UNIT 9 IDEAL GAS

Structure

9.1   INTRODUCTION

The fundamentals of thermodynamics, evaluation of heat and work interactions in various processes and the relationship between these interactions and the properties of the system in question have been discussed in detail in the preceding Units. But, no attempt has ever been made in these Units as to how the properties of a system at different states can be evaluated and also related. Before embarking on thermodynamic analysis of various processes executed by any substance, one must have information on the property relations for these substances. This Unit deals with the p, V, T relations for a particular category of pure substance which is called an IDEAL GAS. This Unit also explains as to how other thermodynamic properties of the ideal gases can be related to the measurable properties such as p, V and T. Also discussed in this unit are the analysis of various processes undergone by an ideal gas. Although an ideal gas is an imaginary gas, most low molecular mass real gases tend to behave as ideal gases at low pressures and high temperatures.

Objectives

After reading this Unit you should be able to

*   write the p, V, T relation for an ideal gas and find the value of the gas constant for each ideal gas,

*   know that internal energy and enthalpy of an ideal gas are functions of temperature only,

*   distinguish between perfect and semiperfect gases and calculate their enthalpy, internal energy and specific heats in terms of temperature,

*   evaluate heat and work interactions in various reversible processes,

*   calculate the entropy change in processes and discuss about the reversibility and irreversibility of each process, and

*   appreciate that the ideal gas temperature scale coincides with the thermodynamic temperature scale.

9.2   IDEAL GAS .

Any equation that represents the relation existing among the three properties; pressure, volume and temperature is called the equation of state of the substance in question. An ideal gas is one that follows the equation of state

$$pv = RT \text{ or } pV = mRT \tag{9.1}$$

at all ranges of pressure and temperature. In this relation p is the pressure in N/m^2, v the specific volume in m^3/kg, V the total volume in m^3, T the temperature in K, m the mass in kg and R the particuar gas constant in J/kg K.

When m kg of an ideal gas undergoes a change from state 1 to 2, irrespective of the path of the process, the properties at these two state are related by:

$$p_1 V_1 = mRT_1 \quad \text{or} \quad \frac{p_1 V_1}{T_1} = mR \text{ and}$$

$$p_2 V_2 = mRT_2 \text{ or } \frac{p_2 V_2}{T_2} = mR \text{ and hence,}$$

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2} \tag{9.2}$$

No gas follows the above pvT relation in equation (9.1) at all ranges of pressure and temperature. The pvT relations for most gases are quite complex and they have been established only for a very few widely used gases.

Although no real gas follows the relation $pv = RT$ at all ranges of pressure and temperature; it is observed that most gases, especially those with low molar mass, tend to follow the ideal gas equation of state at high temperatures or low pressures. Thus, an ideal gas is only a mathematical model used to simplify the thermodynamic analyses in many situations.

9.2.1 Universal Gas Constant

Consider one kmol (M kg, where M is the molar mass of the gas) of any gas at a standard atmospheric pressure of $p_o = 1.01325$ bar and a temperature of $T_o = 273.15$ K. According to Avogadro's principle all gases at this state occupy the same volume of $V_o = 22.416$ m^3. Substitution of these values in the ideal gas equation of state gives the information that

$$MR = \frac{p_o V_o}{T_o} = 8314.3 \text{ J / kmol K.}$$

The value of MR is thus a constant and is the same for all gases. It is for this reason that MR is called the Universal gas constant and is denoted by $\bar{R}$. Although $\bar{R}$ is the same for all gases, the value of the particular gas constant, R, for each gas is different and the two are related by

$$R = \frac{\bar{R}}{M} \text{ or } R = \frac{8314.3}{M} \text{ J / kg K,} \tag{9.3}$$

where M is the molar mass of the gas.

It is not difficult to understand that the ideal gas equation can be written in any one of the following forms:

(i) $pv = RT$

(ii) $pV = mRT$

(iii) $pV = \dfrac{m\bar{R}T}{M}$

(iv) $pV = n\bar{R}T$ where $n = m/M = $ number of kmols.

(v) $p\bar{v} = \bar{R}T$, where $\bar{v} = $ molar specific volume in m^3 / kmol.

SAQ 1

What is the volume occupied by 5 kg oxygen ($M = 32$ kg/kmol) at 1 bar and 27°C.

SAQ 2

Air confined in a piston cylinder mechanism expands from 10 bar, 227°C, 0.01 m³ to 1 bar, 0.06 m³. Find the final temperature.

9.2.2 Internal Energy and Enthalpy of an Ideal Gas.

The internal energy and enthalpy of an ideal gas are functions of temperature only. This is proved here.
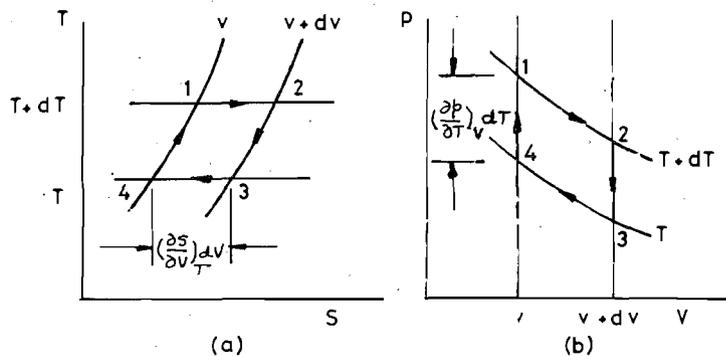

Figure 9.1 : Reversible cycle on T - s and p - v diagrams

Consider an ideal gas undergoing an infinitesimal reversible cycle consisting of two constant volume processes and two constant temperature processes. These cycles are shown in Figure 9.1 on T - s and p - v coordinates. In these figures while 1 - 2 and 3 - 4 are the constant temperature processes, 2 - 3 and 4 - 1 are the constant volume processes. While on the T - s diagram of Figure 9.1 (a) the horizontal distance between two constant volume lines is $\left(\dfrac{\partial s}{\partial v}\right)_T dv$, the vertical distance between two constant temperature lines, on the p - v diagram of Figure 9.1 (b), is equal to $\left(\dfrac{\partial p}{\partial T}\right)_v dT$. The area of the cycle, $\left(\dfrac{\partial s}{\partial v}\right)_T dv\ dT$, on T - S diagram represents the net heat interaction during the cycle. Similarly the area of the cycle, $\left(\dfrac{\partial p}{\partial T}\right)_v dT\ dv$ on p - v diagram represents the net work. According to the first law of thermodynamics, the net heat is equal to net work in a thermodynamic cycle and hence the two areas have to be equal.

Therefore,

$$\left(\frac{\partial p}{\partial T}\right)_v dT\ dv \; = \; \left(\frac{\partial s}{\partial v}\right)_T dv\ dT \quad \text{or} \quad \left(\frac{\partial p}{\partial T}\right)_v = \left(\frac{\partial s}{\partial v}\right)_T \qquad \text{i)}$$

From Tds relation for a pure substance

$$Tds = du + pdv \quad \text{or} \quad du = Tds - pdv$$

Differentiating the above equation with respect to volume at constant T,

$$\left(\frac{\partial u}{\partial v}\right)_T \; = \; T\left(\frac{\partial s}{\partial v}\right)_T - p \qquad \qquad \text{(ii)}$$

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Eliminating $\left(\dfrac{\partial s}{\partial v}\right)_T$ between (i) and (ii)

$$\left(\frac{\partial u}{\partial v}\right)_T = T\left(\frac{\partial p}{\partial T}\right)_v - p \tag{iii}$$

For an ideal gas, $pv = RT$ and hence, $\left(\dfrac{\partial p}{\partial T}\right)_v = \dfrac{R}{v}$ \hfill (iv)

Eliminating $\left(\dfrac{\partial p}{\partial T}\right)_v$ between (iii) and (iv)

$$\left(\frac{\partial u}{\partial v}\right)_T = \frac{TR}{v} - p = p - p = 0 \tag{v}$$

Equation (v) reveals that the internal energy of an ideal gas depends on temperature but not on volume.

By definition, $h = u + pv$ and hence for an ideal gas,

$$h = u + RT \tag{vi}$$

As u does not depend on volume, it can be inferred from equation (vi) that enthalpy h of an ideal gas is also independent of volume but depends only on temperature.

