

$$\frac{100A_s}{bd} = \frac{100 \times 10 \times \frac{\pi}{4} \times 16^2}{300 \times 542.5} = 1.235\%$$

Accordingly,

$$\tau_c = 0.6 + \frac{(0.64 - 0.6)}{(1.25 - 1.0)} \times 0.235 = 0.638$$

$$V_{us} = V_u - \tau_c bd = 140.45 - 0.638 \times 300 \times 542.5 \times 10^{-3}$$

$$= 36.616 \text{ kN}$$

Taking ϕ 8-2 leg. stp

$$V_{us} = \frac{0.8 f_y A_{sv} d}{s_v}$$

$$\text{or; } s_v = \frac{0.87 \times 250 \times 100.53 \times 542.5}{36.616 \times 10^{-3}}$$

$$= 323.95 < 450 < 0.75d(406)$$

From minimum shear reinforcement consideration

$$\frac{A_{sv}}{bs_v} \geq \frac{0.4}{0.87 f_y}$$

$$\text{or } s_v \leq \frac{A_{sv} \times 0.87 \times f_y}{0.4b}$$

$$\leq \frac{100.53 \times 0.87 \times 250}{0.4 \times 300} \leq 182.21 < 323.95$$

Shear reinforcement of cut-off point of (+) tive B.M. reinforcement near intermediate support

S.F. = V_u = 147.9– 39.75 × 1.077 = 105.0 kN

$$\tau_v = \frac{105.09 \times 1000}{300 \times 542.5} = 0.65 \frac{\text{N}}{\text{mm}^2}$$

$$\frac{100A_s}{bd} = \frac{100 \times 2 \times 201}{300 \times 542.5} = 0.25$$

Accordingly τ_c = 0.35 N/mm²

$V_{us} = V_u - \tau_c bd$ = 105.09 – 0.35 × 300 × 542.5 × 10^{-3} = 48.12 kN

$$V_{us} = \frac{0.87 f_y A_{sv} d}{s_v}$$

$$\text{or } A_{sv} = \frac{V_{us} s_v}{0.87 f_y d}$$

$$A_{ex} = \frac{0.4 b s_v}{f_y}$$

∴ Total area of transverse reinforcement.

$$A_s = A_{sv} + A_{ex} = 100.53 = \left(\frac{V_{us}}{0.87 f_y d} + \frac{0.4b}{f_y} \right) s_v$$

$$\text{or; } s_v = \frac{100.53}{\left(\frac{48.12 \times 10^3}{0.87 \times 250 \times 542.5} + \frac{0.4 \times 300}{250} \right)} = 113.23 < 182.2 (s_{v,\,max})$$

$$\beta_b = 0.5$$

$$\therefore \frac{d}{\beta_b} = \frac{542.5}{8 \times 0.5} = 135.625 > 113.5$$

$$\frac{3}{4} d = \frac{3}{4} \times 542.5 = 406.87$$

Hence provided ϕ8 -2 legged step @ 110 for a distance 406 from the cutoff point. (Figure 6.9)

Shear reinforcements for other portion of the beam may be provided in usual way (Unit 3)

SAQ 3

Design and detail a continuous beam of two spans each of 6m between their supports for a total load (excluding self weight) of 30 kN/m. The beam is monolithically supported on columns. Use M 20 concrete and 415 steel

6.5 DESIGN OF A BEAM WITH OVERHANG

Example 6.4

Design an overhang beam loaded with a dead load of 16 kN/m and a live load of 12 kN/m shown in Figure 6.10. Use M 15 concrete and Fe 415 steel.

