UNIT 13 RECIPROCATING COMPRESSORS

Structure

13.1 INTRODUCTION

Compression of gases is an important process in many power plants, refrigeration plants and industrial plants. Industrial uses of gas compression occur in connection with pneumatic tools, air brakes for vehicles, servo-mechanisms, metallurgical and chemical processes, conveying of materials through ducts, production of bottled gases, and transportation of natural gas. The term gas compression applies only to processes involving appreciable change of gas density; this excludes ordinary ventilation and furnace draft processes.

The machinery used in gas compression may be turbine type, such as centrifugal and axial flow machines; or positive displacement type, such as reciprocating machines, meshing rotor or gear machines and vane-sealed machines. In so far as it operates under steady flow conditions, any of these types of machine may have its energy analysis written in the form of the steady flow energy equation. In this unit some general deductions will be made on this basis. A more detailed study will be made of the reciprocating compressor, but not of the other machines.

Objectives

After a study of this unit, you should be able to

* compare reversible adiabatic, reversible isothermal and reversible polytropic processes of compression,
* determine the work of compression in steady flow and reciprocating machines,
* define adiabatic and isothermal efficiencies as also volumetric efficiency of reciprocating compressors,
* evaluate the advantages of multistage compression, and
* determine the saving in work with intercooling.

13.2 RECIPROCATING COMPRESSORS - SOME DEFINITIONS

Bore = Cylinder diameter

Stroke = Distance through which the piston moves

The two extreme positions of the piston are known as head-end and crank-end dead centres

Clearance volume (Cl)	=	Volume occupied by the fluid when the piston is at head-end dead centre.
Piston Displacement (PD)	=	Volume, a piston sweeps through.
Compression ratio (r_v)	=	Ratio of cylinder volume with the piston at crank-end dead centre to the cylinder volume with the piston at head-end dead centre.
Single - acting	=	Where only one side of the piston is used
Double - acting	=	Where both sides of the piston are used.
Mechanical Efficiency	=	$\frac{\text{Brake work}}{\text{Indicated work}}$ which gives an indication of the losses occuring between the piston and driving shaft.
Volumetric efficiency	=	Is a measure of the effectiveness of the machine with regard to gas handling.
η_{vol}	=	$\frac{\text{Vol. of gas actually compressed and delivered as measured at inlet pressure and temperature}}{\text{Piston displacement}}$
	=	$\frac{\text{Mass of gas actually compressed and delivered}}{\text{Mass of gas occupying the piston displacement at inlet pressure and temperature}}$

13.3 COMPRESSION PROCESS

A gas compression process may be designed either to be adiabatic or to involve heat transfer, depending on the purpose for which the gas is compressed. If the compressed gas is to be used promptly in an engine or in a combustion chamber, adiabatic compression may be desirable in order to obtain the maximum possible energy in the gas at the end of compression. In many applications, however, the gas is not used promptly but is stored in a tank for use as needed. The gas in the tank loses heat to the surroundings and reaches room temperature when finally used. In this case the overall effect of compression and storage is simply to increase the pressure of the gas without change of temperature. It can be shown that if the gas is cooled during compression, instead of after the process, the work required will be less than for adiabatic compression. A further advantage of cooling is the reduction of volume and the consequent reduction of pipe line losses. For this reason, since cooling during compression is not very effective, after-coolers are often used to cool the gas leaving a compressor.

In view of the effect of cooling on a compression process, it is customary to investigate two particular idealized cases, namely reversible adiabatic and reversible isothermal as well as a general case of a reversible polytropic process (pv^n = constant). The paths of these processes are plotted in Figure 13.1.

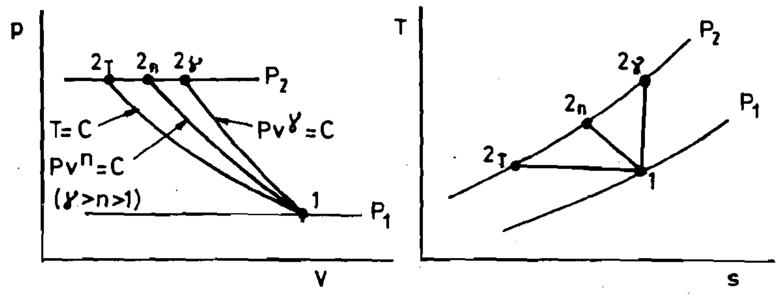


Figure 13.1 : Compression Processes

13.4 WORK OF COMPRESSION IN STEADY FLOW

The steady flow energy equation per unit mass for a compression process, assuming that changes in potential and kinetic energy are negligible may be written (refer Figure 13.2).

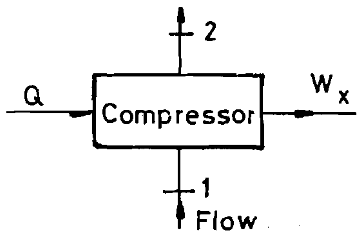


Figure 13.2 : Steady Flow Process

$$Q - W_x = \Delta h = h_2 - h_1 \qquad \text{...(13.1)}$$

we also have the relation $Tds = dh - vdp$ for a reversible process.

$$Q = \Delta h - \int vdp \qquad \text{...(13.2)}$$

Then for any of the idealized cases of Figure 13.1 from equations (13.1) and (13.2)

$$W_x = - \int vdp \qquad \text{...(13.3)}$$

For any gas a compression process may be represented with sufficient accuracy by an equation such as pv^n = constant.

Then
$$v = \frac{p_1^{1/n} \cdot v_1}{p^{1/n}}$$

and
$$W_x = - p_1^{1/n} \cdot v_1 \int_1^2 \frac{dp}{p^{1/n}}$$

$$= - p_1^{1/n} \cdot v_1 \frac{1}{(1 - 1/n)} \left[p_2^{(1 - 1/n)} - p_1^{(1 - 1/n)} \right]$$

$$= - \frac{n}{n - 1} p_1 v_1 \left[\left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right]$$

The work of compression or the steady flow work input to the gas, is the negative of the shaft work W_x.

Therefore W_n = Work of reversible polytropic compression

$$= \frac{n}{n - 1} p_1 v_1 \left[\left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right] \qquad \text{... (13.4)}$$

W_γ = Work of reversible adiabatic compression

$$= \frac{\gamma}{\gamma - 1} p_1 v_1 \left[\left(\frac{p_2}{p_1} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right] \qquad \text{... (13.5)}$$

For isothermal compression of a perfect gas, pv = constant. The work of reversible isothermal compression

$$W_t = -W_x = p_1 v_1 \int_1^2 \frac{dp}{p} = p_1 v_1 \ln \frac{p_2}{p_1} \qquad \text{... (13.6)}$$

In the $p-v$ plot of Figure 13.1, the work of compression for each type of process is represented by the area between the path of that process and the axis of pressure. It is evident that the work of reversible isothermal compression is less than the work of reversible adiabatic compression; the work of reversible polytropic compression is intermediate between the others if n lies between γ and unity. This is the case in reality as the polytropic case will involve some cooling but not enough to obtain isothermal compression. In a real compressor the work will be greater than the work of the reversible compression process because of friction. In such cases the path of compression may be represented by pv^n = constant, but the work of compression is not given by $\int vdp$; the shaft work cannot be determined solely from the properties of the fluid. The friction effects in a reciprocating compressor are often small so that the work may be computed by the integral of $\int vdp$ without great error.

SAQ 1

Tests on reciprocating air compressors with water cooled cylinders show that it is practical to cool the air sufficiently during compression to correspond to a polytropic exponent n in the vicinity of 1.3. Compare the work per kg of air compressed from 100 kPa, 40°C to 600 kPa according to three processes: reversible adiabatic, reversible isothermal and reversible $pv^{1.3}$ = constant. Find the heat transferred from air in each case.