Although by considering an infinitesimal cycle consisting of two reversible isobars and two reversible isotherms it can be shown that $\left(\dfrac{\partial u}{\partial p}\right)_T = 0$ and thus $\left(\dfrac{\partial h}{\partial p}\right)_T = 0$, meaning that internal energy and enthalpy are independent of pressure but depend only on temperature, a little thinking makes it obvious that such an exercise is really unnecessary. Thus for a gas that follows the ideal gas equation of state $pv = RT$, internal energy and enthalpy are functions of temperature only. Mathematically it can be written as

$$u = u(T) \quad \text{and} \quad h = h(T).$$

SAQ 3

It is given that enthapy of air at 1 bar and 27°C is 300.2 kJ/kg. Assuming the air behaves as an ideal gas what is the specific enthalpy of air at (a) 5 bar, 27°C and (b) 27°C and 0.085 m³/kg.

9.2.3 Specific Heats, Internal Energy and Enthalpy of an Ideal Gas

For a pure substance, by definition, the specific heat at constant volume $c_v = \left(\dfrac{\partial u}{\partial T}\right)_v$ and the

specific heat at constant pressure $c_p = \left(\dfrac{\partial h}{\partial T}\right)_p$

An ideal gas is also a pure substance and its internal energy and enthalpy, as discussed above, are functions of temperature only. Hence, the two specific heats can be written as:

$$c_v = \frac{du}{dT} \quad \text{and} \quad c_p = \frac{dh}{dT} \tag{9.4}$$

or $\qquad du = c_v\, dT \quad \text{and} \quad dh = c_p\, dT$ \hfill (9.5)

$$\therefore \Delta u = u_2 - u_1 = \int c_v\, dT \quad \text{and} \tag{9.6}$$

$$\Delta h = h_2 - h_1 = \int c_p\, dT. \tag{9.7}$$

$$\text{Also, } c_p = \frac{dh}{dT} = \frac{d}{dT}(u + RT) = \frac{du}{dT} + R$$

But from equation (9.4) $\frac{du}{dT} = c_v$ and hence,

$$c_p = c_v + R \quad \text{or}$$

$$c_p - c_v = R. \tag{9.8}$$

From the above relation it can be generalised and said that the difference between the two specific heats is always equal to the gas constant of an ideal gas.

SAQ 4

Calcuate the value of c_v for air (M = 28.97 kg/kmol) assuming its c_p to be 1005 J/kg K.

For an ideal gas as u and h are functions of temperatures only, it can be discerned from equation (9.4) that c_v and c_p can either be functions of temperature only or constants depending upon the nature of the function $u(T)$. This is elaborated below:

For example, if $u = A + BT$, where A and B are constants,

$$c_v = \frac{du}{dT} = B, \text{ and hence } c_v \text{ is a constant.}$$

Also, $h = u + pv = u + RT = A + BT + RT = A + (B+R)T$,

$$c_p = \frac{dh}{dT} = (B+R), \text{ and hence } c_p \text{ is also a constant.}$$

Suppose, $u = A + BT + CT^2 + DT^3...,$

$$c_v = \frac{du}{dT} = B + 2CT + 3DT^2 + ... \text{ and hence } c_v \text{ is a function of temperature.}$$

As c_v is a function of temperature c_p will also be a function of temperature.

For an ideal gas whose specific heats are functions of temperature, the changes in internal energy Δu and enthalpy Δh can be calculated using equations (9.6) and (9.7).

For an ideal gas with constant specific heats, equations (9.6) and (9.7) can be written as,

$$\Delta u = c_v \int dT \text{ and } \Delta h = c_p \int dT \text{ or}$$

$$u_2 - u_1 = c_v \left(T_2 - T_1 \right) \text{ and } h_2 - h_1 = c_p \left(T_2 - T_1 \right) \tag{9.9}$$

While an ideal gas whose specific heats are functions of temperature is called a SEMI PERFECT GAS, an ideal gas whose specific heats are constants is called a PERFECT GAS.

SAQ 5

(a) Gaseous nitrogen is compressed in a piston cylinder device from 1 bar, 27°C to 10 bar, 327°C. Calculate the increase in specific enthalpy of nitrogen assuming its c_p is given by

$$c_p = 1.4 - 18.3 \left(\frac{T}{100} \right)^{-1.5} + 38.3 \left(\frac{T}{100} \right)^{-2} - 29.3 \left(\frac{T}{100} \right)^{-3},$$

where c_p is in kJ/kg K and T is in degree Kelvin.

(b) What would be the increase in specific enthalpy, for the same change of state of nitrogen, if nitrogen were to behave as a perfect gas with c_p = 1.04 kJ/kg K ?

9.3 ANALYSIS OF SOME REVERSIBLE PROCESSES

The methodology used in evaluating heat and work interactions and change in properties of an ideal gas with constant specific heats in a few commonly encountered processes are discussed below.

9.3.1 Reversible Constant Pressure Process

A constant pressure process is shown in Figure 9.2 on p - v and T - s coordinates. From data $p_1 = p = p_2 = C$ where C is a constant. An example of this type of process is the expansion of an ideal gas in a vertical frictionless piston cylinder mechanism in which the piston carries a constant weight. In such a case the work interaction is only the displacement work W and hence,
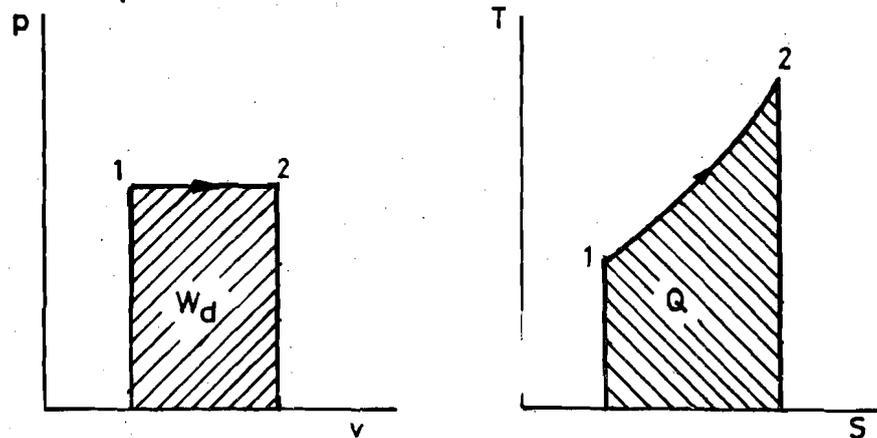

Figure 9.2 : Constant pressure process on p-v and T-s diagrams

$$W = \int p \, dV = \int C \, dV = C(V_2 - V_1)$$

$$\text{or} \quad W = p(V_2 - V_1) = (p_2 V_2 - p_1 V_1)$$

But, as $\quad p_1 V_1 = mRT_1 \quad$ and $\quad p_2 V_2 = mRT_2$,

$$\therefore \quad W = mR(T_2 - T_1) \tag{9.10}$$

$$\text{Also,} \quad \Delta U = mc_v(T_2 - T_1) \tag{9.9}$$

For the process, from the first law of thermodynamics,

$$Q - W = \Delta U \quad \text{and hence,}$$

$$Q = mR(T_2 - T_1) + mc_v(T_2 - T_1) = m(R + c_v)(T_2 - T_1)$$

But, $\quad R + c_v = c_p$, and hence

$$Q = mc_p(T_2 - T_1) = \Delta H \tag{9.11}$$

According to equation (9.11) the heat transfer in a constant pressure process is equal to the change in enthalpy during the process.

Also, graphically, while the area bound by the path 1 - 2 and the extreme ordinates on p - v diagram gives the work interaction, the area on the T - s diagram gives the heat interaction. The process shown here is a process of expansion as $V_2 > V_1$. The analysis is valid even if $V_2 < V_1$, in which case we are dealing with a compression process and hence the directions of heat and work interactions are different.

