UNIT 3 SHEAR

Structure

3.1 INTRODUCTION

When uniform bending moment is applied on a beam, its every cross-section will have the same bending stress distribution (Figure 3.1). There will be no transverse shear force on such beams since applied as well as reactive transverse forces are absent.

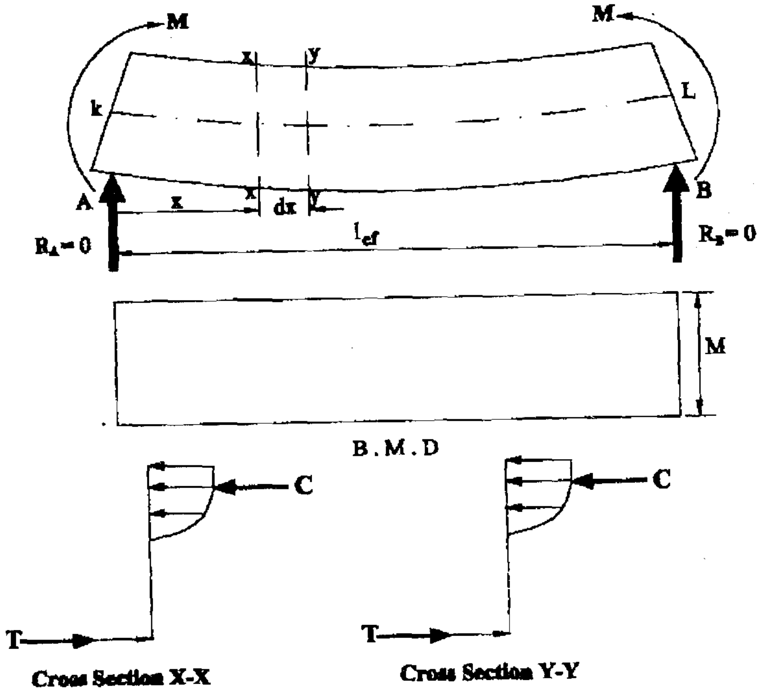

Figure 3.1 : Effect of Uniform B. M. Applied on a Beam

But if a beam is applied with transverse loads, the bending moment distribution along its length will be changing resulting in horizontal as well as vertical shear forces on its different cross-section (Figure 3.2(a), (b) & (c)).

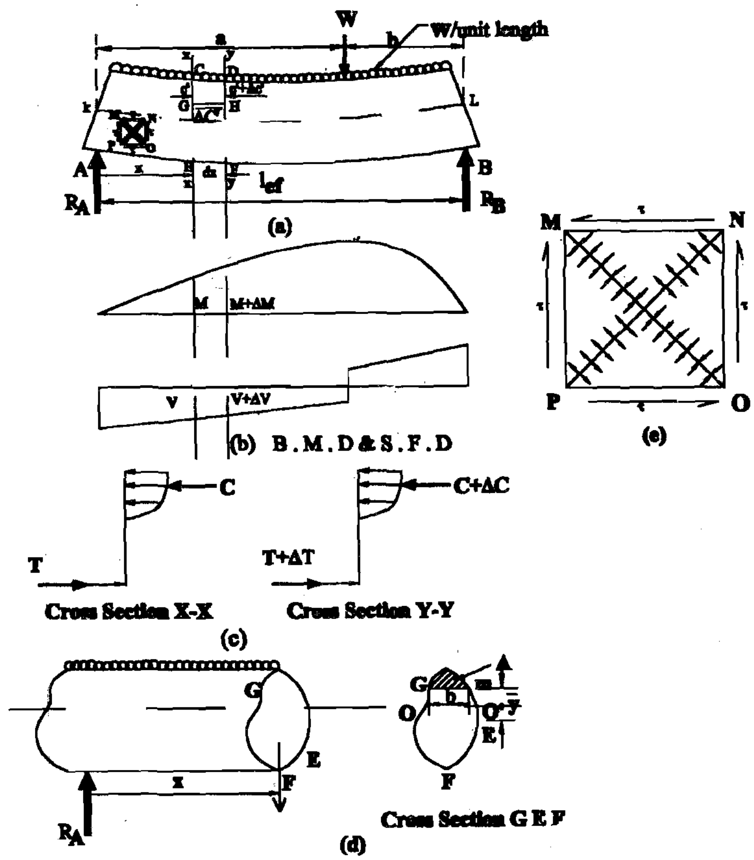

Figure 3.2 : Showing Effect of Varying B. M. D. Along a Beam

Let us assume that M and M + M are bending moments on section X-X and Y-Y respectively. Let there be horizontal forces C' and C' + DC' from left and from right respectively on the block CDHG. The resultant of these two horizontal forces DC' will tend to move the block on the surface containing GH towards left. Since the block is in equilibrium it is clear that a force DC' towards right on the same surface is balancing it. These two horizontal forces, equal and opposite to each other, are called the **Horizontal Shear Force.**

Again due to resultant of R_A and U.D.L. on the L.H.S. of X-X, the beam from A to X-X will tend to move upward. But since it is in equilibrium an *interanl* equal and opposite forces on section X - X is developed. The two balancing forces together are called the **Vertical Shear Force.**

Now according to the principle of Strength of Materials, shear *stress* at any point, say G, (Figure 3.2 (a) & (d)) - both horizontal and vertical - is given by the equation

$$\tau = \frac{VA\bar{y}}{Ib} \qquad \ldots (3.1)$$

where, V = Shear force at any section X - X,

A = Area of cross-section above any horizontal plane containing G m (Figure 3.2(d),

$\bar{y}$ = Distance of c.g. of the area A from n.a., and

I = Moment of Inertia about n.a. of the whole cross-section, and

b = Breadth Gm.

