UNIT 4 TRANSMISSION LINES

Structure

4.1 INTRODUCTION

There are different ways of signal transmission as you have learnt in the last unit. The electrical signals being electromagnetic waves, they can be transmitted in free space. This is commonly used by the radio and TV broadcasters. But this type of transmission is unguided and can be received by any receiver tuned to the signal carrier frequency. In order to make the signal transmission available only to the selected receivers, guided media are necessary. The simplest form of guided medium is a pair of conducting wires which run from transmitting point to the receiving point. This is most commonly observed in telephony, where there is a physical guiding medium in the form of cable to carry the signal from exchange to our telephone set.

There are various types of cables used for transmitting the signal. These are called *transmission lines*. The term transmission line can be used even for a pair of wires conducting electric current at 50 Hz from the electricity board transformer to our home. But the transmission lines we will be dealing with in this unit are the ones used for transmitting the energy at UHF and microwave frequencies.

Table 4.1: Microwave Frequency Bands

Band Designation	Frequency Range (GHz)
L band	1-2
S band	2-4
C band	4-8
X band	8-12
Ku band	12-18
K band	18-27
Ka band	27-40
Millimetre	40-300
Sub-millimetre	> 300

In the first unit you have learnt that the term *microwave* frequency is generally used for the frequency range of electromagnetic spectrum measuring from 1 GHz to 300 GHz. This frequency range is further divided into various bands shown in Table 4.1.

The transmission lines can have different forms, e.g. parallel wires, coaxial cable, stripline, microstripline etc. In this unit you will study the propagation of electromagnetic wave along the transmission line, the characteristics of transmission lines and their different applications.

In Sec. 4.2, we take a brief review of different guided media used in communication. Since the transmission lines are used at high frequency, it is important to learn the behaviour of the conducting wire at these frequencies. A conducting wire, which acts as a resistive component at low frequency, can exhibit capacitive and inductive behaviour at microwave frequencies. In Sec. 4.3 we will discuss representation of the transmission line in terms of conventional lumped components. The basic electrical equations governing transmission line are discussed in Sec. 4.4. For the wave travelling in the transmission line, these equations are interpreted in Sec. 4.5. Various transmission line parameters like reflection coefficient, characteristic impedance, standing wave ratio are discussed in Sec. 4.6. Under different load conditions, the behaviour of the transmission line is described in Sec. 4.7. In order to have the efficient transmission of signal, it is necessary to appropriately match the impedance of transmission line and load. The techniques of impedance matching are discussed in Sec. 4.8. You will learn about some special applications of the transmission line in Sec. 4.9.

Objective

After studying this unit you should be able to:

- describe construction of various types of transmission lines;
- represent the transmission lines in the form of distributed components;
- solve the transmission line equations;
- explain the various transmission line characteristics like Standing Wave Ratio (SWR), characteristic impedance, reflection coefficient;
- describe the behaviour of transmission line with different loads connected to it;
- design some simple impedance matching networks; and
- describe some special applications of the transmission line.

4.2 MEDIA FOR GUIDED TRANSMISSION

For guided signal delivery, the simplest way is to use a pair of conducting wires called **transmission lines**. However, the construction of transmission line decides the frequency range of signal that can be passed through it. Any pair of conducting wires tends to radiate, if the separation between the conductors approaches half the wavelength of operating frequency. At higher frequencies the radiation loss is proportional to the square of frequency. As the frequency goes on increasing, the losses due to radiation in the transmission line increase tremendously. In order to reduce the radiation losses, it is desirable to confine the field within the conductors. It is possible in case of coaxial cable, where one conductor is hollow and the second conductor is placed in the centre of this hollow conductor. This reduces the radiation losses. But to separate the two conductors, some dielectric spacers are used. These dielectric materials have finite resistivity though ideally they should have infinite resistance. The resistivity goes on reducing at increased frequencies, hence the coaxial cables are also useful in only limited range (upto maximum of 40 GHz when used with special dielectric materials). Beyond this limit we have to take the help of **waveguides** for signal transport. You have already learnt in the last unit that the dimensions of waveguide are comparable to the wavelength of their operation. Hence they can be effectively used from 1 GHz to 300 GHz. Beyond this frequency, the dimensions of waveguide become too small (~ 4 mm or less) and it is difficult to handle such guides practically. Above this frequency, the optical range starts and the optical fibre is used for guided wave transmission.

In this Unit, we will be concentrating on the propagation of signal through transmission lines. There are different types of transmission lines depending on their constructions. Fig. 4.1 shows some typical constructions of transmission lines.

a. The simplest form of transmission line is a **pair of parallel conductors** separated by air or any dielectric medium as shown in Fig. 4.1a. These types of lines are used in telephony. Here the spacing between the two conductors is kept constant by separating them with dielectric medium like polyethylene or polyvinyl chloride (PVC). One drawback of these parallel conductor lines is their noise susceptibility. As you can see in Fig. 4.2a, if any other conductor like power line is running parallel to such a transmission line, the signal from this power line may get coupled with the signal on the transmission line by capacitive and inductive coupling. Since the two conductors get different amplitudes of noise signals (V_1 & V_2), the noise does not cancel and cause a distortion in the signal transmission.

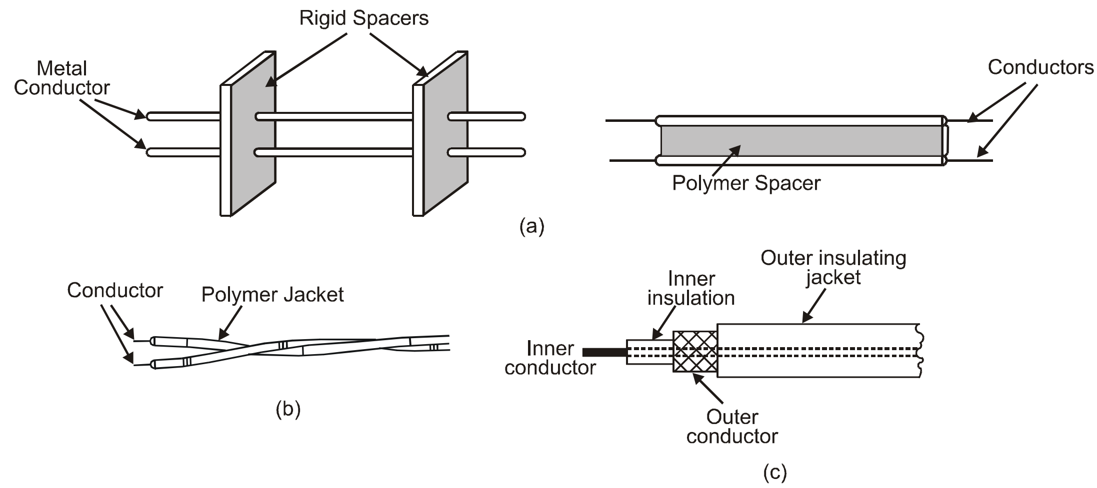

Fig. 4.1: Transmission Lines a) Parallel conductors; b) twisted pair and c) coaxial cable

b. The problem of noise coupling in the earlier case can be reduced by using a **twisted pair** of conductors shown in Fig. 4.1b. In this type, two conductors coated with PVC or polyethylene are twisted together over the entire length of the line. As shown in Fig. 4.2b, the noise coupling in both the conductors is the same and cancel each other completely.

In case of twisted pair, both the cables are exposed to the power line in equal amount and hence both the cables pickup equal noise voltage = $n\ \Delta V_1 + n\ \Delta V_2$ where n is the number of segments of twisted pair facing the noise generating cable.

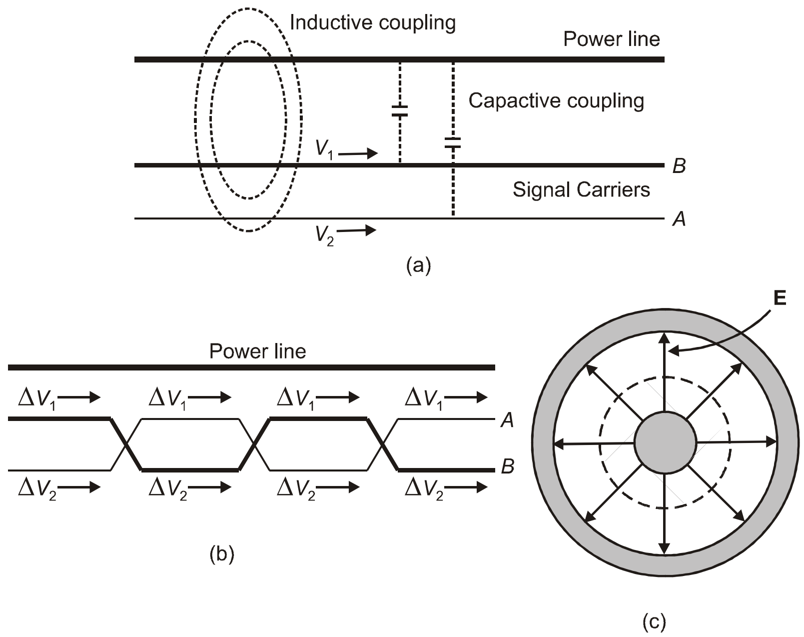

Fig. 4.2: a) Noise coupling in parallel conductor transmission line; b) noise reduction in twisted pair transmission line; and c) electric field confinement in coaxial cable

c. When the signal frequency is higher, **coaxial cables** are used. The construction of coaxial cable is shown in Fig. 4.1c. It consists of a conductor placed inside the large hollow conductor. These two conductors are separated by a dielectric spacer layer. The electric field is confined in the annular space in between the conductors as shown in Fig. 4.2c. This avoids any radiation loss. Normally, coaxial cable can be used for signals with frequencies upto 3GHz. These cables are commonly used for carrying cable TV signals. You have also used these cables for connecting the test circuits to the Cathode Ray Oscilloscope (CRO). There are coaxial cables with a continuous dielectric layer throughout the length of the cable. The resistive losses in dielectric become prominent at higher frequencies. But some modern dielectric materials have less resistive loss and we can use such coaxial cables even upto 40 GHz. Since these materials are very expensive, such cables are used only for short distance applications.

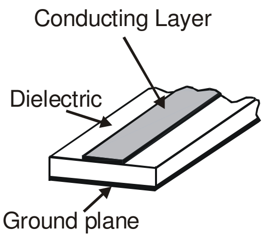

Fig. 4.3: Construction of Microstripline

Another guided medium used at microwave frequency is **microstriplines**. These are strips of conductors supported by a dielectric plate. The ground plane is coated on the other side of the dielectric plate. The construction of microstripline is shown in Fig. 4.3. Most commonly used microstripline structures are made with double sided printed circuit boards (PCB). The striplines are prepared by printing the strip pattern on the PCB. Other methods of preparing microstriplines are thin film deposition of metallic films or thick film deposition of conducting paste on dielectric substrates like alumina. This pattern can be printed using lithography techniques.

The thickness of conductor film and width of the microstripline determine its properties. Design formulae and charts are available commercially which help in designing the geometry of the microstriplines.

Spend 2 *Min.*

SAQ 1

Can you use metallic waveguide at radio frequency? Give reasons.

After understanding the basic construction of transmission line, let us now discuss its behaviour at higher frequencies.