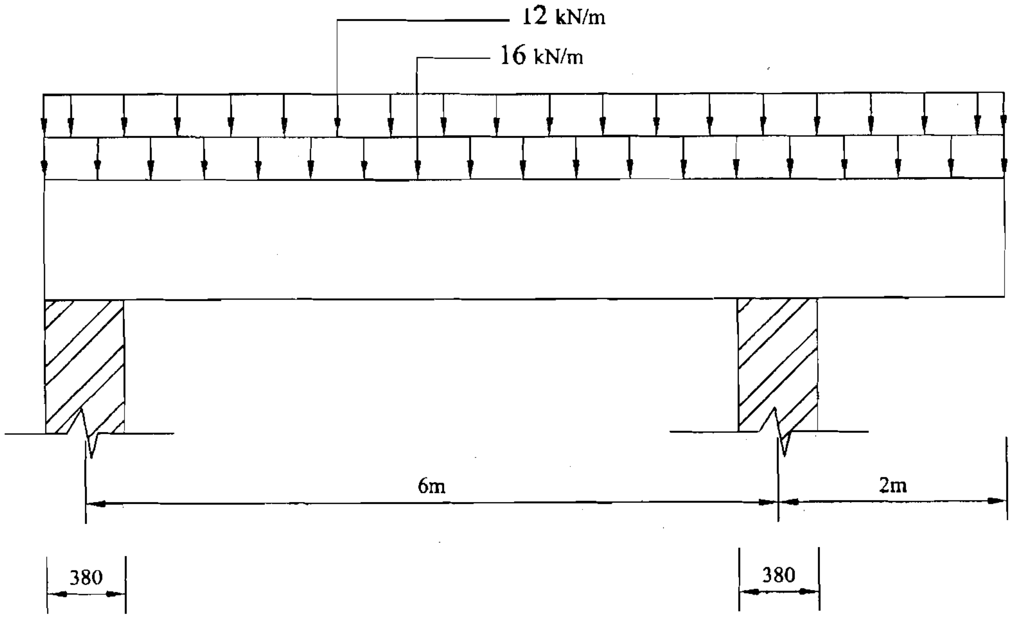


Figure 6.10 : An Overhanging Beam

Solution

Depth (*D*)

(i) *From Thumb Rule*

For simply supported position

$$D \approx \frac{l_{ef}}{10} \text{ to } \frac{l_{ef}}{20}$$

Taking $D = \frac{l_{ef}}{10} = \frac{6000}{10} = 600$ at the start

(ii) *From Deflection Criteria*

For simply supported portion

$$d > \frac{l_{ef}}{K_B K_1 K_2 K_3}$$

$$K_B = \frac{20 + 26}{2} = 23$$

Taking $\frac{l_{ef}}{d} = \left(\frac{20+26}{2}\right) = 23$ since one of the supports is simply supported and the other is continuous.

p_{max} = 0.72% for M 15 concrete and Fe 415 steel accordingly K_1 = 1.05

$K_2 = K_3 = 1$

Substituting these values in the above equation

$$d \nless \frac{6 \times 10^3}{23 \times 1.05 \times 1 \times 1} = 248.45$$

For cantilever portion

$$d \nless \frac{l_{ef}}{K_B K_1 K_2 K_3}$$

where $K_B = 7$

$$\therefore d = \frac{6 \times 10^3}{7 \times 1.05 \times 1 \times 1} = 272.11$$

As maximum (+)tive bending moment in simply supported portion is, to some extent, balanced by the overhang, the depth required will not be the same as that for simply supported beam (Example 6.1). Thus adopted $D = 600$ and $b = 300$

$$d \approx 600 - 50 = 550$$

Check for b from Lateral Stability Consideration

Let the beam be laterally supported at supports

For simply supported portion

$60\,b = 60 \times 300 \times 10^{-3} = 18 > 6$m

$$\frac{250b^2}{d} = \frac{250 \times 300^2 \times 10^{-3}}{550} = 40.91\text{m} > 6\text{m}$$

For cantilever portion

$25b = 25 \times 300 = 7.5$ m > 2m

$$\frac{100b^2}{d} = \frac{100 \times 300^2 \times 10^{-3}}{550} = 16.36\text{m} > 2\text{m}$$

Hence O.K.