9.3.2 Reversible Constant Volume Process

This process is shown on both p - v and T - s planes in Figure 9.3. Here $V_1 = V = V_2 = C$, a constant. Such a process can be visualised as that undergone by a gas in a piston cylinder mechanism in which the piston is held stationary or as that undergone by the gas in any rigid vessel.
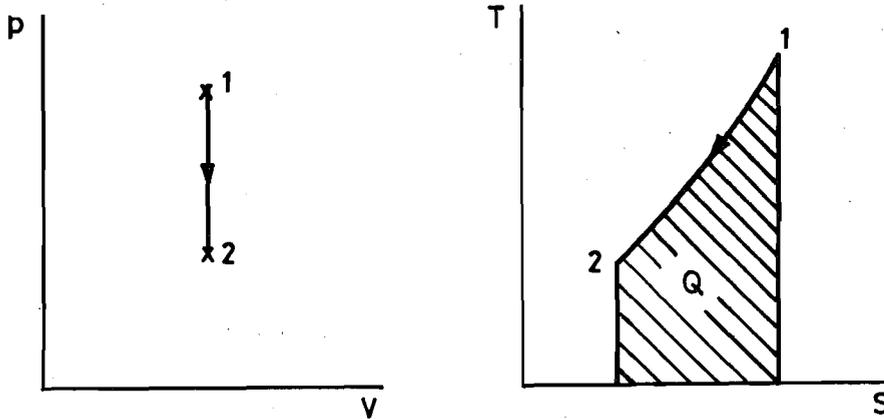

Figure 9.3 : Constant volume process on p-v and T-s diagrams

$$W = \int p \, dV = 0 \text{ as } dV = 0.$$

$$\Delta U = m \, c_v \left(T_2 - T_1 \right) \tag{9.9}$$

$$Q = W + \Delta U = \Delta U = mc_v \left(T_2 - T_1 \right) \tag{9.12}$$

According to equation (9.12) the heat transfer in a constant volume process is equal to the change in internal energy during the process.

9.3.3 Reversible Hyperbolic Process

This process which follows the path pV = constant, is shown in Figure 9.4 on p - v and T - s diagrams. For an ideal gas as $pv = RT$, if during a process pv is constant then the temperature of the gas will also remain a constant. Thus a reversible hyperbolic process in also a constant temperature process or an isothermal process and this is true only for an ideal gas and not for any other substance.

From data, $p_1V_1 = pV = p_2V_2$ = a constant C. This also means that

$$T_1 = T = T_2 = C/mR = \text{ another constant.}$$

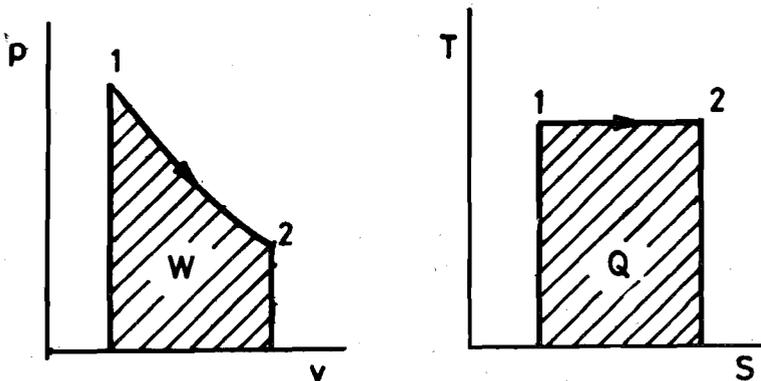$$W = \int pdV = \int \frac{C}{V} dV = C \int \frac{dV}{V} = C \ln \left[\frac{V_2}{V_1} \right]$$

Figure 9.4 : Hyperbolic process on p - v and T - s diagrams

$$\text{or} \quad W = p_1 V_1 \ln\left[\frac{V_2}{V_1}\right] = p_2 V_2 \ln\left[\frac{V_2}{V_1}\right] \tag{9.13}$$

$$\text{or} \quad W = mRT_1 \ln\left[\frac{V_2}{V_1}\right] = mRT_2 \ln\left[\frac{V_2}{V_1}\right] \tag{9.14}$$

Also, as volume is inversely proportional to pressure,

$$W = p_1 V_1 \ln\left[\frac{p_1}{p_2}\right] = p_2 V_2 \ln\left[\frac{p_1}{p_2}\right] \tag{9.15}$$

$$\Delta U = mc_v \ (T_2 - T_1)$$

Here $\Delta U = 0$ as $T_2 = T_1$ and,

$$\therefore \ Q = \Delta U + W = W.$$

The heat transfer in an isothermal process of an ideal gas is equal to the work transfer during the process.

9.3.4 Reversible Polytropic Process

This process, which follows the path $pV^n = $ constant, where n is called the exponent of expansion or compression, is shown in Figure 9.5 on both p - v and T - s coordinates.
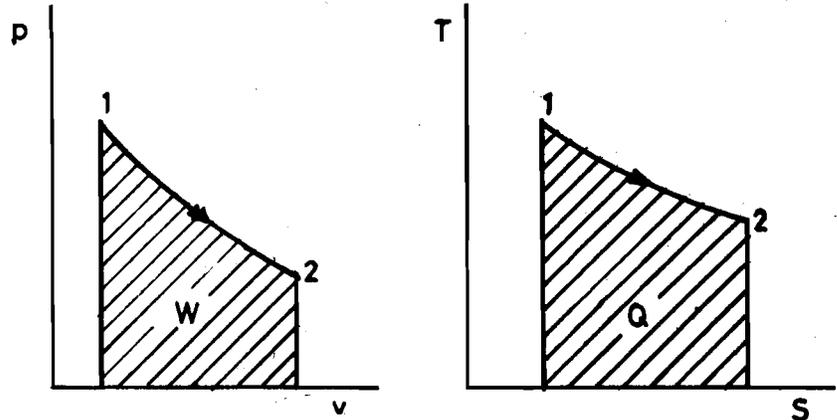

Figure 9.5 : Polytropic process on *p-v* and *T-s* diagrams

$$p_1 V_1^n = pV^n = p_2 V_2^n = \text{a constant } C.$$

$$W = \int_1^2 p \ dV = \int \frac{C}{V^n} dV = C\left[\frac{V^{-n+1}}{-n+1}\right]_1^2$$

$$\text{or} \quad W = C\left[\frac{V_2^{-n+1} - V_1^{-n+1}}{-n+1}\right], \text{ where } C = p_1 V_1^n = p_2 V_2^n.$$

$$\text{Hence,} \quad W = \left[\frac{p_2 V_2^n V_2^{-n+1} - p_1 V_1^n V_1^{-n+1}}{-n+1}\right] = \frac{p_2 V_2 - p_1 V_1}{1-n}$$

But $p_1 V_1 = mRT_1$ and $p_2 V_2 = mRT_2$, and hence,

$$W = \left[\frac{p_2 V_2 - p_1 V_1}{1-n}\right] = mR\left[\frac{T_2 - T_1}{1-n}\right] \tag{9.16}$$

As $\Delta U = mc_v \ (T_2 - T_1)$ and from I law $Q = \Delta U + W$,

$$Q = mc_v \left(T_2 - T_1\right) + mR\left[\frac{T_2 - T_1}{1-n}\right] = m\left(T_2 - T_1\right)\left[c_v + \frac{R}{1-n}\right]$$

But $R = c_p - c_v$ and hence,

$$Q = m \left(T_2 - T_1\right) \left[\frac{c_v - nc_v + (c_p - c_v)}{1 - n}\right] = m \left(T_2 - T_1\right) \left[\frac{c_p - nc_v}{1 - n}\right]$$

i.e., $Q = m \left(T_2 - T_1\right) c_v \left[\frac{(c_p/c_v) - n}{1 - n}\right]$

Representing the ratio of specific heats c_p/c_v by γ

$$Q = mc_v \left(T_2 - T_1\right) \left[\frac{\gamma - n}{1 - n}\right] \tag{9.17}$$

In equation (9.17), which is valid only for a reversible process, the heat transfer becomes zero for $n = \gamma$. If $Q = 0$ the process is adiabatic. Hence, a reversible adiabatic process of an ideal gas is one that follows the path

$$pV^{\gamma} = \text{Constant.} \tag{9.18}$$

Equations (9.16) and (9.17) give the work and heat interactions in a reversible polytropic process. These expressions are general in nature and can be used to evaluate the work and heat interactions in any process using an appropriate value for n. For example in a constant pressure process, $n = 0$. The expressions for work and heat obtained by substituting $n = 0$ in equations (9.16) and (9.17) are the same as those derived in particular for the specific case of the constant pressure process and given by equations (9.10) and (9.11). Similar exercises can be made for the case of constant volume process for which $n = \infty$, and reversible adiabatic process for which $n = \gamma$. The equations (9.16) and (9.17) cannot be used only in the case of an isothermal process, for which $n = 1$, for obvious reasons.
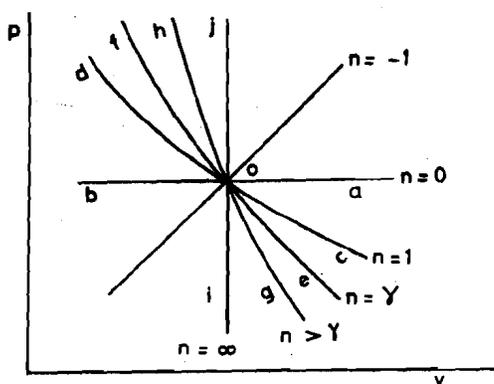
9.3.5 Diagrammatic Representation of Processes