4.3 REPRESENTATION OF TRANSMISSION LINE

As you know, the transmission lines consist of a pair of conductors. The frequencies of operation lie in the range of 1GHz to 300 GHz. The corresponding wavelengths (λ) are from 0.3 m to 10^{-3} m. At these frequencies the inductance associated with the conductors and capacitance between the two conductors cannot be neglected. Since the dielectric between the two conductors has finite conductivity, we also have to consider a shunt resistance between the two conductors. All these components are distributed along the length of the transmission line. Fig.4.4 shows equivalent representation of a transmission line in terms of lumped components *R* (resistance), *L* (inductance), *G* (conductance) and *C* (capacitance). Their values are expressed in terms of per unit length.

The reactance of inductor is $X_L = \omega L$ and the susceptance of capacitor is $B_C = \omega C$. At high frequencies, very small values of *L* and *C* also result into significant values of reactances and susceptances.

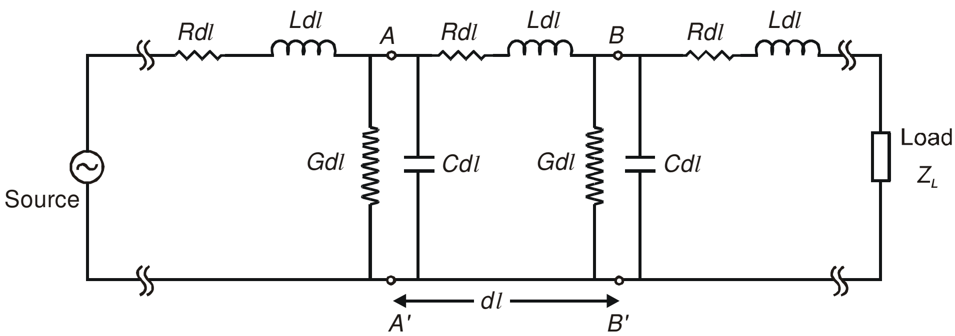

Fig.4.4: Lumped component representation of transmission line

In the short length segment of the transmission line, *dl*, the current and voltage relations are as shown in Fig. 4.5.

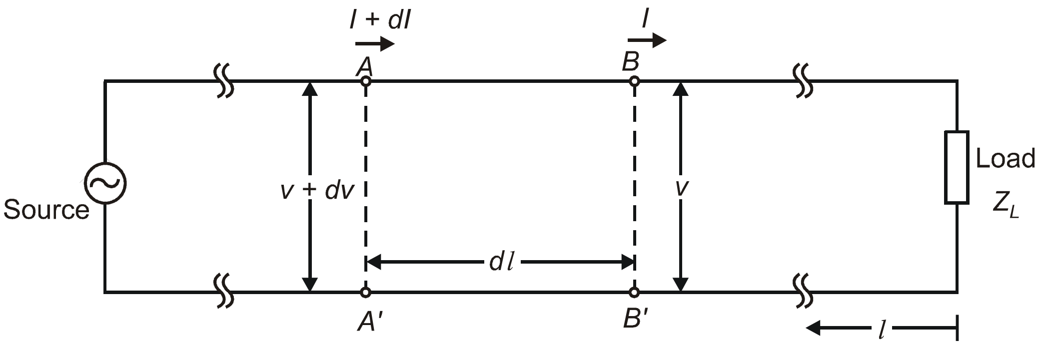

Fig.4.5: Current and voltage relations in a short segment of transmission line

After discussing the distributed nature of electrical components in transmission line at high frequency, let us now derive the basic transmission line equations in terms of current-voltage relationships.

4.4 TRANSMISSION LINE EQUATIONS

The transmission line can be analysed either by the solution of Maxwell's field equations described in the last unit or by the method of distributed circuit theory. We will be following the latter method to analyse a transmission line in terms of voltages, current, impendence and power along the transmission line.

When there is a line current I passing through the transmission line, it produces voltage drop while flowing through the resistance Rdl and reactance $j\omega Ldl$. Let the change in the voltage over a short distance dl be dV. This can be written as:

$$\begin{aligned} dV &= I \times (\text{impedance of length } dl) \\ &= I \times (R + j\omega L) \times dl \end{aligned} \tag{4.1}$$

Due to voltage V existing between the two conductors, small current dI flows through the capacitance Cdl and conductance Gdl. This current can be expressed as:

$$\begin{aligned} dI &= V \times (\text{admittance of length } dl) \\ &= V \times (G + j\omega C)\, dl \end{aligned} \tag{4.2}$$

$G + j\omega C$ is called the *shunt admittance* of the wire. The power is dissipated due to a current flowing between the two conductors of the transmission line in shunt fashion. From Eq (4.1) and (4.2) we get

$$\frac{dV}{dl} = I\,(R + j\omega L) \tag{4.3}$$

and

$$\frac{dI}{dl} = V\,(G + j\omega C) \tag{4.4}$$

In conventional notation, we represent the line series impedance per unit length by

$$Z = (R + j\omega L)$$

and the line shunt admittance per unit length by

$$Y = (G + j\omega C)$$

Then the current-voltage (*I-V*) relations in Eq. (4.3) and (4.4) can be represented as

$$\frac{dV}{dl} = IZ \tag{4.5}$$

$$\frac{dI}{dl} = VY \tag{4.6}$$

where V is the voltage across the line at distance l from the receiving (load) end and I is the current in the line at distance l from the receiving (load) end.

Differentiating Eqs. (4.5) and (4.6) with respect to dl we get

$$\frac{d^2V}{dl^2} = \frac{dI}{dl}Z = VYZ \tag{4.7}$$

Similarly

$$\frac{d^2I}{dl^2} = \frac{dV}{dl}Y = IYZ \tag{4.8}$$

Eq. (4.7) and (4.8) are not independent of each other. They are in the form of standard differential equations. Therefore, the solutions of these equations can be written as

$$V = V_1 e^{\sqrt{ZY}\,l} + V_2\, e^{-\sqrt{ZY}\,l} \tag{4.9}$$

$$I = I_1 e^{\sqrt{ZY}\,l} + I_2\, e^{-\sqrt{ZY}\,l} \tag{4.10}$$

where V_1, V_2, I_1, I_2 are constants of integration. The values of these constants can be found out by applying boundary conditions.

Differentiating Eq.(4.9) we get

$$\frac{dV}{dl} = V_1 \sqrt{ZY}\; e^{\sqrt{ZY}\,l} - V_2 \sqrt{ZY}\; e^{-\sqrt{ZY}\,l} = IZ$$

Rearranging the terms, current I can be written as

$$I = \frac{V_1\sqrt{ZY}\; e^{\sqrt{ZY}\,l}}{Z} - \frac{V_2\sqrt{ZY}\, e^{-\sqrt{ZY}\,l}}{Z}$$

$$= \frac{V_1\, e^{\sqrt{ZY}\,l}}{\sqrt{Z/Y}} - \frac{V_2\, e^{-\sqrt{ZY}\,l}}{\sqrt{Z/Y}}$$

Substituting $\sqrt{Z/Y} = Z_0$ in the above expression, we get

$$I = \frac{V_1\, e^{\sqrt{ZY}\,l}}{Z_0} - \frac{V_2\, e^{-\sqrt{ZY}\,l}}{Z_0} \tag{4.11}$$

Comparing Eq. (4.10) and (4.11) we get

$$I_1 = \frac{V_1}{Z_0} \text{ and } I_2 = -\frac{V_2}{Z_0} \tag{4.12}$$

i.e. only two out of the four (V_1, V_2, I_1 and I_2) constants are independent. Hence, solutions of Eq.(4.5) and (4.6) can be written as

$$V = V_1 \; e^{\sqrt{ZY}\,l} + V_2\, e^{-\sqrt{ZY}\,l} = V' + V'' \qquad (4.13a)$$

and

$$I = \frac{V_1}{Z_0}\, e^{\sqrt{ZY}\,l} - \frac{V_2}{Z_0} e^{-\sqrt{ZY}\,l} = I' + I'' \qquad (4.13b)$$

Eq. (4.13a) and (4.13b) are the **transmission line equations**. $Z_0 = \sqrt{\frac{Z}{Y}}$ is called the **characteristic impedance** of the line.

Hence we can write

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \qquad (4.14)$$

At the radio frequencies, the inductive reactance is much larger than the resistive component and the capacitive susceptance is much larger than the shunt conductance. In such cases the characteristic impendence is:

$$Z_0 = \sqrt{\frac{L}{C}} \qquad (4.15)$$

This characterises a *loss-less transmission line*.

SAQ 2

Spend 2 Min.

Draw the equivalent circuit of a loss-less transmission line at radio frequency.

In practice, the characteristic impendence is determined by the geometry, shape and spacing of the conductors as well as the dielectric constant of the insulator separating them.

If we consider two types of transmission lines as shown in Fig. 4.6 a&b then we can calculate the characteristic impedance by the following formulae:

a. For a pair of parallel wires, with diameter x and inter-conductor separation y, the characteristic impendence

$$Z_0 = 276 \log \frac{2x}{y}\,\Omega \qquad (4.16)$$

b. For a coaxial cable with inner conductor diameter t, outer conductor diameter T, and the dielectric constant of the insulator ε, the characteristic impedance is

$$Z_0 = \frac{138}{\sqrt{\varepsilon}} \log \frac{T}{t}\,\Omega \qquad (4.17)$$

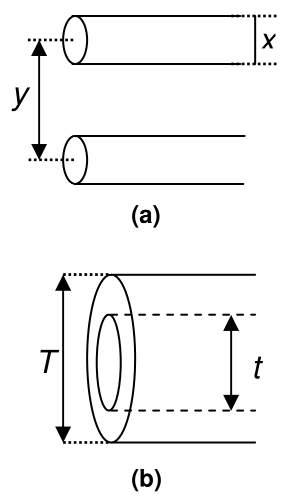

Fig. 4.6: a) Parallel wires; and b) coaxial cable

You may now like to solve one numerical.

SAQ 3

Spend 3 Min.

A coaxial cable with outer diameter 5 mm has a characteristic impedance of 75 Ω. If the dielectric constant of the dielectric is 2, what is the diameter of the inner conductor?

In Eq. (4.13), $\sqrt{ZY} = \gamma$ is called the **propagation constant of the line**. $\sqrt{ZY}$ is a complex quantity, which can be represented as

$$\sqrt{ZY} = \alpha + j\beta = \gamma \tag{4.18}$$

where α is the **attenuation coefficient** that gives measure of decrease in voltage (or current) with length and β is the **phase shift coefficient** which determines phase angle of the voltage (or current) with distance. The phase angle continuously varies along the length of the line and the length required to obtain a phase shift of 2π radians is defined as the *wavelength of the line* λ.

Hence $\beta\lambda = 2\pi$ or

$$\beta = \frac{2\pi}{\lambda} \tag{4.19}$$

For air dielectric, the wavelength λ approximates to the free space wavelength of the propagating wave. However, for the line with dielectric material of dielectric constant ε, the wavelength is

$$\lambda = \frac{\lambda_{free\,space}}{\sqrt{\varepsilon}} \tag{4.20}$$

Let us now interpret the transmission line equations for the travelling wave useful in communications.

4.5 TRANSMISSION LINE EQUATIONS FOR TRAVELLING WAVES

The steady state voltage and current equations of transmission lines are given by Eq. (4.13a) and (4.13b). These equations can be expressed as the sum of voltage and currents of two waves. The wave that is travelling towards receiving end (load end) is called the *incident wave* since it is incident on load. The wave travelling from load to the generator is called the *reflected wave*. It is generated at the load by reflection of incident wave. These two waves are identical in nature but have different directions.

