UNIT 7 REVERSIBILITY - REVERSIBLE AND IRREVERSIBLE CYCLES AND PROCESSES-THERMO-DYNAMIC TEMPERATURE SCALE

Structure

7.1 INTRODUCTION

The second law of thermodynamics is a blunt statement that the most cherished wish of an engineer to convert all the heat transfer to work is unattainable. Laws of thermodynamics can never be violated as they are the laws of nature. Hence, having no alternative but to accept the second law of thermodynamics, an inquisitive engineer would like to probe deeper into the matter and look for answers to several logical questions which arise as a sequel to the restrictions imposed by the second law. Some of these very interesting and important questions are:

1. According to the Kelvin - Planck statement of the second law of thermodynamics, as it is not possible for a system operating in a thermodynamic cycle to completely convert all the heat transfer to it to an equal magnitude of work, what is the maximum conversion possible? Can this limit of conversion be predicted ? Will the prediction be only qualitative or both qualitative and quantitative ? How can such an ideal cycle (heat engine) that gives maximum conversion be conceived ?

2. Can the predicted limit of conversion be realised in practice ? If not why ? What precautions have to be taken in the design of an actual engine so that its efficiency tends towards that of the ideal engine ?

3. According to the Clausius' statement of the second law of thermodynamics, as it is impossible to transfer heat from a low temperature region to a high temperature region in the absence of work, what is the theoretical minimum work that can accomplish the task of transferring heat in the said direction under the given circumstances ? Alternatively, as the COP of the reversed heat engine is inversely proportional to the work supplied to it, what is its maximum performance (COP) ?

4. Can this predicted minimum work, do the job in actual practice ? If not why ? What precautions have to be taken in the design of a heat pump or a refrigerator so that their performance tend towards the maximum predicted performance of an ideal reversed heat engine ?

It is needless to emphasise here that the answers to the above questions have very high economic and practical significance. This Unit is aimed at providing answers to some of these questions. The subsequent Unit on Entropy is meant to answer the remainder.

Objectives

After reading this unit, you should be able to

* understand clearly the concept, importance and superiority of reversible heat engine,
* have a feel for the reversible processes,
* identify qualitatively the factors which make a process irreversible,
* understand and explain the operation of the Carnot Engine,
* realise that under given conditions no heat engine/heat pump can be more efficient than a reversible pump operating between the two given temperature reservoirs,
* appreciate the need for, but also define and understand the Thermodynamic temperature scale,
* evaluate the maximum efficiency and COP under given conditions, and
* identify, quantitatively whether a given thermodynamic cycle is reversible, irreversible or impossible.

7.2 REVERSIBLE HEAT ENGINES

The concept of a 'Reversible heat engine' is highly theoretical and imaginary in nature. A reversible heat engine is one which works most efficiently under the given circumstances. This means, while a reversible direct heat engine converts heat input to the system operating in a cycle to maximum work , a reversible reversed heat engine requires minimum work to cause a given heat transfer from a low temperature to a high temperature region. It is cautioned here that a reversible heat engine is only a mathematical model. We shall see later no practical engine can actually be a reversible heat engine.

What are the specialities of a reversible engine? To discuss this point the reader must be thorough in his understanding of the concept of reversibility. The following crude example is aimed at helping the reader to do so.

Consider a centrifugal gas turbine. High pressure, high temperature gas entering through the inlet pipe expands in the turbine to provide shaft work and then it leaves the turbine as low pressure low temperature gas through the exit pipe. Let the efficiency of the turbine be η_T. Assume now that this turbine is made to run in the opposite direction, i.e. as a centrifugal compressor (this is practically possible with minimal modifications to the machine). In this case the low pressure,low temperature gas enters the compressor, through the original exit of the turbine, and gets compressed because of shaft work input (the direction of work is opposite to that in the turbine) and leaves the compressor as high pressure high temperature gas, through the original entry to the turbine. When such an experiment is conducted, one finds out that the efficiency of the machine while operating as a compressor, η_C, is always less than its efficiency while it operated as a turbine. In other words the work required to drive one kg air through the compressor under the given conditions is much more than the work delivered by the same kg air as it flows through the device when it operates as a turbine. It can be generalised here that devices designed to operate in a particular direction when made to operate in the opposite direction can only work with an efficiency which is less than its value in the designed direction. Suppose, unique engine is imagined which works with identical efficiencies in both the designed and its opposite direction. Such an engine is the REVERSIBLE ENGINE.

With the above understanding one can now imagine a heat engine, direct or reversed (i.e. in general a system operating in a thermodynamic cycle) which would work equally well in both the forward and backward directions. Such a heat engine is the reversible heat engine. The definition of the reversible heat engine can be stated as follows: **A heat engine having heat interactions with two different temperature reservoirs is said to be reversible if its efficiency while operating as a direct heat engine is equal to the reciprocal of its coefficient of performance when it works as a heat pump between the same two temperatures.** In other words, for a reversible heat engine,

with each reservoir and also the magnitude of work interaction it has with the surroundings are the same irrespective of whether the heat engine works as a direct heat engine or as a reversed heat engine (heat pump or refrigerator), but for their directions.

SAQ 1.