The distribution of shear stress τ on a cross-section of a rectangular R. C. beam may be understood as follows (Figure 3.3) :

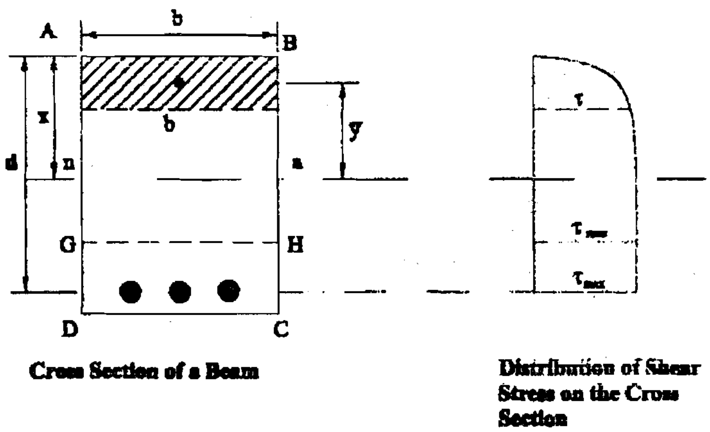

Figure 3.3 : Distribution of Shear Stress on a Cross-section

At the top face AB, the value of shear stress, $\tau = 0$, in Eq. (3.1) since the area A is zero. From surface AB to n.a., the distribution of τ is parabolic as V, I and b are constants and Ay varies as y^2. The area A becomes maximum at n.a. and, hence, τ is the maximum (τ_{max}). Below n.a. concrete area is ineffective; therefore, moment of area, Ay about n.a. for any horizontal surface GH below it remains constants meaning thereby that $\tau = \tau_{max}$ upto the centre of reinforcement as shown in Figure 3.3.

Now total vertical shear force

$$V = b \times d \times \text{average shear stress}$$

$$= b \times d \times \frac{\text{Area of shear stress distribution diagram}}{b}$$

$$= b \times \{\frac{2}{3} \times \tau_{max} + (d - x)\tau_{max}\}$$

$$\text{or, } \tau_{max} = \frac{V}{(d - \frac{x}{3})} = \frac{V}{bjd} \qquad \ldots(3.2)$$

where $(d - \frac{x}{3}) = jd$ = lever arm.

Referring Figure 3.2 (a) & (e) it can be inferred from Principles* of Mechanics of Solids that tensile crack will develop along diagonal PN due to tensile stress of τ_{max} generally called **Diagonal Tension** The above theory presumes that the resistance against shear is developed by concrete in compression *only*.

Test results have amply demonstrated that shear resistance of a section is much more enhanced by aggregate interlocutory effect and dowel action of tensile reinforcement. The phenomena of shear resistance is depicted in Figure 3.4. Based upon the above mentioned behavioural aspect of a R. C. beam under shear, design principles have been developed.

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* When an element is in a state of pure shear (Figure 3.2(e)); maximum direct stresses are induced on mutually perpendicular planes which are sat 45° to the plane of pure shear. One of the maximum direct stresses is tensile while the other is compressive. These direct maximum stress intensities are of the same magnitude as the intensity of shear stress on the plane of pure shear.

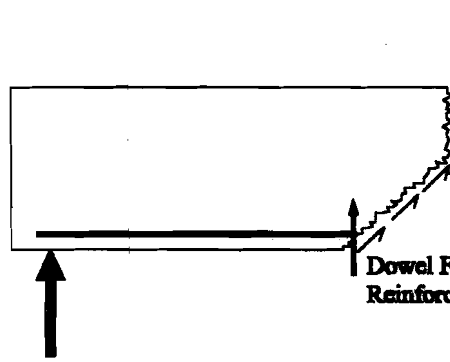


Figure 3.4 : Applied and Resisting Shear Forces

Objectives

After going through this unit a student will learn

- principles underlying shear stress evaluation,
- provision of shear reinforcements,
- analysis for Design, and
- design of shear reinforcement for a beam.

SAQ 1

i) Sketch and explain the theoretical distribution of shear stress on the cross-section of a rectangular R. C. beam.

ii) Explain with diagram, the internal forces developed to resist shear force.

3.2 DESIGN PRINCIPLES

At limit state, τ_{max} which is, henceforth, called **Nominal Shear Stress** τ_υ.

$$\tau_\upsilon = \frac{V_u}{bd} \text{ and } not^* = \frac{V_u}{bjd} \text{ as derived earlier Eq. (3.2)} \qquad \ldots (3.3)$$

where V_u = shear force due to design loads, and

b = breadth of the member, which for flanged sections shall be taken as breadth of web, b_w.

Eq. (3.3) is applicable for rectangular beams of uniform depth. For beams of varying depth (Figure 3.5a), the shear force at a cross-section X - X

$$= V_u - V = V_u - H \tan\beta = V_u - \frac{M_u}{jd} \tan\beta \approx V_u - \frac{M_u}{d} \tan\beta$$

* This simplification is reasonable as the nominal shear stress is a measure of shear resistance offered by the concrete and does not necessarily represent the actual stress conditions.

$$\therefore \tau_v = \frac{V_u - \frac{M_u}{d}\tan\beta}{bd} \qquad \ldots (3.4(a))$$

where β = angle between the top and bottom edges of the beam.