(iii) *D from Moment of Resistance Consideration*

Loads

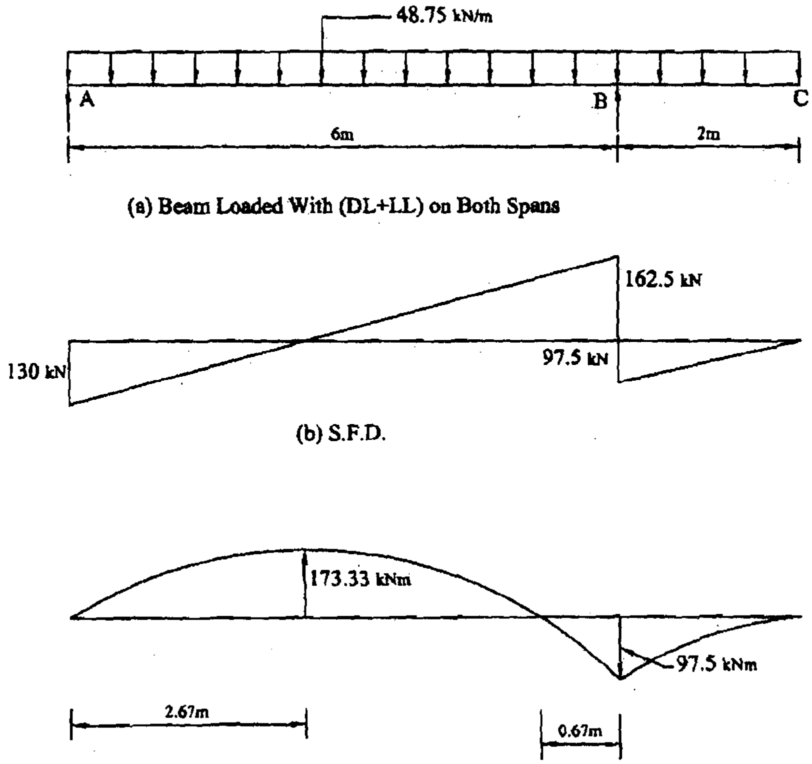
Self = $0.6 \times 0.3 \times 1 \times 25$	= 4.5 kN/m
DL	= 16.0 kN/m
Total *DL*	= 20.5 kN/m
LL	= 12.0 kN/m
Total (*DL* + *LL*)	= 32.5 kN/m
Design Load, $w_u = 1.5 \times 32.5$	= 48. 75 kN/m

$\Sigma M_A = 0$ (Figure 6.11)

or, $48.75 \times 8 \times 4 - R_B \times 6 = 0$

or, $R_B = 48.75 \times 8 \times 4/6 = 260$ kN

$\therefore R_A = 48.75 \times 8 - 260 = 130$ kN



(a) Beam Loaded With (DL+LL) on Both Spans

(b) S.F.D.

(c) B.M.D.

Figure 6.11 : S. F. D and B. M. D. for the Beam

Let $M_{u,max}$ occur at x from A in the span, then S.F. at that section will be zero i.e.,

$$130 - 48.75\,x = 0$$

or, $x = 2.67$ m

$$\text{Thus } M_{u,max} = 130 \times 2.67 - \frac{48.75 \times 2.67^2}{2} = 173.33 \text{kNm}$$

B.M. at support section

$$= \frac{48.75 \times 2^2}{2} = 97.5 \text{kNm}$$

Let the point of inflexion be at x from simple support i.e.

$$130x - \frac{48.75x^2}{2} = 0$$

or x = 5.33 m i.e. 0.67 from support *B*.