13.5 EFFICIENCY OF A COMPRESSOR

The efficiency of a compressor working in a steady flow process may be defined as

$$\eta_c = \frac{h_{2s} - h_1}{w_c} = \frac{w_\gamma}{w_c} \qquad \text{... (13.7)}$$

where w_c = Shaft work supplied to the actual compressor per kg of gas passing through.

w_γ = Shaft work supplied to a reversible adiabatic compressor per kg of gas compressed from the same initial state to the same final pressure as in the actual compressor.

The above efficiency is generally referred to as the **adiabatic efficiency**

If the desirable idealized process is taken to be the reversible isothermal process, then the compressor efficiency is called the **isothermal efficiency**

$$\eta_{c_{isothermal}} = \frac{w_t}{w_c} \qquad \text{... (13.8)}$$

where w_t = Shaft work supplied to a reversible isothermal compressor for compressing 1 kg gas from the same initial state to the actual final pressure.

The normal thermodynamic practice is to use the reversible adiabatic basis. Because of the effects of cooling, the adiabatic efficiency of a real compressor may be greater than unity.

Many turbine - type compressors are essentially adiabatic machines and for these machines the work of compression, $w_c = h_2 - h_1$

Then for an adiabatic compressor the efficiency is

$$\eta_c = \frac{h_{2s} - h_1}{h_2 - h_1} \qquad \text{... (13.9)}$$

As against this, equation (13.7) gives the adiabatic efficiency of any machine.

SAQ 2

For the conditions given in SAQ 1, find the adiabatic efficiency and the isothermal efficiency of the reversible polytropic compressor.

13.6 WORK OF COMPRESSION - RECIPROCATING COMPRESSORS

A typical indicator card obtained from a reciprocating compressor is shown in Figure 13.3.

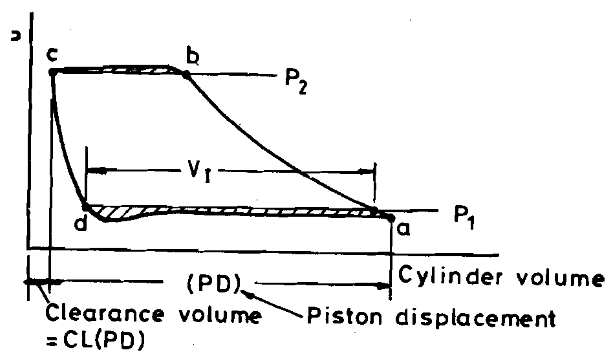


Figure 13.3 : Compressor Indicator Diagram

The sequence of operation in the cylinder is as follows:

1) Compression : Starting at maximum cylinder volume, point *a*, slightly below the inlet pressure *p*, as the volume decreases the pressure rises until it reaches p_2 at *b*; the discharge valve does not open until the pressure in the cylinder exceeds p_2 by enough to overcome the valve spring force.

2) Discharge: Between *b* and *c* gas flows out at a pressure higher than p_2 by the amount of the pressure loss through the valves; at *C*, the point of minimum volume, the discharge valve is closed by its spring.

3) Expansion: From *c* to *d*, as the volume increases, the gas remaining in the clearance volume expands and its pressure falls; the suction valve does not open until the pressure falls sufficiently below p_1 to overcome the spring force.

4) Intake: Between *d* and *a* gas flows into the cylinder at a pressure lower than p_1 by the amount of pressure loss through the valve.

The total area of the diagram represents the actual work of the compressor on the gas. The cross - hatched areas of the diagram above p_2 and below p_1 represent work done solely because of pressure drop through the valves and port passages. This work is called the valve loss.

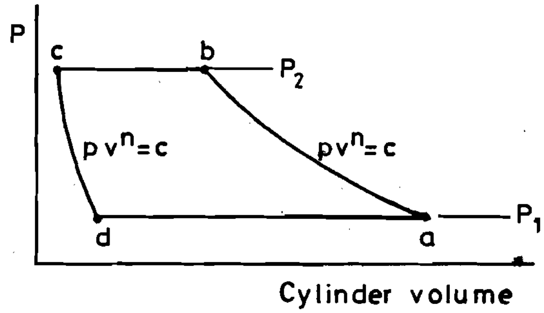


Figure 13.4 : Ideal Indicator Diagram

The idealized machine to which an actual machine is compared has an indicator diagram like Figure 13.4, in which there are no pressure loss effects, and the processes $a-b$ and $c-d$ are reversible polytropic processes. Assuming no state change in the intake $d-a$ and discharge $b-c$ processes, and assuming equal values of the exponent n in the compression $a-b$ and expansion processes $c-d$, the ideal work of compression can be found by taking the integral of pdv around the diagram. If m_f is the mass of fluid taken in and discharged per machine cycle, then the total work interaction per cycle is

$$w = w_{a-b} + w_{b-c} + w_{c-d} + w_{d-a}$$

$$= \frac{p_b v_b - p_a v_a}{1-n} + p_2 (v_c - v_b) + \frac{p_d v_d - p_c v_c}{1-n} + p_1 (v_a - v_d)$$

$$= \frac{p_2 (v_b - v_c)}{1-n} - p_2 (v_b - v_c) + \frac{p_1 (v_d - v_a)}{1-n} - p_1 (v_d - v_a)$$

$$= \frac{n}{1-n} [p_2 (v_b - v_c) + p_1 (v_d - v_a)]$$

$$= \frac{n}{1-n} [p_2 m_f v_2 - p_1 m_f v_1]$$

$$= m_f \frac{n}{1-n} [p_2 v_2 - p_1 v_1]$$

$$= m_f \frac{n}{1-n} p_1 v_1 \left[\frac{p_2 v_2}{p_1 v_1} - 1\right]$$

since pv^n = constant

$$\frac{p_2 v_2}{p_1 v_1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}$$

substituting this in the above expression

$$w = m_f \cdot \frac{n}{1-n} p_1 v_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right]$$

Thus we see that the work per kg of fluid flow is the same as obtained from the steady flow analysis (Equation 13.4). It is therefore unnecessary to make any further analysis of the work of the idealized reciprocating compressor since all desired results have already been obtained by the steady flow analysis.

13.7 VOLUMETRIC EFFICIENCY OF RECIPROCATING COMPRESSORS

The flow capacity of positive displacement compressors is expressed in terms of volumetric efficiency η_{vol}.

... (13.10)

$$\eta_{vol} = \frac{m_f \cdot v_1}{(PD)}$$

where m_f is the mass of fluid flow per machine cycle and (PD) is the piston displacement volume per machine cycle.

The true volumetric efficiency can be determined only by measuring the flow through the machine. An approximate or apparent volumetric efficiency may be obtained from the indicator diagram shown in Figure 13.3. Here the volume V_I is the volume between the point where the cylinder pressure reaches p_1 during the expansion process and the point where it reaches p_1 during the compression process. If the gas remained at constant temperature during the intake process, the volume V_I would be the actual volume taken in at state 1; then the ratio $\frac{V_I}{PD}$ would be the volumetric efficiency. In an actual compressor, because of heat transfer from the cylinder walls, the gas is at higher temperature after entering the cylinder than at state 1. Consequently the volume V_1 is greater than the volume taken in from the supply line, and the ratio $\frac{V_I}{PD}$ is larger than the true volumetric efficiency; hence the name apparent volumetric efficiency.