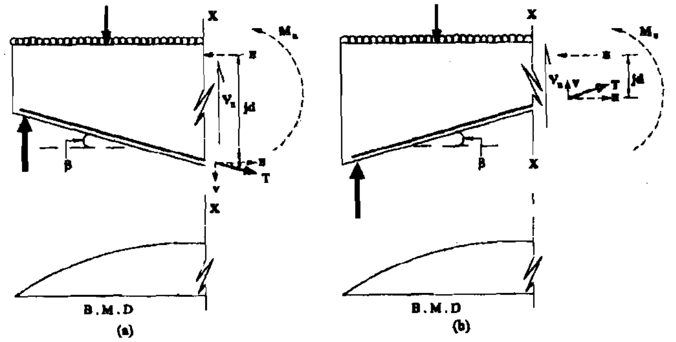


Figure 3.5 : Force Actions for Beams of Varying Depth

It may be noted here that bending moment is increasing as the effective depth is increasing. But for that beam in which bending moment increases as the effective depth decreases

(Figure 3.5b), the value of $\tau_v = \frac{V_u + \frac{M_u}{d}\tan\beta}{bd}$ ... (3.4(b))

The above formulae for evaluation of τ_v are prescribed by the Code.

Shear strength of concrete increases with better grade of concrete and higher percentage of tensile reinforcement. The numerical values of *design shear strength*, τ_c, for different grades of concrete and varying percentages of tensile reinforcement have bean depicted in Table 3.1.

Table 3.1 : Design Shear Strength of Concrete, τ_c, N/mm^2

Percentage of Tensile Reinforcement	Design Shear Strength of Concrete, τ_c					
	Concrete Grades					
$\frac{A_{st}}{bd} \times 100$	M15	M20	M25	M30	M35	M40
0.20	0.32	0.33	0.33	0.33	0.34	0.34
0.25	0.35	0.36	0.36	0.37	0.37	0.38
0.50	0.46	0.48	0.49	0.50	0.50	0.51
0.75	0.54	0.56	0.57	0.59	0.59	0.60
1.00	0.60	0.62	0.64	0.66	0.67	0.68
1.25	0.64	0.67	0.70	0.71	0.73	0.74
1.50	0.68	0.72	0.74	0.76	0.78	0.79
1.75	0.71	0.75	0.78	0.80	0.82	0.84
2.00	0.71	0.79	0.82	0.84	0.86	0.88
2.25	0.71	0.81	0.85	0.88	0.90	0.92
2.50	0.71	0.82	0.88	0.91	0.93	0.95
2.75	0.71	0.82	0.90	0.94	0.96	0.98
3.00 and above	0.71	0.82	0.92	0.96	0.99	1.01

Tensile steel, A_{st}, must continue at least one effective depth beyond the section being considered except at supports where full area of tension reinforcement may be used provided the detailing has been done properly.*

* Detailing has been explained in Unit 5.

SAQ 2

Explain with sketch that for beams of varying cross-section,

$$\tau_c = \frac{V_u \pm \frac{M_u}{d} \tan \beta}{bd} \text{ where } \beta = \text{angle between the top and bottom}$$

edges of the beam.

3.3 PROVISION OF SHEAR REINFORCEMENTS

(i) At a cross-section if $\tau_v < \tau_c$, minimum shear reinforcement in the form of stirrups may be provided (cl. 39.3) such that

$$\frac{A_{sv}}{bs_v} \geq \frac{0.4}{0.87 f_y} \quad . (3.5)$$

where, A_{sv} = cross-sectional area of stirrup legs effective in shear,
s_v = stirrup spacing along the length of the member, and
f_y = characteristic strength of shear reinforcement **which shall not be greater than 415 MPa.**

In minor structural elements, where maximum calculated shear stress is less than half the permissible value, the above provision may not be complied.

The minimum reinforcement provided checks the growth of diagonal tension crack due to shear. Thus, such reinforcements increases the ductility of a flexural member preventing its sudden failure.

The maximum spacing of shear reinforcement, $s_{v,max}$, measured along the axis of the member shall not exceed 0.75 d for vertical stirrups and d for inclined stirrups at 45°. In no case, the spacing will be greater than 450 mm.

ii) If, for a cross-section, $\tau_v > \tau_c$,

i.e. $\tau_v \, bd > \tau_c \, bd$

or, $V_u > V_c$

where V_c = Design shear force to be resisted by concrete,

Shear reinforcement are required only for a shear force of $(V_u - V_c) = V_{us}$

However, if the value of $\tau_v > \tau_{c,max}$ (Table 3.2), the section may be redesigned so that τ_v becomes less than $\tau_{c,max}$ for the revised concrete section. This precaution is necessary in view of the fact that the diagonal compression on diagonal MO (Figure 3.29(e) in combination with normal compression stress due to bending in compression zone, may cross limiting value of concrete in compression causing brittle (sudden) failure.

Table 3.2 : Maximum Shear Stress, $\tau_{c,max} (N/mm^2)$

Concrete Grade	M15	M20	M25	M30	M35	M40
$\tau_{c,max}$ (N/mm^2)	2.5	2.8	3.1	3.5	3.7	4.0

SAQ 3

i) Why minimum shear reinforcement should be provided? How would you comply with the minimum shear reinforcement requirement?

ii) Explain the meaning underlying the assumption that τ_v shall not exceed $\tau_{c,max}$.