The resulting S.F.D. and B.M.D. are as shown in (Figure 6.11)

$$M_u = 0.36\frac{x_{u,\max}}{d}\left(1 - 0.42\frac{x_{u,\max}}{d}\right)bd^2 f_{ck}$$

$$= 0.36 \times 0.48(1 - 0.42 \times 0.48) \times 300 \times d^2 \times 15$$

or, $d = 528.38$

Assuming two layers of # 16 steel,

$d = 600 - 25 - 16 - 25/2 = 546.5 > 528.38$

Hence, provided $b \times D = 300 \times 600$

Provision of Main Reinforcement

(i) For span

$$M_u = 0.87 f_y A_{st} d\left(1 - \frac{A_{st} f_y}{bd.f_{ck}}\right)$$

$$\text{or, } 173.33 \times 10^6 = 0.87 \times 415 \times A_{st} \times 546.5\left(1 - \frac{A_{st} \times 415}{300 \times 546.5 \times 15}\right)$$

or, $33.296833\, A_{st}^2 - 197313.83\, A_{st} + 173.33 \times 10^6 = 0$

or, $A_{st}^2 - 5925.9\, A_{st} + 5205600.1 = 0$

$$\text{or, } A_{st} = \frac{5925.9 \pm \sqrt{5925.9^2 - 4 \times 1 \times 5205600.1}}{2}$$

or, $A_{st} = 1072.6\ \text{mm}^2$

$$A_{st,min} = \frac{0.85}{f_y} \times bd = \frac{0.85}{415} \times 300 \times 600 = 368.67\text{mm}^2 < 1072.6\text{mm}^2$$

Hence provided 6 # 16 (A_{st} = 1206.37 mm² > 1072.6 mm²)

(ii) At support *B*

$$M_u = 0.87 f_y A_{st} d\left(1 - \frac{A_{st} f_y}{bd.f_{ck}}\right)$$

$$\text{or, } 97.5 \times 10^6 = 0.87 \times 415 \times A_{st} \times 546.5\left(1 - \frac{A_{st} \times 415}{300 \times 546.5 \times 15}\right)$$

or, $A_{st}^2 - 5925.9\, A_{st} + 2928206.8 = 0$

$$\text{or, } A_{st} = \frac{5925.9 \pm \sqrt{5925.9^2 - 4 \times 1 \times 2928206.8}}{2}$$

$$\text{or; } A_{st} = \frac{5925.9 \pm 4837.7}{2} = 544.1\text{mm}^2 > 368.67(A_{st,min})$$

Hence provided 3#16 (A_{st} = 603.2 mm²)

Curtailment and Detailing of Tension Reinforcement

(i) At simple support

If three out of six (+)tive reinforceing bars are curtailed, then moment of resistance of the section with remaining three bars,

$$M_u = 0.87 f_y A_{st} d\left(1 - \frac{A_{st} f_y}{bd.f_{ck}}\right)$$

$$= 0.87 \times 415 \times 603.2 \times 546.5\left(1 - \frac{603.2 \times 415}{300 \times 546.5 \times 15}\right)$$

$$= 106.904 \text{ kNm}$$

Let this B.M. be at x from L.H. support, then

$106.904 = 130\,x - 48.75\,x^2/2$

or $x^2 - 5.33\,x + 4.3858 = 0$

$$\text{or } x = \frac{5.33 \pm \sqrt{5.33^2 - 4 \times 4.3858}}{2}$$

or; x = 1.017 m and 4.317 m

(a) These curtailed bars will be extended on both ends by greater of 12ϕ (192) or d(546.5)

(b) The curtailed bars have length greater than L_d (903) towards the maximum bending moment i.e. towards the critical section

(c) The continuing bars, from cut-off point should have total length encluding L_0 greater than L_d and must extend into the support by $\frac{L_d}{3} = \frac{903}{3} = 301$

(d) The length of straight portion of into the support

= 380 − 25 − (4 + 1) × 16 = 275 < 301

Hence, the bars shall be bent with 90° bend.