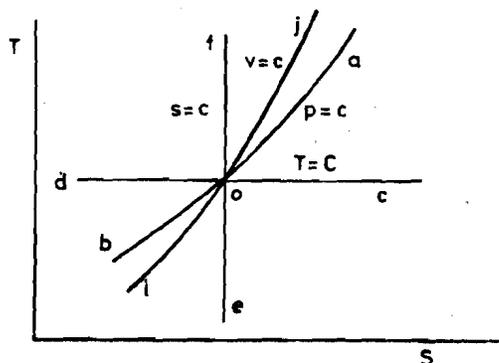Figure 9.6 : Various processes on p - v diagram

Figure 9.7 : Various processes on T-s diagram

All the processes discussed hitherto have been shown together on a single p - v diagram in Figure 9.6. In this diagram a -b represents the constant pressure process for which $n = 0$. Similarly c - d is the reversible isothermal process, for which $n = 1$, e - f is the reversible adiabatic process for which $n = \gamma$, g–h is a general reversible polytropic process for which $n = \gamma$ and i - j is the constant volume process for which $n = \infty$. It is worth noting here that the slope of the paths of various processes change gradually with an increase in the value of n. Also, if it is assumed that each process starts from the initial state 'o', the processes proceeding towards right of 'o', such as o-a, o-c, o-e, o-g and o-i, represent the expansion processes during which there is normally a decrease in pressure and an increase in volume. Similarly the processes proceeding towards left of 'o' represent the compression processes during which normally there is an increase in pressure and a decrease in volume.

The above processes are also shown together on T - s plane in Figure 9.7.

9.4 RELATIONSHIP AMONG P, V & T IN A REVERSIBLE POLYTROPIC PROCESS

An attempt shall now be made to find the relations between p and V, T and V, & T and P during a reversible polytropic process.

Let $pV^n = C$ represent a reversible polytropic process.

For an ideal gas $p = \dfrac{mRT}{V}$, and hence

$$\left(\frac{mRT}{V}\right)V^n = C \text{ or } TV^{n-1} = \frac{C}{mR} = \text{another constant as } m \text{ \& } R \text{ are constants.}$$

Similarly, as $V = \dfrac{mRT}{p}$, $p\left(\dfrac{mRT}{p}\right)^n = C$, another constant.

Thus, the relations among p, V and T during a polytropic are :

(i) $pV^n = $ Constant.

i.e. $p_1 V_1^n = p_2 V_2^n$ or $\left[\dfrac{p_2}{p_1}\right] = \left[\dfrac{V_1}{V_2}\right]^n$ (9.19)

(ii) $TV^{n-1} = $ Constant.

i.e. $T_1 V_1^{n-1} = T_2 V_2^{n-1}$ or $\left[\dfrac{T_2}{T_1}\right] = \left[\dfrac{V_1}{V_2}\right]^{n-1}$ (9.20)

(iii) $p^{1-n} T^n = $ Constant

i.e. $p_1^{1-n} T_1^n = p_2^{1-n} T_2^n$ or $\left[\dfrac{T_2}{T_1}\right] = \left[\dfrac{p_2}{p_1}\right]^{\frac{n-1}{n}}$ (9.21)

SAQ 6

1 kg air at 27°C, 1 bar is expanded reversibly until its volume is doubled according to (a) $p = $ constant, (b) $pV = $ constant and (c) $pV^\gamma = $ constant. For air $\gamma = 1.4$. Find the final temperature of air in each case. Show the processes on a common p - v diagram.

1 kg carbon dioxide ($\gamma = 1.29$) is compressed reversibly from 1 bar, 27°C to 127°C according to (a) $s = $ constant, (b) $V = $ constant and (c) $p = $ constant. Find the final pressure in each case. Show the processes on a common $T - s$ plane.

9.5   CALCULATION OF ENTROPY

It has been explained in Unit 8 that entropy of a pure substance can be calculated using the two Tds relations,

$$Tds = du + pdv \quad \text{and} \tag{8.12}$$

$$Tds = dh - vdp. \tag{8.13}$$

Equation (8.12) can be written as

$$ds = \frac{du}{T} + \frac{p}{T}\, dv \tag{i}$$

For an ideal gas $du = c_v\, dT$ and $\dfrac{p}{T} = \dfrac{R}{v}$ and hence,

$$ds = \frac{c_v\, dT}{T} + \frac{R}{v}\, dv \tag{ii}$$

$$\therefore \ \Delta s = s_2 - s_1 = \int ds = c_v \int \frac{dT}{T} + R \int \frac{dv}{v}$$

i.e. $\quad s_2 - s_1 = c_v \ \ln\left(\dfrac{T_2}{T_1}\right) + R \ \ln\left(\dfrac{V_2}{V_1}\right) \tag{9.22}$

or $\quad S_2 - S_1 = m\left(s_2 - s_1\right) = m\left[c_v \ \ln\left(\dfrac{T_2}{T_1}\right) + R \ \ln\left(\dfrac{V_2}{V_1}\right)\right] \tag{9.23}$

Similarly, using equation (8.13), and that for an ideal gas, $dh = c_p\ dT$ and $v/T = R/p$ it can be easily shown that

$$\Delta s = s_2 - s_1 = \left[c_p \ \ln\left(\frac{T_2}{T_1}\right) - R \ \ln\left(\frac{p_2}{p_1}\right)\right] \tag{9.24}$$

or $\Delta S = S_2 - S_1 = m\left[c_p \ \ln\left(\dfrac{T_2}{T_1}\right) - R \ \ln\left(\dfrac{p_2}{p_1}\right)\right] \tag{9.25}$

Whenever an ideal gas undergoes a process, the change in entropy during the process can be calculated using any one of the equations (9.22) to (9.25).

9.6   IDEAL GAS TEMPERATURE SCALE

It was discussed in Unit 7 that the temperature, T, on Kelvin or Thermodynamic temperture scale is given by

$$T = 273.16 \ \frac{Q}{Q_{TP}} \tag{7.18}$$

Where Q and Q_{TP} are the heat transfer to and from a reversible heat engine operating between the steam point and the triple point of water. It was also observed that, as it was not possible to measure Q and Q_{TP} (but they could be calculated using the properties of water) experimentally, it would be shown that the thermodynamic temperatue scale coincides with the ideal gas temperature scale. To take up this exercise now, the ideal gas temperature scale is described briefly here.

A constant volume ideal gas thermometer is one in which the sensible element is a tiny but rigid cylinder containing a small mass of an ideal gas with a provision to measure the pressure of the gas. The pressure of the gas at any instant is an indicator of the temperature to which the sensing element is exposed to. During any measurement the ideal gas in the sensing element is subjected to a constant volume process. Let the pressure measured by the device at the triple point of water, which has arbitrarily been given the value 273.16 K, be p_{TP}. As the gas follows the relation $pv = RT$, the pressure, p, read by the thermometer at any other temperature, T, is related to P_{TP} by

$$T = 273.16 \left(\frac{P}{P_{TP}} \right)$$

and the temperature so measured is referred to as the ideal gas temperature. This temperature has to exactly coincide with the thermodynamic temperature because of the common assumptions made in establishing the two scales.

But, there are a few problems here from the practical point of view. No gas behaves as an ideal gas in practice at all ranges of pressure and temperature and this poses problems in measuring p_{TP} This problem can be overcome using the fact that all gases behave as ideal gases as their pressure tends to zero.

The pressure indicated by the ideal gas thermometer at any temperature depends upon the mass of the gas in the sensing element. Let it now be assumed that the sensing element could be filled with different masses of gas in steps, and in each case the pressure indicated when the sensing element is exposed to the triple point temperature of water, 273.16 K, is recorded. The pressure decreases with the decrease in the mass of gas. For each measured pressure the indicated temperature is calculated using the ideal gas relation $T = pV/mR$. The indicated temperature (on Y - axis) is plotted against the measured pressure (on X - axis). The procedure is continued to as small a pressure as can be experimentally read. Thereafter the curve obtained is extrapolated to zero pressure to get the correct ideal gas temperature.