3.4 TYPES OF SHEAR REINFORCEMENTS

The reinforcements provided to resist diagonal tension are called **Shear Reinforcements**. Economically, they should be provided perpendicular to the plane of diagonal tension cracks (Figure 3.2(a)); but, generally they are provided in the form of vertical stirrups. These stirrups resist shear as well as contain the concrete in lateral directions from bursting. The stirrups, inclined or vertical, are more effective and, hence, are preferred to single or group of bent bars; because the former type binds the concrete making it stronger. Depending upon the magnitude of shear force and availability of extra tensile reinforcement to be used as shear reinforcements, shear reinforcement may be of any of the following types :

a) Vertical stirrups only

b) Bent-up bars along with stirrups, and

c) Inclined stirrups only.

SAQ 4

Why stirrups are perferred to single or group of bars as shear reinforcements?

3.5 ANALYSIS AND DESIGN OF SHEAR REINFORCEMENTS

a) **Where Vertical Stirrups only are provided**

At a cross-section, where tensile reinforcements are not in excess to be bent for use as shear reinforcement, only vertical stirrups are provided to resist V_{us}.

Let these vertical stirrups be provided at a spacing of s_v (Figure 3.6); then number of stirrups provided in a horizontal distance d (i.e. horizontal distance covered by a diagonal crack inclined

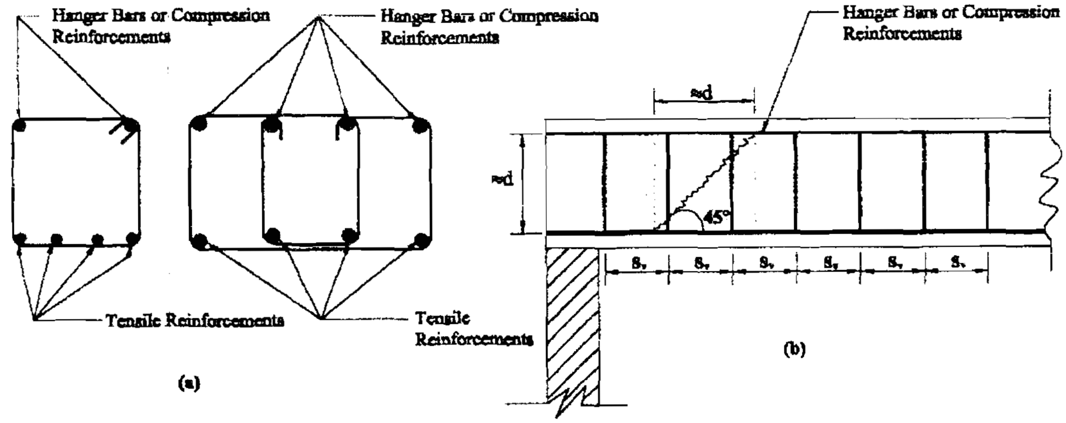


Figure 3.6 : Vertical Stirrups to Resist Shear Force

at $45^0) = \frac{d}{S_\upsilon}$. If $A_{s\upsilon}$ is the area of cross-sectional area of a stirrup (i.e. twice the cross-sectional area of a bar which crosses the plane of a diagonal crack) and V_u is the vertical component of total force of diagonal tension; then equating applied vertical force to the resisting force

$$V_{us} = V_\upsilon = 0.87 A_{s\upsilon} \frac{d}{s_\upsilon}$$

$$\text{or, } V_{us} = \frac{0.87 f_y A_{s\upsilon} d}{s_\upsilon} \qquad \ldots (3.6)$$

b) **Where Bent-up Bars along with Vertical Stirups or Inclined Stirrups at *Different Cross-sections are Provided***

The above mentioned types of provision may be made where the total shear force to be resisted by steel (V_{us}) is born either by bent-up bars and stirrups together or by inclined stirrups only.

The shear force V_{us_1} to be resisted by inclined reinforcement is determined as follows (Figure 3.7).

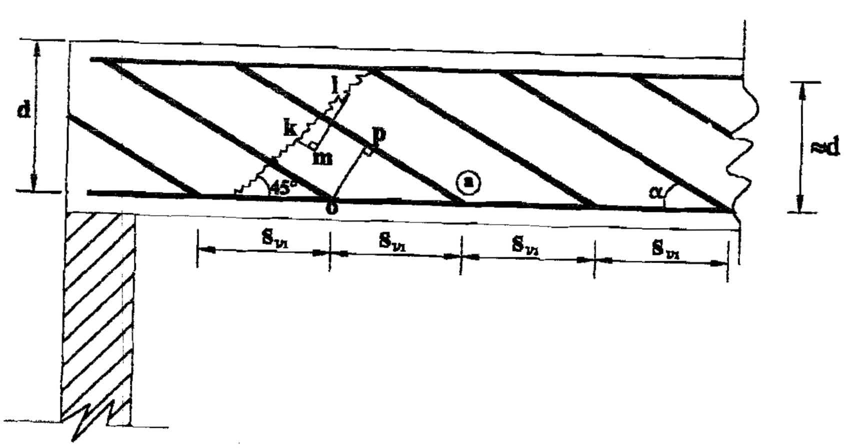


Figure 3.7 : Inclined Reinforcement to Resist a Part of Shear Forces

Let the inclined reinforcement be provided at spacing of s_{υ_1} inclined at an angle α. If on *any* diagonal crack, $k\ell$ is its effective length crossed by *any* **one** bar ⓐ resisting diagonal tension, then vertical component of diagonal tension force $= k\ell \times \tau_{s_1} b \sin 45^0$. Similarly, vertical component of maximum force resisted by bar ⓐ $= 0.87 f_y A_{s\upsilon} \sin\alpha$. Equating the above vertical components of applied and resisting forces, $k\ell \times \tau_{s_1} \times b \times \sin 45^0 = 0.87 f_y A_{s\upsilon_1} \sin\alpha$.