A reversible heat engine operates between two reservoirs at t_1 and t_2. 500 kJ of heat transfer occurs from t_1 to the engine which rejects 187.5 kJ to t_2. (a) Find the work output of the engine. (b) If this engine were to work as a heat pump between the same t_1 and t_2 and that it transfers out 500 kJ to t_1 find the magnitude of heat interaction with the reservoir at t_2 and the work input to the heat pump.

7.2.1. Importance and Superiority of the Reversible Heat Engines

The following exercise brings out clearly the superiority of the reversible heat engine.

Consider a reversible heat engine R and any other heat engine X operating between the same two reservoirs as shown in Fig.7.2. Let the heat transfer to each engine from the high temperature reservoir at t_1 be Q_1. The problem is to analyse which engine is more efficient under the given conditions.

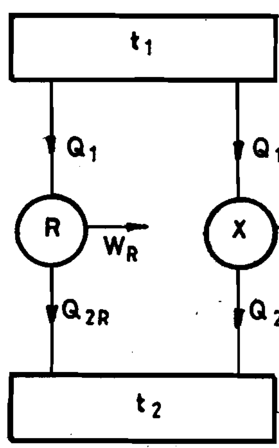

Fig. 7.2 : Heat engines R and X operating between t_1 and t_2

As it is difficult to guess which engine operates better, let it be assumed that engine X is more efficient than engine R.

By definition $\eta_X = \dfrac{W_X}{Q_1}$ and $\eta_R = \dfrac{W_R}{Q_1}$

As per the assumption made here, $\eta_X > \eta_R$.

$$\therefore \quad \frac{W_X}{Q_1} > \frac{W_R}{Q_1}, \text{ or } W_X > W_R.$$

As shown in figure 7.3 let engine R be made to work as a heat pump between the same two temperatures. R is a reversible engine and hence as a heat pump $\mathcal{R}$ it will have the same magnitudes of heat and work interactions with the surroundings as it had as direct heat engine (figure 7.2).

The only possibility for (i) and (ii) to be simultaneously satisfied is that $\eta_{R_1} = \eta_{R_2}$. Therefore, **all reversible heat engines operating between two given temperature limits have the same efficiency.** This statement is known as the corollary to Carnot's theorem or **Carnot's second theorem.**

SAQ 2

A reversible heat engine operating between two reservoirs at t_1 and t_2 has an efficiency of 70 %. A manufacturer of heat engines claims that he has designed an engine which when operating between the above two temperatures develops 80 kJ work when supplied with 100 kJ of heat from the reservoir at t_1. Evaluate the validity of his claim.

7.3 REVERSIBLE PROCESS

Let us now examine what exactly are the features of a reversible heat engine. As has already been made clear, a heat engine is nothing but a system operating in a cycle. If a heat engine is reversible it means the thermodynamic cycle on which it operates is reversible. A thermodynamic cycle consists of more than one non-cyclic process. Hence, if a cycle is reversible all processes that constitute the cycle have to be individually reversible. This makes it necessary to define and understand what exactly a reversible process is.**A process is said to be reversible if, after the process is completed, means can be found to restore the system and all parts of the surroundings to the states they were in at the start of the process.** Although this is the definition of a reversible process further explanation may be necessary to make it clear to the reader. Here is an attempt in this direction.

A process 1 - 2 is reversible only when all changes in the system and all parts of the surroundings during this process are completely undone when process 2 - 1 is carried out. In such a case there can be no evidence left either in the system or in the surroundings to say later that the system had actually been subjected to two processes in opposite directions. If a process is reversible,it is possible to change the direction of the process by making infinitesimal changes in the conditions that control the process. To satisfy all these conditions it is necessary that the reversible process occurs very slowly with no dissipative effects and that the gradients which control the process are infinitesimally small so that the system is always in equilibrium with its surroundings. These explanations make it very obvious that a reversible process is a quasistatic process or a fully resisted process.

The conditions under which a process can be reversible are so unique, it can be said that reversible processes are ideal processes and hence are only mathematical models. All natural processes and almost all man made processes are not reversible in nature. By the extension of the definition of a reversible process it can be said that any process which is not reversible is an **irreversible process.**

7.4 IRREVERSIBLE PROCESS

It is really necessary to know what factors actually make a process irreversible. Once these factors are identified means can be found in practice to either minimise, or eliminate if possible,the effects of these factors so that in the limit the process can tend towards a reversible process. Existence of dissipative effects or lack of equilibrium between the system and the interacting surroundings are responsible for any process to be irreversible. Therefore, generally the factors that make a process irreversible are:

(a) Friction (solid or fluid)

(b) Unresisted expansion

(c) Heat transfer with finite temperature difference

7.4.4 Combustion

Consider a mixture of fuel and air in a rigid insulated container. Ignite the mixture by an infinitesimal spark. Combustion of the fuel occurs instantaneously. There is generally a rise in pressure and temperature of the contents of the container. There is no heat or work interactions with the surroundings. If this process were to be reversible it should be possible in the reverse direction to get back the fuel air mixture at its original pressure and temperature from the hot combustion products, with no heat and work interactions with the surroundings. This is an impossibility and hence combustion makes a process irreversible. The irreversibility here is due to the nonexistence of chemical equilibrium during the process.

7.4.5 Mixing

Gases at different conditions may be mixed with no heat or work interactions but in the reverse direction they cannot be separated out to their initial states without any help from outside. As the system and the surroundings cannot be returned to their initial states a process involving mixing is always an irreversible one.