9.7 ILLUSTRATIVE PROBLEMS

Example 9.1 :

Nitrogen is contained in a rigid tank of volume 2.5 m³ at a pressure of 10 MPa and a temperature of 27°C. Assume nitrogen behaves as an ideal gas. Calculate the mass in kg and also the two specific heats given that their ratio is 1.4.

Solution :

For an ideal gas $pV = mRT$ and hence,

$$m = \frac{pV}{RT} \tag{i}$$

From data, $p = 10 \times 10^6$ N/m², $V = 2.5$ m³ , and

$T = 27 + 273 = 300$ K. Also $R = \dfrac{\bar{R}}{M} = \dfrac{8314.3}{28} = 296.94$ J/kg K.

Substituting these values in (i)

$$m = \frac{10 \times 10^6 \times 2.5}{296.94 \times 300} = 280.64 \text{ kg.}$$

From data, $\dfrac{c_p}{c_v} = 1.4$ \hfill (ii)

As nitrogen is an ideal gas, $c_p - c_v = R = 296.94$ \hfill (iii)

Solving for c_p and c_v between (ii) and (iii)

$$1.4 \, c_v - c_v = 296.94 \text{ , or } c_v = 742.35 \text{ J / kg K.}$$

Substituting for c_v in (ii), $c_p = 1.4 \, c_v = 1039.3$ J/kg K.

Example 9.2 :

One m^3 of air at 5 MPa, 400°C expands in a reversible isothermal process in a cylinder to 0.2 MPa. Calculate for air (a) the heat transfer and (b) change in entropy. Assume air is an ideal gas with $R = 287$ J/kg K.

Solution :

(a) $\qquad m_1 = \dfrac{p_1 V_1}{R T_1} = \dfrac{5 \times 10^6 \times 1}{287 \times (273 + 400)} = 25.9$ kg.

In an isothermal process,

$$W = p_1 V_1 \, \ln \left(\frac{p_1}{p_2} \right) \qquad\qquad (9.15)$$

$$\therefore \; W = 5 \times 10^6 \times 1 \ln \frac{0.2}{5} = -16.094 \times 10^6 \text{ J}$$

$\Delta U = 0$ in an isothermal process as $\Delta T = 0$. Hence $Q = W$

(b) $\qquad \Delta S = m \left[c_p \, \ln \left(\frac{T_2}{T_1} \right) - R \, \ln \left(\frac{p_2}{p_1} \right) \right] \qquad\qquad (9.25)$

Here the process being isothermal $T_2 = T_1$ and hence

$$\Delta S = - mR \, \ln \left(\frac{p_2}{p_1} \right) = - 25.9 \times 287 \ln \left(\frac{0.2}{5} \right) = 23.927 \text{ kJ/K}$$

Example 9.3 :

5 kg of nitrogen is cooled in a rigid tank from 250°C to 27°C. The initial pressure is 25 bar. Calculate the changes in entropy, internal energy and enthalpy. Assume nitrogen to be an ideal gas with $c_p = 1.042$ kJ/kg K and $c_v = 0.745$ kJ/kg K.

Solution :

From ideal gas relation $\qquad \dfrac{p_1 V_1}{T_1} = \dfrac{p_2 V_2}{T_2}$

Cooling occurs in a rigid tank and hence $V_1 = V_2$.

Therefore $\qquad\qquad\qquad \dfrac{p_1}{T_1} = \dfrac{p_2}{T_2}$

Substituting for $p_1 = 25$ bar, $T_1 = (273 + 250) = 523$ K and $T_2 = 300$ K

$$p_2 = \left(\frac{300}{523} \right) 25 = 14.3 \text{ bar.}$$

$$\Delta S = m \left[c_v \, \ln \left(\frac{T_2}{T_1} \right) + R \, \ln \left(\frac{V_2}{V_1} \right) \right] = mc_v \, \ln \left(\frac{T_2}{T_1} \right)$$

$$\therefore \; \Delta S = 5 \times 0.745 \, \ln \left(\frac{300}{523} \right) = - 2.07 \text{ kJ/K}$$

$$\Delta U = mc_v \left(T_2 - T_1 \right) = 5 \times 0.745 \times (300 - 523) = - 830 \text{ kJ.}$$

$$\Delta H = mc_p \left(T_2 - T_1 \right) = 5 \times 1.042 \times (300 - 523) = - 1161.8 \text{ kJ.}$$

Example 9.4 :

An insulated cylinder having an initial volume of 25 lit. contains oxygen at 150 kPa, 227°C. The gas is compressed to 1.5 MPa in a reversible adiabatic process. Calculate the final temperature and work assuming oxygen behaves as an ideal gas with $c_p = 0.922$ kJ/kg K.

Solution :

The process, being reversible and adiabatic, follows the relation $pV^\gamma =$ constant, where $\gamma = c_p/c_v$. Therefore,

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} \quad \text{or} \quad T_2 = T_1\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} \tag{9.21}$$

For an ideal gas, $c_p - c_v = R$ and $c_p/c_v = \gamma$ and hence,

$$\gamma = \frac{c_p}{(c_p - R)} \tag{i}$$

Here, $R = \dfrac{\bar{R}}{M} = \dfrac{8314.3}{32} = 259.82$ J/kg K and

from data $c_p = 0.922$ kJ/kg K. Substituting for c_p and R in (i)

$$\gamma = \frac{0.922}{(0.922 - 259.82 \times 10^{-3})} = 1.392.$$

Substituting for γ, T_1, p_1 and p_2 in the equation (9.21) for T_2

$$T_2 = (273 + 227) \times \left(\frac{1.5 \times 10^6}{150 \times 10^3}\right)^{\frac{0.392}{1.392}} = 956.3 \text{ K}$$

$$W = \frac{p_1 V_1 - p_2 V_2}{\gamma - 1} = \frac{mR\,(T_1 - T_2)}{\gamma - 1} \tag{9.16}$$

Here $$m = \frac{p_1 V_1}{R T_1}$$

$$= \frac{150 \times 10^3 \times 25 \times 10^{-3}}{259.82 \times 500} = 0.02887 \text{ kg.}$$

$$\therefore W = \frac{0.02887 \times 259.82 \,(500 - 956.3)}{0.392} = -8731.4 \text{ J.}$$

Example 9.5 :

Air is contained in a cylinder fitted with a frictionless piston. Initially the cylinder contains 0.1 m³ of air at 1 bar, 27°C. The air is then compressed reversibly according to $pV^n =$ constant. The final pressure is 5 bar at which point the temperature is 141°C. Assume $R = 287$ J/kg K, $c_v = 718$ J/kg K and determine (a) the value of n, (b) final volume of air, (c) Work and heat interactions and (d) change in entropy.

Solution :

(a) For a reversible polytropic process

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} \tag{9.21}$$

Here, $T_1 = 273 + 27 = 300$ K, $T_2 = 273 + 141 = 414$ K and

$p_1 = 1$ bar, $p_2 = 5$ bar. Therefore,

$$\frac{414}{300} = \left(\frac{5}{1}\right)^{\frac{n-1}{n}} \quad \text{or} \quad n = 1.25$$

(b)   As, $\dfrac{p_1 V_1}{T_1} = \dfrac{p_2 V_2}{T_2}$,

$$V_2 = \left(\dfrac{T_2}{T_1}\right) \times \left(\dfrac{p_1}{p_2}\right) V_1 = \left(\dfrac{414}{300}\right) \times \left(\dfrac{1}{5}\right) \times 0.1 = 0.0276 \ m^3$$

(c) $m = \dfrac{p_1 V_1}{R T_1} = \dfrac{1 \times 10^5 \times 0.1}{287 \times 300} = 0.116 \ kg$

$$W = \dfrac{p_1 V_1 - p_2 V_2}{n - 1} = \dfrac{mR \ (T_1 - T_2)}{n - 1} \qquad (9.16)$$

$$= \left[\dfrac{0.116 \times 287 \ (300 - 414)}{0.25}\right] = 15.18 \ kJ.$$

$$\Delta U = mc_v \left(T_2 - T_1\right) = 0.116 \times 0.718 \ (414 - 300) = 9.5 \ kJ$$

$$Q = \Delta U + W = 9.5 + 15.18 = 24.68 \ kJ$$

(d) $$\Delta S = m \ \Delta s = m \left[c_v \ \ln\left(\dfrac{T_2}{T_1}\right) + R \ \ln\left(\dfrac{V_2}{V_1}\right)\right] \qquad (9.23)$$

$$\therefore \ \Delta S = 0.116 \left[0.718 \ \ln\left(\dfrac{414}{300}\right) + 0.287 \ \ln\left(\dfrac{0.0276}{0.1}\right)\right] = -0.016 \ kJ/K.$$

Example 9.6 :

A certain mass of air ($c_p = 1.005$ kJ/kg K, $c_v = 0.718$ kJ/kg K) initially at 120 kPa, 0.15 m^3 is compressed reversibly in a polytropic process to 700 kPa. During this process it is given that heat transfer to air is 15 kJ and the internal energy of the air increases by 45 kJ. Find the final volume of air.