13.8 VOLUMETRIC EFFICIENCY AND CLEARANCE

The volumetric efficiency of an idealized compressor having an indicator diagram like Figure 13.4 can be written directly from equation (13.10).

$$\eta_{vol} = \frac{m_f \cdot v_1}{(PD)}$$

$$PD = V_c - V_a$$

$$\text{Clearance cl} = \frac{V_a}{(PD)}$$

$$m_f = \frac{V_c - V_d}{v_1} \quad \text{and} \quad V_c = (1 + \text{cl})\ PD$$

$$\text{Also} \quad V_d = V_a \frac{v_d}{v_a} \quad (\text{mass being constant})$$

$$= \text{cl}\ (PD)\ \frac{v_1}{v_2}$$

Therefore

$$m_f = \frac{(PD)}{v_1}\left[1 - \text{cl}\left(\frac{v_1}{v_2} - 1\right)\right]$$

Therefore

$$\eta_{vol} = \frac{m_f\ v_1}{(PD)} = 1 - \text{cl}\left[\left(\frac{p_2}{p_1}\right)^{1/n} - 1\right] \quad \text{... (13.11)}$$

Figure 13.5 shows a plot of equation (13.11)

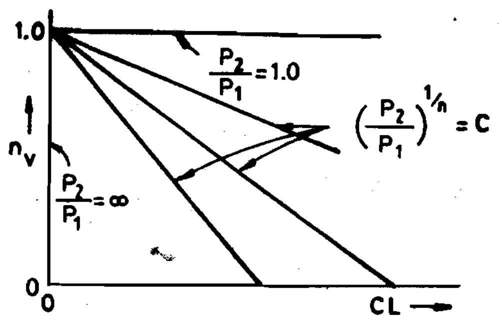


Figure 13.5 : Effect of clearance on volumetric efficiency

Since $\left(\frac{p_2}{p_1}\right)^{1/n}$ is always greater than unity, it is evident that the volumetric efficiency of the idealized compressor decreases as the clearance increases and as the pressure ratio increases.

SAQ 3

A reciprocating air compressor operates between 100 kPa and 500 kPa with a polytropic exponent of 1.3. How much clearance would have to be provided in the ideal case, to make the volumetric efficiency 50 percent? To make it zero?.

Example 13.1 :

An air compressor cylinder has 15 cm bore and 15 cm stroke and 5% clearance. The machine operates between 100 kPa, 27°C and 500 kPa. The polytropic exponent is 1.3.

a) Sketch the idealized indicator diagram, and find

  i) cylinder volume at each corner of the diagram

  ii) mass flow of air, and

  iii) flow capacity in m^3/min at 720 rev.per minute.

b) Find the ideal volumetric efficiency

c) What is the mean effective pressure?

d) Find the heat transferred as a fraction of the indicated work.

Solution :

Referring to **Figure** 13.6

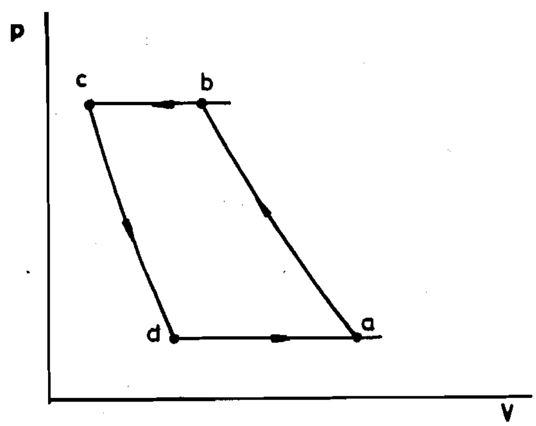


Figure 13.6 : Indicator diagram

Data given : Bore = 15 cm

Stroke = 15 cm

Clearance = 5%

(i) V_a = stroke volume + clearance volume

$$= \frac{\pi}{4} \times \left[\frac{15^2}{(100)^2}\right]\left(\frac{15}{100}\right) + \frac{5}{100}\left[\frac{\pi}{4}\left(\frac{15}{100}\right)^2 \times \frac{15}{100}\right]$$

$$= 26.5 \times 10^{-4} + \frac{5}{100} \times 26.5 \times 10^{-4}$$

$$= 27.83 \times 10^{-4} \text{ m}^3$$

To find V_b

$a - b$ is a polytropic process with $n = 1.3$.

$$p_1 V_a^n = p^2 V_b^n$$

$$\frac{V_b}{V_a} = \left(\frac{p_1}{p_2}\right)^{1/n} = \left(\frac{100\times10^3}{300\times10^3}\right)^{1/1.3} = \left(\frac{1}{5}\right)^{1/1.3} = 0.324$$

Therefore $\quad V_b = V_a \times 0.324 = 27.83\times10^{-4}\times0.324$

$$= 9.03\times10^{-4}\ \text{m}^3$$

To find V_c

V_c = clearance volume = 5% of stroke volume

$$= 0.05\times26.5\times10^{-4}$$

$$= 1.325\times10^{-4}\ \text{m}^3$$

To find V_d

$$p_2 V_c^n = p_1 V_d^n$$

$$\frac{V_d}{V_c} = \left(\frac{p_2}{p_1}\right)^{1/n} = \left(\frac{500\times10^3}{100\times10^3}\right)^{1/n} = (5)^{1/1.3} = 3.46$$

Therefore $\quad V_d = V_c\times3.46 = 1.325\times10^{-4}\times3.46$

$$= 4.57\times10^{-4}\ \text{m}^3$$

ii) Mass flow of air per cycle $= \dfrac{V_a - V_d}{v_1}$

where $\quad v_1$ = specific volume of air entering.

$p_1 v_1 = RT_1$ and $R = 287$ J/kg.K

Therefore $\quad v_1 = \dfrac{RT_1}{p_1} = \dfrac{287\times300}{100\times10^3} = 0.861\ \text{m}^3/\text{kg}$

$$V_a - V_d = 27.83\times10^{-4} - 4.57\times10^{-4} = 23.26\times10^{-4}\ \text{m}^3.$$

Therefore $\quad$ mass flow $= \dfrac{23.26\times10^{-4}}{0.861} = 26.4\times10^{-4}$ kg

iii) Speed = 720 rev./min

Flow capacity $= (V_a - V_d)\times720$

$$= 23.26\times10^{-4}\times720 = 1.68\ \text{m}^3/\text{min}$$

(b) Ideal volumetric efficiency $\eta_{vol} = \dfrac{V_a - V_d}{V_a - V_c}$

$$= \frac{23.26\times10^{-4}}{26.4\times10^{-4}}\times100 = 87.8\%$$

c) Mean effective pressure $= \dfrac{\text{work done per cycle}}{\text{stroke volume}}$

Work done per kg of air $= \dfrac{n}{n-1}\cdot p_1 v_1\left[\left(\dfrac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right]$

$$= \left(\frac{1.3}{1.3-1}\right)\times100\times10^3\times0.861\times[5^{0.3/1.3} - 1]$$

$$= 4.34 \times 0.861 \times 10^5 \, [1.447 - 1]$$

$$= 4.34 \times 0.861 \times 10^5 \times 0.447$$

$$= 1.67 \times 10^5 \text{ N.m}$$

$$\text{Work done per cycle} = 1.67 \times 10^5 \times 26.4 \times 10^{-4}$$

$$= 440.9 \text{ N.m}$$

$$\text{Mean effective pressure} = \frac{440.9}{26.4 \times 10^{-4}} = 1.67 \times 10^5 \text{ N/m}^2$$

d) $Q - w_x = \Delta h$ neglecting changes in kinetic and potential energies

$$Q = \Delta h - w_x = c_p \, \Delta T - w_x$$

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} = \left(\frac{500 \times 10^3}{100 \times 10^3}\right)^{0.3/1.3} = 1.447$$