The above examples and explanations lead one to think that almost all processes are irreversible in nature. However, if one is interested in making a model of a reversible heat engine for achieving maximum efficiency, it must work on a cycle consisting of only reversible processes. Hence, from this point of view it must be possible to conceive some processes which are reversible. Examples of a few reversible processes are:

(i) Frictionless motion of solids, (ii) Slow, frictionless, adiabatic expansion or compression of gases and vapors and (iii) Slow, frictionless, isothermal compression or expansion of gases and vapors.

7.5. CARNOT HEAT ENGINE

By now it is very clear that all processes that constitute a reversible cycle have to be individually reversible. In other words no process can have in it any factor that makes the process irreversible. Such a cycle was conceived as early as 1824 by Sadi Carnot. A heat engine operating on such a cycle is bound to be reversible and is named appropriately as Carnot Heat Engine. Although the Carnot heat engine was never realised completely in practice the modern heat engines (such as the steam power plants and closed cycle gas turbine power plants) approach it as nearly as possible subject to economic and practical constraints. The details of the Carnot Heat Engine are given below.

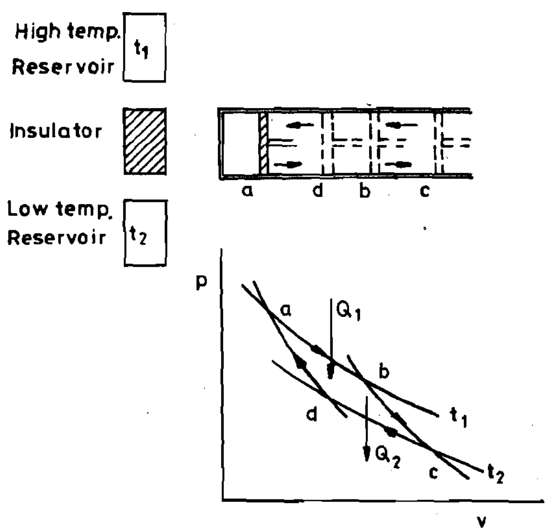

Fig. 7.4 : Carnot Heat Engine

As shown in Fig.7.4, consider a horizontal cylinder with an insulated barrel and a frictionless insulated piston. The left hand face of the cylinder is made out of a thin plate of good conductor of heat. Let the piston cylinder mechanism contain, when the piston is at its left extreme position, a certain mass of a gas at a temperature t_a, and a pressure p_a (shown by state 'a' in the corresponding p - v diagram drawn below the piston cylinder mechanism in the same figure). Assume that a high temperature reservoir at t_1 (which is only infinitesimally higher than t_a so that for all practical purposes it may be assumed that $t_1 = t_a$) is readily available and that it is communicated with the left side conducting end of the cylinder. Because of the slow heat transfer from the source , due to the infinitesimal temperature difference, the gas expands and thus the piston moves out slowly to maintain the gas at the same temperature. During this period the gas undergoes a **reversible isothermal expansion process**, the temperature remaining constant at t_1. Let this expansion be continued until the gas reaches a predetermined state 'b'. Let the heat transfer to the gas during this process be Q_1. When the gas reaches state 'b' let the high temperature reservoir be removed and in its place let an insulating material be brought in instantaneously. The expansion of the gas continues with no heat transfer and consequently the pressure and temperature of the gas decreases. Let this **reversible adiabatic expansion process** be continued until the piston reaches the extreme right position . At this instant let the gas be at state 'c' with temperature t_c and pressure p_c. Assume that a low temperature reservoir at temperature t_2 (which is infinitesimally less than t_c so that for all practical purposes it may be assumed that $t_2 = t_c$) is available and when the gas reaches the state 'c' the insulator is removed and in its place the low temperature reservoir is brought in instantaneously. As the piston moves slowly to the left the gas is compressed and to maintain the gas at the constant temperature t_2, heat is slowly transferred out to the reservoir.This **reversible isothermal compression process** continues until the gas reaches the predetermined state 'd'. Let the heat rejected during this process be Q_2. When the gas reaches the state 'd' the low temperature reservoir is suddenly removed and the insulator is brought in communication with the cylinder head. The compression now continues with no heat interaction. During this **reversible adiabatic compression process** the temperature and pressure of the gas increase. The cycle is completed when the gas reaches the initial state 'a' and the piston is at its extreme left position. The gas is now ready to perform the next cycle in the same sequence.

Thus, the Carnot cycle consists of **two reversible isotherms** (a - b the expansion process at t_1 and c - d the compression process at t_2) and **two reversible adiabatics** (b - c the expansion process and d - a the compression process). The engine operating on this cycle between two reservoirs at different temperatures t_1 and t_2 must have the highest efficiency as it operates on a reversible cycle. This efficiency is given by $(Q_1 - Q_2) / Q_1$.