Solution :

From the I law $Q - W = \Delta U$ and hence,

$$15 - W = 45 \ or \ W = -30 \ kJ.$$

Let the polytropic exponent of compression be 'n'.

$$W = \dfrac{(p_1 V_1 - p_2 V_2)}{(n - 1)} = \dfrac{mR \ (T_1 - T_2)}{(n - 1)} = -30 \qquad (i)$$

Also $$\Delta U = mc_v \left(T_2 - T_1\right) = +45 \ kJ$$

$$\therefore \ m\left(T_1 - T_2\right) = -\dfrac{45}{c_v} \qquad (ii)$$

Between (i) and (ii)

$$\dfrac{(-45 \ R / c_v)}{(n - 1)} = -30, \ where \ R = c_p - c_v = 0.287 \ kJ/kg \ K$$

substituting for R and c_v $$\dfrac{\left(\dfrac{-45 \times 0.287}{0.718}\right)}{(n - 1)} = -30$$

$$\therefore \ n = 1.6$$

Here $$p_1 V_1^{\ n} = p_2 V_2^{\ n} \ or \ V_2 = \left(\dfrac{p_1}{p_2}\right)^{1/n} V_1$$

$$V_2 = \left(\dfrac{120}{700}\right)^{\frac{1}{1.6}} \times 0.15 = 0.05 \ m^3$$

Example 9.7 :

Consider a closed cylinder divided initially into two compartments of equal volume of 25 litres each by an insulated frictionless piston. All the walls of the cylinder are insulated except for its left hand face which is a good conductor of heat. Initially the cylinder contains air and hydrogen as shown in the figure each at a pressure of 150 kPa and 27°C. Air is now heated slowly until the pressure of air increases to 300 kPa. Calculate the final temperatures of air and hydrogen and also the heat transferred to air. Assume air and hydrogen behave as ideal gases. Assume also that $\gamma_{hydrogen}$ = 1.4 and for air c_v = 0.718 kJ/kg K and R = 0.287 kJ/kg K.
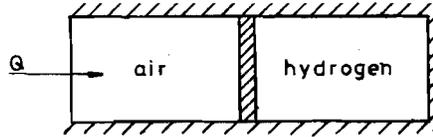
Solution:

Figure 9.8 : Diagram for illustrative prob. 9.7

The pressure and temperature of the air increase during the period it is being heated. Also the piston travels to right slowly thereby causing the air to expand and compress the hydrogen reversibly (frictionless piston) and adiabatically (adiabatic walls and piston). At any instant during the process $P_{air} = P_{hydrogen}$ and at the end of the process both are equal to 300 kPa.

Considering hydrogen in the right hand compartment :

$$p_1 = 15 \text{ kPa}, T_1 = 273 + 27 = 300 \text{ K}, V_1 = 25 \times 10^{-3} \text{ m}^3 \text{ and}$$

p_2 = 300 kPa. Hydrogen is subjected to reversible adiabatic process and hence

$$V_2 = \left(\frac{p_1}{p_2}\right)^{1/\gamma} V_1 = \left(\frac{150}{300}\right)^{\frac{1}{1.4}} \times 25 = 15.24 \text{ lit.}$$

$$\text{and also } \frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} \text{ and hence}$$

$$T_2 = T_1 \left(\frac{V_1}{V_2}\right)^{\gamma-1} = 300 \left(\frac{25}{15.24}\right)^{1.4-1} = 365.7 \text{ K} = 92.7 \text{ °C}$$

Final temperature of hydrogen = 92.7 °C.

Considering air in the left hand side compartment :

The air expands as it is heated. This process can be assumed to follow a reversible polytropic path, pV^n = constant.

For air, p_1 = 150 kPa, T_1 = 300 K, V_1 = 25 lit. , p_2 = 300 kPa
and $V_2 = V_1$ + (decrease in volume of hydrogen)

$$= 25 + (25 - V_2)_{hydrogen} = 25 + (25 - 15.24) = 34.76 \text{ lit.}$$

$$\text{As } p_1 V_1^n = p_2 V_2^n, \quad n = \frac{\ln (p_1/p_2)}{\ln (V_2/V_1)}$$

$$n = \frac{\ln (150/300)}{\ln (34.76/25)} = -2.1$$

$$T_2 = \left(\frac{V_1}{V_2}\right)^{n-1} \times T_1 = 300 \left(\frac{25}{34.76}\right)^{-2.1-1}$$

$$= 833.4 \text{ K} = 560.4° \text{ C.}$$

Final temperature of air $= 560.4° C$.

Also, $m_{air} = \left(\dfrac{p_1 V_1}{R T_1}\right)_{air} = \dfrac{150 \times 10^3 \times 25 \times 10^{-3}}{287 \times 300} = 0.04355$ kg.

$\Delta U_{air} = m c_v (T_2 - T_1) = 0.04355 \times 0.718 \ (833.4 - 300) = 16.68$ kJ

$W_{air} = \left(\dfrac{p_1 V_1 - p_2 V_2}{n - 1}\right) = mR\left(\dfrac{T_1 - T_2}{n - 1}\right)$

$= \dfrac{0.04355 \times 0.287 \ (300 - 833.4)}{-2.1 - 1} = 2.15$ kJ.

From I law, $Q = W + \Delta U$ and thus

$Q = 2.1 + 16.68 = 18.78$ kJ.

Example 9.8 :

(a) Calculate the power required by a compressor if air flowing at a rate of 0.5 kg/s enters at 100 kPa, 27°C with a velocity of 50 m/s and leaves at 200 kPa, 100 °C with a velocity of 100 m/s. Heat transfer from the air to the cooling water circulating around the compressor casing is 1.5 kJ/kg air. (b) Had the air at the same inlet conditions as in (a) been compressed in a reversible adiabatic steady flow device to the same discharge pressure and velocity as in (a) find the exit temperature and power required. Assume air is an ideal gas with $c_p = 1.005$ kJ/kg K and $\gamma = 1.4$.

Solution :

From SFEE $\quad \dot{Q} - \dot{W}_x = \dot{m} \ \Delta \left(h + \dfrac{V^2}{2} + gZ\right)$

Here $\Delta h = c_p \left(T_2 - T_1\right) = 1.005 \ (100 - 27) = 73.365$ kJ

$\dot{Q} = -1.5$ kJ/kg $\times 0.5$ kg/s $= -0.75$ kJ/s

Substituting for $\dot{Q}$, Δh, $\dot{m}$, V_2 and V_1 and neglecting $\Delta \ (gZ)$

$-0.75 - \dot{W}_x = 0.5\left[73.365 + \dfrac{(100^2 - 50^2)}{2000}\right]$

$\therefore \ \dot{W}_x = -39.3$ kJ/s or -39.3 kW.

i.e. Power required $= 39.3$ kW.

(b) Had the expansion been reversible and adiabatic

$$\left(\dfrac{T_2}{T_1}\right) = \left(\dfrac{p_2}{p_1}\right)^{\frac{\gamma - 1}{\gamma}}$$

(9.21)

$\therefore \ T_2 = 300 \left(\dfrac{200}{100}\right)^{\frac{1.4 - 1}{1.4}} = 365.7$ K $= 92.7°$ C.

In this case $\Delta h = c_p \left(T_2 - T_1\right) = 1.005 \ (92.7 - 27) = 66$ kJ/kg.

$Q = 0$ as the process is adiabatic

Substituting in (i)

$$0 - \dot{W}_x = 0.5\left[66 + \dfrac{(100^2 - 50^2)}{2000}\right]$$

or $\qquad\qquad\qquad \dot{W}_x = 34.87$ kJ/s

Power required $\quad = 34.87$ kW.