Therefore $T_2 = T_1 \times 1.447 = 300 \times 1.447 = 434.1$ K

$\Delta h = 1005\,(434.1 - 300)$ since c_p for air = 1005 J/kg.K

$= 1005\,(134.1) = 134.77$ kJ/kg

Δh per cycle $= 134.77 \times 26.4 \times 10^{-4} = 0.356$ kJ

w = indicated work = 440.9 N.m

Therefore Q per cycle $= 0.356 \times 10^3 - 440.9$

$= 356 - 440.9 = -84.9$ J

Q as a fraction of indicated work $= \frac{-84.9}{440.9} = 0.19$

Example 13.2 :

A refrigeration compressor has two single acting cylinders of 7.5 cm bore and 7.5 cm stroke and works as a single stage compressor. The clearance is 4% of piston displacement. Ammonia vapour at – 10°C with a degree of superheat of 5°C, is compressed to a pressure having saturation temperature of 30°C and to a temperature of 105°C. Assuming heat transferred from the compressor to be 25 kJ/kg and the actual volumetric efficiency equal to 85% of the volumetric efficiency based on the idealized indicator diagram, calculate the following:

a) The compressor adiabatic efficiency

b) The flow capacity of the compressor if the compressor runs at 960 rev/min.

Properties of Ammonia

p (N/m^2)	t (°C)	v_g (m^3/kg)	h_g (kJ/kg)
2.28×10^5	– 15	0.5	1442.8
11.3×10^5	30	0.11	1465.1

Assume the average value of c_p of superheated ammonia vapour to be 2.675 kJ/kg.deg and $n = 1.3$.

Solution :

Bore = 7.5 cm

Stroke = 7.5 cm

Clearance = 4%

It is given that ammonia gas at – 10°C has a degree of superheat of 5°C.

Therefore, saturation temperature = – 15°C compressed to 11.3×10^5 N/m^2, and temperature = 105°C and degree of superheat = 105 – 30 = 75°C.

Process 1 – 2 is with heat transfer and process 1 – 2s is reversible adiabatic.

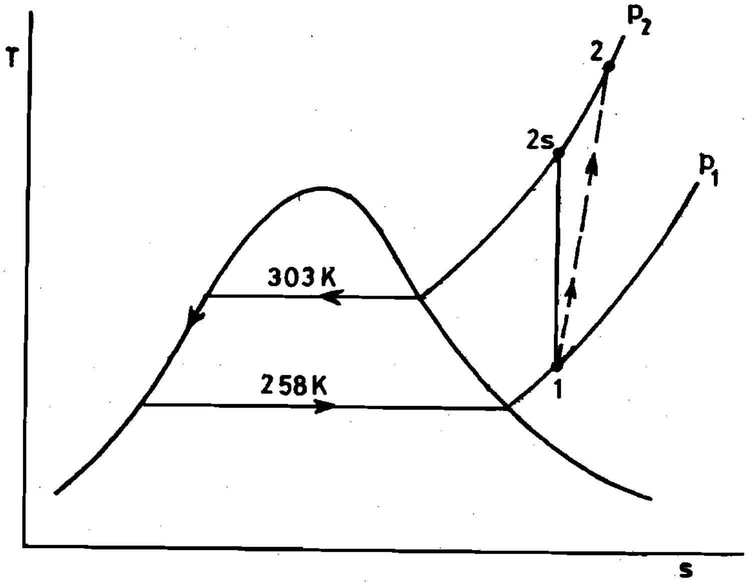


Figure 13.7 : Adiabatic compression on *T-S* coordinates

Applying S.F.E.E., $Q - w_x = \Delta h$ (neglecting changes in K.E. and P.E.).

Treating superheated vapour as an ideal gas

h_1 at -10°C $= 1422.8 + c_p \Delta t$

$= 1442.8 + 2.675 \times 5$

$= 1436.175$ kJ/kg

h_2 at 105°C $= 1465.1 + c_p \times \Delta t$

$= 1465.1 + 2.675 \times 75$

$= 1665.7$ kJ/kg

$w_x = -25 - \Delta h$

$= -25 - (1665.7 - 1436.175)$

$= -254.55$ kJ/kg

If the compression process was reversible adiabatic

$$\frac{T_{2s}}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{500 \times 10^3}{100 \times 10^3}\right)^{0.3/1.3}$$

Therefore $\quad T_{2s} = 381 \text{ K} = 108°\text{C}$

$h_{2s} = 1465.1 + 2.675 \times 78$

$= 1673.75$ kJ/kg

Reversible adiabatic work of compression $= w_{xs}$

$$w_{xs} = Q - \Delta h = 0 - (1673.75 - 1436.175)$$

$$= -237.58 \text{ kJ/kg}$$

Adiabatic efficiency $= \dfrac{237.58}{254.55} = 0.93 = 93\%$

b) Actual Volumetric Efficiency $= 0.85\ \eta_{vol.ideal}$

$$\frac{(V_a - V_d)_{actual}}{(PD)} = \eta_{vol} \times 0.85 \qquad \text{Refer Figure 13.6}$$

$$(V_a - V_d)_{actual} = (PD) \times \eta_{vol} \times 0.85$$

$$PD = \frac{\pi}{4} \times \left(\frac{7.5}{100}\right)^2 \times \left(\frac{7.5}{100}\right)$$

$$= \frac{\pi}{4} \times 56 \times 75 \times 10^{-6} = 3.3 \times 10^{-4}\ m^3$$

$$V_c = 0.04\ (PD) = 0.132 \times 10^{-4}\ m^3$$

$$PD = V_a - V_c = 3.3 \times 10^{-4}\ m^3$$

Therefore $V_a = V_c + 3.3 \times 10^{-4} = 3.432 \times 10^{-4}\ m^3$

To find V_d

During the process $c - d$, $m_c = m_d$

Therefore, $\frac{V_c}{v_c} = \frac{V_d}{v_d}$

Process $b - c$ is at constant pressure

Therefore, $v_b = v_c$

For superheated vapour, v_b at 105°C $= \frac{0.11}{303} \times 378$

$= 0.138\ m^3/kg$

Therefore $v_c = 0.138\ m^3/kg$

Similarly process $d - a$ is a constant pressure process and $v_d = v_a$

v_a for superheated Ammonia Vapour at $-$ 15°C $= 0.5\ m^3/kg$

Therefore $v_d = 0.5\ m^3/kg$

$$V_d = \frac{V_c}{v_c} \cdot v_d = \frac{0.132 \times 10^{-4}}{0.138} \times 0.5 = 0.486 \times 10^{-4}\ m^3/kg$$

Therefore, $\eta_{vol} = \frac{V_a - V_d}{PD} = \frac{3.432 \times 10^{-4} - 0.486 \times 10^{-4}}{3.3 \times 10^{-4}}$

$$= \frac{2.946}{3.3} = 0.893$$

$$(V_a - V_d)_{actual} = (PD) \cdot \eta_{vol}$$

$$= 3.3 \times 10^{-4} \times 0.893 \times 0.85$$

$$= 2.5 \times 10^{-4}\ m^3$$

Therefore the flow capacity of the compressor at 960 rev/min $= (V_a - V_d)_{actual} \times 960$

$= 2.5 \times 10^{-4} \times 960$

$= 0.24\ m^3/min.$

Example 13.3 :

An air compressor has a volumetric efficiency of 70% when tested, the discharge state being 500 kPa, 150°C and the inlet state 100 kPa, 15°C. If the clearance is 4%, predict the new volumetric efficiency when the discharge pressure is increased to 700 kPa. Assume that the ratio of real to ideal volumetric efficiency and the exponent n remain constant.