7.5.1 Remarks on the Carnot Engine

The Carnot Engine can only be treated as a model to which all engines must tend from the point of performance. Realisation of the Carnot Engine is difficult because of several practical constraints. The restriction that the piston should move slowly makes the power output from such an engine very small and of no practical use. During the isothermal processes not only the temperature of the working substance must be uniform throughout the substance but also the difference between the temperatures of the substance and the two reservoirs, must be infinitesimal. The infinitesimal temperature difference demands large engines even for small power outputs. Economic constraints may not permit such large engines. It is assumed that in a Carnot Engine the friction between the piston and cylinder is zero. This condition is highly impossible to comply with in practice. Friction can only be decreased by using precise machining processes and a proper lubrication system. Also, perfect conductors and insulators of heat are not available in practice. This poses material problem in making the cylinder and piston. A careful study of the Carnot cycle reveals that the changes from isothermal expansion process to adiabatic expansion process and from isothermal compression process to adiabatic compression process have to be carried out instantaneously. Any delay may not take the working substance through the required cycle. Practical limitations do not permit such instantaneous changes.

7.6 INTERNAL AND EXTERNAL REVERSIBILITY

During a given process the factors that make the process irreversible may exist either only in the system, or only in the surroundings or in both the system and surroundings. If the factors exist only in the system and not in the surroundings the process is internally irreversible but externally reversible. Similarly if the factors exist only in the surroundings and not in the system then the process is internally reversible but externally irreversible. If the factors exist in both system and surroundings then the process is said to be totally irreversible.

For example consider the system undergoing the cycle in a Carnot heat engine explained above. During the isothermal expansion process the system is not only at steady and uniform temperature but also has heat interaction with a reservoir which is at the same temperature as its own. Similarly during the isothermal compression process also the system is at a steady uniform temperature and is transferring heat to the reservoir at a temperature equal to its own temperature. In other words the heat transfers in a Carnot Heat Engine occur by virtue of practically no temperature difference. Hence the Carnot heat engine is both internally and externally reversible.

Let us discuss another example now. Consider a system undergoing a cycle consisting of two reversible isotherms and two reversible adiabatics. The system may be said to be undergoing a Carnot cycle. During the isothermal expansion process although the system temperature is steady and uniform at t_1 let the high temperature reservoir transferring heat to the system be at t_H where $t_H > t_1$. Similarly during the isothermal compression process let the system be at a steady and uniform temperature of t_2 but let it transfer heat to the low temperature reservoir at t_L where $t_L < t_2$. In such a cycle although there are no factors internally that make the cycle irreversible there are such factors outside the system (heat transfer with finite temperature difference). Hence, although the cycle undergone by the system is internally reversible, it is externally irreversible.

SAQ 3

1 kg water at atmospheric pressure and 100°C is evaporated completely by heating it over an electric heater at a temperature of 700°C. The vapour is simultaneously collected and condensed in a vessel cooled by the surroundings at 27°C such that the water returns to its initial state of 100°C and one atmospheric pressure. As water has been restored to its initial state after two processes in opposite directions, the process undergone by it is reversible. Comment on (a) this statement and (b) on internal and external reversibility.

7.7 THERMODYNAMIC TEMPERATURE SCALE

By now we are certain that it is only the reversible engine that can convert heat transfer into it to maximum work output. In other words the efficiency of the reversible engine is the maximum. However, it has not been made known quantitatively as to what this maximum conversion is ?. The concept of thermodynamic temperature scale provides the answer for this important question. In addition to this it also eliminates the lacunae that exist in the Celsius temperature scale. The thermometers with Celsius temperature scale are not calibrated at each 'degree' marking on them. Generally they are calibrated at certain reproducible temperatures in the required range ,such as the ice point, the steam point etc. What the thermometer reads in between two calibrated markings depends to a large extent on the properties of the thermometric substance and the mode of operation of the thermometer. Hence it is unlikely that two thermometers of different types, or for that matter even of same type, inserted into a system at a particular temperature would give the same reading. There is thus a need for developing a temperature scale obtained through a device whose operation neither depends on the properties of the working substance nor on the mode of its operation. A reversible heat engine, such as the Carnot engine, provides just such a device as its performance does not depend on the properties of the working substance but depends only on the temperature limits between which it operates. For example, under identical conditions, a reversible engine operating with steam as the working substance will have the same efficiency as the one operating on air or any other working substance.

Let an externally reversible engine operate between a high temperature reservoir at a temperature t_H and a low temperature reservoir at a temperature t_L. Let the heat transfer to the engine be Q_H and heat transferred out of the engine be Q_L. According to Carnot theorems, the efficiency of such an engine is the maximum and its value depends only on the temperatures between which the engine is operating and not on the properties of the substance that undergoes the cycle. Hence, the efficiency has to be a function, say , of the two temperatures only. i.e. $\eta_R = 1 - \left(\frac{Q_L}{Q_H}\right) = \psi\left(t_H, t_L\right)$.

$$\text{or} \quad \frac{Q_H}{Q_L} = \left[1 - \psi\left(t_H, t_L\right)\right]^{-1} = \phi\left(t_H, t_L\right). \qquad (7.7)$$

where ϕ is another function related to ψ.

Consider three reversible heat engines A, B and C working between the high temperature reservoir at t_1 and the low temperature reservoir at t_2, as shown in Fig.7.5.