Incident Wave

It is the wave starting from the source and reaching the load. It consists of voltage component $V' = V_1 e^{\sqrt{ZY}\, l}$ and current component $I' = \frac{V_1}{Z_0} e^{\sqrt{ZY}\, l}$. At every point on the transmission line, current and voltage are in phase and $\frac{V'}{I'} = Z_0$.

Now $\quad V' = V_1 e^{\sqrt{ZY}\, l}$

$\qquad\quad = V_1 e^{(\alpha + j\beta)\, l}$

Hence, $\quad |V'| = |V_1|\, e^{\alpha l} \qquad (4.21)$

In this equation, l is measured from the load end, hence as we approach the source, it increases. That is, the voltage of incident wave reduces exponentially as the wave travels towards the load. Here αl is the total attenuation of the line of length l. The units of α are Neper. However, it is a common practice to express the attenuation in dB scale. The relationship between these two units is

$$\text{Attenuation in dB} = 8.686\ \alpha\ (\text{in Neper}) \qquad (4.22)$$

The phase factor β is positive in the present case. It means that, as l increases, i.e. as we travel away from the load, the phase position of incident wave advances the phase position at the load end by βl.

Reflected Wave

When the load termination of the transmission line is open, or the output impedance is not equal to the characteristic impedance of the line, the incident wave gets reflected from the load end. Obviously, this reflected wave travels from the load end to the source end, i.e. in the opposite direction of the incident wave.

The reflected wave consists of following voltage and current components:

$$V'' = V_2\, e^{-\sqrt{ZY}\, l}$$

and $$I'' = -\frac{V_2}{Z_0} e^{-\sqrt{ZY}\, l}$$

and everywhere on the transmission line $\dfrac{V''}{I''} = -Z_0$

The negative sign occurs because the reflected wave travels from the load to the source.

In this case

$$V'' = V_2\, e^{-\sqrt{ZY}\, l} = V_2\, e^{-(\alpha + j\beta) l}$$

Hence $$|V''| = |V_2|\, e^{-\alpha l} \qquad (4.23)$$

It shows that as l increases $|V''|$ decreases, i.e. as the wave travels away from the load, its amplitude decreases. The phase of reflected wave drops by β radians per unit distance from the load. That is the reflected wave at distance l away from the load lags the phase position at the load by βl radians.

You may now like to attempt the following SAQ.

SAQ 4

Spend 3 Min.

A 100 m long transmission line is so terminated that only incident wave is present. The power at load end is 1.2 dB less than the generator end. What is the value of α in Neper per meter?

After discussing the transmission line equations let us now discuss some important parameters characterising a transmission line.

4.6 TRANSMISSION LINE PARAMETERS

You have learnt that a travelling wave along the transmission line has two components, one travelling in positive l direction and the other in negative l direction. The incident component is caused by the source while the reflected component arises due to reflection of the wave from the load end. You know that if the load impedance connected to the line is equal to the characteristic impedance (Z_0) of the transmission line, no reflection occurs. In other cases also, the reflected wave has different amplitude and phase depending on the load value. The incident and reflected waves are related to each other through a transmission line parameter called *reflection coefficient*. When incident and reflected waves add vectorially, standing waves are formed. In such cases, the power transmission is not efficient.

4.6.1 Reflection Coefficient

Consider a transmission line terminated in load impedance Z_L as shown in Fig. 4.7.

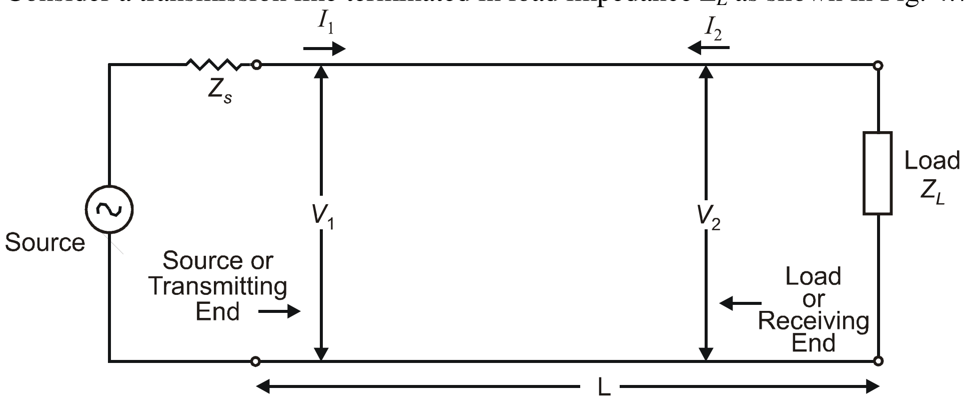

Fig. 4.7: Transmission line terminated in a load impedance

The reflected wave is generated at the load as a result of reflection of the incident wave by the load impedance. Reflection must satisfy the following conditions.

i) For incident wave $\frac{V'}{I'} = Z_0$

ii) For reflected wave $\frac{V''}{I''} = -Z_0$

iii) Load voltage $V_L = V_1 + V_2$ (Sum of incident and reflected voltage at the load, $l = 0$)

iv) Load current $I_L = I_1 + I_2$ (Sum of incident and reflected wave current at $l = 0$)

v) Load impedance $Z_L = \frac{V_L}{I_L}$

If the line has a length L, then from Eq. (4.13a) and (4.13b) the voltage and current at the receiving (load) end can be expressed in terms of propagation constant as:

$$V_L = V_1 e^{\gamma L} + V_2 e^{-\gamma L} \tag{4.24}$$

and

$$I_L = \frac{1}{Z_0}\left(V_1 e^{\gamma L} - V_2 e^{-\gamma L}\right) \tag{4.25}$$

The ratio of voltage to current at the receiving end is the load impedance

$$Z_L = \frac{V_L}{I_L} = Z_0 \frac{V_1 e^{\gamma L} + V_2 e^{-\gamma L}}{V_1 e^{\gamma L} - V_2 e^{-\gamma L}}. \tag{4.26}$$

The **reflection coefficient** is defined as

$$\Gamma = \frac{\text{reflected voltage}}{\text{incident voltage}} = \frac{\text{reflected current}}{\text{incident current}}$$

$$= \frac{V_{refl}}{V_{inc}} = \frac{I_{refl}}{I_{inc}} = \frac{V_2 e^{-\gamma l}}{V_1 e^{\gamma l}} \tag{4.27}$$

The load impedance Z_L can then be expressed in terms of reflection coefficient as:

$$Z_L = Z_0 \frac{1 + \dfrac{V_2\, e^{-\gamma L}}{V_1\, e^{\gamma L}}}{1 - \dfrac{V_2\, e^{-\gamma L}}{V_1\, e^{\gamma L}}} = Z_0 \left[\frac{1+\Gamma}{1-\Gamma} \right] \qquad (4.28)$$

and it can be easily shown that

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} \qquad (4.29)$$

SAQ 5

Spend 3 *Min.*

Using the expression for the reflection coefficient, find out the values of load impedance when you get (i) no reflection; and (ii) when you get complete reflection of the incident wave.

If the load impedance and/or characteristic impedance are complex quantities, then the reflection coefficient is also complex quantity and we can write

$$\Gamma = |\Gamma|\, e^{j\theta} \qquad (4.30)$$

where $|\Gamma|$ is the magnitude of reflection coefficient and is always less than or equal to one. θ is the phase angle between the incident and reflected voltage at the receiving end. It is usually called the *phase angle of reflection coefficient*.

The generalized reflection coefficient is given by Eq. (4.27). If L is the total length of line and d is the distance from the load at which the reflection coefficient is to be determined, then

$$\Gamma_d = \frac{V_2\, e^{-\gamma(L-d)}}{V_1\, e^{\gamma\,(L-d)}} = \frac{V_2\, e^{-\gamma L}\, e^{\gamma d}}{V_1\, e^{\gamma L}\, e^{-\gamma d}} = \Gamma\, e^{2\gamma d}$$

Substituting for γ from Eq. (4.18) we get

$$\Gamma_d = \Gamma\, e^{2(\alpha + j\beta)d} = \Gamma\, e^{2\alpha d}\, e^{2 j\beta d}$$

Now substituting for Γ from Eq. (4.30) we get

$$\Gamma_d = |\Gamma|\, e^{2\alpha d}\, e^{j(\theta + 2\beta d)} \qquad (4.31)$$

This is a very useful equation for determining the reflection coefficient at any point along the line.

Let us now discuss another important transmission line characteristic, viz. *standing wave ratio*.

4.6.2 Standing Wave and Standing Wave Ratio

Just as any ac voltage or current, the two sets of waves, viz. incident and reflected waves add vectorially. As a result of combining these waves, patterns of voltage and current variations are formed on the line. These patterns are stationary with respect to time, and hence they are called **standing waves**.

You have learnt about the standing wave patterns while studying sound/ longitudinal waves in the course on Oscillations and Waves (PHE-02/ BPHE-102).

The general solutions of the transmission line equation consist of two waves travelling in opposite directions with unequal amplitudes given by Eq. (4.13a) and (4.13b). Eq.(4.13a) can be written as

$$\begin{aligned} V_s &= V_1 e^{\sqrt{ZY}\,l} + V_2 e^{-\sqrt{ZY}\,l} \\ &= V_1 e^{(\alpha + j\beta)l} + V_2 e^{-(\alpha + j\beta)l} \\ &= V_1 e^{\alpha l} e^{j\beta l} + V_2 e^{-\alpha l} e^{-j\beta l} \\ &= V_1 e^{\alpha l}(\cos\beta l + j\sin\beta l) + V_2 e^{-\alpha l}(\cos\beta l - j\sin\beta l) \\ &= \cos\beta l\,(V_1 e^{\alpha l} + V_2 e^{-\alpha l}) - j\sin\beta l\,(-V_1 e^{\alpha l} + V_2 e^{-\alpha l}) \end{aligned} \qquad (4.32)$$

Without loss of generality it can be assumed that $V_1\, e^{\alpha l}$ and $V_2\, e^{-\alpha l}$ are real. Then the voltage wave equation can be expressed as

$$V_s = V_0\, e^{-j\phi} \qquad (4.33)$$

This equation is called *voltage standing wave equation*, where V_0 represents the amplitude of the standing wave given by:

$$V_0 = \left[\left(V_1 e^{\alpha l} + V_2 e^{-\alpha l}\right)^2 \cos^2\beta l + \left(V_2 e^{-\alpha l} - V_1 e^{\alpha l}\right)^2 \sin^2\beta l\right]^{\frac{1}{2}} \qquad (4.34)$$

and

$$\phi = \arctan\left[\left(\frac{V_2 e^{-\alpha l} - V_1 e^{\alpha l}}{V_1 e^{\alpha l} + V_2 e^{-\alpha l}}\right)\tan\beta l\right] \qquad (4.35)$$

ϕ being the phase of the standing wave.