(e) $L_d > \frac{1.3 M_1}{V} + L_0$ at L.H. support

$$\text{or; } L_0 < \frac{1.3 M_1}{V} - L_d$$

$$\text{where} \quad M_1 = 0.87 f_y A_{st} d\left(1 - \frac{A_{st} f_y}{bd.f_{ck}}\right)$$

$$= 0.87 \times 415 \times 603.2 \times 546.5 \left(1 - \frac{603.2 \times 415}{300 \times 546.5 \times 15}\right)$$

$$= 106.904 \text{ kNm}$$

V = 130 at centre line of support

$$\text{or; } L_0 < \frac{1.3 \times 106.904}{130} - 0.903 = 0.166 \text{m}$$

Let L_0 = Equivalent length of 90^0 bend + x

or; $166 = 8 \times 16 + x$

or; x = 38

The detailing on L.H. support has been done as shown in (Figure 6.12).

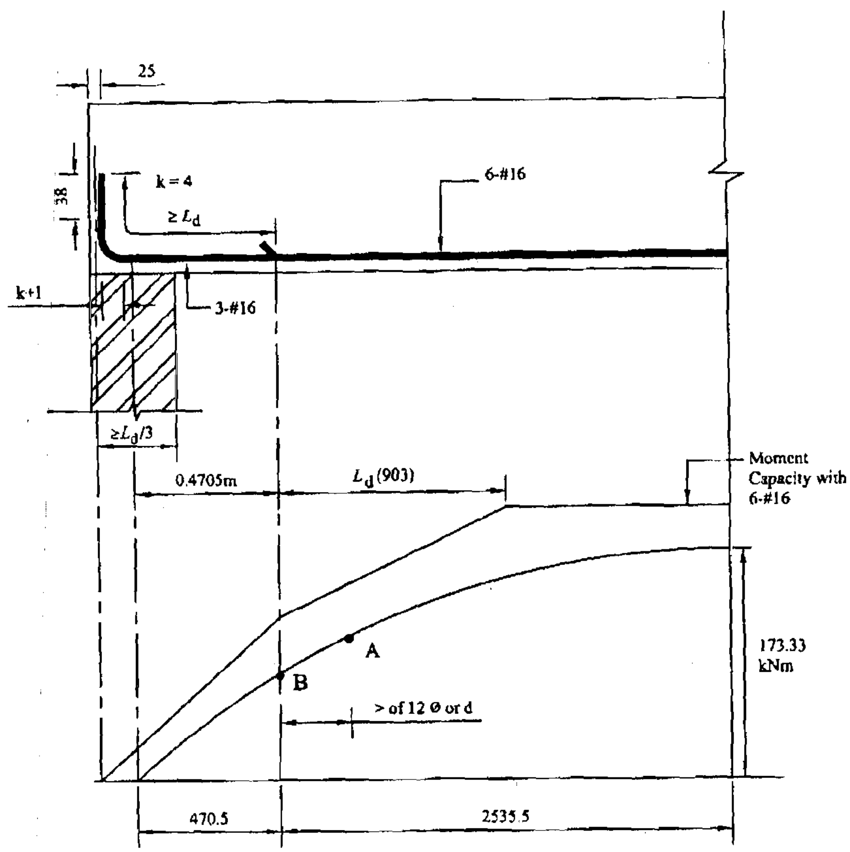


Figure 6.12 : Detailing of the Simple Support

Curtailment of (+)tive Reinforcement towards R.H. Support

(a) The continuing three bars shall extend upto $6 - 4.317 - 0.5465 = 1.1365 \text{ m} > 0.903 \text{ m } (L_d)$ (vide Figure 6.13)

(b) These three bars must extend into the support at length not less than

$$\frac{L_d}{3}(301) \text{ i.e.}$$

upto $\left(301 - \frac{380}{2}\right) = 111$ beyond the centre line of the support.

(c) As the point of inflexion is away from the support, factor 1.3 multiplication for M_1 / V will not apply

Thus $L_d > \frac{M_1}{V} + L_0$; or $L_0 < \frac{M_1}{V} - L_d$

where

M_1 = 106.904 kNm as before, and

V = S.F. at point of inflexion i.e. at 0.67m from centre line of support = 97.5 − 0.67 × 48.75 = 64.838

Substituting M_1 & V in the above equation.

$$L_0 > \frac{106.904}{64.838} - 0.903 = 0.746 > 0.5465(d)$$

The detailing of positive reinforcement of R.H. support is shown in (Figure 6.13).