Engine A operates between the reservoir at t_1 and an intermediate system at a fixed temperature t_x, where $t_2 < t_x < t_1$. Let the heat transfer from the reservoir at t_1 to engine A be Q_1 and the heat transferred out of it to the system at t_x be Q_x. Let the work produced by A be W_A. Engine B operates between the system at t_x and the reservoir at t_2. With the assumption that all the heat transferred by the engine A goes to engine B through the system at constant temperature t_x, heat transferred to B is Q_x. Let the heat transferred by the engine B to the reservoir at t_2 be Q_2, and work produced by it be W_B. Engine C operates between the two reservoirs at t_1 and t_2. Let the heat transfer to engine C be Q_1 which is equal to the heat transfer to engine A. The heat transferred out of this engine has to be Q_2 as, according to Carnot's second statement the efficiency of engine C must be same as the combined efficiencies of engine A and B (note that engine A and B together with the constant temperature system at t_x can be assumed to be another reversible heat engine operating between t_1 and t_2 and hence if this combination transfers out Q_2 to the reservoir at t_2, then engine C must also transfer out Q_2 to the same reservoir).

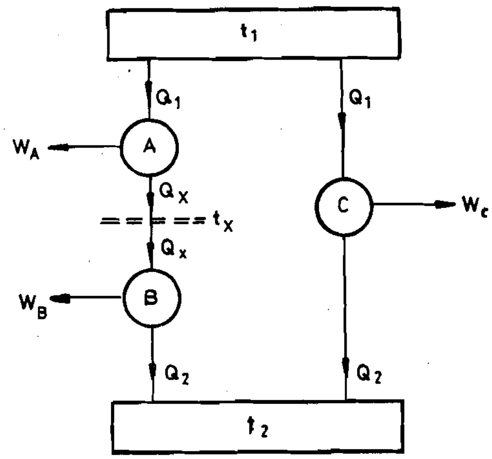

Fig. 7.5 : Reversible heat engines, A, B and C operating between t_1 and t_2

From equation (7.7), the following relations may be stated for the three reversible engines, A,B and C respectively:

$$\frac{Q_1}{Q_x} = \phi\,(t_1\,,\,t_x) \tag{7.8}$$

$$\frac{Q_x}{Q_2} = \phi\,(t_x\,,\,t_2) \tag{7.9}$$

$$\frac{Q_1}{Q_2} = \phi\,(t_1\,,\,t_2) \tag{7.10}$$

But, $$\frac{Q_1}{Q_x} \times \frac{Q_x}{Q_2} = \frac{Q_1}{Q_2}$$

Therefore, combining equations (7.8) to (7.11), it can be shown that

$$\phi\,(t_1\,,\,t_x) \times \phi\,(t_x\,,\,t_2) = \phi\,(t_1\,,\,t_2). \tag{7.12}$$

While the left hand side of equation (7.12) is a function of three temperatures t_1, t_x and t_2 the right hand side is a function of only two temperatures t_1 and t_2. Hence, the value of the right hand side is independent of the temperature t_x. This gives us an insight into the nature of the function which can satisfy such a condition. Therefore,

$$\phi\,(t_1\,,\,t_x) = \frac{f(t_1)}{f(t_x)}\,, \text{ and} \tag{7.13}$$

$$\phi\,(t_x\,,\,t_2) = \frac{f(t_x)}{f\,(t_2)} \tag{7.14}$$

where f is some other function. Therefore

$$\frac{Q_1}{Q_2} = \frac{Q_1}{Q_x} \times \frac{Q_x}{Q_2} = \phi\,(t_1\,,\,t_x) \times \phi\,(t_x\,,\,t_2) \tag{7.15}$$

Combining equations (7.13) to (7.15), $\dfrac{Q_1}{Q_2} = \dfrac{f(t_1)}{f(t_2)}$ (7.16)

Here, f can be any function in general. However it is universally accepted, as suggested by Lord Kelvin, to consider $f(t) = T$ where T is in Thermodynamic or Kelvin Temperature scale.

Therefore

$$\frac{Q_1}{Q_2} = \frac{T_1}{T_2} \tag{7.17}$$

Equation (7.17) defines the thermodynamic temperature scale. **The ratio of two temperatures on thermodynamic temperature scale is equal to the ratio of heat transfers to and from an externally reversible heat engine operating between the given two temperatures.** Although equation (7.17) defines the thermodynamic temperature scale it does not indicate how big or small a degree on this scale is.

It is internationally accepted to assign a value of 273.16, on Thermodynamic or Kelvin temperature scale, to the temperature at triple point of water.

Thus $T_{tp} = 273.16$ K

For a reversible heat engine operating between any temperature T and the triple point temperature T_{tp}, according to equation (7.17),

$$\frac{Q}{Q_{tp}} = \frac{T}{T_{tp}}$$

$$\text{or} \quad T = 273.16\,\frac{Q}{Q_{tp}} \tag{7.18}$$

where Q is the magnitude of heat transfer to the heat engine from the system at temperature T and Q_{tp} is the heat rejected by the heat engine to the system at the triple point temperature T_{tp}. Equation (7.18) gives the information that to get any value of T on Thermodynamic temperature scale a reversible heat engine must be made to operate between T and T_{tp} and the magnitudes of heat transfer to and from the engine are measured. This poses problems as a reversible heat engine is only a mathematical model and it is not possible to measure the heat transfers. It will however be shown later that (refer to Unit 9) the Thermodynamic temperature scale agrees well with the Ideal gas thermometer scale or the absolute temperature scale.

Hereafter while the alphabet t is used to indicate the temperature on Celsius scale, T is used to denote the temperature on Thermodynamic temperature scale.