The maximum and minimum values of Eq. (4.34) can be found by substituting the proper values of βl in the equation. This procedure leads us to the following results:

The maximum voltage amplitude of the standing wave pattern occurs when $\cos^2\beta l = 1$; or $\beta l = n\pi$ where $n = 0, 1, 2,\ldots$ and it can be expressed as

$$V_{max} = V_1 e^{\alpha l} + V_2 e^{-\alpha l} = V_1 e^{\alpha l}\left(1 + \frac{V_2 e^{-\alpha l}}{V_1 e^{\alpha l}}\right) = V_1 e^{\alpha l}\left(1 + |\Gamma|\right) \qquad (4.36)$$

The minimum amplitude occurs when $\sin^2\beta l = 1$ or $\beta l = \dfrac{(2n-1)\pi}{2}$

where $n = 0, 1, 2,\ldots$

The minimum voltage amplitude is:

$$V_{min} = V_1 e^{\alpha l} - V_2 e^{-\alpha l} = V_1 e^{\alpha l}\left(1 - \frac{V_2 e^{-\alpha l}}{V_1 e^{\alpha l}}\right) = V_1 e^{\alpha l}\left(1 - |\Gamma|\right) \qquad (4.37)$$

Spend 2 Min.

SAQ 6

Show that the distance between the two consecutive maxima or minima is equal to half of the wavelength.

The transmission lines can be classified as (i) loss-less when the attenuation coefficient $\alpha = 0$ and (ii) lossy, when $\alpha \neq 0$. For simplicity, in the following discussion we will consider only loss-less lines.

The standing wave pattern is a graph of voltage or current amplitude as a function of distance. The voltage and current standing wave patterns for a loss-less transmission line are shown in Fig.4.8.

The *loss-less transmission line* is the one in which the conductors are ideal with infinite conductivity and the dielectric medium separating the conductors is also ideal with zero conductivity.

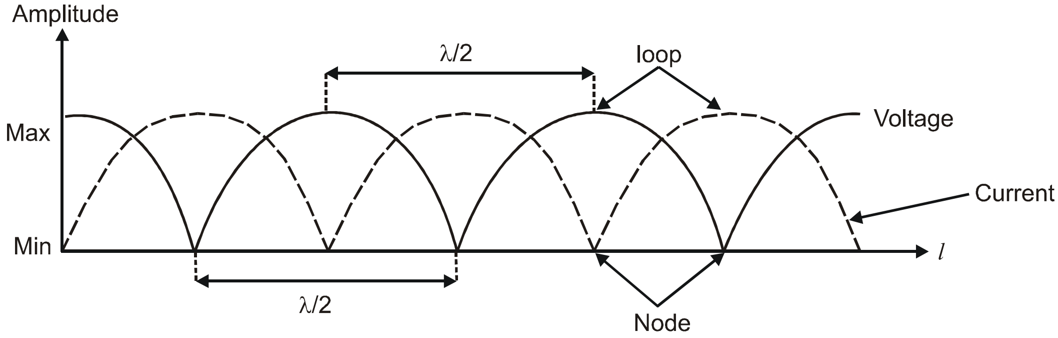

Fig.4.8: Standing wave pattern in a loss-less line

The distance between two consecutive maxima or minima is λ/2. For a loss-less transmission line the maximum and minimum amplitude of the standing wave remains constant throughout the length of the transmission line. You must have also noticed that, wherever there is voltage maxima, there is current minima and vice-versa, i.e. the voltage and current standing waves are 90° out of phase along the line. The points of minimum voltage or current are known as **nodes** while maxima are called the **loops**. Voltage nodes and current nodes are spaced quarter wavelength (λ/4) apart.

Standing Wave Ratio

We have seen that the standing waves are generated due to reflected waves and when the load impedance matches with characteristic impedance of the line, there are no reflected waves. In order to get measure of impedance matching, a ratio of the maximum amplitude of the standing wave pattern to minimum amplitude is taken. It is called the **standing wave ratio** (SWR) and is denoted by *S*.

$$S = \frac{\text{maximum amplitude of voltage}}{\text{minimum amplitude of voltage}} = \frac{\text{maximum amplitude of current}}{\text{minimum amplitude of current}}$$

$$S = \frac{|V_{max}|}{|V_{min}|} = \frac{|I_{max}|}{|I_{min}|} \qquad (4.38)$$

Substituting from Eq. (4.36) and (4.37) we get

$$S = \frac{V_1 e^{\alpha l}\left(1+|\Gamma|\right)}{V_1 e^{\alpha l}\left(1-|\Gamma|\right)} = \frac{1+|\Gamma|}{1-|\Gamma|} \qquad (4.39)$$

If we substitute value of Γ from Eq. (4.29), we get

$$S = \frac{Z_L}{Z_0}. \qquad (4.40)$$

When the load is resistive (R_L) then the SWR takes the following form:

$$S = \frac{R_L}{Z_0} \qquad (4.40a)$$

Since $|\Gamma|$ is always less than or equal to 1 the standing wave ratio is a positive real number which is never less than unity i.e. $S \geqslant 1$. The standing wave ratio of a pure travelling wave, i.e. when no reflected wave exists, is *unity* and that for pure standing wave, i.e. incident and reflected wave having equal amplitudes, is *infinite*. When the standing wave ratio is unity, the transmission line is called *flat line*. The standing wave ratios of voltage and current are identical and are constant throughout the line. The standing wave ratio is normally defined in terms of voltages and is called *voltage standing wave ratio* (VSWR).

Spend
3 *Min.*

SAQ 7

A transmitter is connected via a cable with 50 Ω characteristic impedance to an antenna with input resistance of 300 Ω. Calculate the reflection coefficient and VSWR.

4.6.3 Characteristic Impedance

In the transmission line analysis we make use of a particular line property determined by its circuit components per unit length. This property is known as **characteristic impedance** of the line and is denoted by Z_0. If a voltage source is connected across the input of an infinitely long transmission line, then the impedance offered by the line to this source is its characteristic impendence.

You already know that the infinite line can be represented by a lumped component equivalent circuit as shown in Fig.4.4. We can consider that the infinitely long transmission line is built up from infinite number of sections of finite lengths. If we employ conventional lumped component analysis for each section of the line to determine the equivalent impedance of the line, the value converges to a finite value Z_0 as the line length becomes very large. This means that any addition of further sections would produce only infinitesimal small changes in the impedance.

Determination of Characteristic Impedance (Z_0)

We can derive the general expression for Z_0 in terms of its circuit components from lumped equivalent circuit of a line section shown in Fig. 4.9.

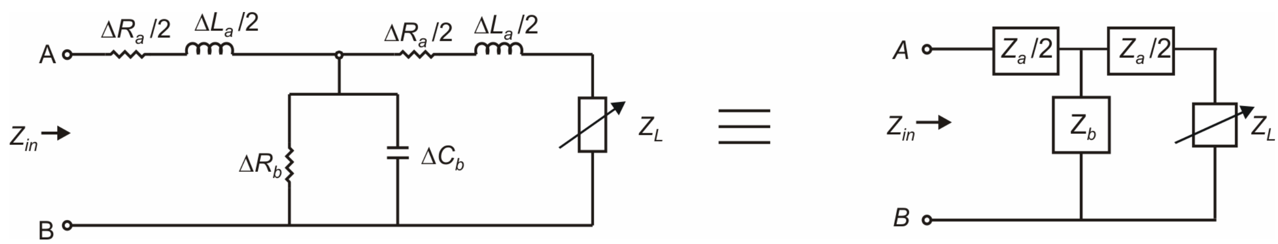

Fig. 4.9: Lumped equivalent circuit of finite line section

For convenience we use different nomenclature in this figure from Fig. 4.4. The circuit is terminated in an adjustable impedance Z_L. Combination of $\Delta R_a/2$ and $\Delta L_a/2$ is denoted by $Z_a/2$ and combination of ΔR_b and ΔC_b is denoted by Z_b.
The general expression for Z_{in} when looked into terminal AB is:

$$Z_{in} = \frac{Z_a}{2} + \frac{Z_b\left(\frac{Z_a}{2} + Z_L\right)}{Z_b + \frac{Z_a}{2} + Z_L} \tag{4.41}$$

Z_L can assume various values. If $Z_{in} = Z_L = Z_0$, then Eq. (4.41) becomes

$$Z_0 = \frac{Z_a}{2} + \frac{Z_b\left(\frac{Z_a}{2} + Z_0\right)}{Z_b + \frac{Z_a}{2} + Z_0} \tag{4.42}$$

Solving this equation we get

$$Z_0 = \frac{Z_a\left(Z_b + \frac{Z_a}{2} + Z_0\right) + 2Z_b\left(\frac{Z_a}{2} + Z_0\right)}{2\left(Z_b + \frac{Z_a}{2} + Z_0\right)}$$

$$\therefore 2Z_0\left(Z_b + \frac{Z_a}{2} + Z_0\right) = \left(Z_a Z_b + \frac{Z_a^2}{2} + Z_a Z_0\right) + \left(Z_a Z_b + 2Z_0 Z_b\right)$$

When simplified, this expression reduces to

$$2Z_0^2 = 2Z_a Z_b + \frac{Z_a^2}{2}$$

i.e.
$$Z_0 = \sqrt{Z_a Z_b + \frac{Z_a^2}{4}} \tag{4.43}$$

But in this expression, values of Z_a and Z_b are:

$$Z_a = R + j\omega L = \frac{\Delta R_a}{2} + j\omega\frac{\Delta L_a}{2} \tag{4.44a}$$

and
$$\frac{1}{Z_b} = G + j\omega C = \frac{1}{\Delta R_b} + j\omega\Delta C_b \tag{4.44b}$$

At radio frequencies series inductive reactance is normally much larger than the series resistance and shunt resistance ($1/G$) is very large in comparison with shunt capacitive reactance, i.e. $\omega L >> R$ and $\omega C >> G$. Therefore R and G can be neglected from Eq. (4.44a & b) and we get

$$Z_a = \frac{j\omega\Delta L_a}{2} \text{ and } Z_b = \frac{1}{j\omega\Delta C_b}$$

Substituting these values in Eq. (4.43) we get

$$Z_0 = \sqrt{\frac{\omega\Delta L_a/2}{\omega\Delta C_b} - \frac{\omega^2(\Delta L_a/2)^2}{4}} \tag{4.45}$$

For a short line section $\frac{\omega^2\Delta L_a^2}{16} << \frac{\Delta L_a/2}{\Delta C_b}$

Hence neglecting $\frac{\omega^2\Delta L_a}{16}$ we get

$$Z_0 = \sqrt{\frac{\Delta L_a/2}{\Delta C_b}} = \sqrt{\frac{L}{C}} \tag{4.46}$$

This is the same result we have obtained in Eq. (4.15). L/C has the dimensions of resistance and therefore from Eq. (4.46) you can see that Z_0 is pure resistance and its magnitude is independent of operating frequency. The magnitudes of L and C for a given transmission line and therefore the characteristic impedance depends upon the dimension and the spacing of the conductors.

Now you may like to attempt the following SAQ.

Spend 3 Min.

SAQ 8

A commonly used coaxial cable has a capacitance of 59 pF m^{-1} and inductance of 147.5 nH m^{-1}. Calculate the characteristic impedances for a 1 m section of wire and for a 100 m length of wire.

From this SAQ you must have noticed that the characteristic impedance in both the cases is same. Hence the characteristic impedance is independent of the length of the line and is in fact a characteristic of the line.

4.7 TRANSMISSION LINE BEHAVIOUR UNDER DIFFERENT LOAD CONDITIONS

We have already discussed that when the load impedance connected to the transmission line is not equal to its characteristic impedance, reflection of wave takes place from the load end and standing wave patterns of voltage and current are formed along the length of the transmission line.