where $K\ell = \ell m \sec K\ell m = \ell m \sec(45^0 - \alpha)$

$$= op \sec(45^0 - \alpha) = s_{\upsilon_1} \sin\alpha \sec(45^0 - \alpha)$$

$$= \frac{\sqrt{2} s_{\upsilon_1} \sin\alpha}{(\sin\alpha + \cos\alpha)}$$

$$\text{or,} \quad \frac{\sqrt{2} s_{\upsilon_1} \sin\alpha}{(\sin\alpha + \cos\alpha)} (\tau_{s_1} b) \sin 45^0 = 0.87 f_y A_{s\upsilon_1} \sin\alpha$$

Multiplying both sindes by d

$$\text{or,} \quad \frac{s_{\upsilon_1} \sin\alpha (\tau_{s_1} bd)}{(\sin\alpha + \cos\alpha)} = 0.87 f_y A_{s\upsilon_1} d$$

since $(\tau_{s\upsilon_1} bd) = V_{us_1}$

$$\therefore \quad V_{us_1} = \frac{0.87 f_y A_{s\upsilon_1} d}{s_{\upsilon_1}} (\sin\alpha + \cos\alpha) \qquad \ldots (3.7)$$

If inclined bars are provided in combination with vertical stirrups, total shear resistance of both types together $= V_{us} = V_{us_1} + V_{us_2}$

where V_{us_1} = shear resistance of inclined bars, and

V_{us_2} = shear resistance of vertical stirrups.

It may be noted here that

i) the area of stirrups shall not be less than the minimum specified in Eq. (3.5), and

ii) where bent-up bars are provided their contribution towards shear resistance shall not be more than *half* that of the **total reinforcement.**

Inclined bars are ineffective in case of reversal of shear force and their exact behaviour in resisting shear is still controversial; that is why contribution of bent up bars towards shear resistance is limited to only half that of total reinforcement.

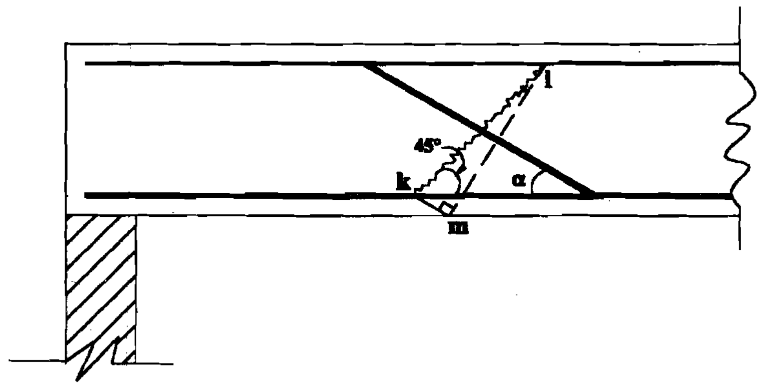
But if *inclined stirrups* are only provided V_{us_1} and $A_{s\theta_1}$ in Eq. (3.7) are to be substituted as V_{us} and $A_{s\upsilon}$ respectively to have Eq 3.8 for such reinforcement i.e.,

$$V_{us} = \frac{0.87 f_y A_{s\upsilon}}{s_\upsilon} (\sin\alpha + \cos\alpha) \qquad \ldots (3.8)$$

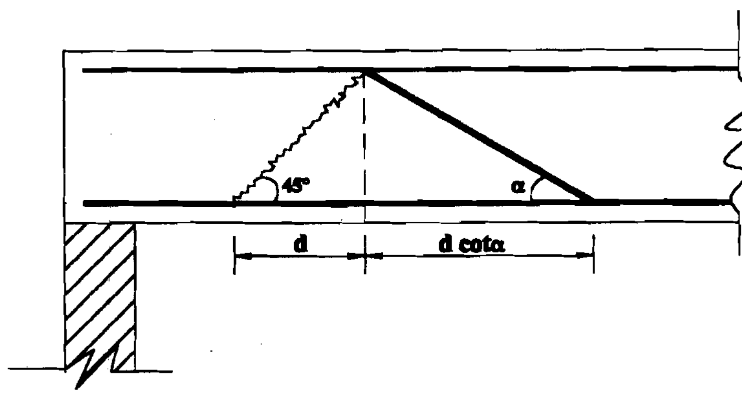
c) **Where Single or Group of Parallel Bars, all bent-up at the *Same* Cross-section, are provided**

If a single or group of parallel bars are bent-up at the *same* cross-section to resist a total shear of V_{us} (Figure 3.8(a)) then

$$V_{us} = 0.87 f_y A_{s\upsilon} \sin\alpha \qquad \ldots (3.9)$$



(a) Single or Group of Parallel Inclined Bars to Resist Sheat at the Same Cross-Section



(b) Maximum Distance Between Bent-up Bars or Inclined Stirrups
Figure 3.8

Such reinforcement is effective* upto a distance (d+ dcot α) along the span as shown in Figure 3.8(b).