Solution :

$\eta_{vol.actual} = 0.7$

$p_1 = 100 \times 10^3\ N/m^2$ $\qquad$ $p_2 = 500 \times 10^3\ N/m^2$

$t_1 = 15°C = 288\ K$ $\qquad$ $t_2 = 150°C = 423\ K$

For polytropic compression process

$$\frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{\frac{n}{n-1}}$$

Therefore $$\frac{5 \times 10^5}{1 \times 10^5} = \left(\frac{423}{288}\right)^{\frac{n}{n-1}}$$

or $$\frac{n}{n-1} = \frac{\ln 5}{\ln (423/288)} = \frac{1.61}{0.147}$$

Ideal volumetric efficiency $$= 1 - \text{cl}\left[\left(\frac{p_2}{p_1}\right)^{\frac{1}{n}} - 1\right]$$

$$= 1 - 0.04[(5)^{1/1.09} - 1]$$

$$= 1 - 0.04[4.37 - 1]$$

$$= 0.865$$

$$\frac{\text{Actual } \eta_{vol}}{\eta_{vol} \text{ ideal}} = \frac{0.7}{0.865} = \text{constant for this compressor}$$

Therefore, when the discharge pressure is increased to 700 kPa.

$$\frac{\text{Actual Vol. Efficiency}}{\text{Ideal Vol. Efficiency}} = \frac{0.7}{0.865} = \text{constant}$$

Ideal Volumetric Efficiency $$= 1 - \text{cl}\left[\left(\frac{p_2}{p_1}\right)^{1/n} - 1\right]$$

$$= 1 - 0.04\left[\left(\frac{700 \times 10^3}{100 \times 10^3}\right)^{1/1.09} - 1\right]$$

$$= 1 - 0.04[5.95 - 1]$$

$$= 0.802$$

Therefore, New Volumetric Efficiency $$= \frac{0.7}{0.865} \times 0.802 = 0.65$$

13.9 VOLUMETRIC EFFICIENCY AND PRESSURE RATIO - MULTISTAGE COMPRESSION

It is evident from Figure 13.5 that as the pressure ratio is increased the volumetric efficiency of a compressor of Fixed clearance decreases, eventually becoming zero. This can also be seen in an indicator diagram shown in Figure 13.8 below. As the discharge pressure is

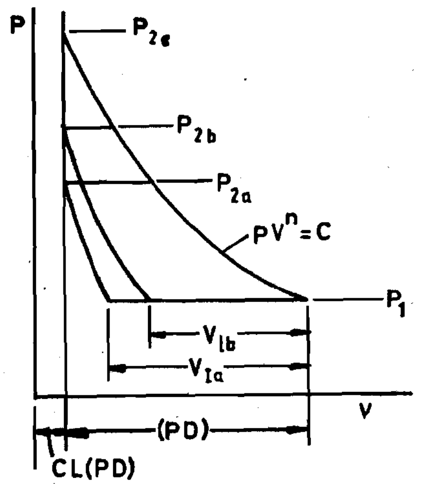


Figure 13.8 : Effect of pressure-on capacity

increased, the volume V_I, taken in at p_1, decreases. At some pressure p_{2c} the compression line intersects the line of clearance volume and there is no discharge of gas. An attempt to pump to p_{2c} (or any higher pressure) would result in compression and re-expansion of the same gas repeatedly, with no flow in or out.

The maximum pressure ratio attainable with a reciprocating compressor cylinder is then seen to be limited by the clearance. There are practical and economic limits to the reduction of clearance; when these limits interfere with the attainment of the desired discharge pressure, it is necessary to use multistage compression. In a multistage compressor the gas is passed in series through two or more compressors, or stages, each of which operates on a small pressure ratio. Disregarding pressure losses between stages, the overall pressure ratio is the product of the pressure ratios of the stages.

Figure 13.9 shows the comparative idealized indicator diagrams for compression of a gas from p_1 to p_2 by a two stage machine or by a single stage machine of the same piston displacement and clearance as the first stage of the two stage machine.

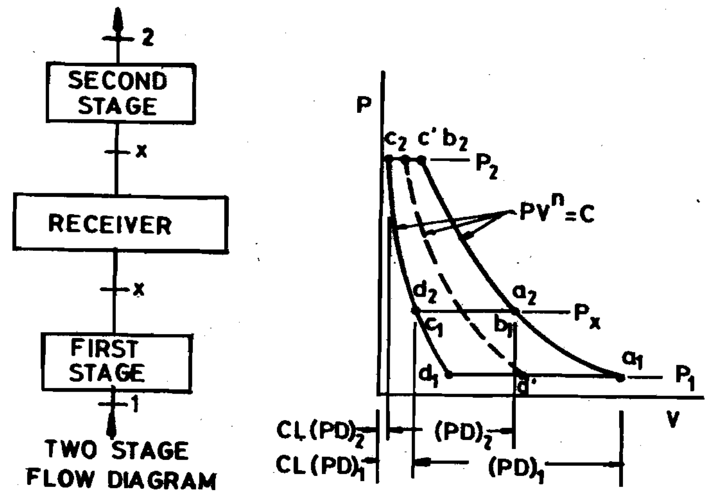


Figure 13.9 : Two-stage compression

The single stage machine compresses gas from a_1 to b_2, discharges at p_2 from b_2 to c^1, expands from c' to d', and take in gas from d^1 to a_1. Thus the capacity per machine cycle is $V_{a1} - V_d'$.

The first stage of the two stage machine compresses gas from a_1 to b_1, discharges at p_x from b_1 to c_1, expands from c_1 to d_1, and takes in gas at p_1 from d_1 to a_1. The capacity per machine cycle is $V_{a1} - V_{d1}$, which is appreciably larger than the capacity of the single stage machine.

The second stage takes in gas from d_2 to a_2 (which coincides with $c_1 - b_1$), compresses from a_2 to b_2, discharges at p_2 from b_2 to c_2, and expands from c_2 to d_2. The flow capacity of the two stage machine is the capacity of the first stage, since all the gas is taken in by the first stage.

The two stage compressor has greater capacity than the single stage compressor of the same clearance, at the same pressure ratio p_2/p_1. This advantage is greater at larger pressure ratios, and at sufficiently large pressure ratios the single stage compressor becomes uneconomical because of low volumetric efficiency.

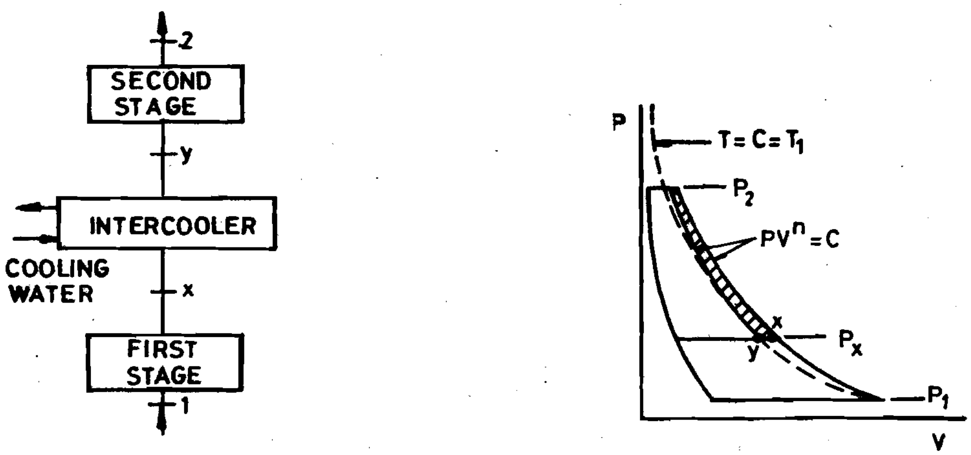
SAQ 4

A gas is to be compressed from 30 kPa to 500 kPa. It is known that cooling corresponding to a polytropic exponent of 1.25 is practical and the clearance of the available compressor is 3 percent. Compare the volumetric efficiencies to be anticipated for (a) single stage compression, and (b) two stage compression with equal pressure ratios in the stages.