The general transmission line equations are given by Eq. (4.24) and (4.25).

$$V = V_{inc}\, e^{\gamma l} + V_{refl}\, e^{-\gamma l}$$

and

$$I = \frac{V_{inc}}{Z_0} e^{\gamma l} - \frac{V_{refl}}{Z_0} e^{-\gamma l}$$

The reflection coefficient Γ is defined by Eq. (4.27) as

$$\Gamma = \frac{V_{refl}}{V_{inc}}$$

Hence we can write

$$V = V_{inc}\left[e^{\gamma l} + \Gamma e^{-\gamma l}\right] \qquad (4.47)$$

and

$$I = \frac{V_{inc}}{Z_0}\left[e^{\gamma l} - \Gamma e^{-\gamma l}\right] \qquad (4.48)$$

The input impedance looking into the transmission line towards the load end will be

$$Z_{in} = \frac{V}{I} = \frac{V_{inc}\left[e^{\gamma l} + \Gamma e^{-\gamma l}\right]}{\frac{V_{inc}}{Z_0}\left[e^{\gamma l} - \Gamma e^{-\gamma l}\right]} \qquad (4.49)$$

$$= Z_0 \frac{e^{\gamma l} + \Gamma e^{-\gamma l}}{e^{\gamma l} - \Gamma e^{-\gamma l}} \qquad (4.50)$$

Substituting the value of Γ from Eq.(4.29) we get

$$Z_{in} = Z_0 \frac{(Z_L + Z_0)e^{\gamma l} + (Z_L - Z_0)e^{-\gamma l}}{(Z_L + Z_0)e^{\gamma l} - (Z_L - Z_0)e^{-\gamma l}} \quad (4.51)$$

$$= Z_0 \frac{(Z_L + Z_0)(\cos\gamma l + j\sin\gamma l) + (Z_L - Z_0)(\cos\gamma l - j\sin\gamma l)}{(Z_L + Z_0)(\cos\gamma l + j\sin\gamma l) - (Z_L - Z_0)(\cos\gamma l - j\sin\gamma l)}$$

$$= Z_0 \frac{2Z_L \cos\gamma l + j2Z_0 \sin\gamma l}{2Z_0 \cos\gamma l + j2Z_L \sin\gamma l}$$

$$= Z_0 \frac{Z_L \cos\gamma l + jZ_0 \sin\gamma l}{Z_0 \cos\gamma l + jZ_L \sin\gamma l} \quad (4.52)$$

$$= Z_0 \frac{Z_L + jZ_0 \tan\gamma l}{Z_0 + jZ_L \tan\gamma l} \quad (4.53)$$

Let us consider some special load conditions. We will consider the transmission line to be loss-less because in this case we obtain steady standing wave pattern over the entire length of the transmission line. In all these cases, the propagation constant γ can be replaced by phase shift coefficient β, since the attenuation coefficient α is zero for the loss-less line.

4.7.1 Open Ended Transmission Line ($Z_L = \infty$)

In case of open ended transmission line shown in Fig. 4.10a there is no load attached i.e. $Z_L = \infty$. From Eq. (4.29) we obtain the value of reflection coefficient Γ to be equal to 1. From this result Eq. (4.39) gives SWR value of infinity.

Substituting the value of $Z_L = \infty$, Eq. (4.53) gives the input impedance value

$$Z_{in_{(open)}} = Z_0 \frac{1}{j\tan\beta l} = -jZ_0 \cot\beta l \quad (4.54)$$

In a single wavelength λ, the phase of the wave is shifted by 2π radians. The standing wave pattern has distance of $\lambda/2$ between consecutive maxima or minima. This corresponds to a phase shift of π radians. The quarter wavelength ($\lambda/4$) corresponds to $\pi/2$ phase shift. Fig. 4.10b shows the variation of input reactance with length l on the open ended transmission line (Eq. 4.54). l is the distance measured from the load end (i.e. at load end, $l = 0$).

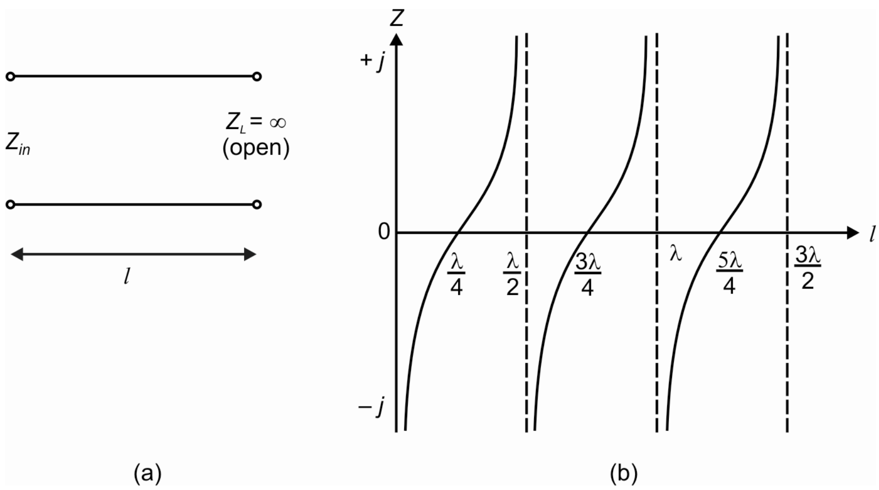

Fig. 4.10: a) Open ended transmission line; and b) reactance variation of open ended transmission line

You must remember here that practically, under Z_{open} case the impedance is not infinite. The free space acts as a load here with impedance of 377 Ω. Hence in reality, $Z_L = \infty$ is actually $Z_L \gg Z_0$.

Here, the reactance of line between $0 \leq l \leq \lambda/4$ is capacitive in nature, while that for $\lambda/4 \leq l \leq \lambda/2$ is inductive in nature. This can be clearly seen from the reactance plot in Fig. 4.10b.

The standing wave pattern for open ended transmission line is shown in Fig. 4.11. Here, at the load end ($l = 0$), since there is a break in the circuit, the voltage and current waves get reflected back completely and the standing wave patterns are generated. Since there is no path for current to flow at the load end, the minimum of current standing wave pattern occurs at $l = 0$; while the entire voltage appears across the open terminals at the load end, the maximum of voltage standing wave pattern occurs.

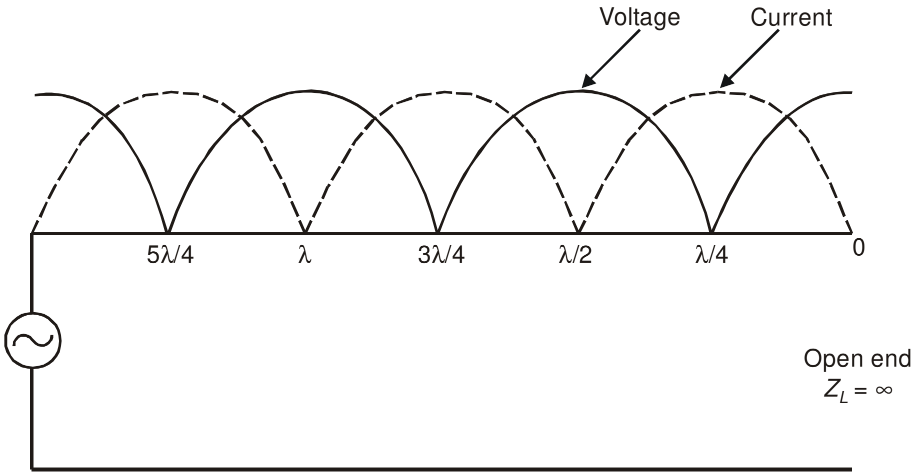

Fig. 4.11: Standing wave patterns of current and voltage on the open ended transmission line

From Eq. (4.36), the amplitude of voltage maximum is given by

$$V_{max} = V_{inc} e^{j\beta l} \left(1 + \left|\Gamma\right|\right) \qquad (4.55)$$

In case of open ended transmission line, $\left|\Gamma\right| = 1$. Substituting this in Eq. (4.55) we get

$$V_{max} = 2V_{inc} e^{j\beta l}$$

i.e. at $\beta l = n\pi$ or $l = 0, \lambda/2, \lambda, \ldots$ the voltage maxima occurs with amplitude = 2 V_{inc}.

Similarly from Eq. (4.37) we can conclude that the amplitude of minimum will be zero and this will occur at $\beta l = \frac{(2n+1)\pi}{2}$ or $l = \lambda/4, 3\lambda/4 \ldots$.

In open ended case the voltage wave is reflected *in-phase* with the incident wave while the reflected current wave is 180° out of phase of incident wave. At $l = 0$, the voltage standing wave has a loop (maxima) while the current has node (minima). In the interval $l = \lambda/4$ to $l = 0$ the current wave decreases from maximum to minimum, at the same time voltage wave increases from minimum to maximum. Hence the current value is leading the voltage by 90° in this interval, which is characteristic of a capacitive reactance. Similarly, you can easily work out that in the interval $l = \lambda/2$ to $l = \lambda/4$, the input impedance is inductive. You must have noticed that for open ended transmission line, at $l = \lambda/4$, there is a voltage node and current loop. i.e. $V = 0$ and $I = I_{max}$, Hence the input impedance

$$Z_{in} = \frac{V}{I} = 0.$$

i.e. the input impedance of an open ended transmission line of length $\lambda/4$ is 0 Ω.

SAQ 9

Spend 2 Min.

What is the input impedance for an open ended transmission line of λ/2 length?

4.7.2 Short Circuited Transmission Line (Z_L = 0)

For the short circuited transmission line $Z_L = 0$ as shown in Fig. 4.12a. Hence $\Gamma = -1$ and SWR = ∞.

Substituting $Z_L = 0$ in Eq. (4.53) results into

$$Z_{in_{(short)}} = jZ_0 \tan \beta l \qquad (4.56)$$

Fig. 4.12b depicts the variation of input reactance with length for the short circuited line.

The condition of $Z_L = 0$ is also very difficult to achieve practically. $Z_L << Z_0$ is considered to be $Z_L = 0$

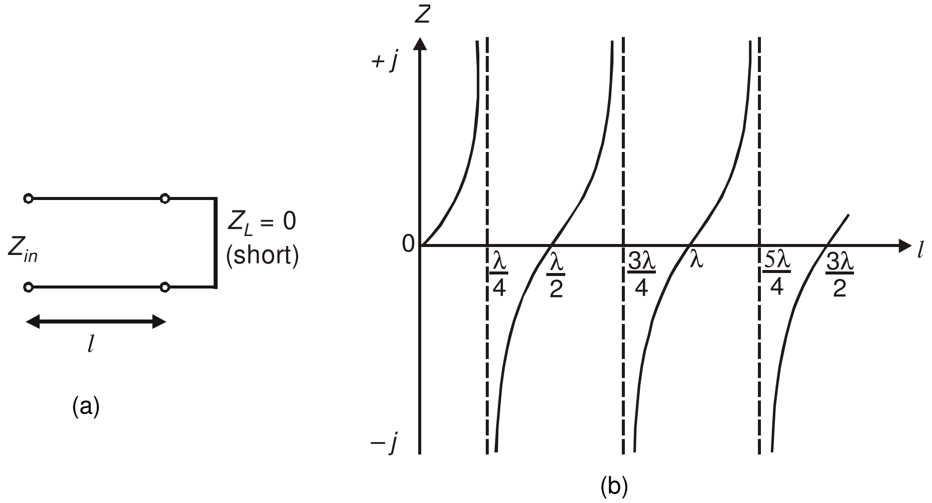

Fig. 4.12: a) A short circuited transmission line; and b) reactance variation of short circuited transmission line with length

The input reactance between $l = 0$ and λ/4 is inductive while between λ/4 and λ/2 is capacitive. The standing wave patterns of current and voltage of a short circuited transmission line is shown in Fig. 4.13.