The triple point temperature for water is 0.01°C (For details on triple point of water refer to Unit 10). Hence the relation between the temperatures on Celsius scale and

Thermodynamic scale is

$$T = t + 273.15 \tag{7.19}$$

For example, at standard atmospheric pressure water boils at 100°C. Hence, the boiling point of water on Celsius scale is 100°C . On Kelvin scale it is equal to 100 + 273.15 = 373.15 K. In engineering applications, it is considered adequately accurate if T and t are related by

$$T = t + 273. \tag{7.20}$$

The thermal efficiency of a heat engine in general is given by

$$\eta_R = 1 - \frac{Q_2}{Q_1}$$

If the engine is externally reversible, according to equation (7.17),

$\frac{Q_2}{Q_1} = \frac{T_2}{T_1}$, and hence the efficiency of a reversible heat engine is given by,

$$\eta_R = 1 - \frac{T_2}{T_1}.$$

It may also be noted that

$$\eta_R \neq 1 - 1 - \frac{t_2}{t_1}, \text{ but } \eta_R = 1 - \frac{(t_2 + 273)}{(t_1 + 273)}$$

It has been made very clear that no engine can be more efficient than a reversible heat engine operating between any two temperatures T_1 and T_2. In general if Q_1 and Q_2 are the heat transfers to and from any heat engine operating between T_1 and T_2, then

$$1 - \frac{Q_2}{Q_1} \leq 1 - \frac{T_2}{T_1} \qquad (7.21)$$

If the equality holds the engine must be a reversible engine. Otherwise the engine is irreversible. However, the LHS quantity can never be greater than the RHS quantity. If so, such an engine cannot operate. Equation (7.21) can thus be used to identify whether a given engine is reversible. irreversible or impossible.

This chapter on Kelvin temperature scale provides answer to one important question posed at the beginning namely, what is quantitatively the maximum possible conversion of heat to work by a heat engine. Supposing the engine is operating between the reservoirs at T_1 and T_2, ($T_1 > T_2$) and the heat transfer to the engine is Q_H then the maximum possible work from the engine is limited to $Q_H\left(1 - \frac{T_2}{T_1}\right)$. No engine can give more work than this under the given circumstances.

SAQ 4

A heat engine operates between 527°C and 27°C. Find its efficiency if (a) the engine is reversible and (b) the engine is irreversible .

SAQ 5

A reversible heat engine is operating between –13°C and 37°C. Find its COP as (a) heat pump and (b)refrigerator.

7.8 ILLUSTRATIVE EXAMPLES

Example 7.1 :

Check for the presence, or otherwise, of all the factors of irreversibility in the following cases and classify them as either reversible or irreversible. Also, wherever possible, comment on the internal and external reversibilities. The system to be considered in each case is underlined. Assume in all cases the properties are uniform throughout the system at any given instant.

(a) **A mass of gas** is compressed slowly in a frictionless leakproof non-conducting piston cylinder mechanism.

(b) **A weight** slides slowly down a rough inclined plane.

(c) **One kg of water** is heated very slowly from 30 to 40°C by an inflow of heat.

(d) **One kg of water** is 'heated' slowly but adiabatically from 30 to 40°C by a paddle wheel mounted on a frictionless pulley.

(e) **One kg of water** is heated rapidly from 30 to 40°C.

(f) **One kg mixture of petrol and oxygen** contained in a sealed vessel immersed in a large pool of water at 27°C reacts spontaneously.

(g) A **stream of steam** at 100°C mixes with **liquid water** at 25°C in a non-conducting heater.

(h) A heavy piston confining **a gas** in a vertical cylinder is held in position by stops. When the stops are removed the piston falls rapidly because it has much more weight than what the initial pressure of the gas can support.

Solution:

(a) The process is reversible as there are no factors of irreversibility in it. It is worth recalling here that this process is actually one of the four processes that constitute the Carnot cycle.

(b) The process is irreversible. The factor that makes the process irreversible is the solid friction between the sliding weight and the inclined plane.

(c) This process can be a reversible process. In such a case the water is heated slowly, step by step, by bringing it in communication successively with several reservoirs each with a temperature infinitesimally higher than the other starting from the one at $(30 + dT)$°C to the last one at $(40 + dT)$°C. In this manner the factor 'heat transfer with finite temperature difference' which makes the process otherwise irreversible is eliminated.

Alternatively it may also be argued that the process is irreversible. Suppose the water is in contact with a single reservoir at 40°C. The heating is slow even under these conditions. In such a case, at the beginning of the process the irreversibility is more as the temperature difference is more and the irreversibility keeps decreasing as the temperature difference reduces with the heating of water. This is a case of external irreversibility as the source of irreversibility is outside the system.

(d) The process is irreversible. The temperature of water increases only because of the fluid friction between the paddle wheel and the water The process is internally irreversible but is externally reversible as there are no factors in the surroundings that makes the process irreversible.

(e) Rapid heating requires a large temperature difference between the water being heated and the external system supplying heat transfer, such as an electric heater or a burner. This temperature difference makes the process irreversible. It can also noticed that this is also an example of external irreversibility.

(f) This is an irreversible process for two reasons. Because of the spontaneous combustion there will be a rise in temperature of the vessel containing the combustion products. The vessel transfers heat to the pool of water due to this temperature difference. The process is internally irreversible due to combustion and heat transfer with finite temperature difference. However the process is externally reversible as the temperature of the large pool of water hardly changes.