13.10 INTERCOOLING

The advantage of multistage compression in itself is primarily that of increased flow capacity or volumetric efficiency for a given pressure ratio. Multistage compression also enables appreciable saving of work if cooling of the gas between stages is resorted to. The cooling is usually done by a water cooled tubular heat exchanger which also serves as a receiver between the stages.

The work saved by intercooling in the idealized two stage reciprocating compressor is illustrated on the indicator diagram of Figure 13.10.



(a) Flow Diagram

(b) Two-stage compression with intercooling

Figure 13.10

Cooling by cylinder water jackets is never very effective. The compression curve is always closer to adiabatic than to isothermal. Therefore the gas discharged from the first stage at state x is at a higher temperature than the inlet temperature T_1; if the gas is then cooled to state y at temperature T_1, the volume entering the second stage will be less than the volume leaving the first stage. The compression in the second stage will proceed along a new polytropic curve at smaller volume. The cross hatched area between the two polytropic curves in Figure 13.10 represents the work saved by interstage cooling to the initial temperature. Actual cooling might be to some other temperature, but it is conventional to discuss cooling to T_1.

The saving of work by two stage compression with intercooling will depend upon the interstage pressure p_x chosen. Obviously, as p_x approaches either p_1 or p_2, the process approaches single stage compression. Any saving of work must increase from zero to a maximum and return to zero as p_x varies from p_1 to p_2.

13.11 MINIMUM WORK IN TWO STAGE COMPRESSION WITH INTERCOOLING

The conditions affecting the work of compression may be studied by use of the steady flow system and $T-s$ diagram of Figure 13.11. As shown in the figure, a perfect gas is compressed from the initial state p_1T_1 to p_x; it is then cooled at constant pressure to T_y, and then compressed from p_x, T_y to p_2. Given p_1, T_1, T_y and p_2, it is desired to find the value of p_x which gives minimum work.

Let the adiabatic compression efficiencies of the two-stages be respectively η_{c1} and η_{c2}.

The work of compression $w_c = w_1 + w_2$

$$w_1 = \frac{1}{\eta_{c1}} \frac{\gamma}{\gamma - 1} RT_1 \left[\left(\frac{p_x}{p_1} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right]$$

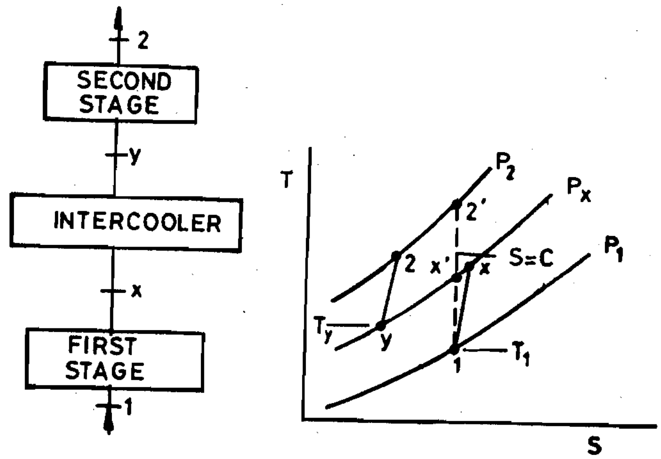


Figure 13.11 : $T-s$ plot of two-stage compression process

and $$w_2 = \frac{1}{\eta_{c2}} \frac{\gamma}{\gamma - 1} RT_y \left[\left(\frac{p_2}{p_y} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right]$$

But $$\left(\frac{p_x}{p_1} \right)^{\frac{\gamma-1}{\gamma}} = \frac{T_x'}{T_1}$$

and $$\left(\frac{p_2}{p_y} \right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{p^2}{p_x} \right)^{\frac{\gamma-1}{\gamma}} = \frac{T_2'}{T_x}$$

Therefore $$w_c = \frac{\gamma R}{\gamma - 1} \left[\frac{T_1}{\eta_{c1}} \left(\frac{T_x'}{T_1} - 1 \right) + \frac{T_y}{\eta_{c2}} \left(\frac{T_2'}{T_x'} - 1 \right) \right]$$

Taking the derivative with respect to T_x' and setting it equal to zero (noting that T_1, T_2' and T_y are constant,

$$\frac{dw_c}{dT_x'} = 0$$

$$\frac{\gamma R}{\gamma - 1} \left[\frac{1}{\eta_{c1}} + \frac{T_y T_2'}{\eta_{c2}} \left(\frac{-1}{(T_x')^2} \right) \right] = 0$$

Then $$(T_x')^2 = \frac{\eta_{c1}}{\eta_{c2}} T_y T_2'$$

and $$\frac{T_x'}{T_1} = \sqrt{\frac{\eta_{c1}}{\eta_{c2}} \frac{T_y}{T_1} \frac{T_2'}{T'}}$$

For minimum work

$$\frac{T_x'}{T_1} = \left(\frac{p_x}{p_1} \right)^{\frac{\gamma-1}{\gamma}}$$

and $$\frac{T_2'}{T_1} = \left(\frac{p_2}{p_1} \right)^{\frac{\gamma-1}{\gamma}}$$

Therefore for minimum work in two stage compression of a perfect gas with intercooling to a fixed temperature T_y,

and $$\frac{p_x}{p_1} = \sqrt{\left(\frac{\eta_{c1}}{\eta_{c2}} \frac{T_y}{T_1} \right)^{\frac{\gamma}{\gamma-1}} \left(\frac{p_2}{p_1} \right)} \quad \text{... (13.12)}$$

For the special case of $T_y = T_1$ and $\eta_{c1} = \eta_{c2}$, which is often taken as standard of comparison, the requirement for minimum work is

$$\frac{p_x}{p_1} = \sqrt{\frac{p_2}{p_1}} \quad \text{... (13.13)}$$

Also for this special case the condition of minimum work is the condition of equal work in the two stages.

When three stages of equal efficiency are used, with intercooling to the initial temperature at two points as shown in Figure 13.12, the condition of minimum work, and of equal division of work among stages is

$$\frac{p_{x1}}{p_1} = \frac{p_{x2}}{p_{x1}} = \frac{p_2}{p_{x_2}} = \sqrt[3]{\frac{p_2}{p_1}} \quad \text{... (13.14)}$$

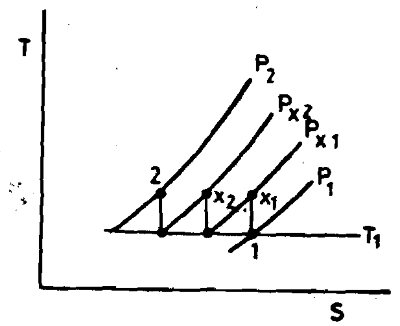


Figure 13.12 : Three stage compression with intercooling

Example 13.4 :

A two stage air compressor is used to compress 10 m^3/min of air from 100 kPa to 1400 kPa. What will be the pressure in the intercooler for the special case of minimum work of compression? If $n = 1.3$, find the percentage saving in work by compressing in two stages compared to single stage compression.