On a short-circuited transmission line, at the load end, ($l = 0$), the current standing wave has maximum (loop) while voltage standing wave has minimum (node).

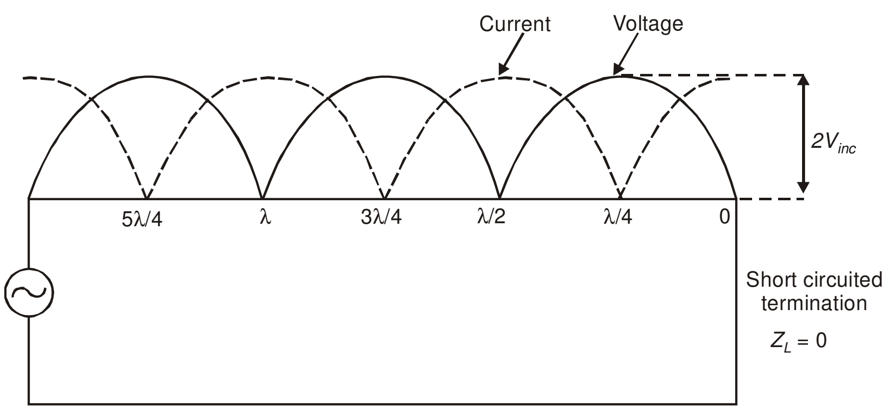

Fig. 4.13: Standing wave patterns in a short circuited transmission line

Similar to the open ended case, here also the amplitude of voltage maxima is $2V_{inc}$ and that of minima is zero. However the voltage maxima occur at $\beta l = \frac{(2n+1)\pi}{2}$ or $l = \lambda/4, 3\lambda/4,..$ while the minima occurs at $\beta l = n\pi$ or $l = 0, \lambda/2, \lambda....$

In the interval $l = \lambda/4$ to 0, the current lags the voltage by 90°, which is the characteristic of a capacitive reactance.

In this case, the voltage of reflected wave is 180° out of phase with respect to incident wave voltage, while the current of reflected wave is *in-phase* with the incident wave current.

From Fig. 4.13 you can see that at $l = \lambda/4$, voltage has maxima and current has zero value. Hence the input impedance offered by a shorted transmission line of length $\lambda/4$ is infinity. You can easily work out that the impedance offered by $\lambda/2$ long short circuited line is zero.

Though open ended and short circuited transmission lines, both find application, it is more practical to use the shorted line because they are easy to handle due to the mechanical support offered by the short circuit. Also, the lengths are accurately measurable in shorted line.

4.7.3 Effect of Load Impedance Z_L on Input Impedance Z_{in}

In a general case, the transmission line need not be open ended or short circuited but may be terminated with a finite load impedance Z_L not equal to Z_0. Let us now discuss the effect of such load impedance on the input impedance of the transmission line.

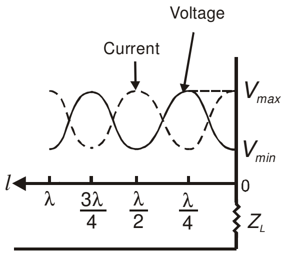

Fig. 4.14: Standing wave pattern for $Z_L = R_L < Z_0$

i) **Z_L purely resistive with $Z_L < Z_0$**: When load impedance is purely resistive, we can denote it as R_L. The conditions for $Z_L = R_L < Z_0$ are similar to short circuited transmission line. We obtain standing wave pattern as shown in Fig. 4.14 where the voltage maxima occur at odd multiple of $\lambda/4$ while voltage minima occur at multiples of $\lambda/2$. Similarly the current maxima and minima also follow the same trend as for short circuited transmission line. The main difference between finite R_L and short circuited transmission line standing wave patterns is the depth of minima and height of maxima of voltage and current. In case of finite load, the amplitude of maxima are not as high as in short circuited transmission line case. Similarly the minimum amplitude is also not as low as zero. This is because of the finite value of R_L. This load dissipates some energy and the incident wave is not reflected back completely i.e. $|V_{inc}| > |V_{refl}|$. In this case $|\Gamma| < 1$, and hence $|V_{\max}| < 2\,|V_{inc}|$.

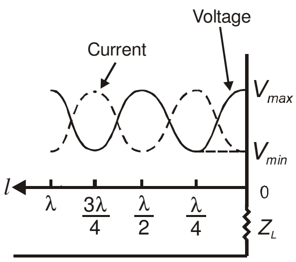

Fig. 4.15: Standing wave pattern for $Z_L = R_L > Z_0$

ii) **Z_L purely resistive with $Z_L > Z_0$**: When load impedance is purely resistive and larger than characteristic impedance, standing wave pattern is similar to the open ended transmission line case. Fig. 4.15 shows this standing wave pattern. Again similar to the earlier case, the amplitudes of V_{max} and V_{min} are different than $Z_L = \infty$ case; however the positions of maxima and minima on the transmission line remain exactly the same as that for open ended case.

iii) **Z_L reactive**: You know that with a pure resistance as a load the voltage minimum or maximum always occurred right at the load end ($l = 0$). But when the terminating impedance has reactance as well as resistance, the maximum or minimum is displaced from the position $l = 0$. The direction and amount of this displacement can be used to determine the sign and magnitude of the reactance of the load.

Eq. (4.52) gives the expression for input impedance Z_{in} in terms of load impedance Z_L, characteristic impedance Z_0 and length of the transmission line l as measured from the load end. Rationalising and separating this expression into real and imaginary parts we get

$$Z_{in}(resistive) = \frac{Z_0^2 Z_L}{Z_0^2 \cos^2 \beta l + Z_L^2 \sin^2 \beta l} \tag{4.57a}$$

$$Z_{in}(reactive) = \frac{Z_0(Z_0^2 - Z_L^2)\sin \beta l \cos \beta l}{Z_0^2 \cos^2 \beta l + Z_L^2 \sin^2 \beta l} \tag{4.57b}$$

From these two equations we can find the resistance and reactance components of the input impedance in terms of Z_L.

The sign of the reactance, i.e. whether inductive (positive) or capacitive (negative), can be obtained with the help of Fig. 4.16. This diagram shows the nature of voltage standing wave beyond the termination point.

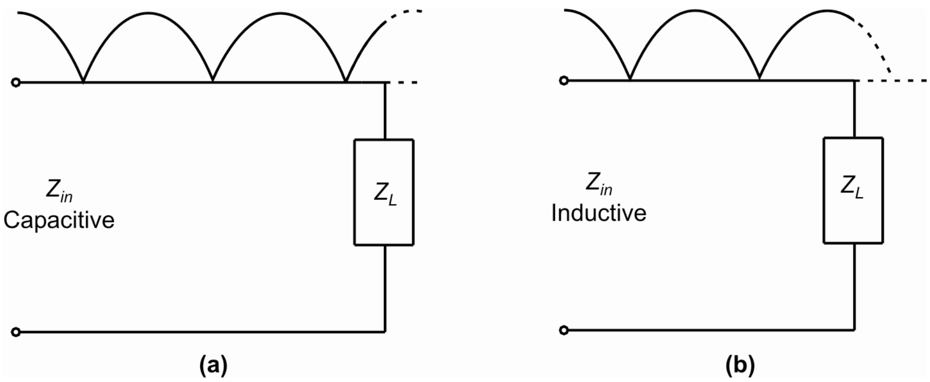

Fig. 4.16: Voltage distributions beyond the load point

Let $Z_L < Z_0$. From Eq. (4.57b) we can conclude that for $l < \lambda/4$, the reactance will be positive (inductive load), while it will be negative for $\lambda/4 < l < \lambda/2$ resulting in a capacitive load. That means, if the standing wave of voltage slopes down towards the load impedance, the input impedance will be inductive and if the slope is up towards the load impedance then it will be capacitive. Of course if the slope is zero at the termination, then the impedance will be a pure resistance.

Thus, for reactive load impedance, minima of the standing wave are displaced with respect to the position of the minima of open circuited line.
If we take open circuit configuration as a reference, then a capacitive load causes the first minimum in the voltage distribution to occur closer to the load end than $\lambda/4$. This is because, for capacitive loads the phase angle of the reflection coefficient is negative, i.e. reflected wave at the load lags behind the incident wave. Thus, with a capacitive load, the distance from the load at which the reflected wave lags 180° behind the incident wave is $< \lambda/4$.

An inductive load causes the first voltage minimum to occur at a distance larger than $\lambda/4$ from the load, because phase angle of Γ is positive.

Both inductive and capacitive loads merely displace the position of minima. The distance between the adjacent minima or maxima still remains $\lambda/2$.

The transmission line characteristics are quite complicated and designing of the transmission line circuits by analysing these characteristics is not trivial. In 1939 Philip H. Smith developed a graphical method for this analysis, which avoided the tedious solution method to calculate the impedance of transmission line along its length. This graphical method is known as **Smith Charts**. The theory behind these charts is too involved and we will not discuss it here.

4.8 IMPEDANCE MATCHING IN TRANSMISSION LINES

Energy is transmitted most effectively by a transmission line when there is no reflected wave giving rise to standing waves. The standing waves increase the line losses. Thus to obtain transmission of energy with maximum efficiency, it is necessary to terminate the line in such a way that the effective load seen at the end of the line equals the characteristic impedance of the line, i.e. to match the actual load impedance with characteristic impedance Z_0 of the line. We shall now discuss some commonly used techniques of impedance matching of the transmission line.

4.8.1 Quarter Wave Line Matching

Transmission lines, along which energy travels away from the signal source end only, i.e. without any reflection, are known as **non-resonant lines**. The lines along which the waves travel in both the directions are called **resonant lines**. Short sections of resonant lines are used as impedance matching devices at microwave frequencies. These resonant line sections may have different dimensions and characteristic impedance than the original line.

For a transmission line of λ/4 length tan $\beta l = \infty$. Then the input impedance Z_L can be obtained from Eq. (4.53).

$\beta = \frac{2\pi}{\lambda}; l = \frac{\lambda}{4}$

$\therefore \tan\beta l = \tan\frac{\pi}{2} = \infty$

$$Z_{in\,(\lambda/4)} = Z_0\left[\frac{Z_L/\infty + jZ_0}{Z_0/\infty + jZ_L}\right] = \frac{Z_0^2}{Z_L} \qquad (4.58)$$

i.e. $$Z_0^2 = Z_{in(\lambda/4)} Z_L$$

Thus, quarter wavelength long transmission line can transform the impedance, hence it is used for impedance matching.

From Eq. (4.46) you will remember that Z_0 is purely resistive. So, you can see that when Z_L is purely resistive then Z_{in} is also purely resistive and if Z_L is purely reactive then Z_{in} is purely reactive with opposite sign.

Let us now discuss the matching mechanism using quarter wave length line with an example.

Example: 1

A transmission line segment of λ/4 length with $Z_0 = 250\ \Omega$ is terminated with pure resistive load of $R_L = 500\ \Omega$. This segment is connected to a transmission line with characteristic impedance of 125 Ω as shown in Fig. 4.17. Are the two impedances matched?