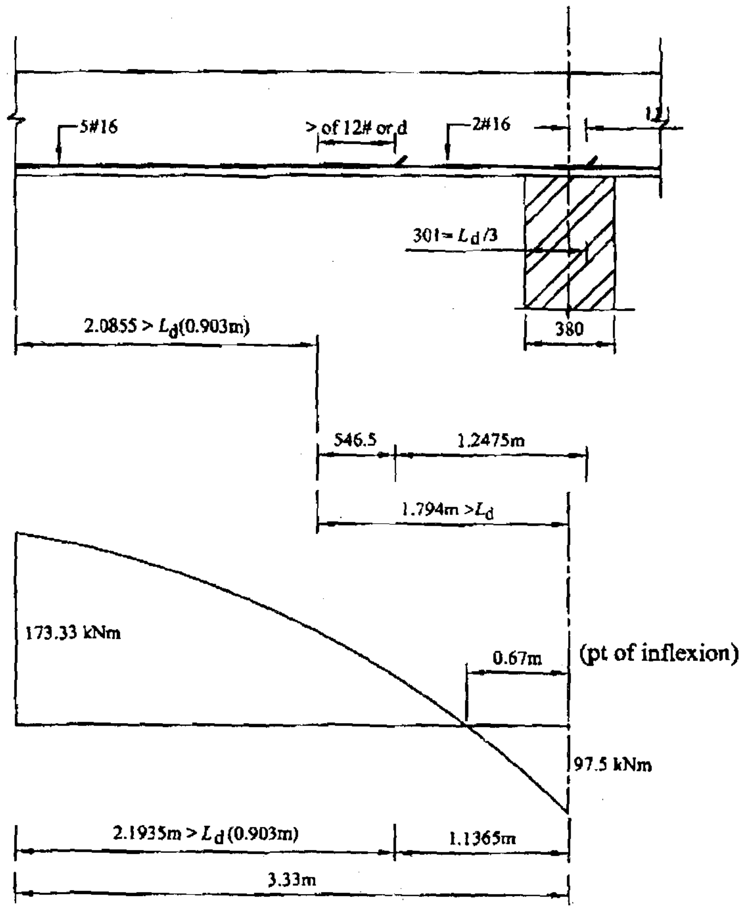


Figure 6.13 : Detailing of Positive Reinforcements at R. H. Supports

Curtailment of (–)tive Reinforcement at R.H. Support

1#16 is curtailed out of 3#16,

(a) The continuing 2#16 shall extend beyond the point of inflexion for a length greater of

(i) d = 546.5

(ii) 12 ϕ = 12 × 16 = 192, and

(iii) $\frac{1}{16}$th of clear span $= \frac{1}{16}(6-0.38)\times 10^3 = 351$

i.e. the continuing bars shall extend (0.67 + 0.5465) ≈ 1.22 m from centre line of R.H. support (vide Figure 6.14).

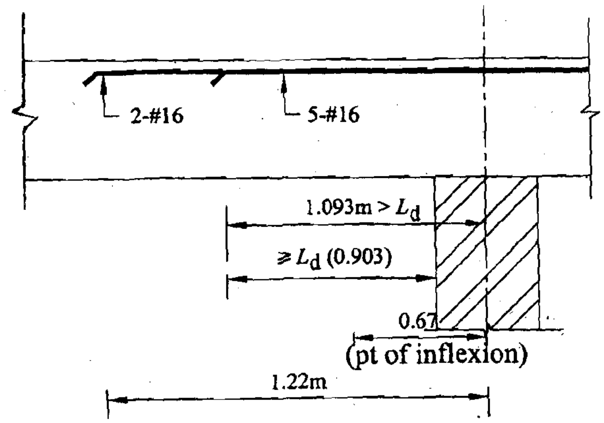


Figure 6.14 : Detailing of Negative Reinforcement over R. H. Supports

(b) out of 3#16 let 1#16 bar be curtailed then M.R. of the section with 2#16

$$M_u = 0.87 f_y A_{st} d\left(1-\frac{A_{st} f_y}{bd.f_{ck}}\right)$$

$$= 0.87\times 415\times 2\times\frac{\pi}{4}\times 16^2\times 546.5\left(1-\frac{\frac{\pi}{4}\times 16^2\times 415}{300\times 546.5\times 15}\right)$$