SAQ 5

i) Derive the expression

$$V_{us} = \frac{0.87 f_y A_{s\upsilon} d}{s_\upsilon} \text{ for vertical stirrups.}$$

* Every potential diagonal crack, assumed inclined at 45°, must be crossed by at least one inclined shear reinforcement; hence maximum spacing of inclined reinforcement in Figure 3.7 is also limited to a distance (d + d cot α) (Figure 3.8(b)).

ii) Derive the expression $V_{us} = \frac{\sigma_{sv} A_{sv} d}{s_v} (\sin\alpha + \cos\alpha)$

for inclined stirrups or a series of bent-up bars at different cross sections.

iii) Explain the reasons behind the fact that the contribution of bent-up bars towards shear resistance is limited to half that of total reinforcement.

3.6 EXAMPLES

Vertical Stirrups only as Shear Reinforcement.

Example 3.1

Design a beam for shear reinforcement having a cross-section of $b \times D = 250 \times 500$ reinforced with $4\phi20$. The factored shear force = 130 kN. Use M15 concrete and Fe250 steel. Provide vertical stirrupts *only* as shear reinforcement.

Solution

Assuming clear cover of 35 mm

$$d = 500 - 35 - \frac{20}{2} = 455$$

Nominal shear stress $= \tau_v = \frac{V_u}{bd} = \frac{130 \times 1000}{250 \times 455} = 1.143$ MPa

$\tau_{c,max}$ for M15 concrete = 2.5 MPa (Table 3.2)

Percentage of tensile reinforcement

$$p\% = \frac{100A_{st}}{bd} = \frac{100 \times 1256.64}{250 \times 455} = 1.1\%$$

Design shear strength for 1.1% tensile reinforcement and M15 concrete

$$= 0.6 + \frac{(0.64 - 0.6)}{(1.25 - 1.0)} \times (1.1 - 1.0)\ 2 = 0.616 \text{ MPa (Table 3.1)}$$

$\because \tau_{c,max} > \tau_v > t_c$; therefore, shear reinforcement is required for $V_{us} = V_u - \tau_c d$

$= 130 \times 1000 - 0.616 \times 250 \times 455$

$= 59930$ N

Providing $\phi 8$ - 2 legged vertical stirrups having

$$A_{sv} = 2 \times \frac{\pi}{4} \times 8^2 = 100.53 mm^2$$

From Eq 3.6, $s_v = \frac{0.87 f_y A_{sv} d}{V_{us}} = \frac{0.87 \times 250 \times 100.53 \times 455}{59930}$

$$= 166 < 0.75d\,(341.25) < 450$$

The maximum spacing, $s_{v,max}$, required for minimum shear reinforcement

according to Eq. (3.5) $= \frac{0.87 f_y A_{sv}}{0.4b} = \frac{0.87 \times 250 \times 100.53}{0.4 \times 250} = 218.65 > s_v(166)$

Hence, provided $\phi 8$-2 legged vertical stirrups at 160 c/c. **Ans**

Bent-up Bars at Different Cross-sections along with Vertical Stirrups as the Shear Reinforcements

Example 3.2

Design the beam for shear at support for the arrangement of tensile reinforcement shown in Figure 3.9. Take b = 300; d = 550; f_y = 250 MPa; f_{ck} = 20 MPa, V_u at support = 250 kN.

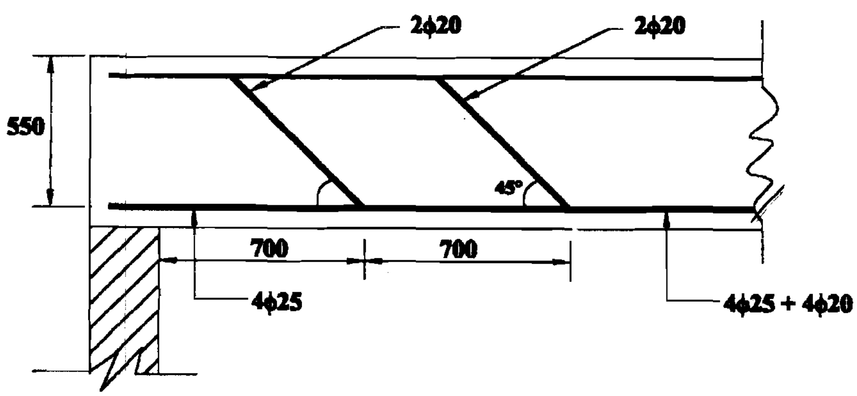


Figure 3.9 : Showing Bent-up Bars at Different Cross-Sections

Solution

Percentage of tensile reinforcement at support = $= \frac{4 \times \frac{\pi}{4} \times 25^2}{300 \times 550} \times 100 = 1.19\%$

$\therefore$ For M20 concrete and 1.19% tensile reinforcement (Table 3.1)

$$\tau_c = 0.62 + \frac{(0.67 - 0.62)}{(1.25 - 1.0)} \times (1.19 - 1.0) = 0.658 N/mm^2$$

For M20 concrete, $\tau_{c,max}$ = 2.8 MPa (Table 3.2)

$$\tau_\upsilon = \frac{V_u}{bd} = \frac{250\times10^3}{300\times550} = 1.52\ \text{MPa}$$

$\because\ \tau_{c,max} > \tau_\upsilon > \tau_c$; therefore shear reinforcement is to be provided for

$$V_{us} = V_u - \tau_c bd = 250\times1000 - 0.658\times300\times550 = 141430\text{N}$$