Example 9.9 :

The gas leaving the turbine of a gas turbine jet engine flows steadily into a jet pipe at a temperature of 900°C, a pressure of 2 bar and a velocity of 270 m/s relative to the pipe. The gas leaves the pipe at a temperature of 815°C and a pressure of 1.1 bar. Heat transfer from the gas is negligible. Evaluate the velocity of the gas leaving the jet pipe. Assume the gas behaves as an ideal gas with $c_p = 1.17$ kJ/kg K.

Solution :

SFEE on unit basis is

$$Q - W_x = \Delta \left(h + \frac{V^2}{2} + gZ \right).$$

Enclosing the jet pipe in a control volume and neglecting the change in potential energy and noting that for the control volume $Q = 0$ and $W_x = 0$, the SFEE reduces to

$$\Delta h = -\frac{\Delta V^2}{2} \text{ or } h_2 - h_1 = \left(\frac{V_1^2}{2} \right) - \left(\frac{V_2^2}{2} \right)$$

The gas is an ideal gas with constant specific heat and hence

$$c_p \left(T_2 - T_1 \right) = \left(\frac{V_1^2}{2} \right) - \left(\frac{V_2^2}{2} \right)$$

$$\text{or } V_2 = \left[c_p \left(T_1 - T_2 \right) \times 2 + V_1^2 \right]$$

$$V_2 = \left[1.17 \times 10^3 \, (815 - 900) \times 2 + 270^2 \right] = 521 \text{ m/s}.$$

Example 9.10 :

An insulated rigid tank of volume 0.5 m³ initially contains air at 250°C and 8 MPa. A valve mounted on the tank is slowly leaking. Because of this the pressure in the tank is slowly decreasing. After a certain time the pressure in the tank is observed to be 500 kPa. Calculate the mass of air leaked out of the tank. Assume air behaves as an ideal gas with $c_p = 1005$ J/kg K and $c_v = 718$ J/kg K.

Solution :

Mass leaked out $= m_1 - m_2$ kg where m_1 is the initial mass in the tank and m_2 is the final mass.

$$m_1 = \left(\frac{p_1 V_1}{R T_1} \right) \text{ where } R = c_p - c_v = 287 \text{ J/kg K.}$$

$$m_1 = \frac{8 \times 10^6 \times 0.5}{287 \times 523} = 26.65 \text{ kg.}$$

Similarly $m_2 = \frac{p_2 V_2}{R T_2}$, Here $p_2 = 500$ kPa, $V_2 = V_1 = 0.5$ m³,

but T_2 is not known. Hence T_2 has to be calculated first to get the value of m_2.

If the tank is considered as a control volume, with respect to time there is decrease in energy of the control volume as air is leaking out of it.

The general energy equation for the control volume is

$$\delta Q - \delta W = d E_c + dm_{out} \left(h + \frac{V^2}{2} + gZ \right) - dm_{in} \left(h + \frac{V^2}{2} + gZ \right)$$

where, dE_c is the change in energy of the control volume.

Here, $dE_c = d(mu)$ where m and u are the mass and internal energy of air in the tank at any instant.

$$\therefore \quad dE_c = dm\, u + m\, du$$

Also, $\delta Q = 0$, $\delta W = 0$, $dm_{in} = 0$. Neglecting changes in kinetic and potential energies. Substituting these in the energy equation,

$$dm\, u + m\, du + dm_{out}\, h = 0.$$

At any instant mass leaving the control volume, $dm_{out} =$ mass depleting in the control volume dm. Therefore, $dm_{out} = -dm$.

$$dm\, u + m\, du - dm\, h = 0$$

For an ideal gas $h = u + RT$ and thus

$$dm\, u + m\, du - dm\, (u + RT) = 0$$

or $m\, du - dm\, RT = 0$. Here $du = c_v\, dT$ and thus

$$mc_v\, dT - dm\, RT = 0 \quad \text{or} \quad \frac{dm}{m} = \frac{\dfrac{c_v}{R}}{\dfrac{dT}{T}}$$

Integrating the above equation

$$\frac{m_2}{m_1} = \left(\frac{T_2}{T_1}\right)^{\frac{c_v}{R}}. \quad \text{But} \quad \frac{c_v}{R} = \frac{c_v}{(c_p - c_v)} = \frac{1}{(\gamma - 1)}.$$

$$\therefore \quad \frac{m_2}{m_1} = (T_2)^{\frac{1}{\gamma - 1}} (T_1)^{\frac{1}{1 - \gamma}}, \quad \text{where } T_2 = \frac{p_2 V_2}{m_2 R}$$

and hence,

$$\frac{m_2}{m_1} = \left(\frac{p_2 V_2}{m_2 R}\right)^{\frac{1}{\gamma - 1}} (T_1)^{\frac{1}{1 - \gamma}} \quad \text{which can be written as}$$

$$m_2^{\frac{\gamma}{\gamma - 1}} = m_1 \left(\frac{p_2 V_2}{R}\right)^{\frac{1}{\gamma - 1}} (T_1)^{\frac{1}{1 - \gamma}}$$

Substituting for m_1, p_2, V_2, R and T_1 and nothing that $\gamma = \dfrac{c_p}{c_v} = 1.4$

$$m_2^{3.5} = 26.65 \left(\frac{500 \times 10^3 \times 0.5}{287}\right)^{1/1.4} \times (523)^{-1/0.4}$$

$$\therefore \quad m_2 = 3.678 \text{ kg.}$$

Therefore, the mass of air leaked out of the tank
$$= m_1 - m_2 = 26.65 - 3.678 = 22.972 \text{ kg.}$$

Alternatively,

m_2 can be calculated by a simpler method through an appropriate imagination. The mass of air finally remaining in the tank (m_2) at p_2, T_2 can also be imagined as that mass which initially could exist at p_1, T_1, occupying a smaller volume. This imagined change from initial state to the final state has to occur adiabatically as the tank is insulated. Also during the process there are no irreversibilities. Hence the assumed process would be reversible and adiabatic. Therefore

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma - 1}{\gamma}}$$. Substituting for T_1, p_2, p_1 and γ

$$T_2 = \left(\frac{500 \times 10^3}{8 \times 10^6}\right)^{\frac{0.4}{1.4}} = 236.85 \text{ K.}$$

Now, $$m_2 = \frac{p_2 V_2}{RT_2} = \frac{500 \times 10^3 \times 0.5}{287 \times 236.85} = 3.678 \text{ kg}$$

Thus. $m_1 - m_2 = 26.65 - 3.678 = 22.972 \text{ kg.}$

Example 9.11 :

At a given temperature, show that for an ideal gas the constant volume line is steeper than the constant pressure line on a T - s diagram.

Solution :

On the T - s diagram, at a given temperature T, while the slope of the constant volume line is $\left(\frac{\partial T}{\partial s}\right)_v$, the slope of the constant pressure line is $\left(\frac{\partial T}{\partial s}\right)_p$.

For a pure substance, $Tds = du + pdv$

For an ideal gas $du = c_v \, dT$ and hence,

$$Tds = c_v \, dT + pdv$$

For a constant volume line $dv = 0$ and therefore,

$$\left(\frac{\partial T}{\partial s}\right)_v = \frac{T}{c_v} \tag{i}$$

Similarly, starting from $Tds = dh - vdp$ and using $dh = c_p \, dT$ it can be shown that for a constant pressure line,

$$\left(\frac{\partial T}{\partial s}\right)_p = \frac{T}{c_p} \tag{ii}$$

For a gas $c_p > c_v$. therefore, it can be inferred from equations (i) and (ii) that

$$\left(\frac{\partial T}{\partial s}\right)_v > \left(\frac{\partial T}{\partial s}\right)_p$$. i.e., slope of the constant volume line is greater than the slope of the

constant pressure line on the T - s diagram. This could be seen in Figure 9.7.

Example 9.12 :

Starting from the Tds relation show that the reversible adiabatic process of an ideal gas with constant specific heats follows the path $pv^\gamma = $ constant, where γ is the ratio of specific heats.