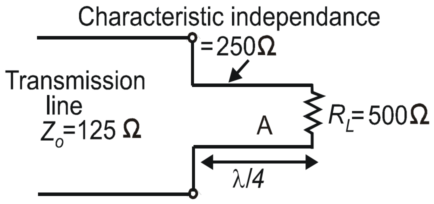

Fig.4.17: Quarter wavelength termination

Solution:

When we use a λ/4 section of $Z_0 = 250\ \Omega$ then the total terminating impedance for the line *A* will be

$$Z_{in(\lambda/4)} = \frac{Z_0^2}{R_L} = \frac{250 \times 250}{500} = 125\Omega$$

It means that the transmission line is effectively terminated in its characteristic impedance and hence is non-resonant.

This technique is commonly used in high frequency transmission lines.

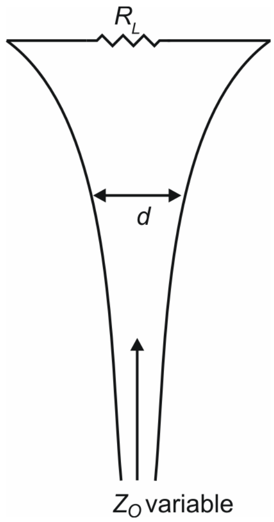

Fig. 4.18: Tapered line

4.8.2 Impedance Matching Using Tapered Line

You have already learnt that the characteristic impedance is a function of the spacing and size of the conductors. If the spacing of a loss-less line is made to vary in a uniform manner along the length of line as shown in Fig. 4.18, then the characteristic impedance Z_0 will vary likewise along the line. Such a transmission line is called as *tapered line*. This variation indicates that it is possible to use tapered lines as matching sections between the line and a load.

4.8.3 Stub Matching

Another method to obtain an impedance matching between the line and its loads is by the use of a shorted piece of transmission line connected in parallel at proper location on the transmission line as shown in Fig. 4.19. This piece is called a **stub line**.

Consider a high frequency transmission line terminated in load resistance $Z_L = R \neq Z_0$. At a point quarter wavelength away from R, the input impedance will be defined by Eq. (4.58). It will be a pure resistance of value $R_{in} = \frac{Z_0^2}{R}$.
If $R < Z_0$ then $R_{in} < Z_0$ while if $R > Z_0$ then $R_{in} > Z_0$.

Somewhere between $l = 0$ and $l = \lambda/4$, there will exist a point where the resistance component of the input impedance equals Z_0. However, there will be some reactive component associated at this point. If this is tuned out by means of an equal and opposite reactance (the stub line), only the resistance component $R_{in} = Z_0$ will remain and the line upto this point will be terminated by proper matching.

Determination of stub position: In Eq. (4.52) for a loss-less line, if we replace Z_L by a resistance R we get

$$Z_{in} = Z_0 \left[\frac{R\cos\beta l + jZ_0 \sin\beta l}{Z_0 \cos\beta l + jR \sin\beta l} \right] \tag{4.59}$$

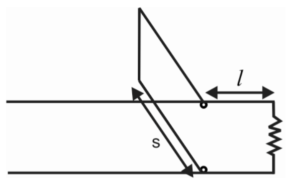

Fig. 4.19: Single stub impedance matching

The input admittance at a point at length l from the load end will be

$$Y_{in} = G_{in} + jB_{in} = \frac{1}{Z_{in}} \tag{4.60}$$

But $$\frac{1}{Z_{in}} = \frac{Z_0 \cos\beta l + jR\sin\beta l}{Z_0 (R\cos\beta l + jZ_0 \sin\beta l)}.$$

After rationalizing this expression we get

$$Y_{in} = \frac{R Z_0}{Z_0 (R^2 \cos^2\beta l + Z_0^2 \sin^2\beta l)} + \frac{j(R^2 - Z_0^2)\sin\beta l \cos\beta l}{Z_0 (R^2 \cos^2\beta l + Z_0^2 \sin^2\beta l)} \tag{4.61}$$

For proper impedance matching, l should be chosen so that real component of admittance equals reciprocal of characteristic impedance.

i.e. $$G_{in} = \frac{Z_0 R(\cos^2 \beta l + \sin^2 \beta l)}{Z_0 (R^2 \cos^2 \beta l + Z_0^2 \sin^2 \beta l)} = \frac{1}{Z_0} \quad (4.62)$$

On rearranging the terms we get

$$\frac{R}{Z_0} \cos^2 \beta l + \frac{Z_0}{R} \sin^2 \beta l = \cos^2 \beta l + \sin^2 \beta l \quad (4.63)$$

By simplification we get expression for tan βl as

$$\boxed{\tan \beta l = \sqrt{\frac{R}{Z_0}}} \quad (4.64)$$

From this equation we can find the distance from the termination R to the point where the input conductance is equal to $\frac{1}{Z_0}$ and this is the correct location of the stub in Fig. 4.19.

Determination of stub length: For tuning away the reactance of the input impedance, the magnitude of the admittance of the stub should be equal to B_{in} while its sign should be opposite, i.e. if B_{in} is having capacitive nature, then the stubs admittance should be inductive and vice-versa.

Let us assume the characteristic impedance of stub to be Z_0.

Hence its reactance will be given by

$$X_s = jZ_0 \tan \beta s$$

where s is the length of the stub.

For impedance matching we need

$$jZ_0 \tan \beta s = -\frac{1}{jB_{in}} \quad (4.65)$$

From Eq. (4.61) we substitute value for jB_{in}.

$$jB_{in} = j\frac{\sin \beta l \cos \beta l\,(R^2 - Z_0^2)}{Z_0\,(R^2 \cos^2 \beta l + Z_0^2 \sin^2 \beta l)} = -\frac{1}{jZ_0 \tan \beta s}$$

Rearranging the terms, we get

$$\tan \beta s = \frac{R^2 \cos^2 \beta l + Z_0^2 \sin^2 \beta l}{\sin \beta l \cos \beta l\,(R^2 - Z_0^2)} \quad (4.66)$$

Dividing the numerator and denominator by ($RZ_0 \cos^2\beta l$)

$$\tan \beta s = \frac{\frac{R}{Z_0} + \frac{Z_0}{R} \tan^2 \beta l}{\tan \beta l \left(\frac{R}{Z_0} - \frac{Z_0}{R}\right)} \quad (4.67)$$

Substituting the value of tan βl from Eq. (4.64) we get

$$\tan \beta s = \frac{\frac{R}{Z_0} + 1}{\sqrt{\frac{R}{Z_0}\left(\frac{R}{Z_0} - \frac{Z_0}{R}\right)}} \tag{4.68}$$

Simplifying this equation we obtain

$$\tan \beta s = \frac{\sqrt{\frac{R}{Z_0}}}{\frac{R}{Z_0} - 1} \tag{4.69}$$

In an actual experimental set up, the value of terminating resistance is generally unknown. Hence it is desirable to determine the dimension of the stub line directly from measurable quantity like standing wave ratio that exits on the transmission line.

From Eq (4.38) and (4.40) we can write

$$SWR = S = \frac{V_{max}}{V_{min}} = \frac{Z_L}{Z_0} = \frac{R}{Z_0} \tag{4.70}$$

Substituting values for $\frac{R}{Z_0}$ we get

$$\tan \beta l = \sqrt{\frac{V_{max}}{V_{min}}} \tag{4.71}$$

and

$$\tan \beta s = \frac{\sqrt{\frac{V_{max}}{V_{min}}}}{\left[\left(\frac{V_{max}}{V_{min}}\right) - 1\right]} \tag{4.72}$$

From these expressions we can find length l, where the stub should be connected, and also the length of the stub.

Stub matching for a complex impedance: In our discussion so far we have considered that the terminating load is purely resistive, i.e. $Z_L = R$. But when a line is terminated in complex impedance, there will be neither a voltage maximum nor a voltage minimum at termination. However, at some point, there will be voltage maximum at which input impedance is a pure resistance.

Hence, if measurements are made from either a voltage maximum or minimum the problem will be reduced to that of matching a line to pure resistance. That is why, the same formulae given by Eq. (4.71) and (4.72) can be used in this case as well.

In the analysis so far, we have worked with a particular value of load resistance. Whenever the load changes, the position and length of the stub needs to be changed. To overcome this problem, the use of multiple stub matching methods is done. The analysis of multiple stub matching is somewhat complex and we will not go into its details here.

Now we shall discuss some applications of transmission lines other than for signal transmission and impedance matching.

4.9 SPECIAL APPLICATIONS OF TRANSMISSION LINES

At high frequencies (above 100MHz) the behaviour of lump components is very different than at low frequencies. For example, the reactance of an inductor (ωL) becomes very large at high frequencies, hence very low L value inductors are needed. Similarly the capacitors of small values are needed. This fact has other effects as well. Normally insignificant values of L and C which may be present in the conductors, become significant at such frequencies. To construct the lumped components at high frequencies is difficult, but the transmission lines behave like inductors and capacitors at these frequencies and can be used as circuit components. The lengths of transmission lines for such purposes are not very long and can be handled easily. These line sections can be used up to approximately 300 MHz because their physical size become too small beyond this frequency. After 300 MHz, *wave guide techniques* are used.

Consider a short circuited line section of length $l < \lambda/4$ as shown in Fig. 4.20a. The input impedance is given by the Eq. (4.52). In this case, $Z_L = 0$. Thus the input impedance becomes

$$Z_{in(short)} = Z_0 \frac{jZ_0 \sin \beta l}{Z_0 \cos \beta l} = jZ_0 \tan \beta l \qquad (4.73)$$

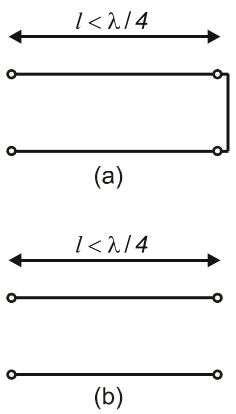

Fig. 4.20: a) λ/4 line as inductor and (b) λ/4 line as capacitor

This is equivalent to an impedance of an inductor.

If we consider an open ended segment of $l < \lambda/4$ as shown in Fig. 4.20b, we get $Z_L = \infty$ and hence

$$Z_{in(open)} = Z_0 \left[\frac{\cos \beta l + \frac{jZ_0}{\infty} \sin \beta l}{\frac{Z_0}{\infty} \cos \beta l + j \sin \beta l} \right] = -jZ_0 \cot \beta l \qquad (4.74)$$

This represents capacitive impedance.

The resistive component of the input impedance is negligible for the usual loss-less lines used at ultra high frequencies. You must remember here that these transmission line inductors and capacitors change their values with frequency. In bulk components, the values are independent of frequency.

Another important application of transmission line segments is their use in filters. Let λ be the wavelength of the signal and let the signal contain noise arising out of second harmonic. In order to filter out this noise, we can use a $\lambda/4$ shorted line segment as a filter shown in Fig. 4.21. In this case, at signal frequency the line segment has infinite impedance and all the signal is passed on to the transmitting antenna. But for the second harmonic noise, this line segment acts as $\lambda'/2$ (where λ' is wavelength of second harmonic i.e. $\lambda' = \lambda/2$) stub. Now, a shorted stub of $\lambda'/2$ length has zero impedance and it dissipates the signal of second harmonic. No second harmonic signal is passed on to the antenna. In this way, the transmission line segment can be used as a filter.