Shear resistance for a series of bent-up bars at different cross-section,

$$V_{us_1} = \frac{0.87 f_y A_{s\upsilon_1} d}{s_\upsilon}(\sin\alpha + \cos\alpha)$$

$$= \frac{0.87\times250\times2\times\frac{\pi}{4}\times20^2\times550}{700}(\sin45^0 + \cos45^0)$$

$$= 151851.39\ \text{N} > V_{us}\ (141430\text{N}) \qquad \ldots(3.7)$$

As the shear resistance of bent-up bars cannot exceed $0.5\times141430 = 70715$ N, vertical stirrups are to be provided for $V_{us_2} = V_u - V_{us_1} = 141430 - 70715 = 70715\text{N}$

Providing ϕ6-2 legged vertical stirrups

$$A_{s\upsilon_2} = 2\times\frac{\pi}{4}\times6^2 = 56.55\text{mm}^2$$

$$\therefore\ s_{\upsilon_2} = \frac{0.87 f_y A_{s\upsilon_2} d}{V_{us_2}}$$

$$= \frac{0.87\times250\times56.55\times550}{70715} = 95.66 < 0.75d(412.5) < 450 \qquad \ldots(3.6)$$

From minimum reinforcement consideration the maximum spacing of vertical reinforcement,

$$s_{\upsilon_{2,max}} = \frac{0.87\times250\times56.55}{0.4\times300} = 102.49 > 95.66 \qquad \ldots(3.5)$$

Hence, **provided ϕ6-2 leggled vertical stirrups @ 95c/c in addition to bent-up bars** at *different* cross-sections to resist total shear force. **Ans**

Bent-up Bars at the Same Cross-section along with Vertical Stirrups as the Shear Reinforcements

Example 3.3

Design a beam for shear reinforcements for the following data :

$$b = 250;\ D = 540;\ d = 500;\ f_y = 415\ \text{MPa and}\ f_{ck} = 15\ \text{MPa}.$$

At a cross-section shear force, V_u, equal to 93 kN is to be resisted by two bent-up bars of ϕ18 inclined at 45° in combination with vertical stirrups. The tensile reinforcement of 2ϕ18 is available at the section.

Solution

$$\text{Nominal shear stress} = \tau_u = \frac{93\times 1000}{250\times 500} = 0.744\text{MPa}$$

$$\text{Percentage of tensile reinforcement} = \frac{2\times\frac{\pi}{4}\times 18^2}{250\times 500}\times 100 = 0.407\ \%$$

$$\text{Accordingly, } \tau_c = 0.35+\frac{(0.46-0.35)}{(0.5-0.25)}\times(0.407-0.25)$$

$$= 0.419\ \text{MPa} \qquad \text{(Table 3.1)}$$

For M15 concrete, $\tau_{c,\max}$ = 2.5 MPa (Table 3.2)

Since $\tau_{c,\max} > \tau_\upsilon > \tau_c$, shear reinforcement is to be designed for

$V_{us} = V_u - V_c = 93000 - 0.419\times 250\times 500 = 40625$ N

Shear resistance of 2ϕ18 at the same cross section

$$= 0.87 f_y A_{s\upsilon} \sin\alpha \qquad \ldots(3.9)$$

$$= 0.87\times 415\times 2\times\frac{\pi}{4}\times 18^2\times \sin 45^\circ$$

$$= 129932.33\ \text{N} > 40625\ (V_{us})$$

The effective shear resistance of bent-up bars, $V_{us_1} = 0.5\times 40625 - 20312.5$ N

Vertical stirrups are to be provided for $V_{us_2} = V_{us} - V_{us_1} = 40625 - 20312.5$
= 20312.5 N

If ϕ6 two-legged stirrups are adopted, spacing of vetical stirrups

$$s_{v_2} = \frac{0.87 f_y A_{su_2} d}{V_{us_2}} \qquad \ldots(3.6)$$

$$= \frac{0.87\times 415\times\left(2\times\frac{\pi}{4}\times 6^2\right)\times 500}{2031.5} = 502.58 > 450 > 375(0.75\,d)$$

The maximum spacing of vertical stirrups vide Eq. (3.5),

$$s_{v_2} = \frac{0.87 f_y A_{su_2} d}{0.4b} = \frac{0.87\times 415\times 56.55}{0.4\times 250} = 204.17 < 375$$

Hence, provided two bent-up bars inclined at 45^0 at the section along with 6 two-legged vertical stirrups @ 200c/c at the section. Ans

Example 3.4 **Shear Reinforcement for Beams of Varying Cross-section**

Design tensile and shear reinforcements at support for a cantilever beam (Figure 3.10) of constant width 300. The depth of a beam is linearly varying from 800 at support to 350 at free end. The beam is loaded with a U.D.L. of 39 kN/m including its self weight. Use M20 concrete and Fe415 steel.