Solution :

Refer Figure 13.13

Data given : Flow rate = 10 m^3/min

$$p_1 = 100 \times 10^3 \text{ N/m}^2$$

$$p_2 = 1400 \times 10^3 \text{ N/m}^2$$

Process 1 – 2 represents single stage compression

For minimum work of compression

$$\frac{p_x}{p_1} = \sqrt{\frac{p_2}{p_1}}$$

Therefore p_x = pressure in the intercooler = $p_1 \sqrt{\frac{p_2}{p_1}}$

$$= 100 \times 10^3 \sqrt{\frac{1400 \times 10^3}{100 \times 10^3}} = 10^5 \sqrt{14}$$

$$= 3.74 \times 10^5 \text{ N/m}^2$$

Work done per minute in two-stage compression is given by

$$W_{1st\ stage} = \frac{nRT_1}{n-1}\left[\left(\frac{p_x}{p_1}\right)^{\frac{n-1}{n}} - 1\right]$$

$$W_{2nd\ stage} = \frac{nRT_y}{n-1}\left[\left(\frac{p_2}{p_x}\right)^{\frac{n-1}{n}} - 1\right]$$

Assuming perfect intercooling ($T_y = T_1$) and

$$\frac{p_x}{p_1} = \frac{p_2}{p_x} \text{ or } p_x = \sqrt{p_2 \cdot p_1}$$

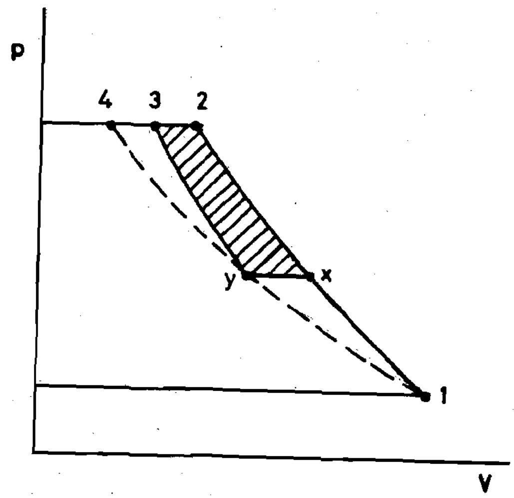


Figure 13.13 : Indicator diagram of two-stage compressor

$$\text{Minimum } W_{total} \text{ per minute} = 2.\frac{n}{n-1} RT_1\left[\left(\frac{p_x}{p_1}\right)^{\frac{n-1}{n}} - 1\right]$$

Substituting the data given

$$\text{Minimum } W_{total} \text{ per minute} = 2 \times \frac{1.3}{0.3} p_1 V_1\left[\left(\frac{p_x}{p_1}\right)^{\frac{n-1}{n}} - 1\right]$$

$$= \frac{2.6}{0.3} \times 10^6 \, [(3.74)^{0.2305} - 1]$$

$$= \frac{2.6 \times 10^6}{0.3} (1.356 - 1)$$

$$= 3085 \text{ kJ/minute}$$

For single stage compression

$$\text{Work of compression per minute} = \frac{n}{n-1} p_1 V_1\left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right]$$

$$= \frac{1.3}{0.3} \times 100 \times 10^3 \times 10[(14)^{0.2305} - 1]$$

$$= 4.33 \times 10^6 \, [1.84 - 1] = 3637 \text{ kJ/min}$$

Therefore saving in work = 3637 – 3085 = 552 kJ/min

$$\text{Percentage saving in work} = \frac{552}{3637} = 15\%$$

Example 13.5 :

A two stage air compressor is to be designed to compress 6 m^3/min of free air (air at ambient conditions) at 100 kPa, 27°C to 900 kPa. The cylinders of the compressor are to be water jacketed and an intercooler provided in between the two stages. From previous experience the following data may be assumed.

i) Index of compression $n = 1.3$

ii) Volumetric efficiency of each cylinder = 80%

iii) Temperature of air leaving the intercooler = 37°C

iv) Overall compression efficiency = 85%

Determine

a) Piston displacement volume for each of the compressors

b) Required size of the electric motor to drive the compressor if the available sizes of induction motors are 22, 30 and 37 kW.

Solution :

Referring to Figure 13.14

$$p_1 = 100 \times 10^3 \text{ N/m}^2$$
$$T_1 = 273 + 27 = 300 \text{ K}$$
$$p_4 = 900 \times 10^3 \text{ N/m}^2$$
$$n = 1.3$$
$$vol = 80\%$$
$$T_3 = 273 + 37 = 310 \text{ K}$$

Overall compressor efficiency = 85%

To find $V_1 - V_6$ and $V_3 - V_5$

$$p_1 v_1 = RT_1$$
$$100 \times 10^3 . v_1 = 287 \times 300$$
$$v_1 = \frac{287 \times 300}{10^5} = 0.861 \text{ m}^3/\text{kg}$$
$$V_1 - V_7 = 6 \text{ m}^3/\text{min}$$
$$\eta_{vol} = \frac{V_1 - V_7}{V_1 - V_6} = \frac{m_f . v_1}{(\text{PD})_1} = 0.8$$

Therefore $$\frac{6}{V_1 - V_6} = 0.8$$

$$V_1 - V_6 = (\text{PD})_1 = \frac{6}{0.8} = 7.5 \text{ m}^3/\text{min}$$

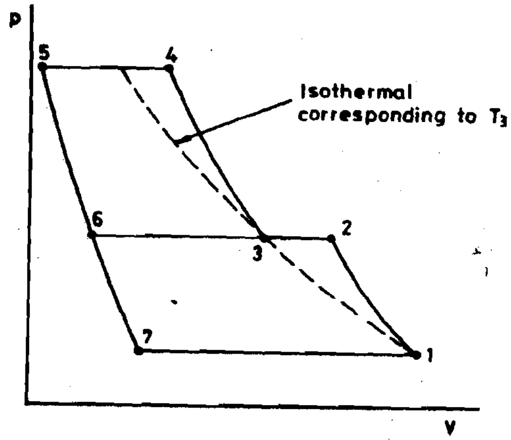


Figure 13.14 : Two-stage process

$$m_f . v_1 = V_1 - V_7 = 6 \text{ m}^3/\text{min}$$

Therefore $$m_f = \frac{6}{0.861} = 6.84 \text{ kg/min}$$

This m_f is the same for the second stage also.

Therefore, $$0.8 = \frac{6.84 \times v_3}{(\text{PD})_2}$$

$$p_3 v_3 = RT_3$$
$$p_2 = p_3 = \sqrt{9 \times 10^5 \times 1 \times 10^5} = 3 \times 10^5 \text{ N/m}^2$$

Therefore $$3 \times 10^5 \times v_3 = 287 \times 310$$

$$v_3 = 0.30256 \text{ m}^3/\text{kg}$$

And $$(\text{PD})_2 = \frac{6.84 \times 030256}{0.8} = 2.585 \text{ m}^3/\text{min}$$

b) Compression work

$$W_{total} = \frac{n}{n-1} p_1 v_1 \left[\left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right] + \frac{n}{n-1} p_3 v_3 \left[\left(\frac{p_4}{p_3} \right)^{\frac{n-1}{n}} - 1 \right]$$

$$= \frac{n}{n-1} \left[\left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right] (p_1 v_1 + p_3 v_3)$$

$$= \frac{n}{n-1} \left[\left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right] m_f . R(T_1 + T_3)$$

$$= \frac{1.3}{0.3} [(3)^{0.3/1.3} - 1] (6.84 \times 287 \times 610)$$

$$= 1493.1 \text{ kJ/min}$$

$$\text{Actual work} = \frac{W_{total}}{\text{Overall Efficiency}} = \frac{1493.1}{0.85}$$

$$= 1756.6 \text{ kJ/min}$$

$$= 29.3 \text{ kW}$$

Therefore a 30 kW electric motor should be used.