Solution :

From Tds relation

$$Tds = du + pdv \tag{i}$$

A reversible adiabatic process is also an isentropic process and hence $ds = 0$. Therefore from (i),

$$du + pdv = 0 \tag{ii}$$

For an ideal gas $du = c_v \, dT$ and hence from (ii)

$$c_v \, dT + pdv = 0 \tag{iii}$$

The differential form of the ideal gas equation $pv = RT$ is

$$pdv + vdp = RdT \tag{iv}$$

Eliminating dT between (iii) and (iv)

$$\frac{c_v \ (pdv + vdp)}{R} + pdv = 0 \quad \text{where} \quad R = c_p - c_v \text{ and hence}$$

$$c_v \, pdv + c_v \ vdp + c_p \, pdv - c_v \, pdv = 0 \text{ or}$$

$c_v \, vdp + c_p \, pdv = 0.$ Dividing this by pv althrough,

$$\frac{dp}{p} + \frac{c_p}{c_v}\left(\frac{dv}{v}\right) = 0 \text{ and therefore}$$

$$\frac{dp}{p} + \gamma\left(\frac{dv}{v}\right) = 0 \text{ which on integration takes the form}$$

$$pv^\gamma = \text{Constant}$$

Example 9.13 :

Air initially at 5 bar, 0.01 m³ and 27°C in a piston cylinder mechanism undergoes a reversible cycle as follows: It expands first at constant pressure and then expands adiabatically to 2 bar. 0.025 m³. The air is then restored to the initial state through an isothermal compression process. Find the net work output and the efficiency of the cycle. Assume for air $\gamma = 1.4$ and $R = 287$ J/kg K.
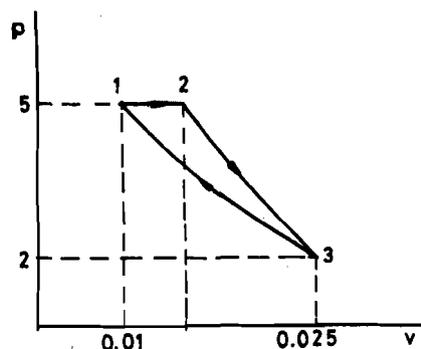

Figure 9.9 : Diagram for illustrative prob. 9.13

Solution :

In the cycle shown on the p - v diagram, 1-2 is the constant pressure expansion process, 2-3 is the adiabatic expansion process and 3-1 is the isothermal compression process. All these processes are reversible. Hence,

$$p_1 = p_2 \tag{i}$$

$$p_2 \, V_2^\gamma = p_3 \, V_3^\gamma \tag{ii}$$

$$p_3 \, V_3 = p_1 \, V_1 \quad \text{or} \quad T_3 = T_1 \tag{iii}$$

From data, $p_1 = 5$ bar, $V_1 = 0.01$ m³, $V_3 = 0.025$ m³, $p_3 = 2$ bar and $T_1 = T_3 = 273 + 27 = 300$ K. Let us take up each process in the sequence and find the heat and work interactions.

Process 1 - 2

$$W_{1-2} = p_1\left(V_2 - V_1\right) \tag{iv}$$

$$\Delta U = mc_v\left(T_2 - T_1\right) \tag{v}$$

$$Q_{1-2} = W + \Delta U \tag{vi}$$

The heat and work interactions during 1-2 cannot be readily evaluated as the values for V_2 and T_2 are not yet known.

Process 2 - 3

$$p_2 V_2^\gamma = p_3 V_3^\gamma \quad \text{or} \quad V_2 = \left(\frac{p_3}{p_2}\right)^{\frac{1}{\gamma}} V_3 \quad \text{and therefore}$$

$$V_2 = \left(\frac{2}{5}\right)^{\frac{1}{1.4}} \times 0.025 = 0.013 \text{ m}^3$$

Substituting for V_2 in (iv)

$$W_{1-2} = 5 \times 10^5 \, (0.013 - 0.01) = 1500 \text{ J} = 1.5 \text{ kJ}$$

$$\text{As} \quad \frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2},$$

$$T_2 = \left(\frac{p_2}{p_1}\right)\left(\frac{V_2}{V_1}\right) T_1 \quad \text{and therefore}$$

$$T_2 = \left(\frac{5}{5}\right) \times \left(\frac{0.013}{0.01}\right) \times 300 = 390 \text{ K}.$$

$$\text{Also } m = \frac{p_1 V_1}{RT_1} = \frac{(5 \times 10^5 \times 0.01)}{(287 \times 300)} = 0.058 \text{ kg}$$

Substituting for m and T_2 in (v)

$$\Delta U = 0.058 \times 0.718 \, (390 - 300) = 3.75 \text{ kJ}.$$

Using this ΔU in (vi), $Q_{1-2} = 1.5 + 3.75 = 5.25$ kJ.

$$W_{2-3} = \frac{(p_2 V_2 - p_1 V_1)}{(\gamma - 1)} \quad \text{and therefore}$$

$$W_{2-3} = (5 \times 10^5 \times 0.013 - 2 \times 10^5 \times 0.025)/0.4 = 3.75 \text{ kJ}.$$

$Q_{2-3} = 0$ as the process is adiabatic.

Process 3 - 1

$$W_{3-1} = p_3 V_3 \ln\left(V_1/V_3\right) = 2 \times 10^5 \times 0.025 \ln\,(0.01/0.025) = -4.58 \text{ kJ}.$$

$\Delta U_{3-1} = 0$ as the process is isothermal.

$$Q_{3-1} = W_{3-1} + \Delta U_{3-1} = -4.58 \text{ kJ}.$$

Net work for the cycle $\quad = W_{1-2} + W_{2-3} + W_{3-1}$

$$= 1.5 + 3.75 - 4.58 = 0.67 \text{ kJ}.$$

Heat transfer to the cycle $= Q_{1-2} = 5.25$ kJ.

Thus the cycle efficiency $= \dfrac{\text{net work}}{Q_{\text{in}}} = \dfrac{0.67}{5.25} = 12.76\%$

9.8 SUMMARY

(i) An ideal gas is one that follows the relation $pv = RT$ or $pV = mRT$ in which R the particular gas constant is $= \bar{R}/M$ where $\bar{R}$ the Universal gas constant is equal to 8314.3 J/kmol K.

(ii) Real gases tend to behave as ideal gas at low pressure and/or high temperatures.

(iii) For an ideal gas the internal energy and enthalpy are functions of temperatures only.

(iv) The specific heats of an ideal gas may either be constants or functions of temperature only, but $c_p - c_v = R$. An ideal gas with constant specific heats is a perfect gas.

(v) For an ideal gas $\Delta h = \int c_p \, dT$ and $\Delta u = \int c_v \, dT$.

(vi) $pV^n =$ constant represents a reversible polytropic process, n being the exponent of expansion or compression.

(vii) For the reversible and adiabtic process undergone by an ideal gas $n = \gamma$, where $\gamma = c_p / c_v$.

(viii) The relation among p, V and T in reversible polytropic processes in general are:

$$\left(\frac{p_2}{p_1}\right) = \left(\frac{V_1}{V_2}\right)^n$$

$$\left(\frac{T_2}{T_1}\right) = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}$$

$$\left(\frac{T_2}{T_1}\right) = \left(\frac{V_2}{V_1}\right)^{1-n}$$

9.9 KEY WORDS AND PHRASES

Ideal Gas	:	A gas that follows the equation of state $pv = RT$ at all ranges of pressure ad temperature.
Gas Constant	:	R in ideal gas equation of state.
Universal Gas Constant	:	Gas constant in the ideal gas equation of state with molar specific volume. This is the same for all gases and is 8314.3 J/kmol K.
Perfect Gas	:	An ideal gas with constant specific heats.
Semi-perfect Gas	:	An ideal gas whose specific heats are functions of temperature only.
Hyperbolic Process	:	A process that follows the relation $pv =$ constant.
Polytropic Process	:	A process that follows the relation $(pv)^n =$ constant.
Isothermal process	:	A process at constant temperature. For an ideal gas it is also an hyperbolic process.
Isentropic Process	:	Process at constant entropy. For an ideal gas it is a polytropic process with $n =$ ratio of specific heats.

9.10 ANSWERS/SOLUTIONS TO SAQs

SAQ 1

$3.9 \, \text{m}^3$

SAQ 2

27°C

SAQ 3

(a) 300.2 kJ/kg,

(b) 300.2 kJ/kg

SAQ 4

718 J/kg K

SAQ 5

(a) 311.74 kJ/kg,

(b) 307.84 kJ/kg

SAQ 6

    (a)   327°C,

    (b)   27°C and

    (c)   - 45.6°C.

SAQ 7

    (a)   3.596 bar,

    (b)   1.33 bar and

    (c)   1 bar.