Solution

i) Design of Tensile Reinforcement at support

Factored B. M. at support

Taking effective cover = 40, d = 800 – 40 = 760 mm.

$$M_u = -1.5 \times \frac{W l^2 ef}{2} = -1.5 \times \frac{30 \times 3^2}{2} = -202.5 \text{ kNm}$$

Tensile reinforcement for M_u is given by the equation

$$M_u = 0.87 f_y A_{st} d \left(1 - \frac{A_{st} f_y}{b d f_{ck}}\right)$$

$$202.5 \times 10^6 = 0.87 \times 415 \times A_{st} \times 760 \left(1 - \frac{A_{st} \times 415}{300 \times 760 \times 20}\right)$$

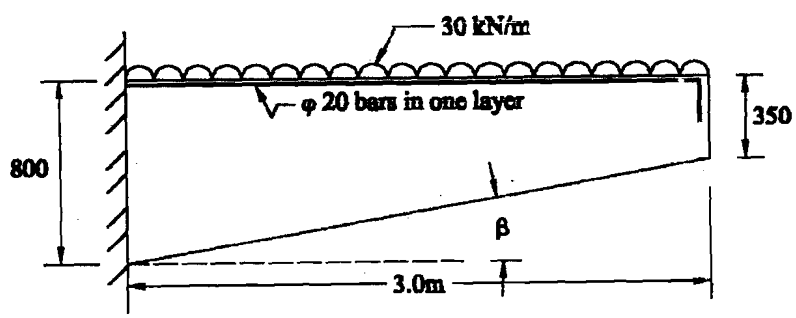


Figure 3.10 : Details of a Cantilever Beam

or, $24.97 A_{st}^2 - 274398 A_{st} + 202.5 \times 10^6 = 0$

or, $A_{st} = 795.58 \text{mm}^2$

Min. tensile reinforcement = $\frac{0.85}{f_y} \times 100 = \frac{0.85 \times 100}{415} = 0.2\%$

i.e. $A_{st} = 0.002 \times 300 \times 760 = 456 \text{ mm}^2 < 795.58 \text{ mm}^2$

Hence provided 3 ϕ 20 (A_{st} = 942 mm²) Ans

ii) Design of Shear Reinforcement at support*

$V_u = 1.5 \times 30 \times 3 = 135$ kN

* The critical section for design of shear reinforcement is the section which is an effective depth away from the support face (Refer clause 21.6.2.1 of the Code)

$$T_v = \frac{V_u - \frac{M_u}{d}\tan\beta}{bd}$$

$$= \frac{135 \times 10^3 - \frac{202.5 \times 10^6}{760} \times \frac{450}{3000}}{300 \times 760} \qquad \text{(Eq. 3.4(a))}$$

$= 0.42$ N/mm^2

$$\% \text{ of tensile steel} = \frac{942 \times 100}{300 \times 760}\% = 0.413\%$$

Accordingly $\tau_c = 0.36 + \frac{0.48 - 0.36}{(0.5 - 0.25)} \times (0.413 - 0.25)$

$= 0.438 \text{ N / mm}^2 > 0.42 (\tau_c)$ (Table 3.1)

Hence only nominal shear reinforcement is to be provided.

Providing $\phi 8$ two legged stirrups

$$A_{sv} = 2 \times \frac{\pi}{4} \times 8^2 = 100 \text{ m}^2$$

$$s_{v\,max} = \frac{0.87 f_y A_{sv}}{0.4\, b} = \frac{0.87 \times 415 \times 100}{0.4 \times 300}$$

$= 300.8 < 450 < 570$ (0.75d) (Eq 3.5)

Hence provided ϕ8 two-legged stirrups @ 300c/c Ans

SAQ 6

i) Provide two-legged vertical stirrups for a beam for the data given below :

b = 300; d = 556; Tensile reinforcement is 5 ϕ 8; V_u = 200 kN; f_y = 415 MPa and f_{ck} = 20 MPa

ii) Design shear reinforcement for a beam for the following data :

b = 400; d = 760; D = 800; V_u = 150 kN; f_y = 250 MPa and f_{ck} = 15 MPa

Out of 5ϕ18 as tensile reinforcement two of them have been bend up at the same cross-section to resist shear force.

iii) Design vertical shear stirrups at a cross section of a beam at which bent-up bars of ϕ16 inclined at 50^0 to the horizontal bars@ 400 c/c are provided to resist a part of total shear force. The tensile longitudinal bars at that cross-section are 3ϕ25. The other design data are as follows :

V_u = 25kN; b = 300; d= 500; f_{ck} = 15MPa and f_y = 250 MPa

3.7 SUMMARY

Shear forces in a beam cause diagonal tension (i.e. tension in planes inclined at 45^0 to the axis of a beam). Concrete being weak in tension cracks. These cracks may propagate into the compression zone and hasten the failure of a beam before flexural collapse load is reached. A part of the total shear force is borne by concrete itself and the rest of it is taken by shear reinforcements.

However, an upper limit to nominal shear stress (i.e. $\tau_v \not> \tau_{c, max}$) restricts, the amount of shear to be resisted by shear reinforcements to a certain datum.

These reinforcements are effective in preventing shear cracks to originate and, thus, ensure the bending failure of the beam. They may be provided in any of the three forms :

i) Vertical stirrups only,

ii) Bent-up bars along with vertical stirrups, or inclined stirrups, and

iii) Inclined stirrups only.

In all flexural members, except in case of minor elements a minimum of shear reinforcement has been prescribed by the code.

3.9 ANSWERS TO SAQs

SAQ 1

i) Refer Section 3.1

ii) Refer Section 3.1

SAQ 2

Refer Section 3.2

SAQ 3

i) Refer Section 3.3

ii) Refer Section 3.3

SAQ 4

Refer Section 3.4

SAQ 5

i) Refer Section 3.5

ii) Refer Section 3.5

iii) Refer Section 3.5

SAQ 6

i) Provided ϕ8 two-legged vertical stirrups @ 190 c/c

ii) Provided two-legged ϕ6 stirrups @ 75 c/c

iii) Refer Section Example 3.2

Provided ϕ8-2 legged vertical stirrups @ 160 c/c in addition to bent-up bars of ϕ16@ 400 c/c to resist total shear force.