$$= 76.652 \text{ kNm}$$

If x be the distance from *free end* to the point where $M_u = -76.652$ kNm, then

$$-76.652 = -\frac{48.75x^2}{2} + 260(x-2)$$

or; $24.375\,x^2 - 260\,x + 443.348 = 0$

$$\text{or; } x = \frac{10.667 \pm \sqrt{113.785 - 72.755}}{2} = \frac{10.667 \pm 6.405}{2}$$

= 2.131 from free end i.e. (2.131m – 2m) = 0.131 from centre line of R.H. support.

The curtailed bars 2#16 shall extend greater of 12φ(196) or d (546.5) beyond the theoretical cut-off point i.e. the bars will extend (0.131 + 0.5465) m = 0.6775m from centre line of support into the simple span or L_d(0.903m) from *face* of support whichever is greater.

Curtailment of Bars in Cantilever Portion

(a) Let 1#16 be curtailed at x from free end then from eqn.

$$M_u = -\frac{w_u x^2}{2}$$

$$\text{or } -38.326 = -\frac{48.75x^2}{2}$$

or x = 1.254 m i.e. 2−1.254 = 0.746 m from support.

The 2#16 bars must extend greater of

(i) $L_d = 903$

(ii) $12\phi = 196$, and

(iii) $d = 546.5$

from centre line of R.H. support.

(b) The continuing 1#16 shall extend beyond cut-off point a distance equal to L_d

The detailing has been shown in Figure 6.15.

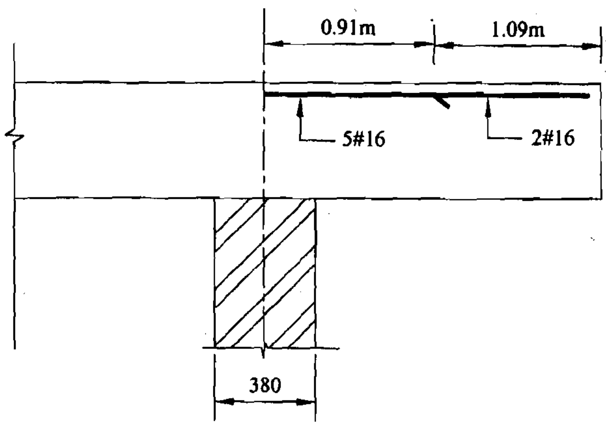


Figure 6.15 : Details of Cantilever Portion

The shear reinforcement for the beam shall be designed as done in the previous examples.

SAQ 4

Design and detail an overhang beam of 5.5 m clear span between the supports and 1.75 m of cantilever portion beyond the centre line of support. It is loaded with 20 kN/m of superimposed load. The supports consists of walls 380 thick. Use M 15 concrete and Fe 415 steel.

6.6 SUMMARY

Design and detailing of a R.C. rectangular beam is a process of fixing size of concrete section, determination of area of tensile and sheer reinforcements of its different cross sections to resist different internal forces due to applied loadings keeping in view its serviceability requirements. The various components of design such as design of beams for flexure, shear, bond & anchorage as well as detailing to meet the serviceability requirements discussed from Unit 1 to 5 have been knitted together to get a complete design and details for beam elements. Design of various types of beams of rectangular cross section have been illustrated with detailed drawings in this unit so that a student may be able to design a beam in a systematic manner and put forward a comprehensive drawing for execution.