(g) This process is also irreversible for two reasons namely mixing and heat transfer with finite temperature difference between the steam and water. The process is only internally irreversible.

(h) The process is possibly irreversible as it is an unresisted process. The system is not in mechanical equilibrium with the surroundings during the process as outside pressure is high compared to the system pressure. The process is internally irreversible.

Example 7.2 :

Consider three hypothetical heat engines, A, B and C, each operating between 1000 K and 300 K. When each engine involves itself with a heat interaction of 1000 kJ with the high temperature reservoir, it is claimed that while A develops a work of 600 kJ, B and C develop 700 and 800 kJ. Use the Carnot statement and identify the engines A,B and C as reversible, irreversible or impossible.

Solution :

As shown in Fig.7.6 the three engines A,B and C are operating between the same two temperature limits of T_1 = 1000 K and T_2 = 300 K. No engine operating between these two temperature can be more efficient than a reversible engine whose efficiency is given by:

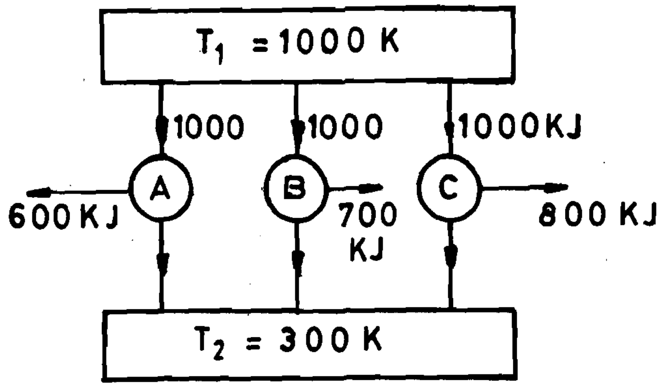

Fig. 7.6 : Diagram for Illustrative Prob. 7.2

$$\eta_{max} = 1 - \left(\frac{T_2}{T_1}\right) = 1 - \left(\frac{300}{1000}\right) = 0.7$$

Considering engine A, Q_1 = 1000 kJ, W = 600 kJ and hence by the First law Q_2 = 1000 - 600 = 400 kJ. Therefore,

$$\eta_A = 1 - \left(\frac{Q_2}{Q_1}\right) = 1 - \left(\frac{400}{1000}\right) = 0.6.$$

As $\eta_A < \eta_{max}$, the heat engine A is irreversible.

Similarly for engine B, $Q_2 = Q_1 - W = 1000 - 700 = 300$ kJ and hence,

$$\eta_B = 1 - \left(\frac{300}{1000}\right) = 0.7.$$

As $\eta_B = \eta_{max}$, the heat engine B is reversible.

Considering lastly the engine C, $Q_2 = Q_1 - W = 1000 - 800 = 200$ kJ Therefore,

$$\eta_C = 1 - \left(\frac{200}{1000}\right) = 0.8.$$

As $\eta_C > \eta_{max}$ the heat engine C is an impossibility.

Example 7.3 :

A direct heat engine A and a reversed heat engine B are operating between 177°C and 27°C. The COP of B as a heat pump is 2.5. A drives B. The magnitudes of heat interaction of A and B with the reservoir at 27°C are 200 kJ and 50 kJ respectively. The combined work output of A & B is 20 kJ. Identify whether the heat engine A is reversible or irreversible.

Solution:

Refer to the schematic diagram of Fig.7.7.

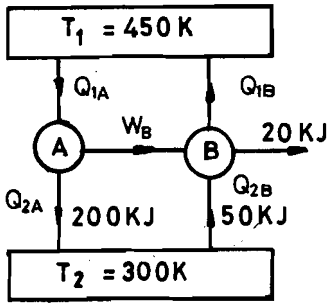

Fig. 7.7 : Schematic diagram for Illustrative Prob. 7.3

From the data, $(COP)_{hp}, B = \frac{Q_{1B}}{W_B} = 2.5$, (i)

where W_B is the work required by the reversed heat engine B.

Applying First Law to engine B,

$$Q_{1B} = W_B + Q_{2B}. \quad \text{(ii)}$$

Between (i) and (ii) $\frac{W_B + Q_{2B}}{W_B}$ and therefore

$$W_B = 33.3 \text{ kJ as } Q_{2B} = 50 \text{ kJ}.$$

$$W_A = W_B + W_{net} = 33.3 + 20 = 53.3 \text{ kJ}.$$

By First law $$Q_{1A} = Q_{2A} + W_A = 200 + 53.3 = 253.3 \text{ kJ}.$$

$$\eta_A = \frac{W_A}{Q_{1A}} = \frac{53.3}{253.3} = 0.21$$

$$\eta_{max} = 1 - \frac{T_2}{T_1} = 1 - \frac{300}{450} = 0.33$$

As $\eta_A < \eta_{max}$, engine A is irreversible.

Example 7.4 :

Consider an engine in outer space which operates between T_1 and T_2 (temperatures are in Kelvin and $T_1 > T_2$). The only way in which heat can be transferred from the engine is by radiation.

The rate at which heat is radiated is directly proportional to fourth power of Kelvin temperature and the area of the radiating surface. Show that for a given power output and a given T_1, the area of the radiator is minimum when $\frac{T_2}{T_1} = 0.75$.