13.12 SUMMARY

The work required to compress a gas from a given initial state to a given final pressure is reduced by removing heat from the gas during compression. In actual machines, the amount of heat which can be transferred during the compression process is limited; so the ideal process for simulating an actual compression may be a polytropic process with the polytropic exponent n closer to γ than to 1. In simple gas turbine plants and in other applications where immediately after compression the gas is to be heated by means of a fuel, adiabatic compression is most desirable from the standpoint of overall plant efficiency, even though the compression work is greater than for compression with cooling.

For an adiabatic compressor, the compressor efficiency is defined as

$$\eta_c = \frac{\text{Work of reversible adiabatic compression from state } p_1 \text{ to } p_2}{\text{Work of actual adiabatic compression from state } p_1 \text{ to } p_2}$$

In a multistage adiabatic compressor, the efficiency of the entire machine is lower than that of the individual stages if they have equal efficiencies of less than 100 per cent.

If isothermal compression is impossible or impractical, as it usually is, a reduction in the work required for given pressure limits can be achieved by cooling the gas at constant pressure between stages. For polytropic compression with the same value of n in each stage and intercooling to the initial temperature, minimum total work is required when the pressure ratio is the same for each stage.

The volumetric efficiency of a reciprocating compressor is an important indicator of its flow capacity. It is defined as $\eta_{vol} = m_f . \frac{v_1}{\text{PD}}$ where m_f is the mass flow per machine cycle and (PD) is the piston displacement. The volumetric efficiency decreases as the clearance increases and as the pressure ratio increases.

Minimum work in two stage compression with perfect intercooling will be achieved for the condition

$$\frac{p_x}{p_1} = \sqrt{\frac{p_2}{p_1}}$$

where p_x is the pressure in the intercooler.

13.13 KEYWORDS AND PHRASES

Reversible adiabatic compression : A slow, frictionless compression process under adiabatic conditions.

Reversible polytropic compression : A slow, frictionless compression process under polytropic (pv^n = constant conditions).

Clearance Volume : volume occupied by the fluid when the piston is at head-end dead centre.

Volumetric efficiency : A measure of the effectiveness of the machine with regard to gas handling.

$$\eta_{vol.} = \frac{\text{Vol. of gas actually compressed}}{\text{Piston displacement volume}}$$

Adiabatic efficiency : It is the ratio of shaft work supplied to a reversible adiabatic compressor per kg. of gas compressed to the shaft work supplied to an actual compressor.

Isothermal efficiency : It is the ratio of shaft work supplied to a reversible isothermal compressor per kg. of gas compressed to the shaft work supplied to the actual compressor.

Multistage compression : In this the gas is compressed in series through two or more compressors or stages each of which operates over a small pressure ratio.

Intercooling : This is cooling of the compressed gas between stages and results in considerable saving of work.

Minimum work with intercooling : For minimum work in two stage compression with intercooling back to the initial temperature the condition is $\frac{P_x}{P_1} = \sqrt{\frac{P_2}{P_1}}$

13.14 FURTHER READING

1. David A.Mooney, Mechanical Engineering Thermodynamics, Prentice-Hall, Inc., 1953, Chapter 21.
2. J.B. Jones and G.A.Hawkins, Engineering Thermodynamics, Second Edition, John Wiley & Sons, Inc., 1986, Chapter 15.
3. Kenneth Wark, Jr., Thermodynamics, 5th Edition, McGraw-Hill Book Company, 1989, Chapter 16.

13.15 ANSWERS /SOLUTIONS TO SAQs

SAQ 1

Considering air as a perfect gas

$$p_1 v_1 = RT_1 = 287 \times 313$$

$$w_{adiabatic} = \frac{\gamma}{\gamma - 1} RT_1 \left[\left(\frac{p_2}{p_1} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right]$$

$$= \frac{1.4}{0.4} \times 287 \times 313 \left[\left(\frac{600 \times 10^3}{100 \times 10^3} \right)^{\frac{1.4-1}{1.4}} - 1 \right]$$

$$= 314408.5\ [1.67 - 1]$$

$$= 210.65 \text{ kJ/kg}$$

$$w_{polytropic} = \frac{n}{n-1} RT_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right]$$

$$= \frac{1.3}{0.3} \times 287 \times 313[(6)^{\frac{0.3}{1.3}} - 1]$$

$$= 389267.6\,[1.52 - 1]$$

$$= 202.42 \text{ kJ/kg}$$

$$w_{isothermal} = RT_1 \ln \frac{p_2}{p_1}$$

$$= 287 \times 313 \ln (6)$$

$$= 287 \times 313 \times 1.7917$$

$$= 160.95 \text{ kJ/kg}$$

The heat transferred in the adiabatic process is zero. In the polytropic process

$$Q = c_v \,.\, \frac{\gamma - n}{1 - n}(T_2 - T_1)$$

where $c_v \,.\, \frac{\gamma - n}{1 - n}$ is called the polytropic specific heat (see unit 9 on ideal gases).

Therefore
$$Q = c_v \,.\, \frac{\gamma - n}{1 - n} \,.\, T_1 \left[\left(\frac{T_2}{T_1}\right) - 1\right]$$

$$= c_v \frac{\gamma - 1}{1 - n} \,.\, T_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right]$$

$$= 718 \left(\frac{1.4 - 1}{1 - 1.3}\right) \times 313\,[1.52 - 1]$$

$$= -\,299645.33 \times 0.52$$

$$= -\,155.82 \text{ kJ/kg}$$

The heat transferred from the air during polytropic compression = 155.82 kJ/kg.

In the isothermal process with a perfect gas the heat transfer is equal to the work; then the heat transferred from the air is 160.95 kJ/kg.

SAQ 2

For the conditions given in SAQ 1, the shaft work actually supplied to the compressor per kg of air compressed $w_c = w_n$.

$$\eta_{c\ Isothermal} = \frac{w_t}{w_c} = \frac{160.95}{202.42} = 0.795$$

Adiabatic efficiency
$$\eta_c = \frac{w_\gamma}{w_c} = \frac{210.65}{202.42} = 1.04$$

SAQ 3

Volumetric efficiency
$$\eta_{vol} = 1 - \text{cl}\left[\left(\frac{p_2}{p_1}\right)^{1/n} - 1\right]$$

$$\left(\frac{p_2}{p_1}\right)^{1/n} = \left(\frac{500 \times 10^3}{100 \times 10^3}\right)^{1/1.3} = 3.45$$

$$\eta_{vol} = 0.50 = 1 - \text{cl}\,[3.45 - 1]$$

Therefore $$cl = \frac{0.5}{2.45} = 0.204$$

If $$\eta_{vol} = 0 = 1 - cl\,(3.45 - 1)$$

$$cl = \frac{1}{2.45} = 0.408$$

SAQ 4

A reasonable comparison can be made here on the idealized basis eventhough the actual volumetric efficiencies may be lower than the ideal.

For the single stage machine

$$\eta_{vol} = 1 - cl\left[\left(\frac{p_2}{p_1}\right)^{1/n} - 1\right]$$

$$= 1 - 0.03\left[\left(\frac{500 \times 10^3}{30 \times 10^3}\right)^{1/1.25} - 1\right]$$

$$= 0.744$$

For the two-stage machine, the pressure ratio in each stage is $\sqrt{\frac{500}{30}}$ and the volumetric efficiency is that of the first stage.

$$\eta_{vol} = 1 - 0.03\,[(4.09)^{1/1.25} - 1]$$

$$= 0.934$$