6.7 ANSWERS TO SAQs

SAQ 1

Refer Section 6.2

SAQ 2

Refer Section 6.3

SAQ 3

Refer Section 6.4. Since the supports are monolithic with the beam, the end supports must be provided with the (–)tive reinforcement and detailed as per rules. The maximum bending moment and shear forces may be calculated as per C 21.4.1

(as of BIS : 456 - 1978)

SAQ 4

Refer Section 6.5

FURTHER READING

SP 16 : 1980 "*Design Aids for Reinforced Concrete to IS : 456-1978*".

BIS : 456-1978, "*Code of Practice for Plain and Reinforced Concrete*". Bureau of Indian Standards, Manak Bhawan, 9 Bahadur Shah Zafar Marg, New Delhi - 110 002.

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SP : 24-1983, "*Explanatory Handbook on Indian Standard Code of Practice of Plain and Reinforced Concrete (IS : 456-1978)*", Burau of Indian Standards, Manak Bhawan, 9 Bahadur Shah Zafar Marg, New Delhi - 110 002.

SP : 34-(S & T)-1978, "*Handbook on Concrete Reinforcement and Detailing*", Bureau of Indian Standards, Manak Bhawan, 9 Bahadur Shah Zafar Marg, New Delhi - 110 002.

Ram Chandra, "*Limit State Design*", Standard Book House, 1705-A, Nai Sarak, Delhi - 110 006.

Arthur H. Nilson & George Winter, "*Design of Concrete Structures*", McGraw-Hill, Inc.

6.9 APPENDIX A

BENDING MOMENT AND SHEAR FORCE

Table A.1 Maximum Bending Moment and Shear in Continuous Beams of Equal Span

		All beams freely supported at ends	
		For all spans equally loaded simultaneously	For incidental load causing the worst effect
Maximum bending moment coefficients	Uniformly distributed load	.125 D.070 D.070 D .100 .100 D.080 D.025 D.080 D .107 .071 .107 D.077 D.036 D.036 D.077 D .105 .080 .080 .105 D.078 D.033 D.046 D.033 D.078 D	.125 D.096 D.096 D .117 .117 D.101 D.075 D.101 D .121 .107 .121 D.099 D.081 D.081 D.099 D .120 .111 .111 .120 D.100 D.080 D.086 D.080 D.100 D

Note : Bending moment = coefficient × w × l

W = Total load on one span

l = effective span

The coefficients written *above* the span are for *negative* moment at supports and those written *below* the span are for *positive* moment at midspan.

Table A.2 : Maximum Shear Force in Continuous Beams of Equal Span

		All beams freely supported at ends	
		For all spans equally loaded simultaneously	For incidental load causing the worst effect
Maximum shear coefficients	Uniformly distributed load	.38 .62 D .62D .38D .40 .50 .60 D .60D .50D .40D .39 .54 .46 .61 D .61D .46D .54D .39D .40 .53 .50 .47 .60 D .60D .47D .50D .53D .40D	.44 .62 D .62D .44D .45 .58 .62 D .62D .58D .45D .45 .60 .57 .62 D .62D .57D .60D .45D .45 .60 .59 .58 .62 D .62D .58D .59D .60D .45D

Note : Shear force = coefficient × total load on *one* span

Note 1. S.F. coefficients above line apply to SF. at right hand side of support.

S.F. coefficients below line apply to S.F. at left hand side of support.

Table A.3 : Bending Moment in Continuous Beams of Equal Spans Freely Supported at Ends due to Unit Moment Applied at Ends

Bending moment = coefficient × applied moment

Bending Moment Coefficients	
Unit moment applied at one end only	Unit moment applied at each of the two ends
1.0 .25	1.0 .50 1.0
1.0 .267 -.067	1.0 .20 .20 1.0
1.0 .268 -.0[illegible]1 .018	1.0 .268 -.1[illegible]2 .286 1.0
1.0 .268 -.072 .019 -.005	1.0 .263 -.053 -.053 -.263 1.0