Solution :

For a given T_2 the radiating area is minimum when heat rejected, Q_2, is minimum. Heat rejected is minimum only if the engine is reversible.

Let Q_1 be the heat transfer to the reversible engine from the high temperature reservoir at T_1 K.

$$\eta = \frac{W}{Q_1} = \frac{(T_1 - T_2)}{T_1} \text{ and hence,} \quad \text{(i)}$$

$$\frac{W}{(T_1 - T_2)} = \frac{Q_1}{T_1} \quad \text{(ii)}$$

For the reversible engine $$\frac{Q_1}{T_1} = \frac{Q_2}{T_2} \quad \text{(iii)}$$

Between (ii) and (iii) $\frac{W}{(T_1 - T_2)} = \frac{Q_2}{T_2}$ (iv)

The heat rejected Q_2, is through radiation only and hence,

Q_2 is proportional to A and fourth power of T_2.

Let the proportionality constant be C. Then,

$$Q_2 = C\,A\,T_2^{\,4} \qquad \text{(v)}$$

Eliminating Q_2 between (v) in (iv)

$$\frac{W}{(T_1 - T_2)} = \frac{C\,A\,T_2^{\,4}}{T_2} = C\,A\,T_2^{\,3} \qquad \text{(vi}$$

$$A = \left(\frac{W}{C}\right)\left\{\frac{1}{\left(T_2^{\,3}\,T_1 - T_2^{\,4}\right)}\right\} \qquad \text{(vii)}$$

For a given W and T_1, A is minimum when $\frac{dA}{dT_2} = 0$, hence

using (vii), $\frac{dA}{dT_2} = \left(\frac{W}{C}\right)\left[\frac{(3T_2^{\,2}\,T_1 - 4\,T_2^{\,3})}{(T_1\,T_2^{\,3} - T_2^{\,4})^2}\right] = 0$.

Here $\left(T_1\,T_2^{\,3} - T_2^{\,4}\right)$ cannot be zero as $T_1 > T_2$

Hence, $\left(3T_2^{\,2}\,T_1 - 4T_2^{\,3}\right) = 0$, or $3T_1 = 4T_2$.

$$\therefore \quad \frac{T_2}{T_1} = 0.75.$$

7.9 SUMMARY

A reversible heat engine is one whose efficiency while operating directly between two given reservoirs at fixed but different temperatures is equal to the reciprocal of the COP while it operates as a heat pump between the same two reservoirs.

According to Carnot's theorems: (a) No engine can be more efficient than a reversible engine, both operating between given reservoirs and (b) all reversible engines operating between given two reservoirs have the same efficiency.

Friction, unresisted expansion, heat transfer with finite temperature difference, combustion and mixing make a process irreversible.

(In the following explanations suffixes 1 and 2 refer to high temperature and low temperature reservoirs respectively)

The thermodynamic temperature scale is defined with respect to a reversible heat engine by

$$\frac{T_1}{T_2} = \frac{Q_{1R}}{Q_{2R}}$$

Maximum efficiency of a heat engine is $= 1 - \left(\frac{T_2}{T_1}\right)$.

In general $1 - \left(\frac{Q_2}{Q_1}\right) \leq 1 - \left(\frac{T_2}{T_1}\right)$.

$$\text{Max (COP)}_{\text{hp}} = \frac{T_1}{(T_1 - T_2)} \text{ and Max (COP)}_{\text{ref}} = \frac{T_2}{(T_1 - T_2)}$$

7.10 ANSWERS/SOLUTIONS TO SAQs

SAQ 1

(a) 312.5 kJ, (b) 187.5 kJ, 312.5 kJ.

SAQ 2

The efficiency of the engine is 0.8 which is greater than the maximum efficiency of 0.7, and hence the manufacturer's claim has to be definitely false.

SAQ 3

Although the water has returned back to its initial state after the two processes in opposite directions, the process is indeed irreversible as during the evaporation there is heat transfer from the heater with finite temperature difference and so also during the condensation between the water and the surroundings. It will be shown in the next chapter on Entropy that there will be permanent changes in surroundings in such cases (b) The process can be internally reversible but externally irreversible.

SAQ 4

(a) 0.625, (b) cannot be evaluated ; however its value is definitely less than 0.625.

SAQ 5

(a) 6.2, (b) 5.2.

7.11 GLOSSARY

Reversible Heat Engine	:	Heat engine with maximum efficiency (if direct) or maximum COP (if reversed)
Carnot Theorem 1	:	No engine can be more efficient than a reversible heat engine operating between two given temperature limits.
Carnot Theorem 2	:	All reversible heat engines operating between two temperatures have the same efficiency.
Reversible Process	:	A process is said to be reversible if, after the process is completed, means can be found to restore the system and all parts of the surroundings to the states they were in at the start of the process.
Carnot Cycle	:	A reversible cycle consisting of two isothermals and two adiabatics.
Thermodynamic Temp. Scale (or) Kelvin Temp. scale	:	The temperature scale in which the ratio of two temperatures is equal to the ratio of heat transfers to and from an externally reversible heat engine operating between the said two temperatures.
Thermodynamic Temperature	:	Temperature on Thermodynamic scale, which coincides with the Absolute or the Ideal Gas thermometer scale and = Temp. in deg. C + 273.15.