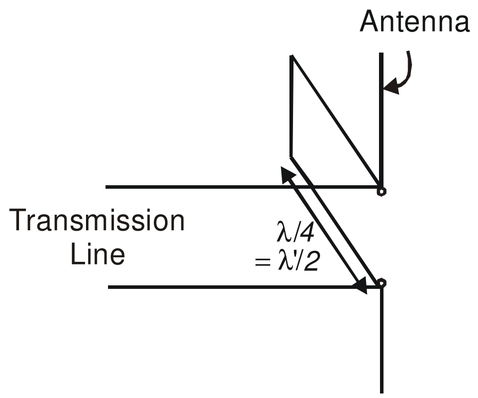

Fig. 4.21: λ/4 Shorted line as low pass filter

Let us now summarize the points you learnt in this unit.

4.10 SUMMARY

- Transmission line consists of two conductors separated by a dielectric.
- It can be in the form of parallel conductors, twisted pair or coaxial cable.
- In coaxial cable the field is confined within the radial region between the two conductors. It reduces radiation losses and can be used even up to 40 GHz frequency over a short distance.
- A transmission line can be represented by a network of series inductors, shunt capacitances, series resistances and shunt conductances, distributed over the entire length of the line.
- The wave travelling on the transmission line gets reflected if the load is not matched.
- The incident and reflected waves give rise to a standing wave pattern.
- The incident wave is represented by

  $$V' = V_1\, e^{(\alpha + j\beta) l}$$

  and the reflected wave is represented by

  $$V'' = V_2\ e^{-(\alpha + j\beta) l}$$

  α is attenuation coefficient while β is the phase shift co-efficient.
- The characteristic impedance of a transmission line is constant through out its length.
- The reflection coefficient Γ is defined by

  $$\Gamma = \frac{\text{reflected voltage (or current)}}{\text{incident voltage (or current)}} = \frac{Z_L - Z_0}{Z_L + Z_0}\ .$$
- The maxima and minima in the standing wave pattern occur at λ/2 distance.
- $SWR = \dfrac{1 + |\Gamma|}{1 - |\Gamma|}$
- The characteristic impedance

  $$Z_0 = \sqrt{\frac{L}{C}}$$
- The open ended transmission line of λ/4 length acts as a short circuited line, while λ/2 length acts as an open circuit.

- For a short circuited line, the load end has current maxima (loop) and voltage minima (node). An open line has voltage maxima and current minima at load end.
- Reactive load impedance shifts the maxima and minima away from load end.
- The transmission line can be matched with load impedance by using quarter wavelength, tapered line matching or stub matching.
- Transmission line segments can be used as reactive components or filters at high frequencies.

4.11 TERMINAL QUESTIONS *Spend* 20 *Minutes*

1. What is the minimum value of characteristic impedance of a parallel wire transmission line?
2. For a transmission line with $Z_0 = 50\Omega$, calculate and plot the reflection coefficient as a function of load resistance for the load ranging from 0 to 250Ω.
3. In a transmission line, if the separation between the two wires is increased, what is the effect on its characteristic impendence?
4. A short-circuited line is 1.5 λ long. Draw the incident, reflected and resultant voltage and current waves at the instant when generator is at its peak positive voltage value.

4.12 SOLUTIONS AND ANSWERS

Self Assessment Questions

1. No. The RF range corresponds to 10^9 to 10^{14} m. since the dimension of waveguide has to be of the order of the wavelength, at RF the waveguide dimensions are not practically realizable.

2. The equivalent circuit of loss-less transmission line is given in Fig. 4.22.

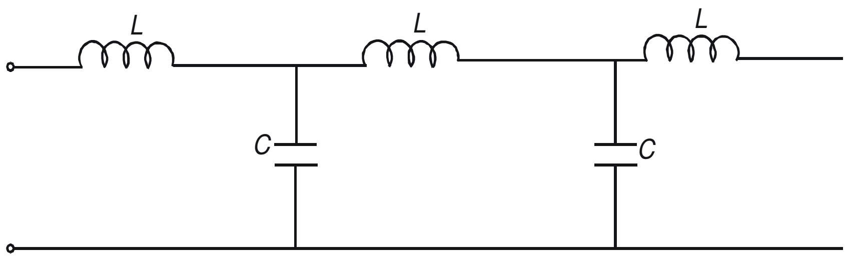

Fig. 4.22

3. For a coaxial cable it is given that $T = 5$ mm. $\varepsilon = 2$ and $Z_0 = 75\ \Omega$. From this we can calculate t by using Eq. (4.17)

$$Z_0 = \frac{138}{\sqrt{\varepsilon}} \log \frac{T}{t}$$

$$\log \frac{T}{t} = \frac{Z_0 \sqrt{\varepsilon}}{138} = \frac{75\sqrt{2}}{138} = 0.77$$

$$\therefore \frac{T}{t} = 10^{0.77} = 5.87.$$

$$\therefore t = \frac{T}{5.87} = \frac{5}{5.87} = 0.85\,mm.$$

4. It is given that $\alpha l = 1.2$ dB with $l = 100$ m. Hence $\alpha = 0.012$ db m^{-1}

From Eq. (4.22)

$$\alpha = \frac{\text{Attenduation in dB}}{8.686} = \frac{0.012}{8.686} = 1.38\times10^{-3}\,\text{Neper m}^{-1}.$$

5. (i) Under no reflection condition, $\Gamma = 0$. This implies $Z_L = Z_0$.

   (ii) For complete reflection of wave, $\Gamma = 1$. Hence $Z_L = \infty$. This is a condition when the ends of the transmission line pair have infinite impedance between them, i.e. they are open.

6. The maxima is obtained when $\beta l = n\pi$ i.e. $l = \dfrac{n\pi}{\beta}$

   Hence the distance between the two consecutive maxima is

$$\Delta l = \frac{(n+1)\,\pi}{\beta} - \frac{n\pi}{\beta} = \frac{\pi}{\beta}.$$

   But $\beta = \dfrac{2\pi}{\lambda}$.

   Hence $\Delta l = \dfrac{\pi}{(2\pi/\lambda)} = \dfrac{\lambda}{2}$

   This is half the wavelength of the signal wave. The same can be shown in case of two consecutive minima as well.

7. $$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{300\Omega - 50\Omega}{300\Omega + 50\Omega} = \frac{250}{350} = \frac{5}{7} = 0.71.$$

$$VSWR = \frac{1+\Gamma}{1-\Gamma} = \frac{1+\frac{5}{7}}{1-\frac{5}{7}} = \frac{12/7}{2/7} = 6.$$

8. For 1 m:

$$Z_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{147.5\times10^{-9}\,\text{Hm}^{-1}\times1\,\text{m}}{59\times10^{-12}\,\text{Fm}^{-1}\times1\,\text{m}}} = \sqrt{2500} = 50\Omega.$$

   For 100 m:

$$Z_0 = \sqrt{\frac{147.5\times10^{-9}\,\text{Hm}^{-1}\times100\,\text{m}}{59\times10^{-12}\,\text{Fm}^{-1}\times100\,\text{m}}} = \sqrt{2500} = 50\Omega.$$

9. In the standing wave pattern of an open ended transmission line, at $l = \lambda/2$, there is voltage loop and current node. Hence

$$Z_{in} = \frac{V}{I} = \frac{V_{\max}}{0} = \infty.$$

   Hence the impedance is infinite.

Terminal Questions

1. The minimum value of characteristic impedance will occur for the minimum value of separation y, which is the case of just touching conductors. This results into $y = x$ where x is diameter of each conductor.

From Eq. (4.16), we get

$$Z_0 = 276 \log 2\frac{x}{y} = 276 \log\ 2 = 83\ \Omega$$

2. From Eq. (4.29)

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$Z_0 = 50\ \Omega$

Z_L	Γ
0 Ω	−1
50 Ω	0
100 Ω	1/3
150 Ω	1/2
200 Ω	3/5
250 Ω	2/3

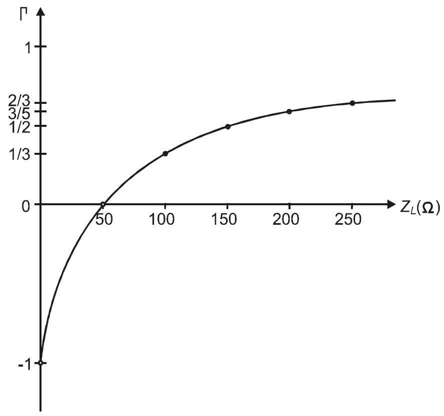

Fig. 4.23

3. The value of Z_0 depends on the ratio of distributed L and C value of the transmission line. The effective inductance is proportional to the flux that may be established between the two wires. If two wire carrying current in opposite direction are separated further apart, more magnetic flux is included between the wires, i.e. they cannot cancel the magnetic effects in each other as effectively as when they were kept closer. This increases the distributed impendence. When the wires are separated further it effectively increases the inter-plate distance of a capacitor and reduces its capacitance. Because of these two factors, the characteristic impedance $Z_0 = \sqrt{\frac{L}{C}}$ increases with increased separation between the wires.

4. For a short-circuited line, the reflected wave voltage is 180° out of phase w.r.t. incident wave voltage. This is shown in Fig. 4.24a. Hence the resultant is zero.

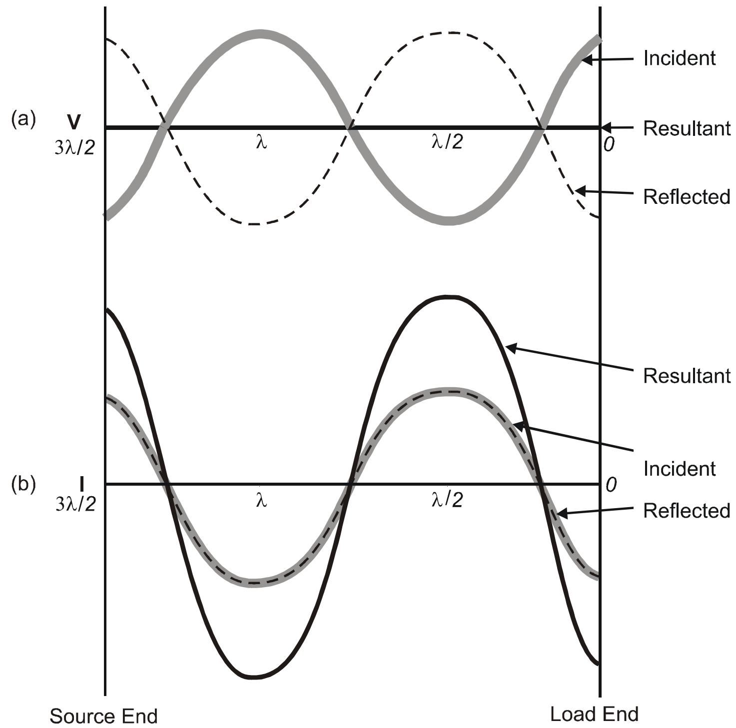

Fig. 4.24: a) voltage and b) current wave on short-circuited transmission line

The current, however, is in phase, hence at maxima, we get double the maximum current amplitude as seen in Fig. 4.24b.

Reference Material:

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3. *Modern Electronic Communication* by Miller, G.M. and Beasley J.S. (VII Edition) (Prentice-Hall of India)
4. *Communication Electronics-Systems, Circuits and Devices* by Barker, Forrest (I Edition) (Prentice-Hall, Inc.)